8.1

$\forall x \forall y \forall z (((Px \wedge Py) \wedge Pz) \rightarrow ((x=y \vee y=z) \vee x=z))$

8.2

I) $D_A = \{1, 2\}$

$|Q^2|_A = \{\langle 1,2\rangle, \langle 2,1\rangle\}$

$|a|_A = 1$

$|b|_A = 2$

II) $D_B = \{1\}$

$|P^1|_B = \emptyset$

8.3 No, from (I), you can do to (II):

$$\dfrac{\dfrac{[a=a]^{=} \qquad Pa}{a=a \wedge Pa}\wedge I}{\exists x(x=a \wedge Px)}\exists I$$

(8.4) I)

$$\frac{[a = a]^{=}}{\exists_y\, 1 = y}\ \exists I$$

II)

$$\dfrac{\exists_x P_x \qquad \dfrac{\exists_x \neg P_x \qquad \dfrac{\dfrac{\dfrac{\dfrac{[P_a] \quad \wedge \quad [a = b]}{P_b}\ {=}E \qquad [\neg P_b]}{\neg a = b}\ \neg I}{\exists_y \neg a = y}\ \exists I}{\exists_x \exists_y \neg x = y}\ \exists I}{\exists_x \exists_y \neg x = y}\ \exists E}{}\ \exists E$$

(8.5)

I)

I)

$$[\forall y(Py \to a=y) \land Pa] \quad \land E$$

$$Pa \qquad [a=b]$$

$$Pb$$

$$[\forall y(Py \to a=y) \land Pa] \quad \land E$$

$$\forall y(Py \to a=y) \quad \forall E$$

$$Pb \to a=b \qquad [Pb] \quad \to E$$

$$a=b$$

$$\leftrightarrow E$$

$$Pb \leftrightarrow a=b \quad \forall I$$

$$\forall y(Py \leftrightarrow a=y) \quad \exists I$$

$$\exists x(\forall y(Py \to x=y) \land Px) \qquad \exists x \forall y(Py \leftrightarrow x=y) \quad \exists E$$

$$\exists x \forall y(Py \leftrightarrow x=y)$$

II)

$$[\forall y(Py \leftrightarrow a=y)] \quad \forall E$$

$$Pb \leftrightarrow a=b \qquad [Pb] \quad \leftrightarrow E$$

$$a=b$$

$$Pb \to a=b$$

$$[\forall y(Py \leftrightarrow a=y)] \quad \forall E$$

$$Pa \leftrightarrow a=a \qquad [a=a]^{=} \quad \leftrightarrow E$$

$$Pa$$

$$\forall I$$

$$\forall y(Py \to a=y)$$

$$\land I$$

$$\forall y(Py \to a=y) \land Pa \quad \exists I$$

$$\exists x \forall y(Py \leftrightarrow x=y) \qquad \exists x(\forall y(Py \to x=y) \land Px) \quad \exists E$$

$$\exists x(\forall y(Py \to x=y) \land Px)$$

8.1

I) $\cancel{\exists x \exists y (Q_1 x \land Q_1 y \land \neg y = x \land \forall z (Q_1 z \to (x = z \lor y = z)))}$

$\exists x \exists y (Q_1 x \land Q_1 y \land \neg x = y \land \forall z (Q_1 z \to (x = z \lor y = z)))$

II)

$\exists x (Qx \land Px \land \forall y ((Qy \land Py) \to x = y) \land \forall z (Q_1 z \to Rxz))$

III)

$\exists x (Q_1 x \land \forall y (Qy \to Rxy) \land \forall z ((Q_1 z \land \forall y (Qy \to Rzy)) \to x = z)$
$\land Px)$

IV $\forall x \forall y \forall z ((Qx \land Qy \land Qz) \to (x = y \lor x = z \lor y = z))$