8.1

$$\forall x \forall y \forall z (((Px \wedge Py) \wedge Pz) \to ((x = y \vee y = z) \vee x = z))$$

✓

8.2

I) $D_{\mathcal{A}} = \{1, 2\}$

$|Q^2|_{\mathcal{A}} = \{\langle 1,2\rangle, \langle 2,1\rangle\}$

$|a|_{\mathcal{A}} = 1$

$|b|_{\mathcal{A}} = 2$

✓

II) $D_{\mathcal{B}} = \{1\}$

$|P^1|_{\mathcal{B}} = \emptyset$

✓

8.3

No, from (I), you can do to (II):

$$\dfrac{\dfrac{[a=a]^{=} \qquad Pa}{a=a \wedge Pa}\wedge I}{\exists x(x=a \wedge Px)}\exists I$$

✓

Very good. For logical equivalence you should also show the other direction:

$$\dfrac{\exists x(x=a \wedge Px) \qquad \dfrac{\dfrac{[b=a \wedge Pb]}{Pb} \qquad \dfrac{[b=a \wedge Pb]}{b=a}}{Pa}{=}\text{Elim}}{Pa}\exists\text{Elim}$$

In case it's of interest to you, Halbach further remarks: "I prefer (i) because it is much simpler. In fact, one could propose to formalise 'x is a philosopher' using an additional existential quantifier."

8.4 I)

$$\dfrac{[a=a]^{=}}{\exists y\; a=y}\;\exists I$$

II)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[Pa] \quad \wedge \quad [a=b]}{Pb}\;{=}E \qquad [\neg Pb]}{\neg a=b}\;\neg I}{\exists y\; \neg a=y}\;\exists I}{\exists x \exists y\; \neg x=y}\;\exists I}{}$$

$$\dfrac{\exists x\, \neg Px \qquad \exists x \exists y\; \neg x=y}{}\;\exists E$$

$$\dfrac{~~a~~\;\exists x\, Px \qquad \exists x \exists y\; \neg x=y}{}\;\exists E$$

8.5 I)

I)

$[\forall y(Py \to a = y) \wedge Pa]$ $\qquad$ $[\forall y(Py \to a=y) \wedge Pa]$

$\wedge E$ $\qquad\qquad$ $\wedge E$

$Pa \qquad [a=b]$ $\qquad$ $\forall y(Py \to a=y)$

$\forall E$

$Pb \qquad\qquad Pb \to a=b \qquad [Pb]$

$\to E$

$a=b$

$\leftrightarrow E$

$Pb \leftrightarrow a=b$

$\forall I$

$\forall y(Py \leftrightarrow a=y)$

$\exists I$

$\exists x(\forall y(Py \to x=y) \wedge Px) \qquad \exists x \forall y(Py \leftrightarrow x=y)$

$\exists E$

$\exists x \forall y(Py \leftrightarrow x=y)$

II)

$[\forall y(Py \leftrightarrow a=y)]$ $\qquad$ $[\forall y(Py \leftrightarrow a=y)]$

$\forall E$ $\qquad\qquad$ $\forall E$

$Pb \leftrightarrow a=b \qquad [Pb]$ $\qquad$ $Pa \leftrightarrow a=a \qquad [a=a]$

$\leftrightarrow E$ $\qquad\qquad$ $\leftrightarrow E$

$a=b$ $\qquad\qquad$ Pa

$Pb \to a=b$

$\forall I$

$\forall y(Py \to a=y)$

$\wedge I$

$\forall y(Py \to a=y) \wedge Pa$

$\exists I$

$\exists x \forall y(Py \leftrightarrow x=y) \qquad \exists x(\forall y(Py \to x=y) \wedge Px)$

$\exists E$

$\exists x(\forall y(Py \to x=y) \wedge Px)$

8.6

I) $\cancel{\exists x \exists y (Q_1 x \land Q_1 y \land \neg y = x \land \forall z (Q_1 z \to (x = z \lor y = z)))}$

$\exists x \exists y (Q_1 x \land Q_1 y \land \neg x = y \land \forall z (Q_1 z \to (x = z \lor y = z)))$

Good. Halbach splits his existential and universal quantifiers:
∃x ∃y (Q1x ∧ Q1 y ∧ ¬x = y) ∧ ∀x∀y∀z((Q1x ∧ Q1 y ∧ Q1z) → (x = y ∨ y = z ∨ x = z))

II)

$\exists x (Q_x \land P_x \land \forall y ((Q_y \land P_y) \to x = y) \land \forall z (Q_1 z \to R_{xz}))$

Again, good. Halbach uses an ↔ instead of → for
the clever tutor expression: ∃x(∀y(Py∧Qy↔x=y)∧∀z(Q1z→Rxz))

III)

$\exists x (Q_1 x \land \forall y (Q_y \to R_{xy}) \land \forall z ((Q_1 z \land \forall y (Q_y \to R_{zy})) \to x = z)$
$\land P x)$

Careful using universal quantifiers and ∧. Here is Halbach:
∃y∀x(((∀z(Qz→Rxz)∧Q1x)↔x=y)∧Py)

IV $\forall x \forall y \forall z ((Q x \land Q y \land Q z) \to (x = y \lor x = z \lor y = z))$