

# Problem Set – 3 Solutions

CS 230, Spring 2023

## Questions

1. Consider the following MIPS program:

```
1. fun:
2.     bne $a1,$zero,lbl
3.     li $v0,1
4.     jr $ra
5. lbl:
6.     addi $sp,$sp,-4
7.     sw $ra,0($sp)
8.     addi $a1,$a1,-1
9.     jal fun
10.    mul $v0,$v0,$a0
11.    lw $ra,0($sp)
12.    addi $sp,4
13.    jr $ra
14. main:
15.    li $t0,2
16.    li $t1,4
17.    move $a0,$t0
18.    move $a1,$t1
19.    jal fun
```

If the program execution begins from main and the initial value of \$sp is 0xf064. Then list all the possible values of \$sp at:

- a. line number 3.
- b. line number 8.
- c. line number 13.

Tracing the sequence of execution:

Code line	Action	Register Values after Action	Call Frame
15. li \$t0,2	Set register \$t0 = 2	t0 = 2 sp = 0xf064	main
16. li \$t1,4	Set register \$t1 = 4	t0 = 2, t1 = 4 sp = 0xf064	main
17. move \$a0,\$t0	Set register \$a0 = \$t0	t0 = 2, t1 = 4 a0 = 2 sp = 0xf064	main

18. move \$a1,\$t1	Set register \$a1 = \$t1	t0 = 2, t1 = 4 a0 = 2, a1 = 4 sp = 0xf064	main
19. jal fun	Jump and link to the label fun. Moves execution to line 2.	t0 = 2, t1 = 4 a0 = 2, a1 = 4 sp = 0xf064	main <sub>19</sub> ->fun
2. bne \$a1,\$zero, lbl	If \$a1 is not zero, then goto line 6 (lbl). Since \$a1 is 4, execution goes to line 6.	t0 = 2, t1 = 4 a0 = 2, a1 = 4 sp = 0xf064	main <sub>19</sub> ->fun
6. addi \$sp,\$sp,-4	Add -4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 4 sp = 0xf060	main <sub>19</sub> ->fun
7. sw \$ra,0(\$sp)	Store the value of register \$ra in memory address stored in \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 4 sp = 0xf060	main <sub>19</sub> ->fun
8. addi \$a1,\$a1,-1	Add -1 to register \$a1	t0 = 2, t1 = 4 a0 = 2, a1 = 3 sp = 0xf060	main <sub>19</sub> ->fun
9. jal fun	Jump and link to the label fun. Moves execution to line 2.	t0 = 2, t1 = 4 a0 = 2, a1 = 3 sp = 0xf060	main <sub>19</sub> ->fun <sub>9</sub> ->fun
2. bne \$a1,\$zero, lbl	If \$a1 is not zero, then goto line 6 (lbl). Since \$a1 is 3, execution goes to line 6.	t0 = 2, t1 = 4 a0 = 2, a1 = 3 sp = 0xf060	main <sub>19</sub> ->fun <sub>9</sub> ->fun
6. addi \$sp,\$sp,-4	Add -4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 3 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
7. sw \$ra,0(\$sp)	Store the value of register \$ra in memory address stored in \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 3 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
8. addi \$a1,\$a1,-1	Add -1 to register \$a1	t0 = 2, t1 = 4 a0 = 2, a1 = 2 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
9. jal fun	Jump and link to the label fun. Moves execution to line 2.	t0 = 2, t1 = 4 a0 = 2, a1 = 2 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
2. bne \$a1,\$zero, lbl	If \$a1 is not zero, then goto line 6 (lbl). Since \$a1 is 2, execution goes to line 6.	t0 = 2, t1 = 4 a0 = 2, a1 = 2 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
6. addi \$sp,\$sp,-4	Add -4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 2 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
7. sw \$ra,0(\$sp)	Store the value of register \$ra in memory address stored in \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 2 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
8. addi \$a1,\$a1,-1	Add -1 to register \$a1	t0 = 2, t1 = 4 a0 = 2, a1 = 1 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
9. jal fun	Jump and link to the label fun. Moves execution to line 2.	t0 = 2, t1 = 4 a0 = 2, a1 = 1 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
2. bne \$a1,\$zero, lbl	If \$a1 is not zero, then goto line 6 (lbl). Since \$a1 is 1, execution goes to line 6.	t0 = 2, t1 = 4 a0 = 2, a1 = 1 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
6. addi \$sp,\$sp,-4	Add -4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 1 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
7. sw \$ra,0(\$sp)	Store the value of register \$ra in memory address stored in \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 1 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun

8.    addi \$a1,\$a1,-1	Add -1 to register \$a1	t0 = 2, t1 = 4 a0 = 2, a1 = 0 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
9.    jal fun	Jump and link to the label fun. Moves execution to line 2.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
2.    bne \$a1,\$zero,lbl	If \$a1 is not zero, then goto line 6 (lbl). Since \$a1 is 0, branch is not taken. Next instruction in sequence is executed.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
3.    li \$v0,1	Set register \$v0 = 1	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 1 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
4.    jr \$ra	Return execution to previous call frame. Moves execution to line 10.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 1 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
10.   mul \$v0,\$v0,\$a0	Multiply \$v0 by \$a0 and store in \$v0	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 2 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
11.   lw \$ra,0(\$sp)	Restore return address of previous call frame from stack	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 2 sp = 0xf054	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
12.   addi \$sp,4	Add 4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 2 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
13.   jr \$ra	Return execution to previous call frame. Moves execution to line 10.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 2 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
10.   mul \$v0,\$v0,\$a0	Multiply \$v0 by \$a0 and store in \$v0	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 4 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
11.   lw \$ra,0(\$sp)	Restore return address of previous call frame from stack	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 4 sp = 0xf058	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
12.   addi \$sp,4	Add 4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 4 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun <sub>9</sub> ->fun
13.   jr \$ra	Return execution to previous call frame. Moves execution to line 10.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 4 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
10.   mul \$v0,\$v0,\$a0	Multiply \$v0 by \$a0 and store in \$v0	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 8 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
11.   lw \$ra,0(\$sp)	Restore return address of previous call frame from stack	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 8 sp = 0xf05c	main <sub>19</sub> ->fun <sub>9</sub> ->fun
12.   addi \$sp,4	Add 4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 8 sp = 0xf060	main <sub>19</sub> ->fun <sub>9</sub> ->fun
13.   jr \$ra	Return execution to previous call frame. Moves execution to line 10.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 8 sp = 0xf060	main <sub>19</sub> ->fun

10. <code>mul \$v0,\$v0,\$a0</code>	Multiply \$v0 by \$a0 and store in \$v0	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 16 sp = 0xf060	main <sub>19</sub> ->fun
11. <code>lw \$ra,0(\$sp)</code>	Restore return address of previous call frame from stack	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 16 sp = 0xf060	main <sub>19</sub> ->fun
12. <code>addi \$sp,4</code>	Add 4 to register \$sp.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 16 sp = 0xf064	main <sub>19</sub> ->fun
13. <code>jr \$ra</code>	Return execution to previous call frame. There is no more code line in main, so execution is halted.	t0 = 2, t1 = 4 a0 = 2, a1 = 0 v0 = 16 sp = 0xf064	main

All possible \$sp values at a given line can be retrieved by looking at above table.

- a. line number 3: 0xf054
- b. line number 8: 0xf060, 0xf05c, 0xf058, 0xf054
- c. line number 13: 0xf058, 0xf05c, 0xf060, 0xf064

2. Consider the previous question (Q1). For all lines of the program, specify the following:

- a. The type of Instruction.
- b. The addressing mode of the Instruction.

### Types of Instructions:

- **R-type:** These are the instructions which use only registers for all operations. Except **sll**, **srl** & **sra** which use 5 bit value for the shift value.
- **I-type:** These instructions include 16 bit immediate values.
- **J-type:** These instructions are used to jump to different locations in the code using 26 bit address.

Type	31	26 25	21 20	16 15	11 10	06 05	00
R-Type	opcode	\$rs	\$rt	\$rd	shamt	funct	
I-Type	opcode	\$rs	\$rt	imm			
J-Type	opcode	address					

### Addressing Modes:

- **Immediate:** Use an immediate value encoded within in the instruction.

- **Register:** Use values from registers.
- **Base:** Base memory address is retrieved from a register. Final memory address is calculated by adding offset to the base address. The final address is used to access memory.
- **PC-relative:** An offset is added to the program counter value to find the target address.
- **Pseudodirect:** Part of the program counter and an specified address is concatenated to calculate the target address.

Code line	Instruction Type	Addressing Mode
2. bne \$a1,\$zero, lbl	I-type (lbl is replaced by a 16-bit immediate value by assembler)	PC-relative
3. li \$v0,1	I-type	Immediate
4. jr \$ra	R-type (return address is stored within a register, while J-type instructions need a 26-bit immediate pseudo-absolute address)	Register
6. addi \$sp,\$sp,-4	I-type	Immediate
7. sw \$ra,0(\$sp)	I-type (Offset is an immediate value)	Base
8. addi \$a1,\$a1,-1	I-type	Immediate
9. jal fun	J-type	Pseudodirect (target address is calculated by concatenating upper 4 bits of (PC + 4) and the specified address.
10. mul \$v0,\$v0,\$a0	R-type	Register
11. lw \$ra,0(\$sp)	I-type	Base
12. addi \$sp,4	I-type	Immediate
13. jr \$ra	R-type	Register
15. li \$t0,2	I-type	Immediate
16. li \$t1,4	I-type	Immediate
17. move \$a0,\$t0	R-type	Register
18. move \$a1,\$t1	R-type	Register
19. jal fun	J-type	Pseudodirect

3. Suppose there is a system which has 32 bit instructions and 32 general purpose registers. Answer the following questions:
- What is the number of bits required for the registers?
  - Is it possible to have various operations consisting of 20 three-address instructions, 22 two- address instructions and 10 one-address instructions?

- There are 32 registers. We need  $\lceil \log_2(32) \rceil = 5$  bits to label each register.

b.

### Three Address Instructions

There are 3 5-bit registers and 20 operations. Therefore, we have  $20 * 2^{15}$  combinations.

### Two Address Instructions

There are 2 5-bit registers and 22 operations. Therefore, we have  $22 * 2^{10}$  combinations.

### One Address Instructions

There is 1 5-bit register and 10 operations. Therefore, we have  $10 * 2^5$  combinations.

$$\begin{aligned}\text{Total number of combinations} &= (20 * 2^{15}) + (22 * 2^{10}) + (10 * 2^5) \\ &= 10599 * 2^6\end{aligned}$$

$$\text{Total bits needed to represent all combinations} = \lceil \log_2(10599 * 2^6) \rceil = 20$$

Since we have 32 bits instructions, it is possible to encode all operations.

4. Consider the following code segment:

```
1.  int a, b, c, d, e, f, g, h, i, j, k, l, m, n, o;
2.  a = b * c;
3.  d = a - e;
4.  f = d + g;
5.  h = f * i;
7.  k = h - l;
8.  m = k + n;
9.  o = m * k;
```

- If you are using temporary registers of MIPS ISA for the code segment, will there be any spill to the memory?
- If you are allowed to change the number of temporary registers, then what is the minimum number of registers required so that there is no spill to the memory for the given code segment?

At a program point a variable is live, if it's value will be used at later point in the program without redefinition.

Code line	Live Variables
	-
int a, b, c, d, e, f, g, h, i, j, k, l, m, n, o;	

	b, c, e, i, g, n, l
a = b * c;	
	a, e, i, g, n, l
d = a - e;	
	d, i, g, n, l
f = d + g;	
	f, i, n, l
h = f * i;	
	n, h, l
k = h - l;	
	k, n
m = k + n;	
	m, k
o = m * k;	
	-

- Before line 2, 7 variables are live. We need 7 registers to store value of each variable. We also need an additional register for variable 'a' to store the value of the expression  $b*c$ . Therefore, **8 registers** are needed. The compiler can reuse registers, assigned to a variable if it is not live. Since MIPS has 10 temporary registers, there will be no spill to memory.
- The minimum number of registers required is 8. But MIPS has 10 temporary registers so no change is needed.

5. Consider the following program, in which the execution starts from main:

```

1. fun3:
2.     move $v0, $ra
3.     jr $ra
4. fun2:
5.     addi $sp, $sp, -4
6.     sw $ra, ($sp)
7.     move $t0, $a0
8.     jal fun3
9.     move $t1, $v0
10.    mul $v0, $t0, $t0
11.    lw $ra, ($sp)
12.    addi $sp, $sp, 4
13.    addi $t1, $t1, 0x4c
14.    sw $t1, ($sp)
15.    jr $ra
16. fun1:
17.    addi $sp, $sp, -4
18.    sw $ra, ($sp)
19.    jal fun2
20.    addi $v0, $v0, 1

```

```

21.    lw $ra, ($sp)
22.    addi $sp, $sp, 4
23.    jr $ra
24. main:
25.    li $t0, 5
26.    li $s1, 0
27.    move $a0, $t0
28.    jal fun1
29.    move $s1, $v0
30.    addi $t2, $s1, 2
31.    nop

```

What will be the value in the register \$t2 after program execution?

Suppose starting stack pointer address is x.

Code line	Action	Register Values after Action	Stack
25. li \$t0, 5	Set register \$t0 = 5	t0 = 5	[]
26. li \$s1, 0	Set register \$s1 = 0	t0 = 5 s1 = 0 sp = x	[]
27. move \$a0, \$t0	Set register \$a0 = \$t0	t0 = 5 s1 = 0 a0 = 5 sp = x	[]
28. jal fun1	Save address of line 29 instruction in \$ra and jump to line 17.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 29 sp = x	[]
17. addi \$sp, \$sp, -4	Decrement stack pointer by 4.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 29 sp = x-4	[]
18. sw \$ra, (\$sp)	Store value of \$ra (addr of line 29) in stack.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 29 sp = x-4	[Addr.line.29]
19. jal fun2	Save address of line 20 instruction in \$ra and jump to line 5.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 20 sp = x-4	[Addr.line.29]
5. addi \$sp, \$sp, -4	Decrement stack pointer by 4.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 20 sp = x-8	[Addr.line.29]
6. sw \$ra, (\$sp)	Store value of \$ra (addr of line 20) in stack.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 20 sp = x-8	[Addr.line.29, Addr.line.20]
7. move \$t0, \$a0	Set register \$t0 = \$a0	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 20	[Addr.line.29, Addr.line.20]



		sp = x-8	
8. jal fun3	Save address of line 9 instruction in \$ra and jump to line 2.	t0 = 5 s1 = 0 a0 = 5 ra = Addr.line 9 sp = x-8	[Addr.line.29, Addr.line.20]
2. move \$v0, \$ra	Set register \$v0 = \$ra	t0 = 5 s1 = 0 a0 = 5 v0 = Addr.line 9 ra = Addr.line 9 sp = x-8	[Addr.line.29, Addr.line.20]
3. jr \$ra	Jump to the address stored in \$ra (line 9)	t0 = 5 s1 = 0 a0 = 5 v0 = Addr.line 9 ra = Addr.line 9 sp = x-8	[Addr.line.29, Addr.line.20]
9. move \$t1, \$v0	Set register \$t1 = \$v0	t0 = 5 t1 = Addr.line 9 s1 = 0 a0 = 5 v0 = Addr.line 9 ra = Addr.line 9 sp = x-8	[Addr.line.29, Addr.line.20]
10. mul \$v0, \$t0, \$t0	Set \$v0 = \$t0 * \$t0 (5 * 5)	t0 = 5 t1 = Addr.line 9 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 9 sp = x-8	[Addr.line.29, Addr.line.20]
11. lw \$ra, (\$sp)	Load address from stack and store in \$ra	t0 = 5 t1 = Addr.line 9 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 20 sp = x-8	[Addr.line.29, Addr.line.20]
12. addi \$sp, \$sp, 4	Increment stack pointer by 4.	t0 = 5 t1 = Addr.line 9 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 20 sp = x-4	[Addr.line.29]
13. addi \$t1, \$t1, 0x4c	Add 0x4c to \$t1. 0x4c is 76 in decimal. Each instruction in mips is of 4 bytes. 76/4 = 19. So, after the addition, \$t1 stores the address of 19 <sup>th</sup> instruction from line 9 which is line 30. Note that lines with labels are not considered.	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 20 sp = x-4	[Addr.line.29]
14. sw \$t1, (\$sp)	Store \$ta in stack.	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 20 sp = x-4	[Addr.line.30]
15. jr \$ra	Jump to the address stored in \$ra (line 20)	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 25 ra = Addr.line 20 sp = x-4	[Addr.line.30]
20. addi \$v0, \$v0, 1	Increment \$v0 by 1	t0 = 5	[Addr.line.30]

		t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 20 sp = x-4	
21. lw \$ra, (\$sp)	Load address from stack and store in \$ra	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 30 sp = x-4	[Addr.line.30]
22. addi \$sp, \$sp, 4	Increment stack pointer by 4.	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 30 sp = x	[]
23. jr \$ra	Jump to the address stored in \$ra (line 30)	t0 = 5 t1 = Addr.line 30 s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 30 sp = x	[]
30. addi \$t2, \$s1, 2	Add 2 to \$s1 and store in \$t2	t0 = 5 t1 = Addr.line 30 t2 = 2 s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 30 sp = x	[]
31. nop	Nada	t0 = 5 t1 = Addr.line 30 <b>t2 = 2</b> s1 = 0 a0 = 5 v0 = 26 ra = Addr.line 30 sp = x	[]

The value in **\$t2** is **2**.

6. Consider the following program:

```

1. li $t0, 0xdecadead
2. li $t1, 0x00000bed
3. add $t3, $t0, $t1
4. sw $t0, 65($t3)
5. lb $s3, 66($t3)
6. lb $s4, 64($t3)

```

What will be the value of the register s3 and s4 (with explanations), if the computer is:

- big-endian
- little-endian

Figure 1 shows how big-endian and little-endian machines store the value 0xdecadead in memory word 0xdecaea9a. After the load byte instruction, lb \$s3, 66(\$t3), and lb \$s4, 64(\$t3) \$s3 would contain 0x000000ca on a big-endian system and 0x000000de on a little-endian system. Whereas the register \$s4 will contain garbage values.

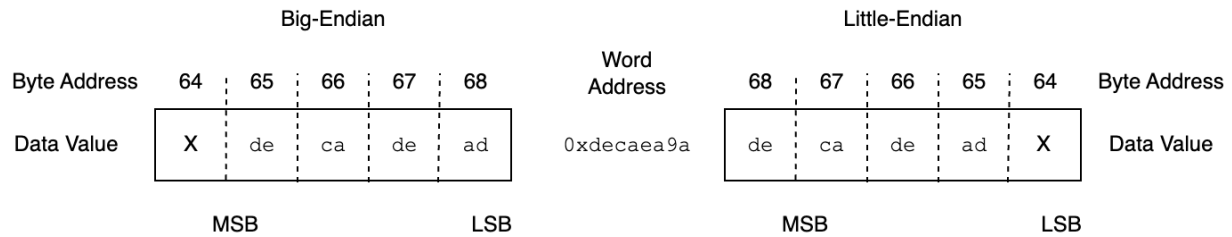


Figure 1