

8.1

$$\forall x \forall y \forall z ((P_x \wedge P_y) \wedge P_z) \rightarrow ((x=y \vee y=z) \vee x=z))$$

8.2

i) $D_A = \{1, 2\}$

$$|Q^2|_A = \{\langle 1, 2 \rangle, \langle 2, 1 \rangle\}$$

$$|a|_A = 1$$

$$|b|_A = 2$$

ii) $D_B = \{1\}$

$$|P|_B = \emptyset$$

8.3

No, from (i), you can do to (ii):

~~Pa~~

Very good. For logical equivalence you should also show the other direction:

$$\frac{\frac{\frac{[b=a \wedge Pb]}{Pb} \quad \frac{[b=a \wedge Pb]}{b=a}}{\exists x(x=a \wedge Px)} \quad Pa}{Pa} \text{=Elim} \quad \exists\text{Elim}$$

$$\frac{[a=a] \quad Pa}{a=a \wedge Pa} \wedge\text{I} \quad \exists\text{I}$$

$$\exists x(x=a \wedge Px)$$

In case it's of interest to you, Halbach further remarks: "I prefer (i) because it is much simpler. In fact, one could propose to formalise 'x is a philosopher' using an additional existential quantifier."

8.4 \pm $\frac{[a=a]}{\exists y \neg y} \rightarrow \exists \pm$ ✓

\pm)

$$\frac{[Pa] \wedge [a=b]}{P_b} = E \quad \frac{[\neg Pb]}{\neg E}$$

$$\frac{\neg a = b}{\neg E}$$

$$\frac{\exists y \neg a = y}{\exists I}$$

$$\frac{\exists x \neg Px \quad \exists x \exists y \neg x = y}{\exists E}$$

$$\frac{\exists x Px \quad \exists x \exists y \neg x = y}{\exists E}$$
 ✓

8.5 \pm)



I)

$$\frac{[\forall y (P_y \rightarrow a=y) \wedge P_a]}{P_a} \wedge E$$

$$\frac{P_a}{P_b} [a=b]$$

$$\frac{[\forall y (P_y \rightarrow a=y) \wedge P_a]}{\forall y (P_y \rightarrow a=y)} \wedge E$$

$$\frac{\forall y (P_y \rightarrow a=y)}{P_b \rightarrow a=b} \forall E$$

$$\frac{P_b \rightarrow a=b}{a=b} [P_b] \rightarrow E$$

$$a=b$$

$\leftrightarrow E$

$$P_b \leftrightarrow a=b \quad \forall I$$

$$\forall y (P_y \leftrightarrow a=y) \quad \forall I$$

$\exists I$

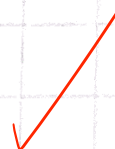
$$\exists x \forall y (P_y \leftrightarrow x=y)$$

$\exists E$

$$\exists x \forall y (P_y \leftrightarrow x=y) \wedge P_x$$

$$\exists x \forall y (P_y \leftrightarrow x=y)$$

~~$\exists x \forall y (P_y \leftrightarrow x=y) \wedge P_x$~~



II)

$$[\forall y (P_y \leftrightarrow a=y)] \quad \forall E$$

$$P_b \leftrightarrow a=b \quad [P_b] \leftrightarrow E$$

$$a=b$$

$$P_b \rightarrow a=b$$

~~$P_b \rightarrow a=b$~~

$\forall I$

$$\forall y (P_y \rightarrow a=y)$$

$\wedge I$

$$\forall y (P_y \rightarrow a=y) \wedge P_a$$

$\exists \neq$

$$\exists x (\forall y (P_y \rightarrow x=y) \wedge P_x)$$

$\exists E$

$$\exists x \forall y (P_y \leftrightarrow x=y)$$

$$\exists x (\forall y (P_y \leftrightarrow x=y) \wedge P_x)$$



8.8

I) $\forall x \exists y (Q_1 x \wedge Q_2 y \wedge \neg y = x \wedge \forall z (Q_3 z \rightarrow (x = z \vee y = z)))$

$\exists x \exists y (Q_1 x \wedge Q_2 y \wedge \neg x = y \wedge \forall z (Q_3 z \rightarrow (x = z \vee y = z)))$

Good. Halbach splits his existential and universal quantifiers:

$\exists x \exists y (Q_1 x \wedge Q_1 y \wedge \neg x = y) \wedge \forall x \forall y \forall z ((Q_1 x \wedge Q_1 y \wedge Q_1 z) \rightarrow (x = y \vee y = z \vee x = z))$

II)

$\exists x (Q_1 x \wedge P_x \wedge \forall y ((Q_1 y \wedge P_y) \rightarrow x = y) \wedge \forall z (Q_3 z \rightarrow R_{xz}))$

III)

Again, good. Halbach uses an \leftrightarrow instead of \rightarrow for the clever tutor expression: $\exists x (\forall y (P_y \wedge Q_y \leftrightarrow x = y) \wedge \forall z (Q_3 z \rightarrow R_{xz}))$

$\exists x (Q_1 x \wedge \forall y (Q_1 y \rightarrow R_{xy}) \wedge \forall z ((Q_3 z \wedge \forall y (Q_1 y \rightarrow R_{zy})) \rightarrow x = z) \wedge P_x)$

Careful using universal quantifiers and \wedge . Here is Halbach:

$\exists y \forall x (((\forall z (Q_3 z \rightarrow R_{xz}) \wedge Q_1 x) \leftrightarrow x = y) \wedge P_y)$

IV $\forall x \forall y \forall z ((Q_1 x \wedge Q_1 y \wedge Q_3 z) \rightarrow (x = y \vee x = z \vee y = z))$

