

EXERCISE 1.5. Consider the relation containing the ordered pairs $\langle \text{Germany, Italy} \rangle$, $\langle \text{Germany, Germany} \rangle$, $\langle \text{Italy, Italy} \rangle$, $\langle \text{France, France} \rangle$ but no other pairs.

- (a) Is this relation reflexive on the set $\{\text{Germany, Italy, France}\}$?
- (b) Is this relation transitive on $\{\text{Germany, Italy, France}\}$?
- (c) Is this relation symmetric on $\{\text{Germany, Italy, France}\}$?
- (d) Is it an equivalence relation on $\{\text{Germany, Italy, France}\}$?
- (e) Is it an equivalence relation on $\{\text{Germany, France}\}$?

EXERCISE 1.6. Consider the following relations, where d and e are persons:

- (i) the set of all ordered pairs $\langle d, e \rangle$ such that d is taller than e
- (ii) $\{\langle d, e \rangle : d \text{ loves } e\}$
- (iii) the relation with all ordered pairs $\langle d, e \rangle$ as members such that d is the father of e
- (iv) the relation with all ordered pairs $\langle d, e \rangle$ as members such that e is the father of d
- (v) the relation of *being of a similar age*

Determine for each of these relations whether it is symmetric, whether it is transitive, and whether it is a function.

EXERCISE 1.7. Identify premisses and conclusions in the following arguments. Are the arguments logically valid?


- (i) All men are mortal, Socrates is a man. Thus, Socrates is mortal. ✓
- (ii) Houses become cheaper only if interest rates are falling. Now houses are actually becoming cheaper, although interest rates are not falling. So the Prime Minister will become the king of France.
- (iii) Tom will move to Edinburgh. This is because he got a job there and he can't find another job where he is living now.
- (iv) Alfred can see the house. So he must have at least one eye.
- (v) If the mind is immortal, it's not identical with the body. So if the mind is identical to the body, the mind is not immortal.
- (vi) This must be a Manx cat: it hasn't got a tail.

yellow = premiss
blue = conclusion
✓ = valid
x = invalid

it's valid - contradictory premisses
valid or invalid?

EXERCISE 1.8. Identify the premisses and the conclusion in the following argument:

Many students will be either in Hegel's or in Schopenhauer's lectures, if they are scheduled at the same time. And of course Schopenhauer will schedule them at the same time as Hegel's. If Hegel's lectures are entertaining, then many students will go to them. That means of course many students will go to Hegel's but not many will go to Schopenhauer's lectures. For if Schopenhauer's lectures are entertaining, Hegel's must be entertaining as well; and of course many students will only come to Schopenhauer's lectures if they are entertaining.



Oliver Berg

Man: Aha. If I didn't pay, why are you arguing? I Got you!

Mr. Vibrating: No you haven't.

Man: Yes I have. If you're arguing, I must have paid.

Mr. Vibrating: Not necessarily. I could be arguing in my spare time.

In this part, the man makes the following argument:

You argue if and only if I paid.

You are arguing.

So: I have paid.

The "if and only if" here is particularly important. It is based on the fact that earlier Mr. Vibrating stopped arguing when he did not pay.

Mr. Vibrating: (*Rings bell*) Good Morning.

Man: What?

Mr. Vibrating: That's it. Good morning.

Man: I was just getting interested.

Mr. Vibrating: Sorry, the five minutes is up.

Man: That was never five minutes!

Mr. Vibrating: I'm afraid it was.

Man: It wasn't. (*Pause*)

Mr. Vibrating: I'm sorry, but I'm not allowed to argue anymore.

Man: What?!

Mr. Vibrating: If you want me to go on arguing, you'll have to pay for another five minutes.

Man: Yes, but that was never five minutes, just now. Oh come on!

Mr. Vibrating: (*Hums*)

Man: Look, this is ridiculous.

Mr. Vibrating: I'm sorry, but I'm not allowed to argue unless you've paid!

Although, at first sight, Mr. Vibrating seems to make a contradiction:

I'm sorry, but I'm not allowed to argue unless you've paid!

[...]

Not necessarily. I could be arguing in my spare time.

Because the first part seems to say, "I argue if and only if you've paid", it is a contradiction for him to say that "he is both unpaid and allowed to argue", these statements are inconsistent. However, the "allowed" in the first sentence might be interpreted as only relating to work and so he bypasses this contradiction by saying he could be arguing in his spare time.

Therefore, the man makes the implicit claim that if Mr. Vibrating is not *allowed* to argue, he will not argue at all.