Assignment 1

(1) $A_1 \oplus A_2 \oplus A_3 = P$

$\oplus$ commutes, associative, $x \oplus x = 0$

$\rightarrow$ $A_1 \oplus A_2 \oplus P = A_3$ (a)

$A_2 \oplus A_3 \oplus P = A_1$ (b)

$A_3 \oplus P = A_1 \oplus A_2$ (c)

$A_1 \oplus A_2 \oplus A_3 = P \oplus A_4 \neq P$

$\oplus A_4$

(2) $f = \overline{\bar{A} + [\overline{B + \bar{C}(\underbrace{\overline{AB + A\bar{C}}}))}]}$

$\bar{C} \cdot \overline{AB} \cdot \overline{A\bar{C}}$

$\|$

$\bar{C} \cdot (\bar{A} + \bar{B})(\bar{A} + C)$

$(\bar{A} + \bar{B})(\bar{A}\bar{C})$

$\|$

$\overline{(B + \bar{A}\bar{C} + \bar{A}\bar{B}\bar{C})}$

$\|$

$\overline{\bar{A} + \bar{B} \cdot (A + C) \cdot (A + B + C)}$

$\|$

$\cancel{\bar{A} + A + B + C}$

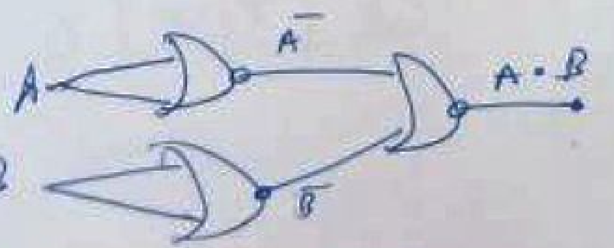
$A \cdot \overline{\bar{B} \cdot (A + C)(A + B + C)}$

$A \cdot (B + \overline{A + C} + \overline{A + B + C})$

$A \cdot (B + \bar{A} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot \bar{C})$

$= \boxed{A \cdot B}$

Now, min-no. of NOR gates $= \boxed{3}$



4-variable gates.

③ $f = \sum(0,2,5,7,8,10,13,15)$

$= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD$

$+ A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C\overline{D} + AB\overline{C}D + ABCD$

~~ABD~~

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

3

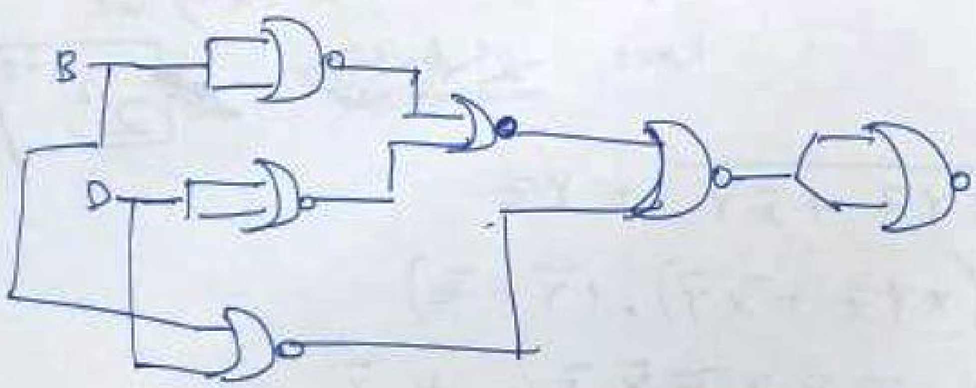
$BD + \overline{B}\,\overline{D}$

1

~~B+D~~ 2

$\boxed{\text{6 gates}}$

~~④~~



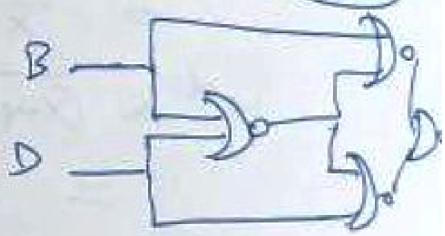
M2 :

$BD + \overline{B}\,\overline{D}$

$= B \oplus \overline{D}$

$= \overline{B \oplus D}$

4 gates

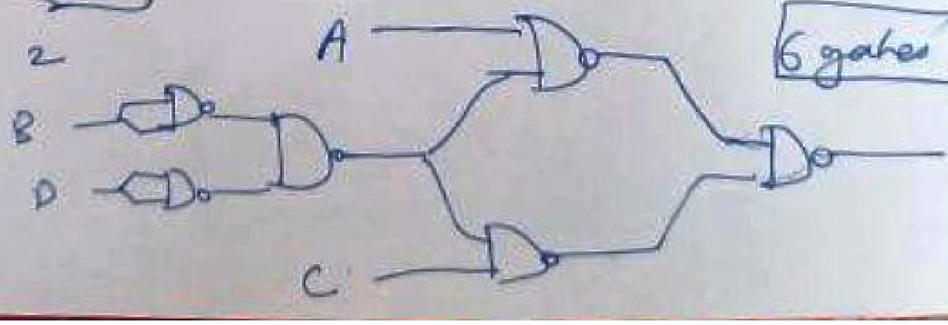


④ $f = (A+C)(B+D)$

M1 : $\underbrace{\overline{\underbrace{\overline{A}}_{1}\cdot\underbrace{\overline{C}}_{1}}}_{1} \cdot \underbrace{\overline{\underbrace{\overline{B}}_{1}\cdot\underbrace{\overline{D}}_{1}}}_{1}$ ⇒ 8 gates

2

M2 :



$\boxed{\text{6 gates}}$

(5) f s.t. $f^d = f^c = \overline{f}$

in a system of n literals, how many such f^n ?

$$\sum_i \cancel{\prod} \prod_{j \in I_i} x_j \to \prod_i \sum_{j \in I_i} x_i$$

$$= \prod_i M_i \text{ where}$$

$$M_i = \sum_{j \in I_i} x_j$$

$\Rightarrow$ all 0s and 1s are interchanged in the LHS of the boolean map

for f^c 0s & 1s on RHS are interchanged.

$$\Rightarrow \quad f(k) = f(2^n - k) \ \forall k$$

$\Rightarrow$ we have a map on 2^{n-1} space i.e., we have ~~2^n functions~~ $\boxed{2^{2^{n-1}}}$ fine.

(6) $M = \overline{XYZ + \overline{X}\,\overline{Y}} + YZ$

$$\overline{M} = (XYZ + \overline{X}\,\overline{Y}) \cdot (\overline{Y} + \overline{Z})$$

$$= \overline{X}\,\overline{Y} + \overline{X}\,\overline{Y}\,\overline{Z} = \overline{X}\,\overline{Y}$$

$$M^d = \overline{(X+Y+Z) \cdot (\overline{X} + \overline{Y})} + (Y+Z)$$

$$= \overline{X+Y+Z} + \overline{\overline{X}+\overline{Y}} + Y + Z$$

$$= \overline{X}\,\overline{Y}\,\overline{Z} + Y + Z$$

Hence (e) is correct

(7) $f(a,0,0,d) = 1$

$f(1,b,1,d) = b + d$

$f(a,1,c,d) = ad + c$

~~f(a,0,c,d)~~

ab \ cd	00	01	11	10
00	1	1	X	X
01	0	0	1	1
11	1	1	1	1
10	1	1	1	0

$$f = cd + \bar{b}\bar{c} + ab + bc$$

$\Rightarrow$ $\boxed{\text{4 literals}}$

(8)

CD \ AB	00	01	11	10
00	1	1		1
01	X			
11	X			
10	1	1		X

$\bar{B}\bar{D} + \bar{A}\bar{D}$

~~minimal form~~!

$= (\bar{A} + \bar{B})\bar{D}$

2 gates (NAND, NOR)

(minimal form.