# Learning From Crowdsourced Noisy Labels: A Signal Processing Perspective

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#### **Abstract**

One of the primary catalysts fueling advances in artificial intelligence (AI) and machine learning (ML) is the availability of massive, curated datasets. A commonly used technique to curate such massive datasets is crowdsourcing, where data are dispatched to multiple annotators. The annotator-produced labels are then fused to serve downstream learning and inference tasks. This annotation process often creates noisy labels due to various reasons, such as the limited expertise, or unreliability of annotators, among others. Therefore, a core objective in crowdsourcing is to develop methods that effectively mitigate the negative impact of such label noise on learning tasks. This feature article introduces advances in learning from noisy crowdsourced labels. The focus is on key crowdsourcing models and their methodological treatments, from classical statistical models to recent deep learning-based approaches, emphasizing analytical insights and algorithmic developments. In particular, this article reviews the connections between signal processing (SP) theory and methods, such as identifiability of tensor and nonnegative matrix factorization, and novel, principled solutions of longstanding challenges in crowdsourcing—showing how SP perspectives drive the advancements of this field. Furthermore, this article touches upon emerging topics that are critical for developing cutting-edge AI/ML systems, such as crowdsourcing in reinforcement learning with human feedback (RLHF) and direct preference optimization (DPO) that are key techniques for fine-tuning large language models (LLMs).

### 1 Introduction

Artificial Intelligence (AI) and machine learning (ML) have made significant strides over the past few decades, expanding the potential of models and learning algorithms. These advancements have revolutionized natural language processing by enhancing language generation and understanding, transformed computer vision with superior image recognition and analysis, and enabled complex decision-making capabilities. A key driving factor behind such successes of AI/ML is the availability of large-scale, accurately labeled training data. In fact, data annotation has become an indispensable integrating part of the AI industry. In a recent market report from *Grand View Research*, it was stated that "the global data collection and labeling market size was valued at \$2.22 billion in 2022, and it is expected to expand at a compound annual growth rate of 28.9% from 2023 to 2030" [1].

A prominent data annotation paradigm is crowdsourcing. In crowdsourced data annotation systems, data items are dispatched to a group of annotators for labeling. For each item, multiple annotators provide their individual, possibly noisy, annotations. These annotations are then integrated to improve the overall label accuracy. Leveraging the "wisdom of crowd" in data labeling makes sense, as the label accuracy provided by individual annotators is sensitive to many factors. If the annotators are human workers, their

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annotation accuracy is limited by their expertise level, background knowledge, and personal experience; when the annotations are given by automated machine annotators, the accuracy is affected by their model expressiveness and the quality/amount of their training data. The idea of systematic crowdsourcing can be dated back to more than a century ago; see the inserted box "Crowdsourcing in History". Over the past two decades, many commercial crowdsourcing platforms have become widely used, including notable ones such as Amazon Mechanical Turk (AMT) and CrowdFlower [2]. These platforms were instrumental in creating many highly influential datasets in modern AI history. For example, the ImageNet dataset, that has driven significant research advancements in AI, contains approximately 14 million annotations provided by approximately 25,000 AMT annotators. More recently, large language models (LLMs), such as ChatGPT, Gemini, and Llama, used substantial amounts of crowdsourced labels for fine-tuning [3].

Beyond AI, crowdsourcing has also left significant footprints in a gamut of real-world applications in science and engineering. Popular citizen science projects like GalaxyZoo and eBird are byproducts of crowdsourcing, which provided effective and economical solutions to the ever-lingering data scarcity problem in life and physical sciences. Crowdsourcing techniques have also shown great promise in medical imaging-based diagnosis, producing reliable results with minimal expert supervision [4]. In addition, crowdsourcing is widely utilized in sensing applications through *crowdsensing* [5], where a large group of mobile users or devices collectively share real-time data. An example of crowdsensing is the crowd-contributed real-time information on road conditions, which is now widely available in applications such as Google Maps and Waze. Various data fusion applications in remote sensing and healthcare that aggregate information from multiple sources of varying reliability also have strong connections to crowdsourcing techniques [6]. Furthermore, crowdsourcing is conceptually similar to ensemble learning methods, such as bagging, boosting, and stacking, that combine multiple models to improve overall predictive performance.

Despite the remarkable achievements of crowdsourcing, the fundamental challenge is that annotator-provided labels can be substantially noisy, and such noisy labels are detrimental to downstream tasks' performance. For example, in deep learning systems, noisy labels cause overfitting to noise and poor generalization. Early approaches for alleviating the negative impacts of crowdsourced label noise focused on improving the quality of annotators' responses during label collection. These approaches include various project management-based strategies, such as designing proper labeling workflow, querying proper experts, enhancing supervision mechanisms, and using effective incentive methods—see [7] and references therein. However, implementing such complex quality control mechanisms has become increasingly difficult and less cost-effective as data volume increases. For massive data annotation tasks faced by modern crowdsourcing systems, automated annotation integration and label correction algorithms advocated by the machine learning community are much more relevant. In this article, the term "crowdsourcing" refers to this type of automated systems and algorithms, unless otherwise specified.

**Our Goal.** In this feature article, we aim to provide insights into key developments of learning from crowd-sourced noisy labels. As one will see, designing crowdsourcing mechanisms and algorithms presents a series of challenges in problem distillation, optimization, and performance characterization. Interestingly—but not very surprisingly—many design considerations and solutions in crowdsourcing are deeply intertwined with theory and methods of signal processing (SP). We are particularly interested in viewing crowdsourcing from an SP perspective. Therefore, we will emphasize problem distillation (modeling), learning criterion formulation, algorithm design, and theoretical advancements, rather than presenting an exhaustive survey.

Crowdsourcing in History. The term "crowdsourcing" was first introduced by business journalist Jeff Howe in his 2006 *Wired* magazine article on task outsourcing to Internet users, yet the concept has deep historical roots. One of the earliest examples of a successful crowdsourcing project is the compilation of the Oxford English dictionary back in the nineteenth century, where hundreds of English speaking readers' efforts were utilized to collect words and their meanings. Years later in 1907, a statistical perspective to this paradigm—"the wisdom of the crowd"—was first postulated by the famous statistician Sir Francis Galton, when he observed a crowd at an auction accurately predicting the weight of an ox through their collective guessing. In the late 1990s, many large-scale annotation projects, such as

TreeBank, FrameNet, and PropBank, began to emerge, driven by the need for large amounts of curated data in natural language processing (NLP) tasks. Another notable early effort in crowdsourcing was pioneered by Luis von Ahn in 2004, who introduced an online game to generate image annotations, which sparked substantial interest in utilizing online workers for large-scale annotation tasks.

**Notation.** We use x, x, and X to denote scalar, vector, and matrix, respectively; both  $[x]_i$  and x(i) refer to the ith element of vector x;  $[X]_{i,j}$  or X(i,j) is the element in the ith row and jth column of X; X(i,:) or  $[X]_{i,:}$  denote the ith row of X; X(i,:) or  $[X]_{i,:}$  denote the jth column of X;  $\mathbb{I}[A]$  is the indicator function, i.e.,  $\mathbb{I}[A] = 1$ , if the event A occurs, otherwise  $\mathbb{I}[A] = 0$ ;  $I_K$  denotes the identity matrix of size K;  $\|x\|_2$  denotes the  $\ell_2$  norm;  $\|X\|_F$  and  $\|X\|_2$  denote the Frobenius norm and the spectral norm of X, respectively;  $[N] := \{1, \ldots, N\}$  is the set of natural numbers from 1 to N;  $\operatorname{krank}(X)$  denotes the Kruskal rank of the matrix X;  $\operatorname{trace}(X)$  denotes the trace, i.e., the sum of the diagonal entries, of matrix X;  $\circ$  denotes the outer-product;  $\operatorname{Pr}$  denotes probability or probability mass function (PMF) or probability density function (PDF), and  $\operatorname{Pr}(X; \theta)$  denotes a distribution parametrized by  $\theta$ .

## 2 Problem Settings

In this section, we formally define the problem statement of crowdsourcing. In addition, we introduce two major paradigms of the crowdsourcing system design.

#### 2.1 Problem Statement

As mentioned, crowdsourcing can be used to integrate various types of annotations (e.g., class labels [8], traffic information [9], and bird counts); that is, in principle, the annotations in crowdsourcing can be both discrete and continuous values. Nonetheless, for simplicity, we use categorical class label annotations as the primary working example to introduce crowdsourcing ideas and algorithms. Consider a dataset consisting of N data items, i.e.,  $\mathcal{X} := \{x_n\}_{n=1}^N$ , where  $x_n \in \mathbb{R}^D$  is the feature vector representing the nth data item. Assume that each data item belongs to one of K classes. Let  $\{y_n\}_{n=1}^N$  denote the set of ground-truth labels, where  $y_n \in [K]$ ,  $\forall n$ ; i.e.,  $y_n = k$  if the data item  $x_n$  belongs to class k. Note that the ground-truth labels  $\{y_n\}_{n=1}^N$  are unknown, and M crowdsourced workers (annotators) are employed to label the dataset  $\mathcal{X}$ , i.e., to give their estimates of  $\{y_n\}$ . We use  $\widehat{y}_n^{(m)} \in [K]$  to denote the label assigned to the nth data item by the mth annotator. Note that the annotators may be human, pre-trained ML algorithms, or predefined decision rules. Crowdsourced labels are often noisy, which means that we have  $\widehat{y}_n^{(m)} \neq y_n$  for many (m,n) pairs.

### 2.2 Crowdsourcing Systems: Two Paradigms

The developments of crowdsourcing methods can be roughly classified into two categories, namely, label integration approaches (see, e.g., [8, 10–12]) and end-to-end (E2E) learning approaches (e.g., [13–16]). The former isolates noisy label correction from downstream tasks, yet the latter directly uses the noisy labels to train systems serving for specific tasks.

**Label Integration.** Fig. 1 shows the schematics of the first type of crowdsourcing system, where label integration is the key module. These systems consist of two stages, where the label integration stage is not affected by the subsequent downstream task stage. In the label integration stage, the labels  $\hat{y}_n^{(m)}$ 's are fused to produce an estimated label  $\hat{y}_n$  in the hope that  $\tilde{y}_n = y_n$ . Oftentimes, the characteristics of the annotators, i.e. "annotator confusions" in the figure, are also estimated in the process. Then, the estimated labels  $\{\tilde{y}_n\}$  are used for training the downstream machine learning system, e.g., training a neural network function to predict ground-truth labels. The training stage does not need specific design to handle multiple annotators or noisy labels.

**End-to-End (E2E) Learning.** Fig. 2 illustrates a typical E2E crowdsourcing system. This type of systems considers both data and noisy labels as input and directly trains the learning systems on the target downstream tasks. As there is no stage breakdown, these systems are called end-to-end (E2E) systems. When the

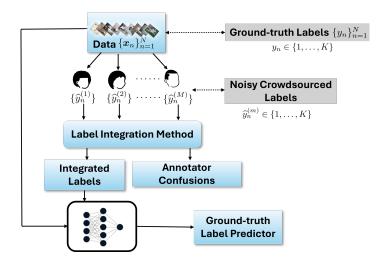


Figure 1: The two-stage strategy with label integration and annotator confusion estimation followed by learning the ground-truth label predictor.

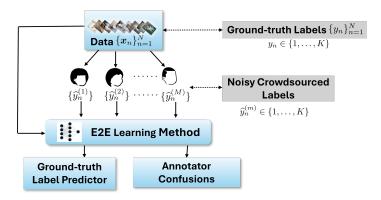


Figure 2: The E2E learning strategy with joint estimation of the ground-truth label predictor and the annotator confusions.

target is to train a classifier  $f_{\theta}: \mathbb{R}^D \to \mathbb{R}^K$ , E2E systems design a loss function  $\ell$  such that

$$\widehat{f}_{\boldsymbol{\theta}} \leftarrow \arg\min_{\boldsymbol{\theta}, \boldsymbol{\eta}} \ \ell(\{\boldsymbol{x}_n\}, \{\widehat{y}_n^{(m)}\}, \boldsymbol{\theta}, \boldsymbol{\eta}),$$

where  $\eta$  represents additional model parameters according to specific loss designs. The learned  $\hat{f}_{\theta}$  is expected to be a good ground-truth label predictor. For example, denote the ground-truth label posterior distribution as a function  $f^*: \mathbb{R}^D \to \mathbb{R}^K$  where

$$[\mathbf{f}^{\star}(\mathbf{x}_n)]_k = \Pr(y_n = k|\mathbf{x}_n), \quad \forall (\mathbf{x}_n, y_n). \tag{1}$$

The goal of many E2E approaches (see, e.g., [13–15]) is to learn  $\hat{f_{\theta}}$  such that  $\hat{f_{\theta}} \approx f^{\star}$ .

Generally speaking, label integration approaches can feature more lightweight and tractable algorithms, as they do not require training a learning system  $f_{\theta}$  that is usually parameterized by a complex function class, such as kernels or neural networks. On the other hand, E2E methods often exhibit more appealing performance, possibly because they naturally exploit information from data features and the underlying structure of the specific tasks in a joint way. Both crowdsourcing approaches can be considered *unsupervised*,

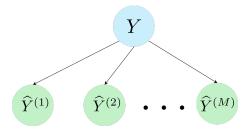


Figure 3: Graphical model of the DS model [8]. The green colored circles indicate observed variables, whereas the blue one indicate latent variable.

as there are no ground-truth labels available to guide the learning process. The developments in label integration and E2E learning based crowdsourcing, are reviewed in Sec. 3 and Sec. 4, respectively.

## 3 Label Integration Approaches

In this section, we focus on approaches designed for the label integration module in Fig. 1.

### 3.1 Majority Voting and Challenges

The arguably simplest approach for label integration is *majority voting* (MV), where the estimated label for each data item is the one voted by most annotators, i.e.,

$$\tilde{y}_n = \arg\max_k \sum_{m=1}^M \mathbb{I}[\hat{y}_n^{(m)} = k]. \tag{2}$$

However, MV is not always effective, as it implicitly assumes equal reliability among all annotators. Furthermore, due to the significant time and effort required, it is not economical to ask all annotators to label every data item. This means that many items may not have sufficient annotations that merit a voting process. Under such circumstances, MV can be far from optimal for label integration [17]. To take unequal reliability of the annotators into consideration, *weighted majority voting* (WMV) schemes were proposed. For example, a non-negative scalar weight  $w^{(m)}$  representing the labeling accuracy of annotator m was considered in [18]:

$$\tilde{y}_n = \arg\max_k \sum_{m=1}^M w^{(m)} \mathbb{I}[\hat{y}_n^{(m)} = k]. \tag{3}$$

Nonetheless, accurate weight estimation *per se* is a challenging task. Due to these reasons, MV schemes are often only used as the basic baselines in label integration.

#### 3.2 Label Noise Models in Label Integration

A notable milestone in label integration research occurred when the MV approaches were superseded by probabilistic modeling approaches. By explicitly modeling noisy labels using probabilistic generative models, these approaches often provide more reliable label integration performance. In this section, we introduce a number of representative models in this genre.

**Dawid-Skene (DS) Model.** The seminal work of Dawid and Skene introduced one of the most influential probabilistic models for crowdsourcing in the late 1970s [8]. Under the Dawid-Skene (DS) model, the ground-truth label is a latent discrete random variable, denoted as Y, and each label  $y_n \in [K]$  is drawn independently from the random variable Y. The annotator's responses are the observed random variables,

denoted as  $\widehat{Y}^{(1)}, \dots, \widehat{Y}^{(M)}$  with  $\widehat{y}_n^{(m)}$  representing the nth realization of  $\widehat{Y}^{(m)}$ . The key assumption of the DS model is that, given the ground-truth label, annotators' responses are conditionally independent; see Fig. 3. In other words, the DS model is a *naive Bayes* model.

Under the DS model, the joint probability of all the annotator responses can be expressed as follows:

$$\Pr(\widehat{Y}^{(1)} = k_1, \dots, \widehat{Y}^{(M)} = k_M) = \sum_{k=1}^K \prod_{m=1}^M \Pr(\widehat{Y}^{(m)} = k_m | Y = k) \Pr(Y = k), \tag{4}$$

where  $k_1, ..., k_M \in [K]$  are the annotator responses, and we have used the law of total probability, Bayes rule, and the conditional independence of annotator responses given the ground-truth label. From the relation in (4), the following terms are defined:

$$\boldsymbol{A}_{m}(k',k) \triangleq \Pr(\widehat{Y}^{(m)} = k'|Y = k), \quad \boldsymbol{d}(k) \triangleq \Pr(Y = k), \ \forall k, k' \in [K], \tag{5}$$

where  $A_m \in \mathbb{R}^{K \times K}$  is called the *confusion matrix of* the annotator m and  $d \in \mathbb{R}^K$  is the PMF of the ground-truth label distribution. As the name suggests, the off-diagonal elements of  $A_m$  characterize the probabilities of annotator m making mistakes. An ideal annotator would have  $A_m = I$ . Also note that, as each column of a confusion matrix is a PMF, it holds that  $A_m \geq 0$ ,  $\mathbf{1}^{\top}A_m = \mathbf{1}^{\top}$ , with 1 denoting the all ones vector of appropriate dimension. Thus, both d and columns of  $A_m$  belong to the so-called (K-1) dimensional probability simplex,  $\Delta_K = \{x \in \mathbb{R}^K : x \geq 0, \mathbf{1}^{\top}x = 1\}$ . Under the DS model, if the  $A_m$ 's and d are known, one can construct optimal ground-truth label es-

Under the DS model, if the  $A_m$ 's and d are known, one can construct optimal ground-truth label estimators using the *maximum a posteriori* (MAP) principle, i.e. maximizing the posterior distribution of the unknown label, given annotator responses:

$$\tilde{y}_{n} = \arg \max_{k \in [K]} \Pr(y_{n} = k | \widehat{y}_{n}^{(1)} = k_{1}, \dots, \widehat{y}_{n}^{(M)} = k_{M}) 
= \arg \max_{k \in [K]} \Pr(\widehat{y}_{n}^{(1)} = k_{1}, \dots, \widehat{y}_{n}^{(M)} = k_{M} | y_{n} = k) \Pr(y_{n} = k) 
= \arg \max_{k \in [K]} \log \mathbf{d}(k) + \sum_{k'=1}^{K} \sum_{m=1}^{M} \mathbb{I}[\widehat{y}_{n}^{(m)} = k'] \log \mathbf{A}_{m}(k', k),$$
(6)

where we have successively used Bayes rule, the properties of the logarithm and the definitions in (5). Hence, the label integration problem under the DS model amounts to estimating  $\{A_m\}$  and d accurately.

Connections to Communications and Information Theory. The DS model can also be interpreted as single-input multiple-output (SIMO) channel from information theory. The common input is the single ground-truth label per data item  $y_n$ , the multiple outputs are the M annotator responses  $\{\widehat{y}_n^{(m)}\}_{m=1}^M$ , and the annotator confusion matrices  $A_m$ 's can be associated with the unknown "channel" characteristics. A similar problem to crowdsourcing is decentralized detection [19], where a distributed network of sensors (annotators) observe the same environmental phenomenon, and send their observations (labels) to a fusion center, and the fusion center has to recover the underlying hypothesis of the environment. The key difference between decentralized detection and crowdsourcing is that in the former case, the fusion center can control the characteristics of both the annotators and the fusion rule. The crowdsourcing setting with real-valued annotations also bears resemblance to classical problem of blind multichannel deconvolution [20], where the unobserved input signal and the channel characteristics are inferred from multiple noisy observed signals. Furthermore, the crowdsourcing problem exhibits similarities to the *chief executive officer* (CEO) problem in information theory [21], where M agents observe noisy sequences and the CEO aims to recover the ground truth within a communication constraint, similar to the labeling cost budget in crowdsourcing.

**Special Cases of the DS Model.** The DS model serves as a foundation for different models that are frequently used in the literature. The so-called "one-coin" model [22] is a simplified DS model that encodes

each annotator's reliabilities using only one parameter such that

$$\Pr(\widehat{Y}^{(m)} = k'|Y = k) = \begin{cases} p_m, & k' = k, \\ \frac{1 - p_m}{K - 1}, & k' \neq k, \end{cases}$$

i.e., annotator m determines their response with a single biased coin flip. For each data item, the annotator assigns the correct label with probability  $p_m$ , and gives a wrong label with equal probabilities across the remaining K-1 classes. Other popular DS model variants include the *spammer-hammer* model [23] and the *confusion vector* model [24]. In the spammer-hammer model, each annotator is a "hammer" with probability q, providing correct labels, or a "spammer" with probability 1-q, providing random labels. The confusion vector model employs a parameter vector  $\mathbf{a}_m \in \mathbb{R}^K$ , per annotator m, to characterize their reliabilities:

$$\Pr(\widehat{Y}^{(m)} = k'|Y = k) = \begin{cases} \boldsymbol{a}_m(k), & k' = k, \\ \frac{1 - \boldsymbol{a}_m(k)}{K - 1}, & k' \neq k, \end{cases}$$

i.e., unlike the one-coin model, the probability that the annotator m chooses the correct answer or a wrong answer varies across different classes.

These special cases of the DS model offer succinct characterizations of annotator confusions. Yet, they are less general than the DS model. Nevertheless, these models often admit more tractable algorithms and reasonable performance guarantees; see, e.g., [22,23,25].

**Extended DS Model: Incorporating Item Difficulties.** The DS model introduced in [8] assumes that annotator behavior is the same across all data items, i.e.,  $A_m$  is the same for all n. Nonetheless, it may be more realistic to assume that the difficulty of labeling varies across data items. To capture both annotator behavior and item difficulty [26] introduced the following model:

$$\Pr(\widehat{Y}^{(m)} = k'|Y = k) = \frac{\exp(\boldsymbol{A}_m(k',k) + \boldsymbol{B}_n(k',k))}{\sum_{k'} \exp(\boldsymbol{A}_m(k',k) + \boldsymbol{B}_n(k',k))},$$

where  $A_m \in \mathbb{R}^{K \times K}$  is defined as before, and  $B_n \in \mathbb{R}^{K \times K}$  is an item-specific confusion matrix that reflects item difficulty. Following the same spirit, the one-coin model was also generalized to incorporate item difficulty; see the *generative model of labels*, *abilities*, *and difficulties* (GLAD) model in [27].

**Bayesian Models.** To incorporate prior information, reduce the number of parameters, and enhance model interpretability, Bayesian models were also introduced on top of the DS model (see, e.g., [16, 28]) Using the fact that both d and  $A_m(:,k)$  are PMFs, [28] imposes Dirichlet priors on to d and  $A_m(:,k)$ 's, while [16] assumes a Beta prior for the confusion parameters, in the binary classification case.

#### 3.3 Methods for Noise Model Learning

Under the aforementioned noise generation models, label integration boils down to learning the key model parameters, e.g., the confusion matrices and the prior probability vector in the DS model. In this subsection, we briefly review some representative methods for learning these models.

**Expectation Maximization (EM).** The seminal work by Dawid and Skene [8] sought a maximum-likelihood estimator (MLE) of annotator confusion matrices and class priors of the DS model. Collect all ground-truth labels and annotator responses in  $\mathcal{Y}=\{y_n\}$  and  $\widehat{\mathcal{Y}}=\{\widehat{y}_n^{(m)}\}$ , respectively, and all unknown model parameters in  $\psi=\{A_1,\ldots,A_M,d\}$ . The MLE of  $\psi$  is given by

$$\widehat{\psi} = \underset{\psi}{\operatorname{arg max}} \log \Pr(\widehat{\mathcal{Y}}; \psi) = \underset{\psi}{\operatorname{arg max}} \sum_{n=1}^{N} \log \Pr(\widehat{y}_{n}^{(1)}, \dots, \widehat{y}_{n}^{(M)}; \psi)$$

$$= \underset{\psi}{\operatorname{arg max}} \sum_{n=1}^{N} \log \sum_{k=1}^{K} d(k) \prod_{m=1}^{M} \sum_{k'=1}^{K} \mathbb{I}[\widehat{y}_{n}^{(m)} = k'] A_{m}(k', k), \tag{7}$$

where  $\log \Pr(\widehat{\mathcal{Y}}; \psi)$  is the log-likelihood function of observed annotator labels, parametrized by  $\psi$ . As directly optimizing the log-likelihood is often intractable, Dawid and Skene introduced an EM-based algorithm to learn the DS model parameters. The EM algorithm is the workhorse for learning Naive Bayes and mixture models. Using the EM algorithm, the log-likelihood function is maximized iteratively, with the following steps performed at each iteration:

(i) The "Expectation" (E) step: The E-step in iteration t is performed as follows:

$$Q(\boldsymbol{\psi}; \boldsymbol{\psi}^t) = \mathbb{E}_{\mathcal{Y} \sim \mathsf{Pr}(\mathcal{Y}; \widehat{\mathcal{Y}}, \boldsymbol{\psi}^t)}[\log \mathsf{Pr}(\widehat{\mathcal{Y}}, \mathcal{Y}; \boldsymbol{\psi})] = \sum_{n=1}^N \sum_{k=1}^K q(y_n = k; \boldsymbol{\psi}^t) \log \mathsf{Pr}(\widehat{y}_n^{(1)}, \dots, \widehat{y}_n^{(M)}; \boldsymbol{\psi}),$$

where the superscript t denotes iteration index, and the expectation is taken with respect to (w.r.t.) the posterior probability  $\Pr(\mathcal{Y}; \widehat{\mathcal{Y}}, \psi^t)$  based on the current estimates  $\psi^t = \{A_1^t, \dots, A_M^t, d^t\}$ . Note that,  $Q(\psi; \psi^t)$  is essentially a lower bound of the log-likelihood. The E-step boils down to estimating  $q(y_n = k; \psi^t) = \Pr(y_n = k | \widehat{y}_n^{(1)}, \dots, \widehat{y}_n^{(M)}, \psi^t)$  given  $A_m^t$ 's and  $d^t$ . Under the DS model and using Bayes rule,  $q(y_n = k; \psi^t)$  admits a simple closed form, i.e.,

$$q(y_n = k; \boldsymbol{\psi}^t) = \frac{\exp\left(\log \boldsymbol{d}^t(k) + \sum_{m=1}^M \sum_{k'=1}^K \log \boldsymbol{A}_m^t(k', k) \mathbb{I}[\widehat{y}_n^m = k']\right)}{\sum_{k'=1}^K \exp\left(\log \boldsymbol{d}^t(k') + \sum_{m=1}^M \sum_{k''=1}^K \log \boldsymbol{A}_m^t(k'', k') \mathbb{I}[\widehat{y}_n^m = k'']\right)}.$$
 (8)

Note that this posterior has a similar form to the MAP estimator of (6).

(ii) The "Maximization" (M) step: The M-step refines model parameters by maximizing  $Q(\psi; \psi^t)$ , which also admits analytical updates:

$$\begin{split} \boldsymbol{A}_{m}^{t+1}(k',k) &= \frac{\sum_{n=1}^{N} q(y_{n}=k;\boldsymbol{\psi}^{t})\mathbb{I}[\widehat{y}_{n}^{m}=k']}{\sum_{k''=1}^{K} \sum_{n=1}^{N} q(y_{n}=k;\boldsymbol{\psi}^{t})\mathbb{I}[\widehat{y}_{n}^{m}=k'']},\\ \boldsymbol{d}^{t+1}(k) &= \frac{\sum_{n=1}^{N} q(y_{n}=k;\boldsymbol{\psi}^{t})}{\sum_{k'=1}^{K} \sum_{n=1}^{N} q(y_{n}=k';\boldsymbol{\psi}^{t})}. \end{split}$$

The EM scheme can also be readily adapted to learn the special cases of the DS model, i.e., one-coin, confusion vector, spammer-hammer, and GLAD models as discussed in 3.2. A similar EM strategy is also employed to learn the Bayesian model proposed in [16].

One salient feature of the EM algorithm is its scalability—it enjoys a computational complexity that is linear in N and M, which is appealing for large-scale crowdsourcing problems. However, it was also observed that the EM algorithm does not converge well, if the initialization is not carefully chosen [11,29], perhaps due to the nonconvexity of the MLE loss.

**Spectral Methods.** One of the notable spectral methods for label integration is the eigendecomposition-based approach proposed in [22] under the one-coin binary model. Consider the binary classification problem with unobserved ground-truth labels  $y_n \in \{-1, +1\}$  and the one-coin model parameters  $p_m$ 's, where  $p_m$  is the probability that annotator m provides the correct label. Let  $z_n^{(m)}$  be the correctness indicator of annotator m on item n; i.e.,  $z_n^{(m)} = 1$  if annotator m provides the correct label and  $z_n^{(m)} = -1$  otherwise. Then, it can be shown that

$$z_n^{(m)} z_{n'}^{(m)} = \begin{cases} 1, & \text{w.p } p_m^2 + (1 - p_m)^2, \\ -1, & \text{w.p } 1 - p_m^2 - (1 - p_m)^2. \end{cases}$$

Since  $\widehat{y}_n^{(m)} = z_n^{(m)} y_n$  holds, we further obtain:

$$\mathbb{E}\left[\sum_{m=1}^{M} \widehat{y}_{n}^{(m)} \widehat{y}_{n'}^{(m)}\right] = y_{n} y_{n'} \mathbb{E}\left[\sum_{m=1}^{M} z_{n}^{(m)} z_{n'}^{(m)}\right] = \begin{cases} y_{n} y_{n'} \kappa, & n \neq n', \\ M, & n = n', \end{cases}$$
(9)

where  $\kappa = \sum_{m=1}^{M} (2p_m - 1)^2$ . By defining the annotator response matrix  $\widehat{U}$  with entries  $\widehat{U}(n,m) = \widehat{y}_n^{(m)}$ , the relation in (9) can also be expressed as

$$\mathbb{E}[\widehat{\boldsymbol{U}}\widehat{\boldsymbol{U}}^{\top}] = \kappa \boldsymbol{y} \boldsymbol{y}^{\top} + (M - \kappa) \boldsymbol{I}_{N}, \tag{10}$$

where  $y = [y_1, \dots, y_N]^{\top}$ . Based on the above, [22] proposed an intuitively simple spectral algorithm where the ground-truth labels y are extracted from the top eigenvector of the empirical estimate of  $\mathbb{E}[\hat{U}\hat{U}^{\top}]$ . Consequently, the annotator confusions p can be inferred from the estimated y. Nonetheless, the approach requires that the annotator response matrix is fully observed, meaning that all annotators provide labels for all data items. To accommodate the incomplete labeling scenarios, [25] extended this strategy by considering an annotator-item binary matrix alongside the annotator response matrix and performing a joint singular value decomposition (SVD) operation, still under the one-coin model.

Moment-Based Approaches: From Tensor Decomposition to Nonnegative Matrix Factorization. The rank-one plus diagonal structure in (10) is only applicable under the one-coin model. To identify the parameters of the general DS model, [10] proposed a moment matching approach that considered the third-order moments of the annotator responses as follows:

$$\mathbb{E}[\widehat{\boldsymbol{Y}}^{(m)} \circ \widehat{\boldsymbol{Y}}^{(i)} \circ \widehat{\boldsymbol{Y}}^{(j)}] = \sum_{k=1}^{K} \boldsymbol{d}(k) \boldsymbol{A}_{m}(:,k) \circ \boldsymbol{A}_{i}(:,k) \circ \boldsymbol{A}_{j}(:,k), \ \forall m \neq i \neq j,$$
(11)

where  $\circ$  denotes the outer product (i.e.,  $\underline{X} = a \circ b \circ c \Leftrightarrow \underline{X}(m,i,j) = a(m)b(i)c(j)$ ), and  $\widehat{Y}^{(m)}$  denotes the K-dimensional one-hot encoding of the annotator response random variable  $\widehat{Y}^{(m)}$ , i.e., if  $\widehat{Y}^{(m)} = k$ , then  $\widehat{Y}^{(m)} = e_k$ , where  $e_k \in \mathbb{R}^K$  is a unit vector with  $[e_k]_k = 1$  and zeros elsewhere. In general, the conditional independence assumption of the DS model induces the outer product expression of the higher-order moments. To see why, consider the second-order moment

$$\begin{split} \left[\mathbb{E}[\widehat{\boldsymbol{Y}}^{(m)} \circ \widehat{\boldsymbol{Y}}^{(i)}]\right]_{k_1,k_2} &= \Pr(\boldsymbol{Y}^{(m)} = k_1, \boldsymbol{Y}^{(i)} = k_2) \\ &= \sum_{k=1}^K \underbrace{\Pr(\boldsymbol{Y} = k)}_{\boldsymbol{d}(k)} \underbrace{\Pr(\boldsymbol{Y}^{(m)} = k_1 | \boldsymbol{Y} = k)}_{\boldsymbol{A}_m(k_1,k)} \underbrace{\Pr(\boldsymbol{Y}^{(i)} = k_2 | \boldsymbol{Y} = k)}_{\boldsymbol{A}_i(k_2,k)}, \end{split}$$

which implies  $\mathbb{E}[\hat{\boldsymbol{Y}}^{(m)} \circ \hat{\boldsymbol{Y}}^{(i)}] = \sum_{k=1}^{K} \boldsymbol{d}(k) \boldsymbol{A}_m(:,k) \circ \boldsymbol{A}_i(:,k) = \boldsymbol{A}_m \operatorname{diag}(\boldsymbol{d}) \boldsymbol{A}_i^\top$ ; a similar derivation holds for the third-order moments.

Using third-order moments, a *coupled tensor factorization* (CTD) criterion can be used to identify d and  $\{A_m\}$  [10]:

$$\underset{\substack{\{A_m\},d\\i>m\\j>i}}{\text{minimize}} \sum_{\substack{m=1\\i>m\\j>i}}^{M} \|\underline{\boldsymbol{T}}_{m,i,j} - [\![\boldsymbol{d},\boldsymbol{A}_m,\boldsymbol{A}_i,\boldsymbol{A}_j]\!]\|_{\mathrm{F}}^2 \tag{12}$$

subject to 
$$A_m \geq \mathbf{0}, \mathbf{1}^{\top} A_m = \mathbf{1}^{\top}, d \geq \mathbf{0}, \mathbf{1}^{\top} d = 1,$$

where  $[\![d,A_m,A_i,A_j]\!]$  is the shorthand notation for  $\sum_{k=1}^K d(k)A_m(:,k)\circ A_i(:,k)\circ A_j(:,k)$ , and  $\underline{T}_{m,i,j}$  is the empirical version of  $\mathbb{E}[\widehat{Y}^{(m)}\circ\widehat{Y}^{(i)}\circ\widehat{Y}^{(j)}]$ . The term "coupled" is due to the fact that the tensors  $\underline{T}_{m,i,j}$  share one or two latent factors. To further regularize the objective function first- and second-order moments were used in [10]. The problem in (12) is nontrivial to solve. An alternating direction method of multipliers (ADMM)-based optimization algorithm was employed in [10] to handle this criterion. Another third-order moment-based DS learning approach was introduced in [11] that employs an orthogonal tensor decomposition via a robust power method. Nonetheless, higher-order moments like  $\underline{T}_{m,i,j}$  in general require a significant amount of samples (annotator labels) to be accurately estimated.

To avoid the sample complexity required for estimating third-order moments, [12,30] proposed *using* only second-order statistics, leading to a coupled nonnegative matrix factorization (CNMF) criterion:

$$\underset{\{A_m\}, d}{\text{minimize}} \sum_{\substack{m=1\\i>m}}^{M} \text{KL}\left(S_{m,i} || A_m \text{diag}(d) A_i^{\top}\right) 
\text{subject to } A_m \ge \mathbf{0}, \mathbf{1}^{\top} A_m = \mathbf{1}^{\top}, d \ge \mathbf{0}, \mathbf{1}^{\top} d = 1,$$
(13)

where  $S_{m,i}$  is the empirical estimate of  $\mathbb{E}[\widehat{Y}^{(m)} \circ \widehat{Y}^{(i)}]$ , and the KL divergence was used in the loss due to the PMF estimation nature.

The moment-based methods in [10–12, 30] come with interesting theoretical support, reminiscent of signal processing research on tensor and nonnegative matrix factorization; see also Sec. 3.5.

**Bayesian Methods.** Under the Bayesian paradigm, [28] considered the following joint probability under presumed priors of  $A_m$  and d:

$$\Pr(\{\boldsymbol{A}_m\}, \boldsymbol{d}, \mathcal{Y}|\widehat{\mathcal{Y}}) \propto \prod_{n=1}^{N} \boldsymbol{d}(y_n) \prod_{m=1}^{M} \boldsymbol{A}_m(\widehat{y}_n^{(m)}, y_n) \cdot \Pr(\boldsymbol{d}|\boldsymbol{\nu}) \prod_{m=1}^{M} \sum_{k=1}^{K} \Pr(\boldsymbol{A}_m(:, k) | \boldsymbol{\pi}_k^{(m)})$$
(14)

where  $\nu, \pi_k^{(m)} \in \mathbb{R}^K$ ,  $\forall m, k$  are Dirichlet priors for d and  $A_m(:,k)$ 's, respectively. Compared to the plain-vanilla DS model learning approaches, Bayesian approaches enjoy more succinct models by treating  $A_m$  and d as random quantities. However, this comes at the cost of computational intractability; the posterior in (14) is often not easy to evaluate, as it involves marginalization of this joint probability distribution over all parameters. To circumvent this issue, sampling techniques are often adopted. From the posterior in (14), inference of unknown parameters, i.e,  $\{A_m\}, d, \mathcal{Y}$ , is performed using the Gibbs sampling technique by iteratively sampling each parameter from its conditional density function [28]. Other Bayesian approaches include variational inference techniques by approximating the conditional probability densities using belief propagation and mean field assumptions [31].

**Other Methods.** Beyond methods based on the DS model and its extensions, there are also numerous alternative label integration methods that provide interesting insights and simple implementations. Some examples are introduced below:

Label integration can be formulated as an optimization problem as follows [32]:

$$\underset{\boldsymbol{w},\boldsymbol{y}}{\text{minimize}} \sum_{m=1}^{M} w_m \sum_{n=1}^{N} \mathsf{dist}(y_n, \widehat{y}_n^{(m)})$$
(15a)

subject to 
$$R(\boldsymbol{w}) = 1$$
, (15b)

where  $\boldsymbol{w} = [w_1, \dots, w_M]^{\top}$  and  $\boldsymbol{y} = [y_1, \dots, y_n]^{\top}$  are the vector of annotator reliabilities (similar to the confusion parameter in the one-coin model or the weight values in weighted majority voting) and the vector of ground-truth labels, respectively. The distance measure  $\operatorname{dist}(y_n, \widehat{y}_n^{(m)})$  captures the deviation from the ground-truth and the annotator responses, e.g., 0-1 loss for the classification case.  $R(\boldsymbol{w})$  is a regularization term, which ensures that the annotator reliabilities  $w_m$  remain bounded. For example,  $R(\boldsymbol{w}) = \sum_{m=1}^M \exp(-w_m)$  is employed in [32], rendering the overall optimization problem convex and allowing it to be effectively handled using the method of Lagrange multipliers.

A minimax conditional entropy-based label integration was introduced in [24], which considered the sum of the entropies of the observed crowdsourced labels. Let  $a_n^{(m)}$  denote the PMF of the noisy label  $\hat{y}_n^{(m)}$ , i.e.,  $[a_n^{(m)}]_k = \Pr(\hat{y}_n^{(m)} = k)$ . Note that this PMF is dependent on the data item index n. Then, following

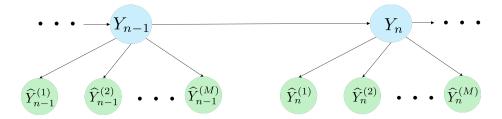


Figure 4: Graphical representation of a hidden markov model for the sequentially dependent data. The green colored circles indicate observed variables, whereas the blue one indicate latent variable.

minimize-maximize formulation is considered in [24]:

$$\min_{\boldsymbol{y}} \max_{\{\boldsymbol{a}_{n}^{(m)} \in \boldsymbol{\Delta}_{K}\}} - \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} [\boldsymbol{a}_{n}^{(m)}]_{k} \log[\boldsymbol{a}_{n}^{(m)}]_{k}$$
subject to 
$$\sum_{m=1}^{M} [\boldsymbol{a}_{n}^{(m)}]_{k} = \sum_{m=1}^{M} \mathbb{I}[\widehat{y}_{n}^{(m)} = k], \ \forall n, k,$$

$$\sum_{n=1}^{N} \mathbb{I}[y_{n} = k'][\boldsymbol{a}_{n}^{(m)}]_{k} = \sum_{n=1}^{N} \mathbb{I}[y_{n} = k'] \mathbb{I}[\widehat{y}_{n}^{(m)} = k] \forall m, k, k'.$$

Here, the parameters  $a_n^{(m)}$  are learned by maximizing the entropy of the observations. Simultaneously, the ground-truth labels y are inferred by minimizing the entropy, ensuring that the observed labels are the "least random" choice given the ground-truth label. The constraints enforce that the learned PMFs  $a_n^{(m)}$  align with the empirical observations of the annotator responses. Specifically, the first constraint means that the entries of  $a_n^{(m)}$  match to the numbers of votes obtained per class per data item collectively from all annotators; the second constraint ensures that the learned parameters of each annotator m agree with the empirical estimate of the confusions derived from all the responses of that annotator.

Another notable approach was proposed in [33] that adopted a geometric interpretation to design a label integration algorithm. To illustrate their idea, consider an M-dimensional indicator vector  $\mathbf{g}_{n,k}$  for each  $n \in [N]$  and  $k \in [K]$ , with entries  $\mathbf{g}_{n,k}(m) = \mathbb{I}[\widehat{\mathbf{y}}_n^{(m)} = k]$ . Then,  $\mathbf{w}^{\top}\mathbf{g}_{n,k}$  corresponds to the aggregated score of weighted majority voting with  $\mathbf{w} = [w_1, \dots, w_M]^{\top}$  denoting the vector of weight values [c.f. (3)]. Inspired by the notion of maximum margin in multiclass support vector machines (SVMs), the approach in [33] seeks a hyperplane that separates the point  $\mathbf{g}_{n,y_n}$  from other points  $\mathbf{g}_{n,k}$ ,  $k \neq y_n$  by the maximum margin. Consequently, the annotator-specific weights and the ground-truth labels are learned using the following constrained optimization problem:

where  $\beta$  represents the maximum margin hyperparameter.

Other interesting developments in crowdsourced label integration include graph-based approaches, e.g., the message passing-based algorithm proposed in [17] and its multi-class extension in [34].

#### 3.4 Label Integration in More Complex Scenarios

**Dependent Data.** The models discussed so far consider that the data samples are drawn i.i.d. from some unknown distribution. However, structured data often arise, e.g., the words in a text and the frames in

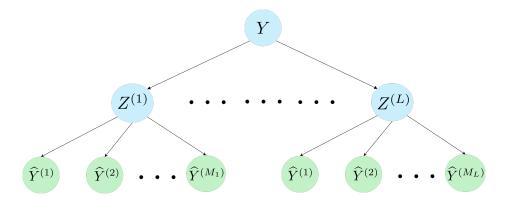


Figure 5: Graphical representation of an extended DS model with *L* annotator groups. The green colored circles indicate observed variables, whereas the blue one indicate latent variable.

a video. In these cases, the data samples show temporal dependencies, i.e.,  $x_n$  is dependent on  $x_{n-1}$  and  $x_{n+1}$ . Modeling these dependencies can be useful especially in natural language processing (NLP)-related crowdsourcing tasks, such as part-of-speech tagging and named-entity recognition, that have recently gained more popularity with the advent of large language models.

While dealing with sequential data, the DS model can be extended using a hidden Markov model (HMM) [35]. A simple extension involves a one-step, time-homogeneous Markov structure is employed to model the sequence of labels  $y_1, y_2, \ldots, y_n$  such that nth label depends only on its immediate predecessor, i.e.  $\Pr(Y_n|Y_{n-1},\ldots,Y_1)=\Pr(Y_n|Y_{n-1})$ —also see Fig. 4. In addition to annotator confusion matrices  $A_m$ 's and the prior probability vector d, the model is also characterized by a  $K \times K$  transition matrix T that describes the transitions between labels, i.e.,  $T(k,k')=\Pr(Y_n=k|Y_{n-1}=k')$ . Under this DS-HMM model, the joint probability of the observed crowdsourced labels  $\widehat{\mathcal{Y}}=\{\widehat{y}_n^{(m)}\}$  is given by

$$\Pr(\widehat{\mathcal{Y}}) = \sum_{k} d(k_1) \prod_{n=2}^{N} T(k_n, k_{n-1}) \prod_{m=1}^{M} A_m(\widehat{y}_n^{(m)}, k_n),$$
(16)

where  $k = [k_1, ..., k_N]^{\top} \in [K]^N$ .

To estimate model parameters, an EM algorithm, similar to the one outlined in Sec. 3.3, can be derived [35]. The key difference is that the algorithm incorporates a forward-backward algorithm in the E-step due to the causal nature of the ground-truth labels. The EM algorithm can also be initialized with the solutions obtained from the moment matching strategy, which is reminiscent of the method described in (12). Here, the moments of annotator responses are characterized including the transition probabilities as well, e.g., the second-order moments are given by

$$\mathbb{E}[\widehat{\boldsymbol{Y}}^{(m)}\widehat{\boldsymbol{Y}}^{(i)\top}] = \boldsymbol{A}_{m}\boldsymbol{T}\operatorname{diag}(\boldsymbol{d})\boldsymbol{A}_{i}^{\top}, \forall m \neq i.$$
(17)

Once the parameters are estimated, a MAP estimate of the ground-truth labels  $\mathcal{Y}$  can be obtained via the Viterbi algorithm.

A Bayesian alternative is introduced in [36] that characterizes annotators as follows:

$$C_m(k, k', k'') = \Pr(\hat{Y}_n = k | \hat{Y}_{n-1} = k', Y_n = k'''),$$

where  $C_m$  is a  $K \times K \times K$ -sized tensor that incorporates the label dependencies as well for the annotator confusions. Under Dirichlet prior assumptions on the model parameters, the approach maximizes the posterior probability and adopts a variational Bayes-based algorithm for inference.

Empirical studies of these approaches [35,36] show that considering the dependencies of the data is beneficial and the proposed algorithms are much promising than those designed for i.i.d. data—see Tab.

Table 1: Real-data results for the sequential data. Table from [35]. The asterisk \* indicates that results are

from a subset of available data.

Dataset	K	M	N	Metric	Single best	MV	DS-EM	SeqMM	SeqMM + EM	MV + EM
				Precision	0.23	0.22	0.23	0.24	0.25	0.23
POS	12	10	100,676	Recall	0.25	0.23	0.25	0.24	0.26	0.24
				F-score	0.23	0.22	0.22	0.23	0.24	0.23
NER	9	47	78, 107	Precision	0.90*	0.79	0.77	0.74	0.77	0.75
				Recall	0.24*	0.59	0.66	0.89	0.69	0.66
				F-score	0.89*	0.68	0.71	0.62	0.72	0.70
				Precision	0.94*	0.89	0.81	0.75	0.69	0.62
Biomedical IE	2	120	7,880,254	Recall	0.76*	0.45	0.57	0.60	0.68	0.74
				F-score	0.84*	0.60	0.66	0.67	0.68	0.67

1. Here MV denotes majority voting, DS-EM denotes the EM algorithm by Dawid and Skene, described in Sec. 3.3. In the table, the methods SeqMM, SeqMM + EM, and MV + EM consider data dependencies. Specifically, SeqMM denotes a moment matching method designed for the DS-HMM model, while SeqMM + EM denotes an EM algorithm tailored to the DS-HMM model and initialized with SeqMM. MV + EM denotes the DS-HMM-based EM algorithm initialized with MV.

Alternative dependency structures can also be considered, e.g., networked or graph data using a dependency graph that encodes the pairwise relations is handled in [35].

**Dependent Annotators.** Recall the key assumption of the DS model, i.e., the annotator responses are conditionally independent. Nevertheless, this assumption may not hold in some cases. For instance, annotators who underwent similar training may respond similarly to the same tasks or spatially close sensors that are observing the same phenomenon may capture correlated measurements. As the conditional independence no longer holds in these cases, the previously introduced methods and algorithms may yield sub-optimal outcomes, due to model misspecification.

Extending ideas from the distributed detection, [37] introduced a variant to the DS model where dependencies are captured by assigning highly correlated or dependent annotators into one group. Assuming that there are L such groups, annotator responses in the  $\ell$ th group are conditionally independent given a latent variable  $Z^{(\ell)} \in [K]$ , where  $z_n^{(\ell)}$  denotes the nth realization of  $Z^{(\ell)}$ . In addition,  $Z_1,\ldots,Z_L$  are conditionally independent given the ground-truth label Y, yielding a hierarchy of DS models—see Fig. 5. Similar to the original DS model, annotator behavior is characterized by confusion matrices w.r.t. different groups, i.e., if an annotator m belongs to group  $\ell$ , then its confusion matrix  $\tilde{A}_m$  is defined as  $\tilde{A}_m(k',k) = \Pr(\hat{Y}^{(m)} = k'|Z^{(\ell)} = k)$ . The labeling behavior of a group is captured by defining a confusion matrix  $\mathbf{\Xi}^{(\ell)}$  such that  $\mathbf{\Xi}^{(\ell)}(k',k) = \Pr(Z^{(\ell)} = k'|Y = k)$ .

Under the described model, [37, 38] proposed to estimate  $\tilde{A}_m$ 's and  $\Xi^{(\ell)}$ 's using a hierarchical algorithm. The fist step is to estimate the annotators' group membership. In the case of binary classification, the approach in [37] used spectral clustering onto the cross-correlation matrix between different annotator responses in order to assign the annotators to L different groups. This approach was later extended for K>2 classes in [38]. Once the annotator's group membership is estimated, the following step can employ any DS model learning algorithm within each group  $\ell$  to estimate the latent observations  $\{z_n^{(\ell)}\}_{n=1}^N$ . Using the estimated latent variables  $z_n^{(\ell)}$ 's, the unknown ground-truth labels  $\{y_n\}$ 's can be estimated.

Indeed, when annotator dependencies are present, the aforementioned methods show noticeable performance gains compared to methods that are agnostic of said dependencies. Fig. 6 shows the classification accuracies (i.e., the percentage of the data items that are classified correctly) of various methods from a synthetic experiment, for various values of N. The majority voting, denoted as MV, and the EM algorithm [8], denoted as DS-EM are compared against their counterparts that estimate annotator groupings prior to DS model estimation, denoted as Group-aware MV and Group-aware DS-EM, respectively. From the results, one can note that the "group-aware" algorithms outperform their group-agnostic counterparts. Finally, note that, similar to dependent data, alternative dependency structures between annotators, encoded in a

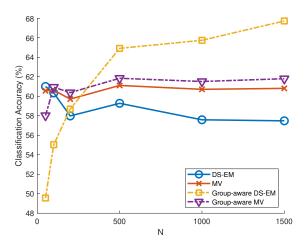


Figure 6: Simulated tests on a synthetic dataset with K=3 classes, M=80 annotators and L=4 groups. "Group-aware" algorithms denote algorithms that estimate annotator groups prior to label integration.

known graph, were considered in [28].

### 3.5 Performance Characterizations of Label Integration

A key metric that quantifies the deviation of the corrected labels  $\tilde{y}_n$ 's from the ground-truth ones  $y_n$ 's is given by the probability of error (error rate)  $P_e$ , defined as follows:

$$P_e = \Pr(\tilde{Y} \neq Y),\tag{18}$$

with  $\tilde{Y}$  denoting the random variable corresponding to the corrected label  $\tilde{y}_n$ . It was shown in [10,39] that under the DS model, the error rate  $P_e$  of the MAP rule in (6) decreases exponentially as the number of annotators M increases, i.e.,

$$P_e \le \alpha \exp(-\beta M),$$
 (19)

for some constants  $\alpha>0, \beta>0$ . Similar exponential decrease w.r.t. increasing M was reported for the one-coin, confusion vector models as well as for the majority voting rule [39]. The theoretical results reveal the importance of "crowd wisdom". Importantly, these results only hold when the model parameters are known. This underscores the significance of having accurate parameter estimation. In this subsection, we review some notable theoretical advancements in label integration.

**Identifiability of Noise Models.** If the joint PMF  $Pr(Y^{(1)}, ..., Y^{(M)})$  is available, the identifiability of  $A_1, ..., A_M$  and d under the DS model trivially holds. This is because the naive Bayes model is also a CPD model with order M and rank K. To be more specific, the DS model in (4) can be re-written as

$$\underline{\boldsymbol{P}} = \sum_{k=1}^{K} \boldsymbol{d}(k) \boldsymbol{A}_{1}(:,k) \circ \dots \circ \boldsymbol{A}_{M}(:,k),$$
(20)

where  $\underline{P}(i_1,\ldots,i_K)=\Pr(Y^{(1)}=i_1,\ldots,Y^{(M)}=i_K)$ , which is an Mth-order rank-K tensor. The essential uniqueness of d and  $\{A_m\}$  in this tensor model holds under mild conditions; see [40] and the inserted box "Identifiability of CPD and NMF". Nonetheless,  $\Pr(Y^{(1)},\ldots,Y^{(M)})$  is almost impossible to directly estimate, as it requires  $\Omega(K^M)$  colabeled samples by all annotators to reach a reasonable accuracy.

To circumvent this "curse of dimensionality", the identifiability of the DS model using joint distributions of three annotators was established in [10,11]. These smaller joint distributions are much more realistic to estimate in practice. In particular, [10] showed that the third-order moment term in (17) is also a rank-K

tensor under the CPD model. Therefore, the optimal solutions to (12) reveal the ground-truth  $A_m$ ,  $\forall m$  up to a unified permutation ambiguity under mild conditions, e.g.,  $\operatorname{rank}(A_m) = K$  for all  $m \in [M]$  as  $\operatorname{rank}(A_m) = \operatorname{krank}(A_m)$  in this case (see [41] and the inserted box "Identfiiability of CPD and NMF"). Similar results were derived in [11] using a different moment construction. These results are significant, meaning that if at least three annotators co-label a sufficient amount of data (so that the third-order moments can be reliably estimated), then the DS model is likely identifiable.

The work in [12] argued that one only needs second-order moments (or, joint distributions of two annotators' outputs) to establish identifiability of the DS model. To be specific, consider a case where we wish to find  $A_m$  and d such that  $S_{m,i} = A_m \operatorname{diag}(d) A_i^{\top}$  for  $m \in \mathcal{M}$  and  $i \in \mathcal{I}$ , where  $\mathcal{M} \cap \mathcal{I} = \emptyset$ . Take a simple case where  $\mathcal{M} \cup \mathcal{I} = [M]$ . Then, this is equivalent to

$$egin{aligned} egin{bmatrix} egin{aligned} egin{aligned\\ egin{aligned} egi$$

where Q + T = M. Fitting the above model using a KL divergence loss leads to a CNMF formulation like that of (13). As both W and H are nonnegative, the identifiability of these factors holds if both W and H satisfy the separability condition from the NMF literature [42] (see the inserted box "Identifiability of CPD and NMF"). The separability condition holds if K rows of W are close to all K unit vectors. This means that there exist (mutually non-exclusive) annotators  $m_1, \ldots, m_K$  such that

$$\Pr(Y^{(m_k)} = k | Y = k) \approx 1 \Longrightarrow A_{m_k}(k, :) \approx e_k;$$

i.e., annotator  $m_k$  is an expert of recognizing items from class k, leading to the existence of a unit vector in W. In other words, if there are K annotators, who are experts of class k = 1, ..., K respectively, then W satisfies separability—and the same argument applies to H; see [12, 30] for more relaxed conditions and more advanced settings (e.g., where not all  $S_{m,i}$ 's are observed).

Beyond the DS model, identifiability was also studied for other noise models. As discussed, for the one-coin model, model identifiability was established by using the uniqueness (up to a scaling ambiguity) of the principle eigenvector of  $\mathbb{E}[\widehat{U}\widehat{U}^{\top}]$  [22]—see "Spectral Methods" and Eq. (10) in Sec. 3.3. When dependent data are present, the identifiability of both the confusion matrices and the HMM were established in [35], also using tensor-based arguments.

**Identifiability of CPD and NMF.** In the context of crowdsourcing, some classical results from the signal processing literature are particularly relevant:

CPD Uniqueness. Let us denote the Mth-order rank-K tensor in (20) using  $\underline{P} = \llbracket d, A_1, \ldots, A_M \rrbracket$ . Note that the expression in (20) is reminiscent of SVD of matrices. That is,  $A_m \in \mathbb{R}^{I_m \times K}$  has normalized columns, and the term d(k) is analogous to the kth singular value. The model in (20) is referred to as the canonical polyadic decomposition (CPD), which is known to be essentially unique under mild conditions. Specifically, for any alternative  $\widetilde{d}$  and  $\widetilde{A}_m$  for  $m=1,\ldots,M$  such that  $\underline{P}=\llbracket \widetilde{d},\widetilde{A}_1,\ldots,\widetilde{A}_M \rrbracket$ , it must hold that  $A_m=\widetilde{A}_m\Pi$  and  $\widetilde{d}=\Pi d$ , under mild conditions. A widely used condition is

$$\sum_{m=1}^{M} \operatorname{krank}(\boldsymbol{A}_{m}) \geq 2K + M - 1,$$

where krank denotes the Kruskal rank; see [41].

*NMF Uniqueness.* Consider a low-rank nonnegative matrix X = WH where  $W \in \mathbb{R}_+^{M \times K}$  and  $H \in \mathbb{R}_+^{K \times N}$ . Then, any nonnegative solution satisfying  $X = \widehat{W}\widehat{H}$ , W and W has to have the form  $\widehat{W} = W\Pi\Sigma$  and  $\widehat{H} = \Sigma^{-1}\Pi^{\top}H$  under reasonable conditions, where  $\Sigma$  is a diagonal matrix and  $\Pi$  is

Table 2: Classification Error (%) and Run-time (sec): AMT Datasets; Table is from [12].

Alaarithma		TREC	Bl	uebird	RTE	
Algorithms	(N = 19033, M = 762, K = 2)		(N = 108, M = 39, K = 2)		(N = 800, M = 164, K = 2)	
	(%) Error	(sec) Time	(%) Error	(sec) Time	(%) Error	(sec) Time
CNMF-SPA [12]	31.47	50.68	13.88	0.07	8.75	0.28
CNMF-OPT [12]	29.23	536.89	11.11	1.94	7.12	17.06
CNMF-EM [12]	29.84	53.14	12.03	0.09	7.12	0.32
Spectral-EM [11]	29.58	919.98	12.03	1.97	7.12	6.40
CTD [10]	N/A	N/A	12.03	2.74	N/A	N/A
DS-EM [8]	30.02	3.20	12.03	0.02	7.25	0.07
Minimax-Entropy [26]	30.89	352.36	8.33	3.43	7.50	9.10
SparseSpectralSVD [25]	43.95	1.48	27.77	0.02	9.01	0.03
KOS [17]	51.95	9.98	11.11	0.01	39.75	0.03
SpectralSVD [22]	43.03	11.62	27.77	0.01	49.12	0.03
MV	34.85	N/A	21.29	N/A	10.31	N/A

a permutation matrix. The term  $\Sigma$  can be removed if the column norm and row norm of W and H, respectively, are known. Assume that there exist index sets  $\Lambda_i$  for i=1,2 such that  $W(\Lambda_1,:)=D_1$  and  $H(:,\Lambda_2)=D_2$ , where  $D_i$  for i=1,2 are full rank diagonal matrices; i.e., both W and H satisfy the separability condition. Then, the NMF model is essentially unique. More relaxed conditions for NMF uniqueness exist, e.g., that both W and H satisfy the so-called sufficiently scattered condition (SSC). The readers are referred to [42] for a tutorial on NMF identifiability.

**Algorithms - Tractability and Scalability.** Establishing identifiability of the noise model is only the first step towards successful model learning—as identifiability does not guarantee the existence of a tractable or scalable algorithm. Some criteria, e.g., the CTD and CNMF objectives in (12) and (13), respectively, present NP-hard optimization problems. These objectives are handled by standard non-convex optimization tools. Although the empirical results of these algorithms are often acceptable, the quality of the solution is not theoretically guaranteed.

Nonetheless, some progress has been made towards performance-guaranteed algorithm design. For example, the eigen-decomposition based method for the one-coin model admits tractable algorithms, as eigen problems are solvable in polynomial time [22]. If the power method is used, the computational complexity is  $O(\text{nnz}(\widehat{U}))$  per iteration (where  $\text{nnz}(\cdot)$  counts the number of nonzero elements), and the algorithm converges at an exponential rate. For learning the DS model, similar results were shown in [11], where the tensor power method was used. If class experts with  $\Pr(Y^{(m)} = k | Y = k) \approx 1$  exist for every class, [12] showed that there is a Gram-Schmidt-like NMF algorithm that recovers  $\widehat{A}_m \approx A_m \Pi$ . This second-order moment matching-based algorithm is also scalable, with a per-iteration complexity of at most  $O(MK^3)$ , where K is often small.

There are also a number of algorithms that are "locally convergent" to the parameters of the DS model. Given sufficiently good initialization, [11] established that the EM algorithm improves the solution towards the ground-truth confusion matrices. There, the tensor power iterations are combined with the scalable EM algorithm (which takes O(NMK) flops per iteration) to provide an overall tractable solution. A similar result for a variational inference algorithm was shown in [43]. Again if reasonably initialized, [30] showed that a symmetric NMF algorithm that leverages lightweight Procrustes projection converges exponentially to the ground-truth DS model parameters.

**Takeaways.** We use some numerical evidence to conclude our discussion of this section. Table 2 shows the performance of a series of label integration methods on real datasets annotated by AMT workers. The annotations are fairly noisy. One can see that majority voting could only correct the labels up to 34.85%, 21.29% and 10.31% error rates for the TREC, Bluebird, and RTE datasets, respectively. However, methods that learn annotator parameters, namely, DS-EM [8], Spectral-EM [11], CNMF [12], and CTD [10] methods, all improve upon the result of MV by large margins. As we discussed, the EM algorithm enjoys a low computational complexity. If it is initialized by a reliable algorithm, e.g., Spectral-EM and CNMF-EM, the

The complete management approaches and their characteristics							
Methods	Model-type	Method-type	Identifiability	Tractabiliy	Scalability	Conditional Independence	
DS-EM [8]	DS	EM	×	×	<b>✓</b>	<b>√</b>	
CNMF-SPA [12]	DS	Moment-based Fitting	✓	✓	✓	✓	
CNMF-OPT [12]	DS	Moment-based Fitting	✓	×	×	✓	
SymNMF [30]	DS	Moment-based Fitting	<b>✓</b>	<b>✓</b>	<b>✓</b>	✓	
CTD [10]	DS	Moment-based Fitting	<b>✓</b>	×	×	<b>√</b>	
Spectral-EM [11]	DS	Moment-based Fitting + EM	✓	✓	×	✓	
Minimax-Entropy [24]	Extended DS	Entropy	×	×	×	✓	
SpectralSVD [22]	One-coin	SVD	✓	✓	✓	<b>√</b>	
SparseSpectralSVD	One-coin	SVD	✓	✓	✓	✓	
GLAD [27]	Extended One-coin	EM	×	×	✓	✓	
KOS [17]	Spammer-hammer	Graph-based	×	✓	✓	✓	
IBCC [28]	Bayesian-DS	MCMC	X	×	×	✓	
OPT-Crowd [32]	Weighted-MV	Optimization	×	✓	✓	×	
MaxMarginMV [33]	Weighted-MV	SVM	×	×	✓	×	
VariationalBayes [31]	Spammer-hammer	Variational Inference	×	×	×	<b>√</b>	
SeqEM [35]	DS-HMM	Moment-based Fitting + EM	×	×	×	✓	
BayesSeq [36]	Extended DS-HMM	Variational Inference	×	×	×	✓	
GroupAware-EM [38]	Hierarchical-DS	Spectral Clustering + EM	×	X	✓	X	

Table 3: Different label integration approaches and their characteristics

EM algorithm often outshines other methods in both accuracy and speed. These results also attest to the importance of identifiability guarantees—the CNMF and CTD methods learn more accurate labels relative to other methods.

Table 3 presents a summary of the several methods discussed in this section along with their characteristics. The *key takeaways* are as follows: First, identifiability of the noise models is often a key performance indicator. The empirical evidence strongly suggests that approaches with identifiability guarantees consistently work well for label integration. Second, sample (annotator label) complexity is a key consideration when selecting an algorithm in practice. Since collecting more labels incurs extra costs and increases the annotators' workload, methods that perform well with fewer annotations are more economical. Third, methods need to be scalable as real-world label integration often involves a large number of data items and annotators. Hence, it is essential for methods to scale gracefully as both *N* and *M* increase.

## 4 End-to-End (E2E) Learning from Crowdsourced Labels

Compared to the label integration paradigm, the E2E approaches of Fig. 2 have shown more appealing performance over various datasets—see some numerical evidence in Fig. 7. This may be due to the fact the E2E approaches directly work with data features. They are also often a one-stage approach that can avoid error accumulation and propagation among stages.

## 4.1 E2E Learning via Maximum Likelihood and EM

Let us denote a dataset with crowdsourced labels as  $\mathcal{D} = \{x_n, \{\widehat{y}_n^{(m)}\}_{m=1}^M\}_{n=1}^N$ . Under similar assumptions to the DS model, i.e. data items are sampled independently, and annotator responses are conditionally independent, given the ground-truth label, the joint likelihood of the data can be expressed as follows:

$$\Pr(\mathcal{D}) = \prod_{i=1}^{N} \Pr(\boldsymbol{x}_n, \widehat{y}_n^{(1)}, \dots, \widehat{y}_n^{(M)}) = \prod_{i=1}^{N} \sum_{y_n=1}^{K} \Pr(y_n | \boldsymbol{x}_n) \prod_{i=1}^{M} \Pr(\widehat{y}_n^{(m)} | y_n, \boldsymbol{x}_n),$$
(21)

$$= \prod_{i=1}^{N} \sum_{y_n=1}^{K} \underbrace{\Pr(y_n | \boldsymbol{x}_n)}_{[\boldsymbol{f}^*(\boldsymbol{x}_n)]_k} \prod_{i=1}^{M} \underbrace{\Pr(\widehat{y}_n^{(m)} | y_n)}_{A_m(\widehat{y}_n^{(m)}, y_n)}. \tag{22}$$

Here, (21) used the conditional independence of annotators' outputs given the ground-truth label, and (22) used the assumption that annotator confusion is independent from data items. In the above,  $f^*$  and  $A_m$ 

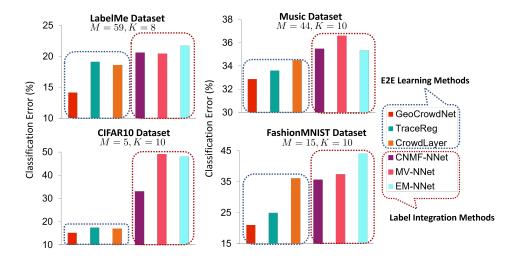


Figure 7: The performance of E2E methods, GeoCrowdNet [13], TraceReg [15], and CrowdLayer [14] and the two stage approaches (label correction methods CNMF [30], MV, and DS-EM [8] followed by a neural network (NNet) classifier training) over four different datasets.

are defined as before [cf. (1) and (5)]. Again, the confusion matrix  $A_m$  is assumed to be the same across all n, as  $\widehat{y}_n^{(m)}$  and  $y_n$  for all n are i.i.d. samples.

The goal of E2E learning is to find  $Pr(y_n|x_n)$ . Under classification settings, the posterior distribution maps any data item to a PMF over the class labels  $1, \ldots, K$ . Therefore, let a function  $f_{\theta} : \mathbb{R}^D \to \mathbb{R}^K$ parametrized by  $\theta$  to represent the ground-truth label posterior  $f^{\star}$ . Collecting all model parameters in  $\psi = (A_1, \dots, A_M, \theta)$ , we have

$$\mathsf{Pr}(\mathcal{D}; oldsymbol{\psi}) = \prod_{i=1}^N \sum_{n=1}^K [oldsymbol{f_{oldsymbol{ heta}}} [oldsymbol{f_{oldsymbol{ heta}}}(oldsymbol{x}_n)]_{y_n} \prod_{i=1}^M oldsymbol{A}_m(\widehat{y}_n^{(m)}, y_n).$$

Similar to the label integration setting in Sec. 3.3, the MLE problem is formulated as

$$\widehat{\psi} = \underset{\psi}{\operatorname{arg max}} \log \left( \Pr(\mathcal{D}; \psi) \right), \tag{23}$$

and as with the label integration case, optimization w.r.t.  $\psi$  is not trivial. To tackle the optimization problem in (23), [14,16] adopted the EM strategy. Specifically, by considering the unobserved ground-truth labels  $\mathcal{Y} = \{y_n\}_{n=1}^N$  as the latent variables, the expected value of the complete log-likelihood  $\log(\Pr(\mathcal{D}, \mathcal{Y}; \psi))$ under the current estimate of  $\psi$  is computed in the E-step:

$$Q(\boldsymbol{\psi}; \boldsymbol{\psi}^t) = \mathbb{E}_{\mathcal{Y} \sim \mathsf{Pr}(\mathcal{Y}; \mathcal{D}, \boldsymbol{\psi}^t)}[\log \mathsf{Pr}(\mathcal{D}, \mathcal{Y}; \boldsymbol{\psi})] = \sum_{n=1}^{N} \sum_{k=1}^{K} q(y_n = k; \boldsymbol{\psi}^t) \log \mathsf{Pr}(\boldsymbol{x}_n, \widehat{y}_n^{(1)}, \dots, \widehat{y}_n^{(M)}; \boldsymbol{\psi})$$

with  $q(y_n=k; \boldsymbol{\psi}^t) = \frac{1}{Z} [\boldsymbol{f}_{\boldsymbol{\theta}^t}(\boldsymbol{x}_n)]_k \prod_{i=1}^M \boldsymbol{A}_m^t(\widehat{y}_n^{(m)}, k)$  and Z being the normalization constant [cf. (8)]. The M-step estimates  $\psi$  by maximizing  $Q(\psi; \psi^t)$ . This can be done by alternating between the following

updates:

$$\mathbf{A}_{m}^{t+1}(k',k) = \frac{\sum_{n=1}^{N} q(y_{n} = k; \boldsymbol{\psi}^{t}) \mathbb{I}[\widehat{y}_{n}^{m} = k']}{\sum_{k''=1}^{K} \sum_{n=1}^{N} q(y_{n} = k; \boldsymbol{\psi}^{t}) \mathbb{I}[\widehat{y}_{n}^{m} = k'']}$$

$$\boldsymbol{\theta}^{t+1} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}; (\boldsymbol{\theta}^{t}, \{\boldsymbol{A}_{m}^{t+1}\})). \tag{24b}$$

$$\boldsymbol{\theta}^{t+1} = \arg\max_{\boldsymbol{\theta}} \ Q(\boldsymbol{\theta}; (\boldsymbol{\theta}^t, \{\boldsymbol{A}_m^{t+1}\})). \tag{24b}$$

This EM formulation is very similar to the one advocated for the DS model in Sec. 3.3, albeit with the classifier  $f_{\theta}$  taking the role of the prior class probabilities d. The EM framework is flexible in terms of incorporating various  $f_{\theta}$  function classes. For binary classification, [16] advocated for a logistic regression model, where  $[f_{\theta}]_1 = \sigma(\theta^{\top}x_n)$  and  $[f_{\theta}]_2 = 1 - \sigma(\theta^{\top}x_n)$ , and  $\sigma$  denotes the sigmoid function. As a result, a Newton-Raphson algorithm can be implemented for (24b). Neural networks were used in [14] to serve as  $f_{\theta}$ , where (24b) was updated by back-propagation based stochastic gradient. To deal with sequence-type data, [44] used a condition random field (CRF) function as the classifier, which uses of the Viterbi algorithm and the limited-memory Broyden–Fletcher–Goldfarb–Shannon (BFGS) algorithm for the M-step. A Bayesian method was adopted in [45] that used a Gaussian process to model a binary classifier and adopted an expectation propagation (EP)-based algorithm for the inference that involves EM-like iterative steps.

## 4.2 Deep Learning with "Crowd Layer"

Among all the functions that can be used as  $f_{\theta}$ , deep neural networks (DNNs) naturally attract a lot of attention, due to their remarkable empirical success in various domains. While [14] showed that EM can be used together with DNNs, the EM framework has some limitations. First, the EM framework is based on multi-class classification, yet it is not straightforward to extend it to cover other problem settings, e.g., when sequence data is involved—the E-step could quickly become intractable. Second, the derivation of the EM framework relies on the conditional independence of the annotators, which may not be always a valid assumption, as discussed in Sec. 3.4. Third, the function  $f_{\theta}$  needs to be trained in each M-step, which may be computationally demanding.

An alternative approach to incorporate DNNs in crowdsourcing was advocated in [14]. Consider the probability of m-th annotators' response to the data item  $x_n$  as follows:

$$\Pr(\widehat{y}_n^{(m)} = k | \boldsymbol{x}_n) = \sum_{k'=1}^K \Pr(\widehat{y}_n^{(m)} = k | y_n = k') \Pr(y_n = k' | \boldsymbol{x}_n), \ k \in [K], \tag{25}$$

where we used the law of total probability and the assumption that annotator responses are instance-independent, given the label  $y_n$ . Upon defining a K-dimensional vector  $\boldsymbol{p}_n^{(m)}$  such that  $[\boldsymbol{p}_n^{(m)}]_k \triangleq \Pr(\widehat{y}_n^{(m)} = k|\boldsymbol{x}_n)$ , Eq. (25) can be expressed as follows:

$$\boldsymbol{p}_n^{(m)} = \boldsymbol{A}_m \boldsymbol{f}^*(\boldsymbol{x}_n), \forall m, n, \tag{26}$$

where  $[f^*(x_n)]_k = \Pr(y_n = k|x_n)$  is as defined in (1). Under this model, observations can be understood as realizations of a categorical random variable, i.e.  $\widehat{y}_n^{(m)} \sim \operatorname{categorical}(p_n^{(m)})$ . To estimate  $A_m$  and  $f^*$ , a commonly used criterion in machine learning is *cross entropy* (CE), i.e.,

$$ext{CE}(\widehat{oldsymbol{p}}_n^{(m)}, oldsymbol{A}_m oldsymbol{f}_{oldsymbol{ heta}}(oldsymbol{x}_n)) = -\sum_{k=1}^K [\widehat{oldsymbol{p}}_n^{(m)}]_k \log [oldsymbol{A}_m oldsymbol{f}_{oldsymbol{ heta}}(oldsymbol{x}_n)]_k,$$

where  $\widehat{p}_n(k)=1$  if  $\widehat{y}_n^{(m)}=k$  and  $\widehat{p}_k(k')=0$  for  $k'\neq k$ , and  $f_{\theta}$  is the learning function for approximating  $f^{\star}$  as before. In a nutshell, CE seeks a model  $\{A_m,f_{\theta}(x_n)\}$  that matches the "empirical PMF"  $\widehat{p}_n^{(m)}$ . It can be shown that when  $N\to\infty$ , the minimum of CE is attained at  $Af_{\theta}(x_n)=p_n^{(m)}$ . Collecting all annotator responses, [14] used the following *coupled cross-entropy minimization criterion* (CCEM) [14]:

$$\underset{\boldsymbol{f_{\theta}} \in \mathcal{F}, \{\boldsymbol{A}_m \in \mathcal{A}\}}{\text{minimize}} - \frac{1}{|\mathcal{S}|} \sum_{(m,n) \in \mathcal{S}} \sum_{k=1}^{K} \mathbb{I}[\widehat{y}_n^{(m)} = k] \log[\boldsymbol{A}_m \boldsymbol{f_{\theta}}(\boldsymbol{x}_n)]_k$$
 (27)

where  $S \subseteq [M] \times [N]$  is the index set of annotator-labeled samples,  $\mathcal{F} \subseteq \{f(x) \in \mathbb{R}^K | f(x) \in \Delta_K, \forall x\}$  is a function class parameterized by  $\theta$ ,  $\Delta_K$  represents the (K-1)-probability simplex, and A is the constrained

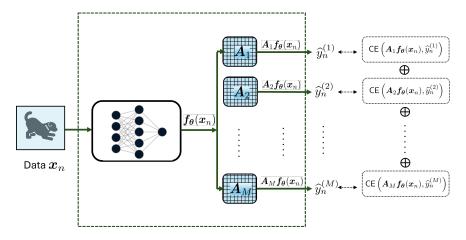


Figure 8: The "crowdlayer"-based architecture for deep learning-based E2E crowdsourcing

set of confusion matrices  $\{A \in \mathbb{R}^{K \times K} | A \geq 0, \mathbf{1}^{\top} A = \mathbf{1}^{\top} \}$ . In practice,  $f_{\theta} \in \mathcal{F}$  can be approximately enforced by using a softmax layer as its output. The constraints ensure that the output of  $f_{\theta}$  and columns of  $A_m$ 's are PMFs. The term "coupled" refers to the fact that the expressions of  $p_n^{(m)}$  for all m are coupled by  $f_{\theta}(x_n)$ .

Fig. 8 illustrates the loss function of CCEM. Here, confusion matrices  $A_m$ 's act as additional annotator-specific layers of the neural network [14], hence this approach is termed *crowdlayer*. The CCEM type formulation is arguably more versatile in terms of modeling and computation relative to the EM-type E2E algorithms. First, regularization terms on  $A_m$  and  $\theta$  can be easily added for various purposes, e.g., incorporating prior knowledge and enhancing identifiability [13,15]. Second, as we have seen, the CCEM approach does not require that annotators are conditionally independent (also see details in [13]). Third, the empirical distribution  $\hat{p}_n^{(m)}$  needs not be categorical, opening doors for continuous measurement-based problems, e.g., regression. Finally, the crowdlayer architecture can be easily trained via backpropagation and any gradient decent-based optimization algorithms like Adam, using off-the-shelf deep learning libraries such as PyTorch and TensorFlow.

### 4.3 Model Identifiability under CCEM

Issues of model identifiability also arise in the E2E context. Under the CCEM criterion, and the generative model  $\widehat{y}_n^{(m)} \sim \operatorname{categorical}(A_m f^*(x_n))$ , it is essential to identify  $A_m$ 's and the ground-truth  $f^*(x_n)$  (i.e.,  $\Pr(y_n|x_n)$ ). Note that identifying  $f^*(x_n)$  (i.e., to attain  $f_{\theta}(x_n) \approx f^*(x_n)$ ) over all the seen (training) data  $\{x_n\}_{n=1}^N$  allows the learned model  $f_{\theta}$  to generalize over unseen (test) data as well. This turns out to be non-trivial. In the ideal case where  $N \to \infty$ , the CCEM criterion returns  $\widehat{f}_{\theta}$  and  $\widehat{A}_m$  such that  $p_n^{(m)} = \widehat{A}_m \widehat{f}_{\theta}(x_n)$ . Nevertheless, one can easily note that such a relation is highly non-unique since there exists an infinite number of nonsingular matrices  $Q \in \mathbb{R}^{K \times K}$  such that  $p_n^{(m)} = (\widehat{A}_m Q)(Q^{-1}\widehat{f}_{\theta}(x_n))$ . However, CCEM-type approaches seem to always learn reasonable  $A_m$  and  $f_{\theta}$  in practice. To understand this phenomenon, [13] provided performance characterizations of CCEM, from an NMF identifiability viewpoint. To be specific, when  $N \to \infty$ , the CCEM criterion can be understood as finding solutions to fit the following model:

$$\underbrace{\begin{bmatrix} \boldsymbol{p}_{1}^{(1)} & \dots & \boldsymbol{p}_{N}^{(1)} \\ \vdots & \ddots & \vdots \\ \boldsymbol{p}_{1}^{(M)} & \dots & \boldsymbol{p}_{N}^{(M)} \end{bmatrix}}_{\boldsymbol{P} \in \mathbb{R}^{MK \times N}} = \underbrace{\begin{bmatrix} \boldsymbol{A}_{1} \\ \vdots \\ \boldsymbol{A}_{M} \end{bmatrix}}_{\boldsymbol{W} \in \mathbb{R}^{MK \times K}} \underbrace{\begin{bmatrix} \boldsymbol{f}^{\star}(\boldsymbol{x}_{1}) & \dots & \boldsymbol{f}^{\star}(\boldsymbol{x}_{N}) \end{bmatrix}}_{\boldsymbol{H} \in \mathbb{R}^{K \times N}}, \tag{28}$$

where the factors W and H are both nonnegative per their physical meaning. Clearly, if W and H both satisfy the separability condition (or the sufficiently scattered condition (SSC)), as discussed in "Identifiability of CPD and NMF" (also see [42]), the factorization model in (28) is essentially unique; i.e., there exists a permutation matrix  $\Pi$  such that

$$\|\widehat{\boldsymbol{A}}_{m}\boldsymbol{\Pi} - \boldsymbol{A}_{m}\| \to 0, \ \|\boldsymbol{\Pi}^{\top}\widehat{\boldsymbol{f}}_{\boldsymbol{\theta}}(\boldsymbol{x}) - \boldsymbol{f}^{\star}(\boldsymbol{x})\| \to 0,$$
 (29)

when  $N \to \infty$ . For W, the separability condition still requires the existence of class experts for each of the K classes. For H, separability implies the existence of K indices,  $\{n_1, \ldots, n_K\} = \{1, \ldots, K\}$  such that  $f^*(x_{n_k}) = e_k \Leftrightarrow \Pr(y_{n_k} = k | x_{n_k}) = 1$ . Such points  $x_{n_k}$  are called "anchor points" for the classes and are often found useful in noisy label learning.

Aiming to relax the conditions on one of W and H, [13] proposed a regularization-based CCEM. Specifically, a regularization term to maximize the volume of  $H = [f_{\theta}(x_1), \dots, f_{\theta}(x_N)]$  was added:

$$\underset{\{\boldsymbol{A}_{m} \in \mathcal{A}\}, \boldsymbol{f}_{\boldsymbol{\theta}} \in \boldsymbol{\Delta}_{K}}{\text{minimize}} \, \mathcal{L}_{\text{CCEM}} - \beta \log \det(\boldsymbol{H} \boldsymbol{H}^{\top}), \tag{30}$$

where  $\mathcal{L}_{\text{CCEM}}$  is defined as the objective function in (27) and  $\beta \geq 0$ . This formulation leverages the *simplex volume minimization*-based structured matrix factorization [42] to establish identifiability as shown in (29). This way, the separability or SSC condition imposed onto W can be removed. This substantially reduces the expertise requirement for the annotators to establish identifiability of the model.

### 4.4 Other E2E Crowdsourcing Approaches

**Agreement-Based Model.** Besides the confusion matrix-based models and formulations as in (23) and (27), some other treatments for E2E learning were also introduced [46]. The approach in [46] proposes to use an annotator "aggregator" denoted as  $g_{\phi}$  that measures the "average agreement" between different annotator responses, and is modeled as an affine mapping followed by a so-called softmax operation:

$$oldsymbol{g_{\phi}(\{\widehat{oldsymbol{p}}_{n}^{(m)}\})} = \operatorname{softmax}\left(\sum_{m=1}^{M} oldsymbol{W}^{(m)} \widehat{oldsymbol{p}}_{n}^{(m)} + oldsymbol{b}
ight).$$

Here,  $\phi = \{ \boldsymbol{W}^{(1)}, \dots, \boldsymbol{W}^{(M)}, \boldsymbol{b} \}$  and  $\widehat{p}_n^{(m)}$  denotes the one-hot embedding of the noisy label  $\widehat{y}_n^{(m)}$  as before. Note that  $f_{\theta}(\boldsymbol{x}_n)$  and  $g_{\phi}(\{\widehat{p}_n^{(m)}\})$  can be understood as two label predictors using data features and annotator-produced labels as inputs, respectively. The idea in [46] is to maximize the "agreement" (measured by f-mutual information gain (MIG $^f$ )) between these two hypotheses  $f_{\theta}$  and  $g_{\phi}$ :

$$\underset{\boldsymbol{\theta}, \boldsymbol{\phi}}{\text{maximize MIG}} f(\boldsymbol{f}_{\boldsymbol{\theta}}, \boldsymbol{g}_{\boldsymbol{\phi}}; \{\boldsymbol{x}_n\}, \{\widehat{\boldsymbol{p}}_n^{(m)}\}). \tag{31}$$

In essence,  $\mathrm{MIG}^f$  measures the agreement between  $f_{\theta}(x_n)$  and  $g_{\phi}(\{\widehat{p}_n^{(m)}\})$  averaged over all the data items. The measure of agreement is an f-divergence loss function, such as the KL divergence. It was also shown in [46] that the objective (31) finds optimal solutions (i.e., the solutions that extract maximum information from their inputs to predict the ground-truth) in the asymptotic case, provided there are conditionally independent expert annotators.

Instance-dependent Confusion Matrix. Both EM and CCEM based E2E methods assume that  $A_m$  remains identical across all  $x_n$  [cf. (26)]. Recent works studied crowdsourcing approaches under an instance-dependent setting:

$$\boldsymbol{p}_n^{(m)} = \boldsymbol{A}_m(\boldsymbol{x}_n) \boldsymbol{f}^*(\boldsymbol{x}_n), \forall m, n,$$
(32)

where  $A_m(x_n)$ 's are instance-dependent annotator confusions with entries  $[A_m(x_n)]_{k,k'} = \Pr(\widehat{y}_n^{(m)} = k|y_n = k', x_n)$  [cf. (21)]. To estimate the instance-dependent confusion matrices, a common approach

is to use two learnable functions, i.e.,  $f_{\theta}: \mathbb{R}^D \to \mathbb{R}^K$  and  $A_m^{\phi}(\cdot): \mathbb{R}^D \to \mathbb{R}^{K \times K}$  (e.g., neural networks) to parameterize  $f^*$  and  $A_m(\boldsymbol{x}_n)$ , respectively.

E2E learning of the model in (32) was considered in [47], which extended [15] as follows:

$$\underset{\{\boldsymbol{A}_{m}^{\phi} \in \mathcal{A}\}, \boldsymbol{f}_{\boldsymbol{\theta}} \in \boldsymbol{\Delta}_{K}}{\text{minimize}} - \frac{1}{|\mathcal{S}|} \sum_{(m,n) \in \mathcal{S}} \sum_{k=1}^{K} \mathbb{I}[\widehat{y}_{n}^{(m)} = k] \log[\boldsymbol{A}_{m}^{\phi}(\boldsymbol{x}_{n}) \boldsymbol{f}_{\boldsymbol{\theta}}(\boldsymbol{x}_{n})]_{k} + \beta \sum_{m=1}^{M} \sum_{n=1}^{N} \operatorname{trace}(\boldsymbol{A}_{m}^{\phi}(\boldsymbol{x}_{n})). \tag{33}$$

However, establishing identifiability of the two functions  $A_m(\cdot)$  and  $f^*(\cdot)$  from their product using E2E learning is fundamentally challenging. A common workaround is to use a multi-stage strategy, i.e., first learning  $A_m^{\phi}(\cdot)$  using some pre-selected data items and then using the learned  $A_m^{\phi}(\cdot)$  to train  $f_{\theta}$  using losses similar to (33). For instance, [48] parametrizes the instance-dependent annotator confusions using a *mixed* effects neural network model (MNN):

$$[\boldsymbol{A}_{m}^{\boldsymbol{\phi}}(\boldsymbol{x}_{n})]_{:,k} = \operatorname{softmax}(\underbrace{\boldsymbol{B}^{(m)}\boldsymbol{g}_{\boldsymbol{\phi}_{1}}(\boldsymbol{x}_{n})}_{\text{annotator-specific}} + \underbrace{\boldsymbol{C}^{(k)}\boldsymbol{g}_{\boldsymbol{\phi}_{2}}(\boldsymbol{x}_{n})}_{\text{class-specific}}), \tag{34}$$

where  $g_{\phi_1}$  and  $g_{\phi_2}$  are two neural networks parameterized by  $\phi_1$  and  $\phi_2$ , respectively. The model aims to capture the the annotator-specific and class-specific effects on  $A_m(x_n)$ . In this work, the first step selects some "anchor data points" for each class k. The anchor points satisfy  $\Pr(y_n = k|x) = 1$ , which means  $[A_m(x_n)]_{:,k} = \Pr(\hat{y}_n^{(m)} = k|x_n)$ . Using such selected data items  $\{x_s, \{\hat{y}_n^{(m)}\}\}_{s=1}^S$ , the MNN parameters  $\varphi = \{\phi_1, \phi_2, \{B^{(m)}\}, \{C^{(k)}\}\}$  are learned first using a regression approach. The next step involves predicting the ground-truth labels through a pairwise likelihood ratio test using the learned  $A_m^{\phi}(\cdot)$ . These predictions are then used to train  $f_{\theta}$ .

### 4.5 Other Types of Annotations

While the discussions in Sec. 3 and 4 focus on the classification setting where the annotations are categorical, similar modeling and algorithm design ideas can be applied for other types as well.

**Regression.** Consider the case where the ground-truth label and the corresponding annotations take continuous real values, i.e.,  $y_n \in \mathbb{R}$  and  $\widehat{y}_n^{(m)} \in \mathbb{R}$ . The most naive aggregation scheme is averaging, i.e.

$$\tilde{y}_n = \frac{1}{|\mathcal{M}_n|} \sum_{m \in \mathcal{M}_n} \widehat{y}_n^{(m)}$$

with  $\mathcal{M}_n \subseteq [M]$  denoting set of annotators who have provided a response for the n-th data item. A more robust alternative, that is often used in federated learning, is the median integration rule.

Both average and median can be regarded as continuous extensions of majority voting. The DS model can also be extended to continuous annotation cases. For example, one can assign continuous distributions to the ground-truth labels and annotator responses, i.e.,  $y_n \sim \mathcal{N}(\mu_n, \sigma_n^2)$  where  $\mu_n, \sigma_n^2$  are the mean and variance of the labels, respectively, while the conditional distributions of annotator responses given the ground-truth label  $y_n = \alpha$  are  $\widehat{y}_n^{(m)}|y_n = \alpha \sim \mathcal{N}(\alpha, \sigma_m^2)$ , with  $\sigma_m^2$  being annotator specific variance [49]. In the E2E learning-based setting, linear regression, Gaussian Process or deep learning models can be readily employed, with  $f_{\theta}(x_n)$  being the continuous distribution  $\Pr(y_n|x_n)$  [14].

**Ranking.** In some cases, annotators are asked to provide ranking of data items. Two types of ranking are often considered, namely, pairwise preference and ordinal ranking. In the pairwise preference case, the ordinal ranks of all the data items are sought by comparing pairs. Given a pair  $x_n$  and  $x_{n'}$ , the preference of  $x_n$  over  $x_{n'}$  is denoted as  $x_n \succ x_{n'}$ . One of the popular models for preference is the the Bradley-Terry (BT) model, in which the probability of  $x_n \succ x_{n'}$  is modeled as follows:

$$\Pr(x_n \succ x_{n'}) = \frac{e^{s_n}}{e^{s_n} + e^{s_{n'}}},\tag{35}$$

where  $s_n$  for  $n \in [N]$  is a so-called ranking score for  $x_n$ . In the crowdsourcing setting, each annotator indicates their preference over  $x_n$  and  $x_{n'}$ . That is, if annotator m prefers  $x_n$  over  $x_{n'}$ , it is denoted as  $x_n \succ_m x_{n'}$ . Given a ground-truth preference relation  $x_n \succ x_{n'}$ , annotator m's correctness and confusion probabilities can be expressed using  $w_m = \Pr(x_n \succ_m x_{n'} | x_n \succ x_{n'})$  and  $1 - w_m$ , respectively. Under this model, [50] used the MLE principle to jointly learn  $w_m$  and ranking  $s_n$ . In E2E crowdsourcing,  $s_n$  is expressed as  $s_n = f_{\theta}(x_n)$ , where  $f_{\theta} : \mathbb{R}^D \to \mathbb{R}$  learns a real-valued score function from the data features. Note that the BT model and such function-based score representation were used in LLM fine-tuning, specifically, by the *direct preference optimization* (DPO) approach [51]. In the ordinal ranking case, [16] converted the annotator-provided preference order of the data items into binary labels. This also allows the associated MLE formulation to be combined with E2E learning method.

Similarity Annotations. Crowdsourced E2E learning was also formulated as a graph clustering problem. In this case, the annotators are asked to indicate the similarity of two data items. This way, an (incomplete)  $N \times N$  binary adjacency graph G is constructed, where G(i,j)=1 means that  $x_i$  and  $x_j$  are regarded as "similar" or from the same class and G(i,j)=0 means otherwise. Similarity-based annotation requires substantially lower expertise level from the annotators and thus is a promising paradigm for extra-large scale data annotation. The labels can be modeled as Bernoulli samples as follows:

$$G(i,j) \sim \text{Bernoulli}\left(f^{\star}(x_i)^{\top} f^{\star}(x_j)\right),$$
 (36)

where  $f^*$  such that  $[f^*]_k = \Pr(y_n = k | x_n)$  for k = 1, ..., K following the same realizability assumption before. The term  $f^*(x_i)^{\top} f^*(x_j) \in [0,1]$  naturally models the similarity of  $x_i$  and  $x_j$ . Let a function  $f_{\theta}$  be the learner of the ground-truth  $f^*$  as in the aforementioned E2E approaches, the MLE under (36) is a logistic regression objective (see [52] and some predecessors):

$$\underset{\boldsymbol{f_{\theta}} \in \boldsymbol{\Delta}_K}{\text{minimize}} \sum_{(i,j) \in \boldsymbol{\Omega}} \left[ \mathbb{I}[g_{ij} = 1] \log \left( \boldsymbol{f_{\theta}}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{f_{\theta}}(\boldsymbol{x}_j) \right) + \mathbb{I}[g_{ij} = 0] \log \left( 1 - \boldsymbol{f_{\theta}}(\boldsymbol{x}_i)^{\mathsf{T}} \boldsymbol{f_{\theta}}(\boldsymbol{x}_j) \right) \right], \tag{37}$$

where  $\Omega$  is the index set of annotated pairs. The identifiability of  $f^*$  was established by treating the model in (36) as a quantized nonnegative matrix factorization problem. Similarity annotations can be easily acquired by the crowd (despite no annotator-specific models used in (36)), and thus this type of approaches are often referred to as *crowdclustering* in the literature.

## 5 Emerging Topics in crowdsourcing

As discussed in the previous sections, the research in crowdsourcing has made impressive progress in the last several decades. Next, we will explore some additional contemporary topics in this domain that are garnering increasing attention from the machine learning community.

#### 5.1 Bias and Fairness

Machine learning and statistical learning algorithms are increasingly being applied in areas with significant human and societal impact, such as credit scoring, loan underwriting, job applications, and the penal system. While these algorithms often achieve overall good performance for the underlying task, they can also perpetuate unfairness and bias by discriminating against certain *sensitive attributes*, such as race, age, or affiliation. In crowdsourcing, the presence of unfair workers, who provide more inaccurate labels to the data items belonging to a particular group, can significantly impact the overall system performance w.r.t. the sensitive attributes. For instance, a recent empirical study [53] observed that the performance of the label integration approaches including majority voting, DS model-based EM, and the E2E approach Crowdlayer, all are impacted by the presence of unfair annotators. An approach to select fair workers for labeling tasks was advocated in [54]. They proposed an annotator assignment search algorithm such that the overall label integration accuracy is maximized while satisfying certain notions of fairness. Consider *sensitive attribute-specific confusion matrix* per annotator, similar to the instance-dependent confusion matrix

of (32), with entries  $[A_m(Z=z)]_{k,k'}=\Pr(\widehat{Y}^{(m)}=k|Y=k',Z=z)$  where  $z\in\{0,1\}$  and Z is the random variable denoting the sensitive attribute (e.g., gender). Suppose that  $s_m$  denotes the probability that any data item is assigned to mth annotator, regardless of the sensitive attribute. Under this setting, [54] sought an optimal annotator-assignment policy  $s=[s_1,\ldots,s_M]$  by maximizing the expected labeling accuracy, i.e.,  $\sum_{z\in\{0,1\}}\Pr(Z=z)\sum_{k\in[K]}\Pr(Y=k)\sum_{m=1}^Ms_m[A_m(Z=z)]_{k,k}$ . Nevertheless, confusion matrices and the prior probabilities are assumed to be estimated a priori using some limited "gold" data items with known ground-truth labels and sensitive attributes. In addition, the maximization is performed under certain fairness constraints such as false positive rate parity on  $A_m(Z=z)$ 's and diversity constraints on s to balance the efforts across more annotators.

#### 5.2 Adversarial Attacks

In crowdsourced settings, label noise sometimes stems from undesirable annotator behaviors rather than unintentional errors. Such undesirable behaviors may be caused by annotators who provide random responses with no effort (spammers) and those who intentionally give incorrect responses (adversaries). In these cases, identifying and excluding spammers and adversaries is important for performance enhancement. One approach to identify spammers is through examining annotator confusion matrices. Consider a spammer annotator m. Then, the entries of the corresponding confusion matrix are  $A_m(k',k) = \Pr(\widehat{y}_n^{(m)} = k'|y_n = k) = \Pr(\widehat{y}_n^{(m)} = k'|y_n = k'') = \Pr(\widehat{y}_n^{(m)} = k'), \forall k, k''$ , i.e., the annotator response does not depend on the ground-truth label of the item. As such, all columns of  $A_m$  are the same and  $\operatorname{rank}(A_m) = 1$ . Based on this observation, [55] derived a "spammer score" that was then used in a modified EM algorithm to integrate labels and eliminate spammers. Using similar principles, a spectral approach based on the second-order moments  $\left[\mathbb{E}[\widehat{Y}^{(m)} \circ \widehat{Y}^{(i)}]\right]$  can be used to identify spammers prior to label aggregation.

A particularly challenging setting involves colluding adversaries, i.e., annotators that cooperate to degrade the performance of a crowdsourcing system. Such a scenario can be particularly detrimental to label integration algorithms that rely on the conditional independence between annotators. A few recent works have attempted to tackle this issue. A robust rank-one matrix completion method was advocated in [56]. Specifically, the rank-one plus identity structure, as given by (10) in Sec. 3.3, is exploited assuming that the reliabilities of the adversaries deviate from this structure. For the more general DS model, [57] advocated a spectral method that leverages the particular structure of the annotator agreement matrix. Note that the methods for dependent annotators in 3.4 attempt to unveil the dependencies between annotators. Here, while there are dependencies between colluding annotators, adversarial crowdsourcing algorithms treat adversaries as "outliers", and seek to filter them.

When training deep learning systems, effective adversarial attack can be realized by adding designated noise to  $x_n$ . Under such circumstances, [58] introduced an E2E method, based on EM, that can simultaneously learn annotator confusion matrices and a robust classifier.

## 5.3 Reinforcement Learning with Human Feedback

A recent field gaining significant attention is *reinforcement learning with human feedback* (RLHF), particularly for its application in fine-tuning LLMs [59]. In classical reinforcement learning (RL), an agent interacts with an environment over T time steps. Per time step, the environment is at a state  $s_t$  and the agent chooses an action  $a_t$ , and receives a reward  $r_t$ . The next environment state  $s_{t+1}$  is affected by the current state  $s_t$  and the agents action  $a_t$ , via a so-called transition probability  $p(s_{t+1}|s_t,a_t)$ . The goal of the agent is to learn a policy (i.e., a function mapping states to actions) that will maximize their rewards. RL has been extensively studied and a plethora of algorithms are available to train the agent. Nevertheless, a key component of RL is the reward function, whose design is a non-trivial task. RLHF circumvents the task of designing a reward function a priori; instead of using a pre-defined reward function, a reward model is learned via human feedback. In RLHF, the agent can issue queries to one or more humans and receive labels in response. Furthermore, queries can be specific actions and states, or trajectories  $\tau$ , which are sequences of states and

actions  $\tau = \{s_0, a_0, s_1, a_1, \ldots\}$ . Typically, feedback is received asynchronously from the main RL agent-environment interaction. Human feedback can be in the form of classifying the queries as "good" or "bad", or as preferences between a pair of trajectories  $(\tau_1, \tau_2)$ . In the former case, the label integration approaches of the previous sections can be utilized to denoise the feedback from multiple human annotators. The latter can be treated using preference-based learning as in (35), and in RLHF settings  $x_n$  and  $x'_n$  represent two trajectories. Indeed, the Bradley-Terry model for RL was recently extended to the crowdsourcing case in [60]. All in all, the field of RL with its' multitude of applications, presents exciting opportunities to apply the principles and ideas of crowdsourcing.

### 5.4 Connections to Active Learning, and Transfer Learning

#### 5.4.1 Active learning

Most crowdsourcing jobs submitted to services such as AMT, operate within a predetermined budget. This budget can be easily translated into the total number of queries that can be asked from the annotators. Thus, efforts that reduce the overall amount of annotator queries, while maintaining classification performance are well motivated. These goals can be formulated as an active learning problem. Under a classification setup, active learning iteratively re-trains a learner by gradually selecting and adding samples to the training set in each iteration. Central components to any active learning system are the method that data is selected to augment the training set, and efficient retraining of the classifier as new data arrive. If these components are designed properly, the samples needed for learning the classifier can be drastically reduced relative to ordinary training processes. Data selection is typically facilitated by choosing data for which the current classifier is most "uncertain" of. Furthermore, (re-)training of f as new data arrive can be facilitated via online optimization methods such as stochastic gradient descent (SGD).

To minimize the number of queries to annotators, while maintaining high classification accuracy, active learning can be retooled and incorporated in the label integration procedure. Notably, in addition to selecting which datum to acquire labels for, one must also select which annotator to query. Ideally, the annotator selected should be the one with the highest probability of providing the correct label for a given datum. As discussed in Secs. 3 and 4, popular label integration algorithms compute the posterior probability for each datum  $Pr(y_n|\widehat{\mathcal{Y}})$  at each iteration. These posteriors can then be used to quantify the "uncertainty" of the label integration for each data point, given currently available annotations. At the same time, the available estimates of annotator confusion matrices may not be reliable and introduce additional uncertainty into the model. Annotators can be selected using the current estimates of confusion matrices. However, one should be careful when selecting which annotator to query, as the "best" annotator based on current parameters may be suboptimal. Thus, randomized exploration strategies may be beneficial, especially when only a few labels are available. A few recent methods utilize active learning in the pure label integration [61] and E2E cases [45]. In addition, it can be noted that the annotator selection problem can be cast as a Multi-armed Bandit (MAB) problem, with each annotator corresponding to an "arm" or "action" in the bandit setting [62]. In all these cases, active learning-based sampling of data and annotators outperforms randomly selected ones, indicating the significance of active methods in crowdsourcing.

#### 5.4.2 Transfer learning

Another set of techniques designed to decrease the amount of annotations are based on *Transfer Learning* (TL). In TL, one is interested in training a model for a machine learning task of interest, termed the *target task*. However, the target task does not have enough labeled data available for training. Instead, another related *auxiliary task* has plenty of labeled data. TL seeks to *transfer* knowledge from the auxiliary task, such that a high-performance model can be trained for the target task. In the crowdsourcing setup, annotators can be regarded as "vehicles" to transfer knowledge across tasks. Indeed an interesting question that arises is: "Which annotators would perform well on a low-resource target task?". In [63], a probabilistic model that captures annotator ability and task specific parameters was developed to estimate annotator reliability

across multiple different tasks. When data features are available, [64] advocated for a logistic regression-based model that can transfer knowledge across multiple tasks.

#### 6 Conclusions and future directions

Crowdsourcing-based data annotation is pivotal in the AI era. Since crowdsourced annotators are often unreliable, effectively integrating multiple noisy labels to produce accurate annotations stands as arguably the most important consideration for designing and implementing a crowdsourcing system. In this feature article, we reviewed key milestones in crowdsourcing research, including models, methods, theoretical advancements and emerging topics. We also reviewed the intimate connections between crowdsourcing and signal processing theory and methods, such as NMF, tensor decomposition, distributed detection, and optimization techniques—showing how SP perspectives could offer principled design and enhanced performance for crowdsourcing systems.

Together with the AI boom and the wide adoption of large, complex models that have high overfitting risks, the relevance of crowdsourced data annotation continues to rise. We highlight several worthwhile future research directions below:

- Understanding of crowdsourcing under realistic and challenging scenarios, such as those involving
  instance-dependent label noise, imbalanced data distribution, or dynamic and evolving tasks, is still
  limited. Existing approaches either require stringent conditions or rely on heuristics. Principled solutions with performance guarantees—such as model identifiability, generalization, and sample complexity—are highly desirable in these cases.
- Crowdsourcing approaches are a key component of the training and fine tuning foundation models—see, e.g., our introduction on using crowdsourced labels to fine tune LLMs via RLHF and DPO. These directions have just begun to be explored, and thus many research questions, such as those regarding model building, method design, and performance characterization, remain wide open.
- In-depth annotator behavior models, which consider aspects like bias and adversarial tendencies, have proven beneficial. However, these complex models often involve nontrivial and multi-faceted design considerations, e.g., fairness metrics effectiveness, parameter parsimony, and algorithm scalability. Despite some advancements in the past decade, unified design frameworks and standards have yet to be established.
- Crowdsourcing offers solutions to a wide range of problems in science and engineering. However, there remains significant scope for the joint design of crowdsourcing algorithms and specific applications, such as medical diagnosis, environmental monitoring, and disaster response. This can be achieved by using cutting-edge models and algorithms tailored to the problem at hand, especially in light of the emergence of large-scale foundation models. Additionally, the collaboration between human and machine annotators, including LLM agents, can enhance data annotation. These opportunities present many unique challenges to be addressed in system design, data and computational resource management, and performance characterization.
- The scope of classical applications such as distributed detection, remote calibration, the CEO problem from information theory, and blind multichannel deconvolution can be broadened via cross-polination with crowdsourcing methods. For instance, blind deconvolution can be extended to nonlinear settings, using ideas from E2E crowdsourcing, whereas rate distortion ideas from information theory can be potentially applied to study the cost-performance tradeoff in crowdsourcing. Another exciting avenues of research include game theoretic approaches in adversarial crowdsourcing and robust data fusion tenchniques that learn from noisy multi-view data.

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