8.1

$$\forall x \forall y \forall z (((Px \land Py) \land Pz) \to ((x = y \lor y = z) \lor x = z))$$

8.2

I) $D_{\mathcal{A}} = \{1, 2\}$

$|Q^2|_{\mathcal{A}} = \{\langle 1,2\rangle, \langle 2,1\rangle\}$

$|a|_{\mathcal{A}} = 1$

$|b|_{\mathcal{A}} = 2$

II) $D_{\mathcal{B}} = \{1\}$

$|P^1|_{\mathcal{B}} = \emptyset$

8.3 No, from (I), you can do to (II):

$$\dfrac{\dfrac{[a=a]^{=} \qquad Pa}{a = a \land Pa}\land I}{\exists x (x = a \land Px)}\exists I$$

8.4 I)

$$\dfrac{[a = a]^{=}}{\exists y\ a = y}\ \exists I$$

II)

$$\dfrac{[Pa] \quad \wedge \quad [a=b]}{P_b}\ {=}E$$

$$\dfrac{P_b \qquad [\neg P_b]}{\neg a = b}\ \neg E$$

$$\dfrac{\neg a = b}{\exists y\ \neg a = y}\ \exists I$$

$$\dfrac{\exists y\ \neg a = y}{\exists x \exists y\ \neg x = y}\ \exists I$$

$$\dfrac{\exists x\ \neg P_x \qquad \exists x \exists y\ \neg x = y}{\exists x \exists y\ \neg x = y}\ \exists E$$

$$\dfrac{\exists x\ P_x}{}\ \exists E$$

8.5 I)

I)

$[\forall y(Py \to a=y) \land Pa]$ $\land E$

Pa $\quad [a=b]$

Pb

$[\forall y(Py \to a=y) \land Pa]$ $\land E$

$\forall y(Py \to a=y)$ $\forall E$

$Pb \to a=b$ $\quad [Pb]$ $\to E$

$a=b$

$\leftrightarrow I$

$Pb \leftrightarrow a=b$ $\forall I$

$\forall y(Py \leftrightarrow a=y)$ $\exists I$

$\exists x(\forall y(Py \to x=y) \land Px)$ $\quad \exists x \forall y(Py \leftrightarrow x=y)$ $\exists E$

$\exists x \forall y(Py \leftrightarrow x=y)$

II)

$[\forall y(Py \leftrightarrow a=y)]$ $\forall E$

$Pb \leftrightarrow a=b$ $\quad [Pb]$ $\leftrightarrow E$

$a=b$

$Pb \to a=b$

$\forall I$

$\forall y(Py \to a=y)$

$[\forall y(Py \leftrightarrow a=y)]$ $\forall E$

$Pa \leftrightarrow a=a$ $\quad [a=a]$ $\leftrightarrow E$

Pa

$\land I$

$\forall y(Py \to a=y) \land Pa$ $\exists I$

$\exists x \forall y(Py \leftrightarrow x=y)$ $\quad \exists x(\forall y(Py \to x=y) \land Px)$ $\exists E$

$\exists x(\forall y(Py \to x=y) \land Px)$

8.7

I) $\cancel{\exists x \exists y (Q_1 x \wedge Q_1 y \wedge \neg y = x \wedge \forall z (Q_1 z \rightarrow (x = z \vee y = z)))}$

$$\exists x \exists y (Q_1 x \wedge Q_1 y \wedge \neg x = y \wedge \forall z (Q_1 z \rightarrow (x = z \vee y = z)))$$

II)

$$\exists x (Q x \wedge P x \wedge \forall y ((Q y \wedge P y) \rightarrow x = y) \wedge \forall z (Q_1 z \rightarrow R x z))$$

III)

$$\exists x (Q_1 x \wedge \forall y (Q y \rightarrow R x y) \wedge \forall z ((Q_1 z \wedge \forall y (Q y \rightarrow R z y)) \rightarrow x = z) \wedge P x)$$

IV $\forall x \forall y \forall z ((Q x \wedge Q y \wedge Q z) \rightarrow (x = y \vee x = z \vee y = z))$