

(6.7)

$$\begin{array}{c} \text{I)} \\ \frac{[P] \quad \vee \text{Intro } 2}{R \vee P} \\ \hline P \rightarrow (R \vee P) \rightarrow \text{Intro} \end{array}$$

$$\begin{array}{c} \text{II)} \\ \frac{R \wedge Q \quad R \wedge Q}{Q \wedge R} \wedge \text{Elim } 2 \quad \frac{R \wedge Q}{R} \wedge \text{Elim } 1 \\ \hline Q \wedge R \rightarrow \text{Intro} \end{array}$$

$$\begin{array}{c} \text{III)} \\ \frac{P \rightarrow Q \quad [P]}{Q} \rightarrow \text{Elim} \\ \hline [ \neg Q ] \\ \hline \neg P \rightarrow \text{Intro} \\ \hline \neg Q \rightarrow \neg P \end{array}$$

$$\begin{array}{c} \text{IV)} \\ \frac{[P \rightarrow \neg P] \quad [P]}{\neg P} \rightarrow \text{Elim} \\ \hline \neg P \quad [P] \\ \hline \neg P \rightarrow \text{Intro} \\ \hline (P \rightarrow \neg P) \rightarrow \neg P \end{array}$$

$$\begin{array}{c} \text{V)} \\ \frac{P \leftrightarrow Q \quad [P]}{Q} \leftrightarrow \text{Elim } 1 \\ \hline Q \quad \neg Q \\ \hline \neg P \rightarrow \text{Intro} \end{array}$$

VI)

$$\begin{array}{c}
 \frac{[P] \quad [Q]}{P \wedge Q} \quad \wedge \text{Intro} \\
 \frac{P \wedge Q}{(P \wedge Q) \rightarrow R} \rightarrow \text{Elim} \\
 \frac{}{R} \\
 \frac{}{Q \rightarrow R} \rightarrow \text{Intro} \\
 \frac{Q \rightarrow R}{P \rightarrow (Q \rightarrow R)} \rightarrow \text{Intro}
 \end{array}$$

VII)

$$\begin{array}{c}
 \frac{[ \neg P ]}{\neg Q \rightarrow \neg P} \rightarrow \text{Intro} \\
 \frac{\neg Q \rightarrow \neg P \quad [ \neg Q ]}{\neg P} \rightarrow \text{Elim} \\
 \frac{\neg P \quad [ P ]}{Q} \neg \text{Elim} \\
 \frac{Q}{P \rightarrow Q} \rightarrow \text{Intro} \\
 \frac{P \rightarrow Q \quad \neg (P \rightarrow Q)}{P} \neg \text{Elim}
 \end{array}$$

A faster way would be:

$$\begin{array}{c}
 \frac{[ \neg P ] \quad [ P ]}{Q} \neg \text{Elim} \\
 \frac{Q}{P \rightarrow Q} \rightarrow \text{Intro} \\
 \frac{P \rightarrow Q \quad \neg (P \rightarrow Q)}{P} \neg \text{Elim}
 \end{array}$$

↑  
this is how  
Hilbert  
does it



[illegible]
$$\begin{array}{c}
 \frac{R \rightarrow P2 [R]}{P2 \rightarrow Q1} \rightarrow Elim \\
 \frac{P2 \rightarrow Q1}{Q1 \rightarrow Q} \rightarrow Elim \\
 \frac{Q1 \rightarrow Q}{Q} \rightarrow Elim \\
 \frac{Q}{Q \wedge R} \wedge Intro \\
 \frac{Q \wedge R}{-R} \neg Elim \\
 \frac{-R}{\neg Intro} \neg Intro \\
 \frac{\neg Intro}{Q \wedge \neg R} \wedge Intro
 \end{array}$$

Lively! Fed free to sled to the easier way.