2 Syntax and Semantics of Propositional Logic

EXERCISE 2.1. Add quotation marks to the following sentences so that true English sentences are obtained. In some cases there is more than one solution. Try to find all solutions.

(i) Potassium designates a chemical element. “Potassium” designates a chemical element.
(ii) Snow is white is true if and only if snow is white. “Snow is white” is true if and only if snow is white.
(iii) John, Jane and Jeremy all have J as their first letter. “John”, “Jane” and “Jeremy” all have “J” as their first letter.
(iv) George is the quotation of George. “‘George’” is the quotation of “George”.
(v) Tom is monosyllabic and Reginald is polysyllabic. “Tom” is monosyllabic and “Reginald” is polysyllabic.

EXERCISE 2.2. Check whether the following expressions are sentences of $\mathcal{L}_1$.

(i) $(((P_1 \to P_1) \to P_1) \lor Q)$
(ii) $(((P_2 \land R)) \to Q_4)$
(iii) $(P \to \neg P)$
(iv) $(P\neg \to P)$
(v) $(\neg P \to P)$
(vi) $(P \to \neg\neg\neg(R \lor \neg R))$
(vii) $\neg((P \to (P \to \neg Q)) \leftrightarrow \neg\neg(R_2 \leftrightarrow \neg(P \lor R_7)))$

No bracketing conventions are applied in the expressions.

EXERCISE 2.3. The following expressions are abbreviations of $\mathcal{L}_1$-sentences. Restore the brackets that have been dropped in accordance with the Bracketing Conventions of Section 2.3.

(i) $\neg P \land Q$ (¬P ∧ Q)
(ii) $P \land \neg Q \land R \leftrightarrow \neg P_3 \lor P \lor R_5$ (((P ∧ ¬Q) ∧ R) ↔ ((¬P3 ∨ P) ∨ R5))
(iii) $\neg\neg\neg(P \to Q) \leftrightarrow P$ (¬¬¬(P → Q) ↔ P)

EXERCISE 2.4. Drop as many brackets as possible from the following $\mathcal{L}_1$-sentences by applying the Bracketing Conventions from Section 2.3.

(i) $(((\neg P \to \neg Q) \vee Q_2) \wedge P)$ ((¬P →¬Q)∨Q2)∧P
(ii) $(((\neg P \to \neg Q) \wedge Q_2) \wedge P)$ (¬P→¬Q)∧Q2∧P
(iii) $\neg(((P \wedge (P \to \neg Q)) \wedge Q_1) \wedge P)$ ¬(P∧(P→¬Q)∧Q1∧P)

EXERCISE 2.5. Show that the following sentences are tautologies. You may use partial truth tables. Examples of calculations of partial truth tables can be found on WebLearn.

(i) $P \wedge (P \to Q) \to Q$ (modus ponens)
(ii) $\neg Q \wedge (P \to Q) \to \neg P$ (modus tollens)
(iii) $P \vee \neg P$ (law of excluded middle)
(iv) $\neg(P \wedge \neg P)$ (law of contradiction)
(v) $(\neg P \to P) \to P$ (consequentia mirabilis)
(vi) $(P \to Q) \wedge (\neg P \to Q) \to Q$ (classical dilemma)
(vii) $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$ (de Morgan-law)
(viii) $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$ (de Morgan-law)
(ix) $P \wedge \neg P \to Q$ (ex falso quodlibet)

EXERCISE 2.6. Classify the following $\mathcal{L}_1$-sentences as tautologies, contradictions or as sentences that are neither.

(i) $P \wedge P$
(ii) $((P \to Q) \to R) \leftrightarrow (P \to (Q \to R))$
(iii) $(P \leftrightarrow (Q \leftrightarrow R)) \leftrightarrow ((P \leftrightarrow Q) \leftrightarrow R)$
(iv) $\neg(P \to Q) \leftrightarrow (P \wedge \neg Q)$

3 Formalisation in Propositional Logic

EXERCISE 3.1. Discuss whether the following argument is propositionally valid.

> If Jones arrives at the airport after the scheduled departure time, the plane will wait for him. Therefore, if Jones arrives at the airport after the scheduled departure time and nobody notices that he arrives at the airport after the scheduled departure time, the plane will wait for Jones.

EXERCISE 3.2. Determine the scopes of the underlined occurrences of connectives in the following sentences, which have been abbreviated in accordance with the bracketing conventions.

(i) $P \to \underline{\neg}(P_{44} \vee \neg(Q_3 \wedge \neg Q_3))$
(ii) $P \underline{\wedge} Q \wedge \neg R \wedge Q$
(iii) $P \to Q \wedge \neg R_2 \underline{\wedge} \neg(P_2 \leftrightarrow P_1)$

EXERCISE 3.3. Draw truth tables for the following English expressions in the style of the truth table for '*A* because *B*' in Section 3.1 of the Manual. That is, determine for (i)–(iv) below whether substituting a true sentence for *A* yields only true sentences or only false sentences or true and false sentences. Then check the result of substituting false sentences. Proceed in a similar way for phrases (v)–(vi), which contain *A* and *B*.

(i) Robin believes that *A*
(ii) Robin knows that *A*
(iii) Robin knows that *A*, but it's not true that *A*
(iv) The infallible clairvoyant believes that *A*
(v) *A*, but *B*
(vi) Suppose *A*; then *B*

EXERCISE 3.4. Formalise the following sentences as accurately as possible using the arrow →.

(i) If God can create the soul without the body, then soul and body are different.
(ii) The rise in interest rates is a sufficient reason for a house price crash.
(iii) The boy and the general are the same person, only if the general can remember what he did as a boy.
(iv) My believing that the wall is yellow is a necessary condition for my knowing that the wall is yellow.

A: God can create the soul without the body
B: soul and body are different
$A \to B$
A: interest rates crash
B: house prices crash
$A \to B$
A: the boy and the general are the same person
B: the general can remember what he did as a boy
$A \to B$
A: My believing that the wall is yellow
B: my knowing that the wall is yellow
$B \to A$

EXERCISE 3.5. Formalise the following sentences in the language of propositional logic. Your formalisations should be as detailed as possible.

(i) Russell and Whitehead wrote *Principia Mathematica*.
(ii) The traffic light turned green, and Bill pulled away.
(iii) Ben, who hates logic, is a philosophy student.

A: Russel wrote Principia Mathematica
B: Whitehead wrote Principia Mathematica
$A \land B$
A: The traffic light turned green
B: Bill pulled away
$A \land B$
A: Ben hates logic
B: Ben is a philosophy student
$A \land B$

EXERCISE 3.6. Show that the following argument becomes propositionally valid after adding assumptions upon which the speaker might naturally be expected to be relying. Note any difficulties or points of interest.

> Many students will be either in Hegel's or in Schopenhauer's lectures, if they are scheduled at the same time. And of course Schopenhauer will schedule them at the same time as Hegel's. If Hegel's lectures are entertaining, then many students will go to them. That means of course many students will go to Hegel's but not many will go to Schopenhauer's lectures. For if Schopenhauer's lectures are entertaining, Hegel's must be entertaining as well; and of course many students will only come to Schopenhauer's lectures if they are entertaining.