2

PERT Network and Time Estimates

2.1 EVENTS AND ACTIVITIES

Here, we shall concentrate on the PERT network. As noted in Chapter 1, the PERT network is event-based. Let us recall the meaning of an event:

(a) It must indicate a noteworthy or significant point in the project.

(b) It is the start or completion of a job.

(c) It does not consume time or resources.

Examples of what an event is and what it is not are:

Foundation digging started : is a PERT event
Foundation is being dug : is not a PERT event
Assemble parts A and B : is not a PERT event
Electrical design completed : is a PERT event

In practice, the words "start" and "complete" are, respectively, shortened to S and C. Further, the bubble or the circle denoting an event is given shortened descriptions, such as "S foundation". In a network, the events fall in a logical sequence, and therefore the person preparing a network must ask himself the following questions regarding the sequence:

(a) What event or events must be completed before the particular event can start?

(b) What event or events follow this?

(c) What activities can be accomplished simultaneously?

Event or events that immediately come before another event without any intervening events are called *predecessor events* to that event.

Event or events that immediately follow another event without any intervening events are called *successor events* to that event.

Consider the PERT network shown in Fig. 2-1. The events are numbered 1 through 8 and the activities are designated A through L. Remember that a PERT activity is the actual performance of a task. It is the time-consuming portion of a PERT network and requires manpower, material, facilities, space, and other resources. In this network:

Event 5 cannot take place until activities A and D have been completed. Event 7 cannot take place until activities A, B, C, G, J, and H have been completed. Note that no event can be considered reached until *all* activities leading to the event are completed, and no activity may be completed until the event preceding it has occurred. Following this, event 7 cannot occur until activities *J* and *H* are complete and these activities cannot take place until their preceding events 4 and 3 have occurred. Event 4 cannot take place until activity *B* has been completed and event 3 cannot take place until activities *C* and *G* are complete. Similarly, we can work backwards until we come to event 1. Hence, event 7 cannot take place until activities *A*, *B*, *C*, *G*, *J*, and *H* have been completed.

These distinguish between a successor event and a predecessor event.

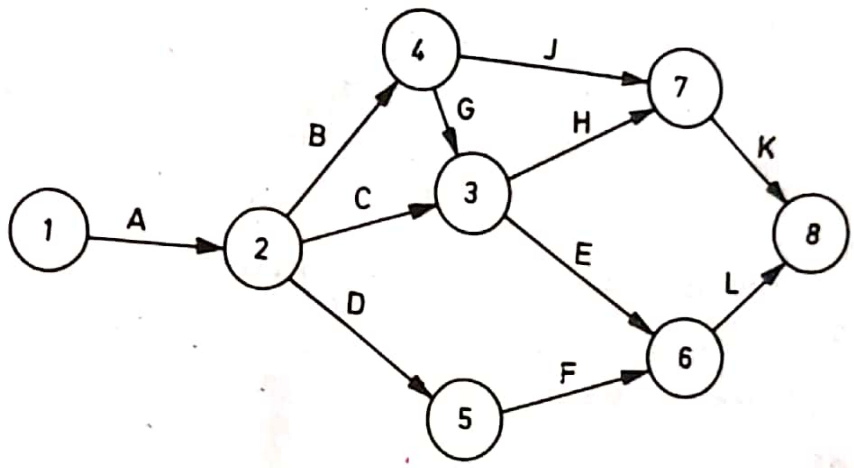

FIGURE 2-1

Consider now the problem of drawing a network for a particular project. Let the problem be of opening a new office for a commercial airline and let a few specific events be considered. First, we look for a site or location for the office. After looking over several areas near the busy localities, we select a particular street where several buildings may be available on rent. A particular building is selected and the following events are assumed to take place in some sequence. We shall number these events as:

Event 1 : Location of site started
Event 2 : Location of site completed
Event 3 : Building for office selected
Event 4 : Cleaning of office building started
Event 5 : Interior decorator starts work
Event 6 : Interior decorator finishes work
Event 7 : Opening of new office advertised or announced
Event 8 : List of invitees for the opening day prepared
Event 9 : Invitations sent
Event 10 : Office formally opened

The PERT network for this might appear as in Fig. 2-2. Assuming that activity 6-10 represents some inaugural functions, events 9 and 7 can be connected to event 6.

It should be noted that the network representing a project may not look unique. Another planner may conceivably arrive at a different type

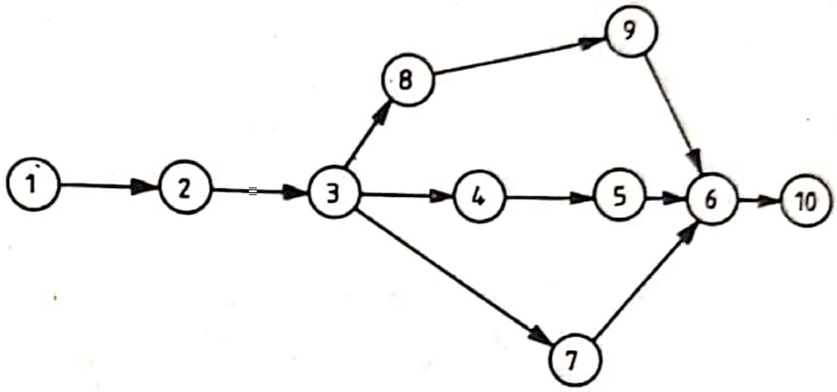

FIGURE 2-2

of network based upon his decision as to which events should precede or occur concurrently. This is perfectly all right.

2.2 HINTS FOR DRAWING NETWORKS

GENERAL POINTS

Many of the rules for drawing the networks are based on common sense. A few examples which belong to this category are shown in Fig. 2-3. In Fig. 2-3a, the arrows cross each other. This should be avoided, if possible, as shown in Fig. 2-3b. In Fig. 2-3c, it is stressed that the arrows should

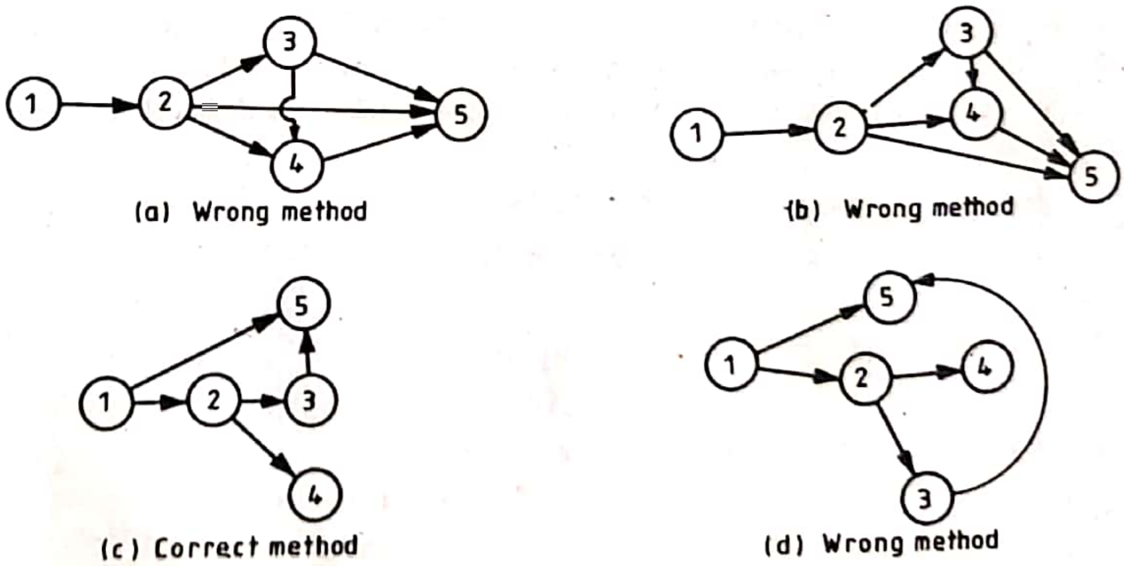

FIGURE 2-3

be straight and not curved as in Fig. 2-3d. It is a good practice not to keep the angles enclosed between events too small.

LOOP NETWORK

One of the important points to keep in mind is to avoid a loop network. This may occur in complicated networks. An example is shown in Fig. 2-4. Here, event 2 cannot occur until activity *E* is over which in turn cannot take place until event 5 has taken place. Event 5 cannot occur until event 2 has taken place. Hence, there is a kind of going back on time or the formation of a loop. This may occur inadvertently from duplicating event

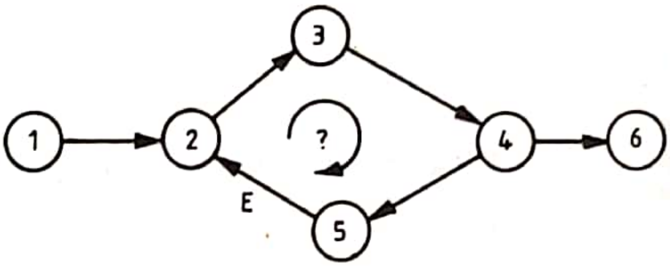

FIGURE 2-4

numbers or repetition of a particular activity, or while transcribing data inaccurately. A method by which events are numbered in a logical sequence and the possibilities of loop networks are reduced will be described in Section 2.4.

DUMMIES

In connecting events by activities showing their interdependencies, very often a situation arises where a certain event *j* cannot occur until another event *i* has taken place; but, the activity connecting *i* and *j* will not involve any real time or expenditure of other resources. In such a case, *i* is a constraint upon *j* with a *dummy activity* connecting the two events. Consider the example of a car taken to a garage for cleaning. The inside as well as the outside of the car is to be cleaned before it is taken away from the garage. The events can be put down as follows:

Event 1 : Start car from house
Event 2 : Park car in garage
Event 3 : Complete outside cleaning
Event 4 : Complete inside cleaning
Event 5 : Take car from garage
Event 6 : Park car in house

Consider the network shown in Fig. 2-5 for this project. It is assumed that inside cleaning and outside cleaning can be done concurrently by two assistants. Activities *B* and *C* represent these cleaning operations. What do activities *D* and *E* stand for? Their time consumptions are zero but they express the condition that events 3 and 4 must occur before event 5 can take place. Activities *D* and *E* are called the *dummy activities* and are usually indicated by dashed lines to distinguish them from real time-consuming activities. However, the network can be drawn in a better manner

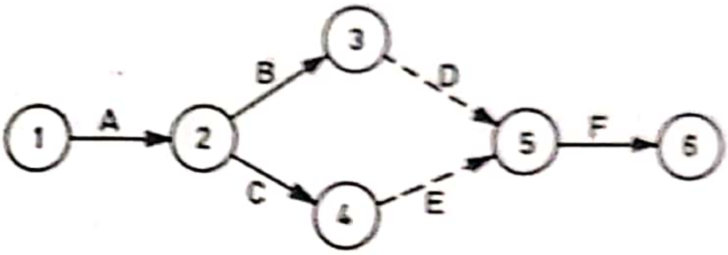

FIGURE 2-5

as shown in Fig. 2-6. The improvements in this figure are obvious. The two dummy arrows have been reduced to one. Event 4 does not occur

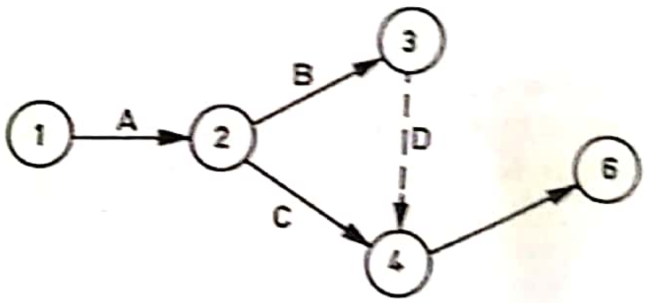

FIGURE 2-6

until event 3 has occurred—they may occur concurrently. Event 5 has been removed since it is obvious that the car has to be taken out of the garage before it can be brought back home.

In the majority of the PERT networks where ambiguities are not caused, the start bubble or circle is omitted and only the completed event is recorded, it being understood that the *start* event must have taken place before the *completed* event can occur.

PARTIAL CONSTRAINT

There is another case where the occurrence of an event is partially dependent on another activity being completed. By this, we mean that event j cannot occur until at least a part of activity J is over. Consider, for example, a foundation laying project; the erection of side boards or shuttering need not wait until the excavation is completed. While this can definitely be done, it would be economically poor planning. Figure 2-7 shows this poor planning graphically. In Fig. 2-8, the same network is repeated but without the "Shuttering Started" operation since "Shuttering Completed" indicates that it must have started. Figure 2-9 shows the improved planning where the shuttering operation can begin after part A of excavation is completed. The dummy activities are shown by dashed lines.

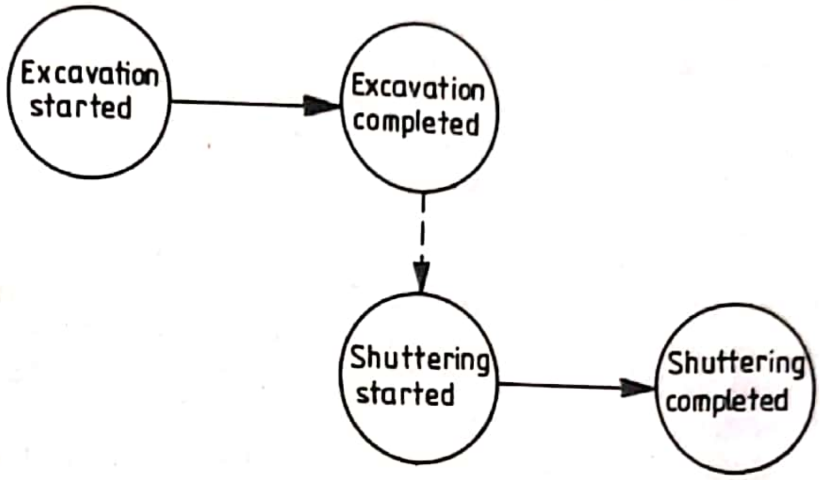

FIGURE 2-7

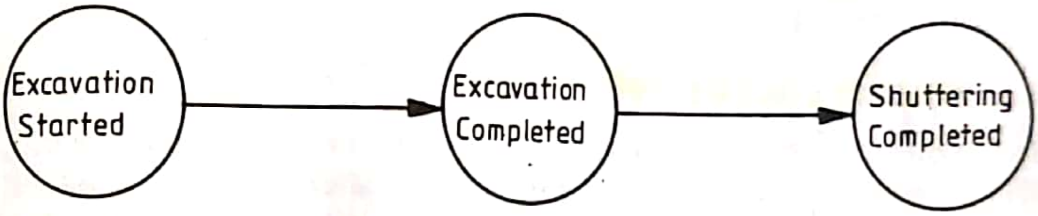

FIGURE 2-8

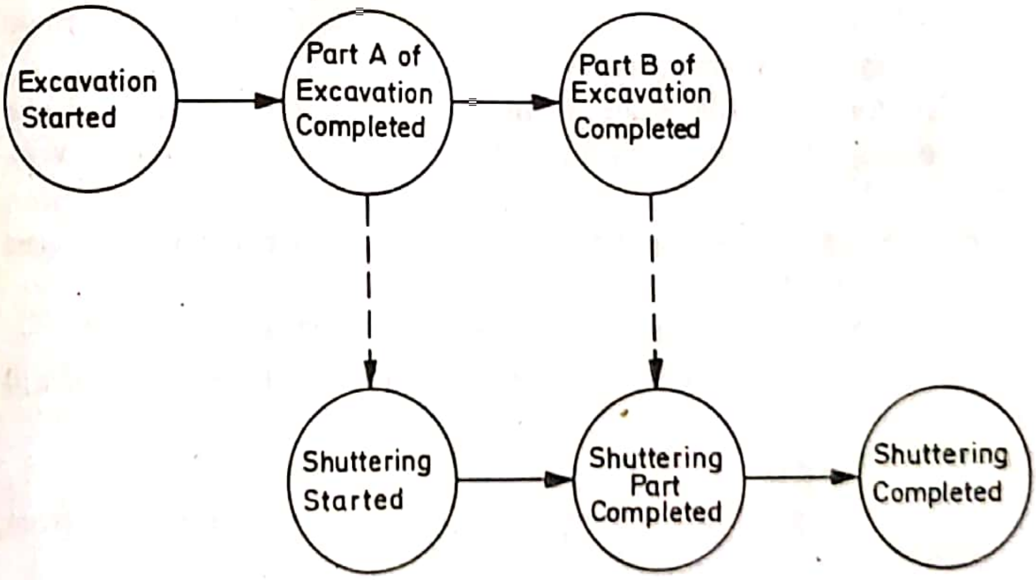

FIGURE 2-9

2.3 FORWARD AND BACKWARD PLANNING

The PERT planner can build his network in one of two ways. One approach would be to start from an initial or a starting event and build up the events and activities until the end event is reached. In this process, the planner keeps on thinking "what event comes next?" and "what events can take place concurrently?". In a complex situation, this is not an easy task because the activities in this process are not end-event oriented. A majority of persons would like to think backward. This means, having the goal in view, we work backward, thinking "if we want to achieve this, what should have taken place?". In this manner, we come to the initial event. This is in a certain sense system oriented, and the indenture level concept or the hierarchical breakdown, discussed in Section 1.5, will aid this process. Many a time, it may be easier to think: "Well, here is an aircraft as the final system. It consists of the propulsion system, the control system, the landing-gear system, and the structural system. The propulsion system consists of the engine, the fuel-delivery system, the temperature-indicating system, ... The landing-gear system has wheels, retracting unit, locking unit, ..." In this way, we can work backward, starting from the final goal, to several earlier events. In practice, however, we cannot strictly adhere to either the forward planning or backward planning procedure. A combination of both will be adopted, the network being traversed back and forth several times until it is found to be satisfactory to the PERT planner.

2.4 NUMBERING THE EVENTS

The event numbers should in some respect reflect their logical sequences. When a complicated network has been formed after numerous additions and deletions (which are unavoidable before a final acceptable network is obtained), the problem of assigning numbers to the events has to be considered. A rule devised by D. R. Fulkerson reduces this sequential numbering to the following steps:

(i) An "initial" event is one which has arrows coming out of it and none entering it. In any network, there will be one such such event. Number it "1".

(ii) Delete all arrows emerging from event 1. This will create at least one more "initial event".

(iii) Number these new initial events as "2, 3, . . .".

(iv) Delete all emerging arrows from these numbered events which will create new initial events.

(v) Follow step (iii).

(vi) Continue until the last event which has no arrows emerging from it is obtained.

Let us consider the network shown in Fig. 2-10 and apply Fulkerson's rule to number the events:

(i) Event A has no arrows entering, but only emerging arrows. Hence, we shall number it 1.

(ii) There are two arrows a and b emerging from this numbered event. Deletion of these arrows yields two events B and H with no entering arrows, but only emerging arrows.

(iii) These will be numbered 2 and 3, respectively.

(iv) From these newly-numbered events 2 and 3, arrows c, d, e, and f

emerge. Deleting these, we get events E and J which have only emerging arrows. *Notice carefully that event K will have an arrow entering after deleting e and f, and hence cannot form initial events.* Number events E and J as 4 and 5, respectively.

(v) Delete arrows g, d, and h. These give events F and K which will be numbered 6 and 7, respectively.

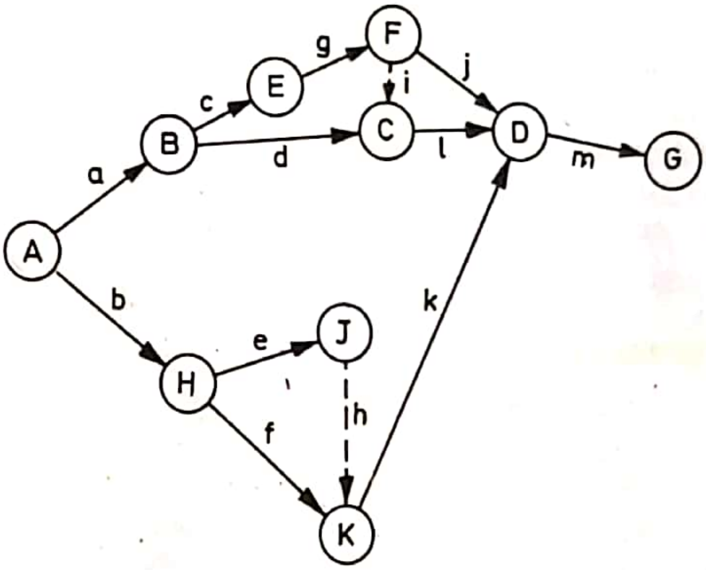

FIGURE 2-10

(vi) Delete arrows i, j, and k. Events C and D will get numbers 8 and 9, respectively.

(vii) Finally, Event G which has no emerging arrows will form the end event and is numbered 10.

The network with events numbered is shown in Fig. 2-11.

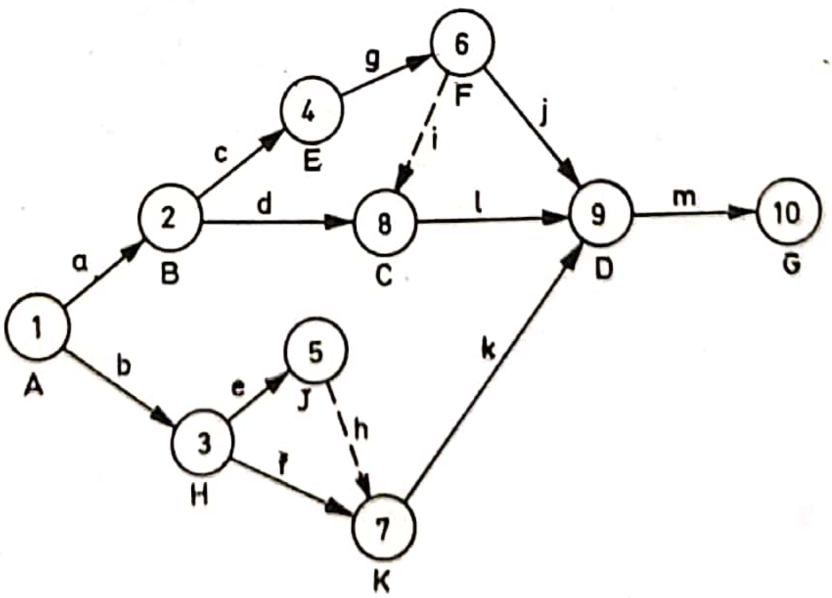

FIGURE 2-11

In the foregoing example where there are only ten events, numbering them serially 1–10 may be all right. But in large networks, where extensive modifications may have to be made either before the network is finalized or as the project progresses, freedom must be kept to add new events and number them without causing inconsistencies or loops. This can be achieved by what is known as *skip numbering*. Here, in the initial event numbering, use is made of only every tenth number, such as 10, 20, 30, 40, and 50. Any event added later would be assigned a number which lies between the number of the immediately preceding event (predecessor event) and that of the immediately succeeding event (successor event). Another practice would be to leave out such numbers as 8, 9; 18, 19; 28, 29 in the initial network and allot the left-out numbers to the newly-added events. Obviously, we can devise a variety of ways by which this kind of freedom is built into the system.

2.5 TIME ESTIMATES

After the planner has decided on the network, the next attention is on the time required for the execution of each activity or job. Unfortunately, today's environment, because of the various uncertainties involved, is such that an exact estimation of the time a job will take is very difficult. There is always a pressure to get the job done in the minimum interval of time. This is the usual impatience of the manager. The person immediately in charge of a particular operation, while being quite willing to put an effort to get it done with minimum delays, unfortunately, is dependent on so many uncertainties that, in the majority of cases, he is pressed for time. In complex programmes such as weapon systems, we are always looking for a technological breakthrough that can be incorporated to meet the schedule. To take these uncertainties into account, three kinds of time estimates are generally obtained. These are:

(a) *The Optimistic Time Estimate* This is the estimate of the shortest possible time in which an activity can be completed under ideal conditions. In arriving at this estimate, no provisions are made for delays or setbacks. Better than normal conditions are assumed to prevail during the execution of the job. We shall denote this estimate by t_0.

(b) *The Pessimistic Time Estimate* This is the maximum possible time it could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate for the activity. Of course, major catastrophes such as labour strikes or unrest and acts of God are excluded from this estimate. This time estimate is denoted by t_p.

(c) *The Most Likely Time Estimate* This is the time estimate which lies between the optimistic and pessimistic time estimates. It assumes that things go in the normal way, with a few setbacks, usual lapses in deliveries, no dramatic breakthroughs, and so on. It reflects a situation where "things are as usual, nothing exciting". The most likely estimate is represented by t_L.

These three estimates are stated in days, weeks or months and represent calendar dates and not actual working days. Once these estimates are made, they should be held firm unless a change in resources demands a corresponding alteration in them.

2.6 SINGLE-VERSUS-MULTIPLE TIME ESTIMATES

We can appreciate the multiple time estimates in projects where research and development activities, technological breakthroughs, political pressures have a considerable effect. However, in the networks used in process and construction industries, where vast experience has provided a basis for reliable and accurate time estimates, a single time estimate appears to be more reasonable. Such a single time estimate will make calculations much simpler and small networks can be analyzed by hand calculations. Using single time estimate in large networks will make it possible to employ smaller and more readily available computers without resorting to larger capacity computers. This will reduce both time and money.

2.7 FREQUENCY DISTRIBUTION

The three time estimates are provided by the person immediately in charge of a particular job and his estimation is based upon his information and previous experience. This experience is in regard to the time taken for completion of activities of a particular type under diverse conditions. This means that some sort of a relationship exists or has been established between the jobs of a particular type and the various times or durations they have consumed for completion. Such a relationship expressed by a curve is called the *frequency distribution curve* (Fig. 2-12). This curve gives the information as to how many jobs were finished in a given duration.

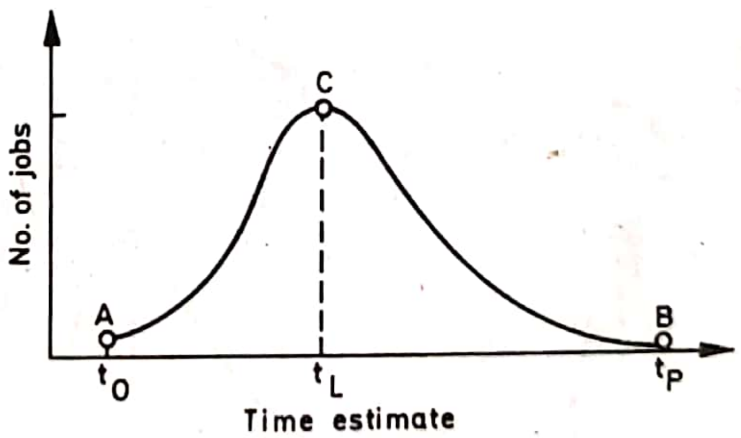

FIGURE 2-12

In estimating the optimistic time t_O, we assume better than normal conditions. The number of cases when such situations exist is obviously not large. Similarly, the pessimistic estimate assumes unusually bad conditions and such instances are also small. There will be a large number of cases which fall under the category of the most likely time estimate. If we were to draw a distribution curve, it would roughly have a shape shown in Fig. 2-12. Point A would correspond to t_O, point B to t_P, and point C to t_L. Frequency distribution curves having a single hump (as shown in Fig. 2-12) are generally called *unimodal curves*. If the curve is symmetrical on either side of t_L and exhibits certain distribution properties, it is known as a *normal curve*; otherwise, it is said to have a *skew* which could be left- or right-sided.

Consider the four distribution curves shown in Fig. 2-13. Suppose, for four different activities of a PERT network, the following time estimates were obtained:

Number	t_O	t_L	t_P
Activity 1	3	6	9
Activity 2	5	6	9
Activity 3	3	6	7
Activity 4	5	6	7

Obviously,

the curve in Fig. 2-13a corresponds to activity 2,
the curve in Fig. 2-13b corresponds to activity 1,
the curve in Fig. 2-13c corresponds to activity 4,
the curve in Fig. 2-13d corresponds to activity 3.

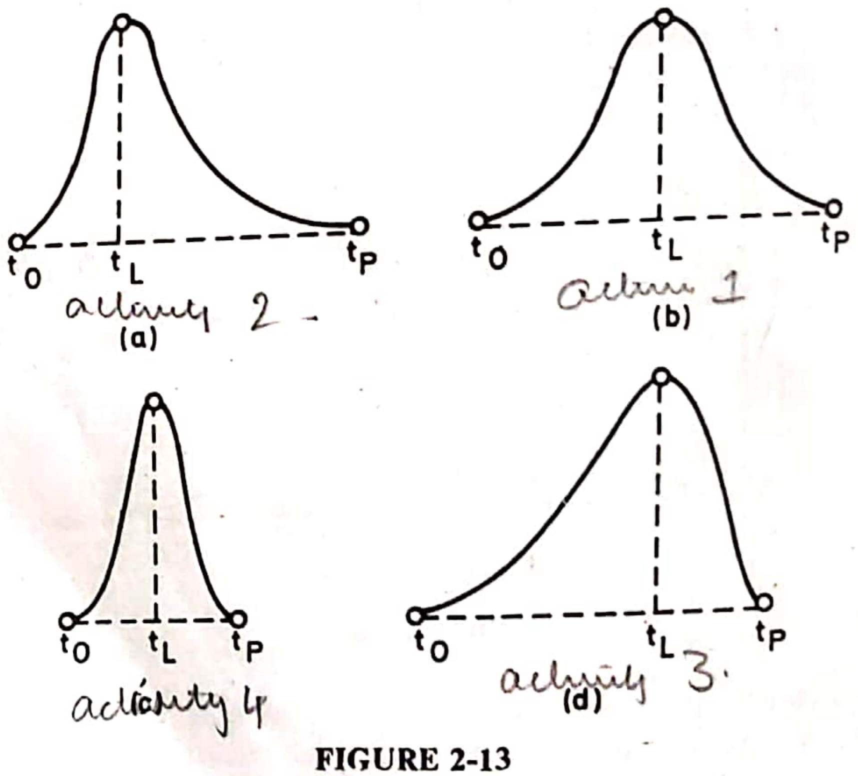

FIGURE 2-13

Consider the case of 70 measurements, in metres, of the stopping distances of cars. The cars are assumed to be travelling at the same speed before the brakes are applied. The road conditions are assumed to be the same for all cars. The data shown in Table 2-1 gives the stopping distances.

TABLE 2-1

76	53	64	40	56	60	61
62	30	34	44	38	58	72
39	43	44	54	76	38	42
36	46	63	57	27	48	59
45	53	35	32	47	58	36
63	55	53	44	52	46	51
47	64	54	65	56	65	68
56	66	69	59	67	52	58
44	55	21	64	22	72	37
81	74	84	42	41	75	55

The interest now is on the data which tells how many cars stopped in a given interval of distance. The statistical pattern which gives this information is called the frequency distribution, i.e., this tells how frequently a car can be stopped within a given range. The table gives the information that the stopping distances vary from about 20 metres to about 85 metres. Let us divide this range into 10 equal divisions and count the number of cars falling into each division as shown in Table 2-2.

TABLE 2-2

Class interval (metres)	*Tally*	*Frequency*
20 to 29	///	3
30 to 39	///// //////	11
40 to 49	///// ///// /////	15
50 to 59	///// ///// ///// /////	20
60 to 69	///// ///// ////	14
70 to 79	///// /	6
80 to 89	/	1

The information given in Table 2-2 can be indicated on a bar chart. If the ranges chosen are equal, then the area of each rectangle will be proportional to the frequency distribution. Such a bar chart is also called a *histogram* which, for this case, is shown in Fig. 2-14. The vertical axis gives the number of cars stopping in a given range. This is called the frequency per given range. In our case, the axis will read frequency per 10 metres.

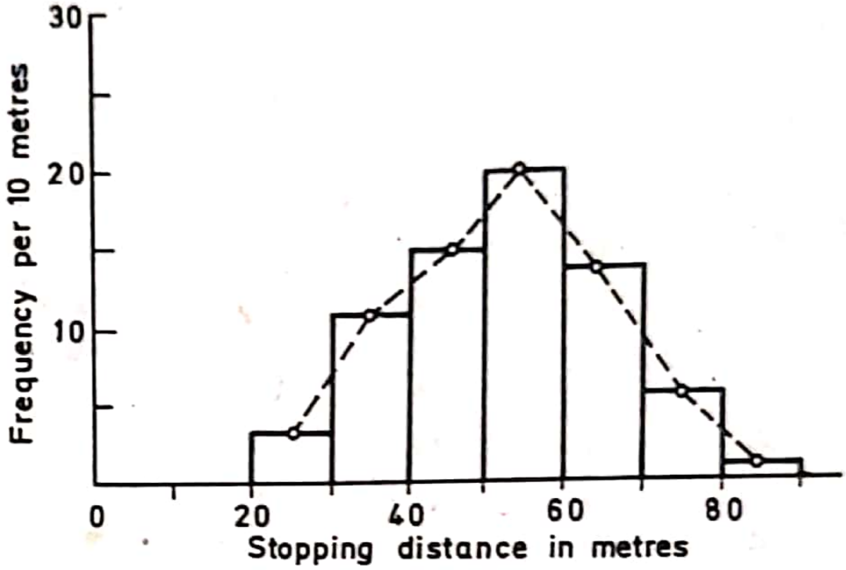

FIGURE 2-14

In drawing the histogram, Fig. 2-14, the two adjacent ranges such as 20 to 29 and 30 to 39 are made to touch each other, i.e., no gap is left between 29 and 30. If the range is changed from 10 metres to some other value, the data will give a different histogram. Smaller the range, more prominent will be the differences in heights between the bars in that histogram.

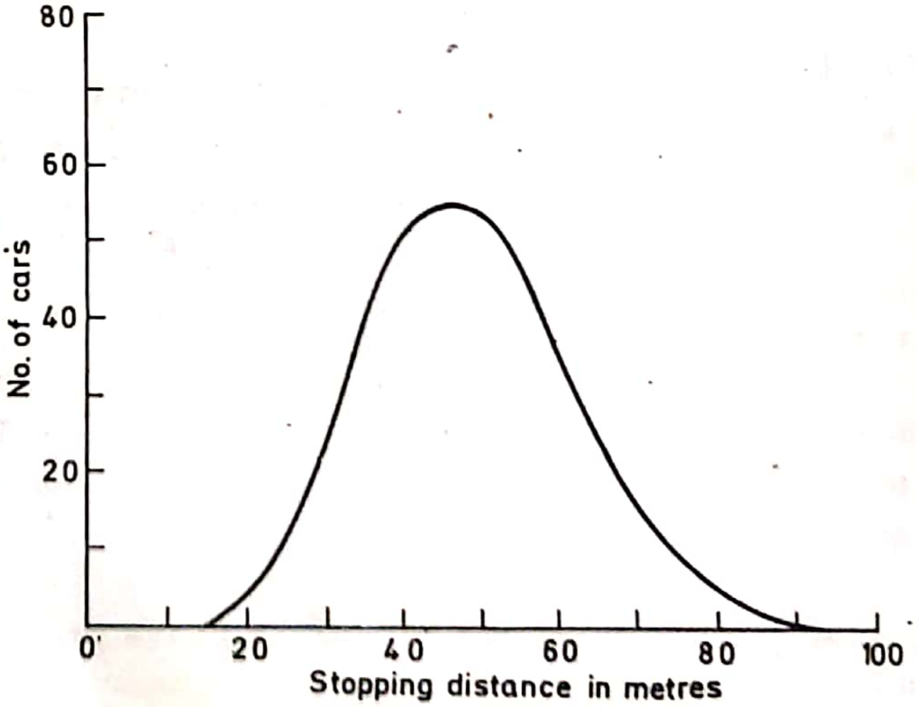

FIGURE 2-15

This means that individual properties will be shown more prominently. For our analysis, we are interested in group behaviour, and hence a suitable range should be chosen to give the correct picture.

If the midpoints of the top sides of the rectangles are joined by straight lines, we get what is known as the *frequency polygon*, Fig. 2-14. If the number of cases considered (here, the number of cars) is large, we can reduce the interval (from the present 10 metres), and the frequency polygon will consist of short but a large number of straight lines. In the limit, the polygon will assume the shape of a smooth curve, also known as the frequency distribution curve, Fig. 2-15.

PROBLEMS

1 Number the events in the network in Fig. 2-16 according to the Fulkerson rule, in steps of 10. The start event is numbered 10.

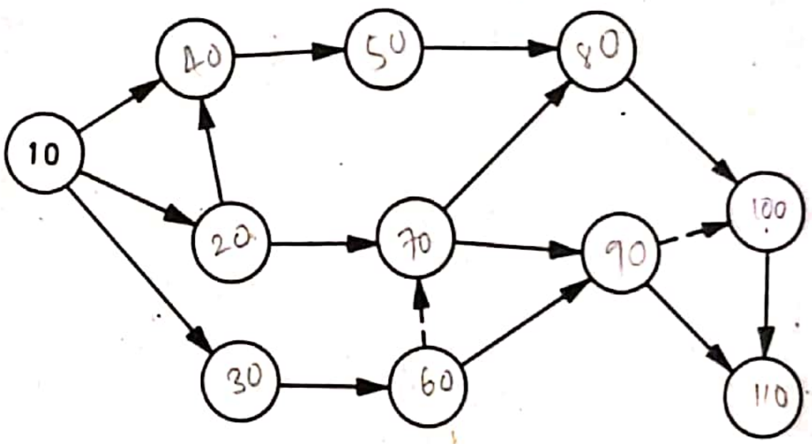

FIGURE 2-16

2 Draw the network for the following project and number the events according to Fulkerson's rule:

Event number	*Preceded by*
A	Start event
B	*A*
C	*B*
D	*B*
E	*D*
F	*B*
G	*E*
H	*G*, *E*
J	*D*, *F*, *H*
K	*C*, *J*
L	*K*

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equal to σ. It is worth remembering that the value of $f(x)$ is negligible for such values of x as are more than 3σ away from μ.

3.4 THE BETA-DISTRIBUTION

The PERT analysts have found that the *beta*-distribution curve happens to give fairly satisfactory results for most of the activities. This distribution curve is different from the normal curve and has a shape as shown in Fig. 3-4. We shall assume that the β-distribution curve will fit into our

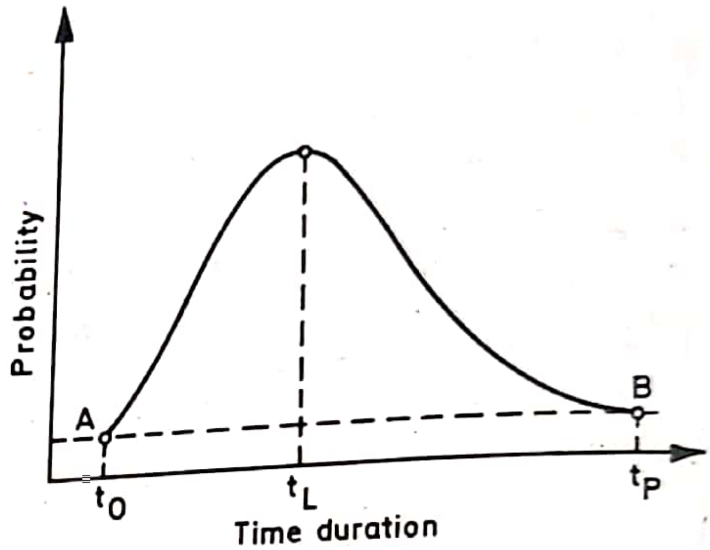

FIGURE 3-4

study. In order to make use of its general characteristics, we take, in the theoretical curve (Fig. 3-4), point A to coincide with the optimistic time t_O and point B, with the pessimistic time t_P, i.e., the range is equal to $(t_P - t_O)$. The mode is made to correspond with the most likely time t_L.

For a distribution of this type, the standard deviation is approximately one-sixth of the range, i.e.,

$$\sigma = \frac{t_P - t_O}{6}.$$

The variance therefore is

$$\sigma^2 = \left(\frac{t_P - t_O}{6}\right)^2.$$

Variance, as explained earlier, is a measure of the dispersion. Since it depends on the range $(t_P - t_O)$, larger the variance, greater will be the uncertainty.

Let us consider the time estimates obtained from two persons, X and Y, for the execution of a particular job. Suppose that the estimates given are as follows:

	t_O	t_L	t_P
Estimate by X	6	8	10
Estimate by Y	5	7	11

Calculating the variance, we have

$$\sigma^2 = \left(\frac{10-6}{6}\right)^2 = 0.44 \quad \text{for } X,$$

$$\sigma^2 = \left(\frac{11-5}{6}\right)^2 = 1.00 \quad \text{for } Y.$$

Hence, Y was more uncertain about his estimate than X.

3.5 EXPECTED TIME OR AVERAGE TIME

After having obtained—using the β-distribution—the variance and the standard deviation from the optimistic and pessimistic time estimates, the next task is to get some idea about the average time taken for the completion of the job. This average time is called by the PERT analysts the *expected time* and is denoted by t_E. Our good friend, Mr. Statistician, once again comes to our rescue and suggests that in β-distribution we can get the average by adding together one-sixth of the optimistic, two-thirds of the most likely, and one-sixth of the pessimistic time estimates. That is,

$$t_E = \frac{1}{6}t_O + \frac{2}{3}t_L + \frac{1}{6}t_P$$

or

$$\boxed{t_E = \frac{t_O + 4t_L + t_P}{6}.}$$

It is one of the most important equations in PERT analysis. It shows us how to calculate the average or expected time from the three time estimates. The average time indicates that there is a fifty-fifty chance of getting the job done *within* that time. The importance of the average time or the expected time with reference to a network is shown in Fig. 3-5. For each activity, the optimistic, most likely, and pessimistic time estimates have been given. 10 is the start event and 100 the end event. There are four paths from the start event to the end event. These are:

A : 10-20-50-80-100
B : 10-20-50-70-100
C : 10-30-70-100
D : 10-40-60-90-100

The time taken from the start to the end by each of these paths will be