Kalman Filter Implementation for State Estimation with Noisy Measurements (2023)

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Introduction

This report describes the implementation of the Kalman filter for estimating the true state of a system from noisy measurements. The Kalman filter is a recursive algorithm that efficiently estimates the mean and covariance of a Gaussian distribution over time. It is widely used in various applications, including tracking missiles, computer vision, navigation, economics and control systems.

Kalman filter is a set of mathematical equation that provides an efficient (recursive) means to estimate the state of a process which minimizes the mean squared error otherwise is a recursive solution to discrete data-linear filtering problem.

System Model

The Kalman filter is based on a linear system model, which describes the relationship between the true state of the system and the noisy measurements. The system model is represented by the following equations:

Consider discrete time process

$$x_k = A x_{k-1} + B u_{k-1} + \underbrace{w_{k-1}}_{\text{process noise}}$$

$$z_k = H x_k + \underbrace{v_k}_{\text{measurement noise}}$$

Where,

$\hat{x}_k^-$ – Priority state estimation given knowledge of process prior to step k.

$\hat{x}_k$ – Posterior state estimate at k given measurement z_k

$$e_k^- = x_k - \hat{x}_k$$

$$P_k^- = E[e_k^- e_k^{-T}]$$

Priority estimate error covariance

$$P_k = E[e_k e_k^T]$$

Posterior estimate error covariance

$$E[x_k] = \hat{x}_k$$

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = P_k$$

$$P[x_k | z_k] = N(\hat{x}_k, P_k)$$

Kalman Filter Algorithm

The Kalman filter algorithm consists of two main steps: prediction and update.

Prediction Step:

The prediction step estimates the mean and covariance of the state distribution based on the previous state estimate and the system model. The equations for the prediction step are:

• update equations (Predict)

$$\hat{X}_K^- = A\hat{X}_{K-1} + B U_{K-1} \quad (3)$$

$$P_K^- = A P_{K-1} A^T + Q \quad (4)$$

where,

- X_K_hat- is the predicted state estimate at time step k
- P_K_hat- is the predicted state covariance matrix at time step k

Using matlab to calculate the predicted state covariance matrix

```
x_hat_pred = A * x_hat(k-1);
P_pred = A * P(k-1) * A' + Q;
```

Update Step:

The update step corrects the predicted state estimate and covariance based on the new noisy measurement. The equations for the update step are:

Measurement update equation (correct)

$$K_K = P_K^- H^T (H P_K^- H^T + R)^{-1} \quad (5)$$

$$\hat{X}_K = \hat{X}_K^- + K_K (Z_K - H\hat{X}_K^-) \quad (6)$$

$$\hat{P}_K = (I - K_K H) P_K^- \quad (7)$$

where:

- K_K is the Kalman gain matrix
- I is the identity matrix

Using matlab to calculate the updated state covariance matrix.

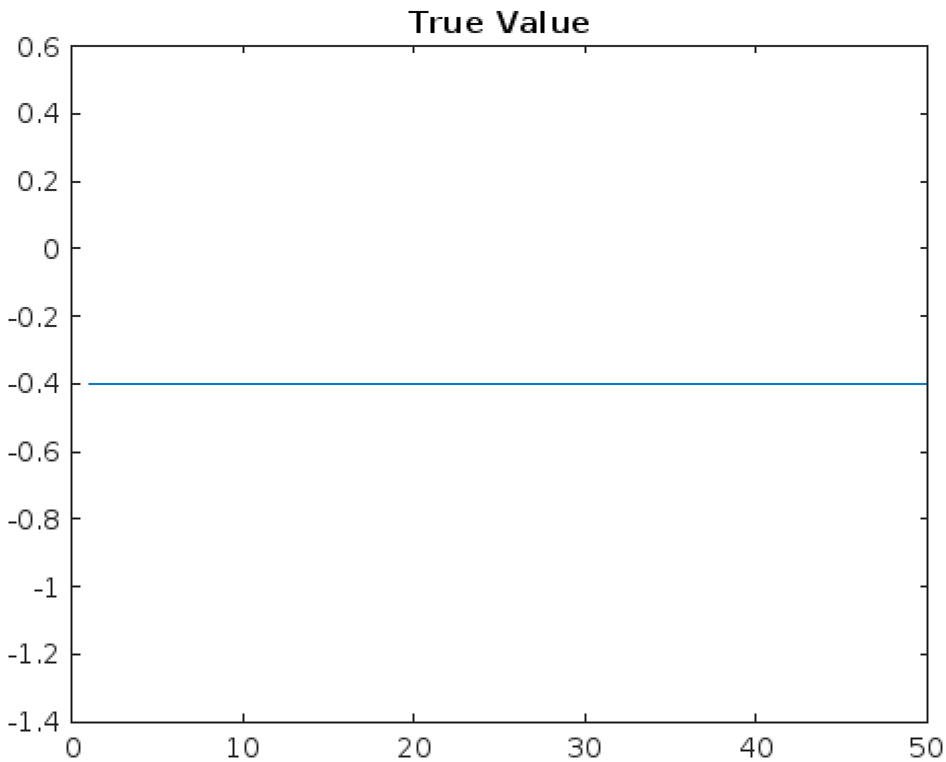
```
K = P_pred * H' / (H * P_pred * H' + R);
x_hat(k) = x_hat_pred + K * (measurements(k) - H * x_hat_pred);
P(k) = (eye(size(P_pred)) - K * H) * P_pred;
```

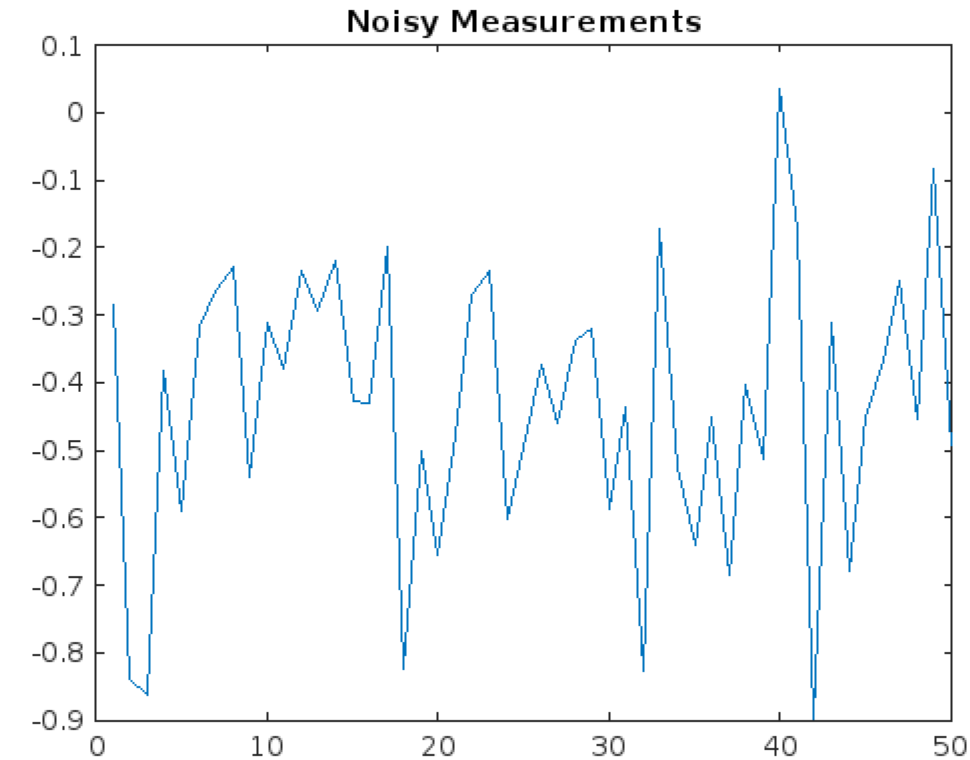
Implementation

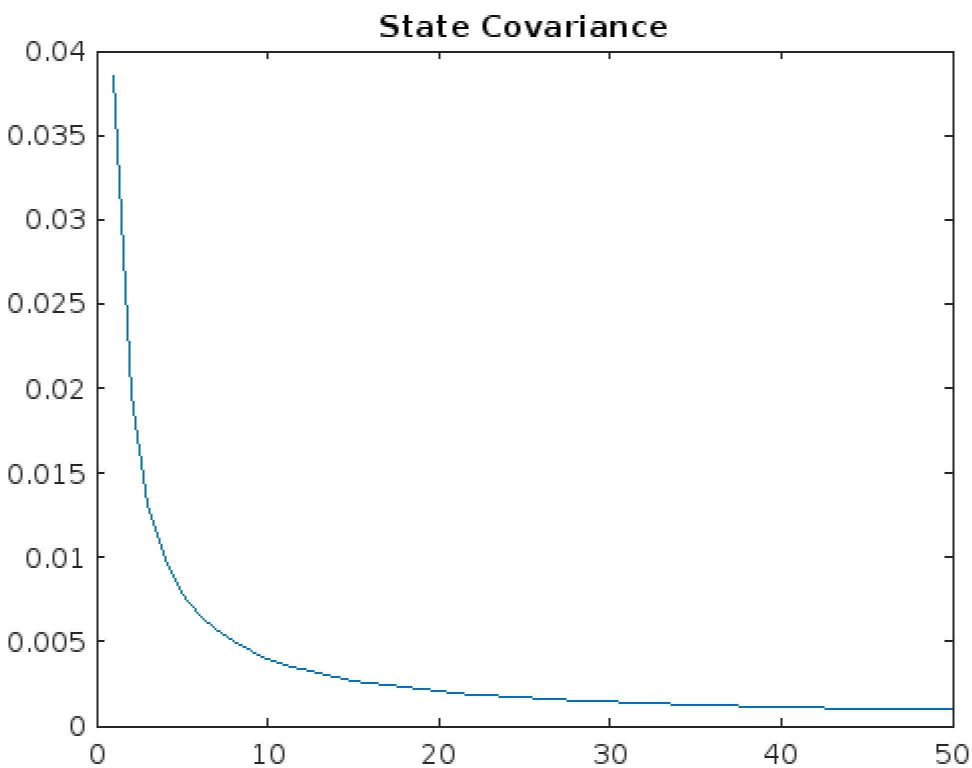
The provided code in the following page implements the Kalman filter for estimating the true voltage from noisy measurements. The system model assumes that the voltage changes linearly over time and that the measurements are corrupted by additive Gaussian noise. The code iterates for a specified number of time steps, updating the state estimate and covariance at each step.

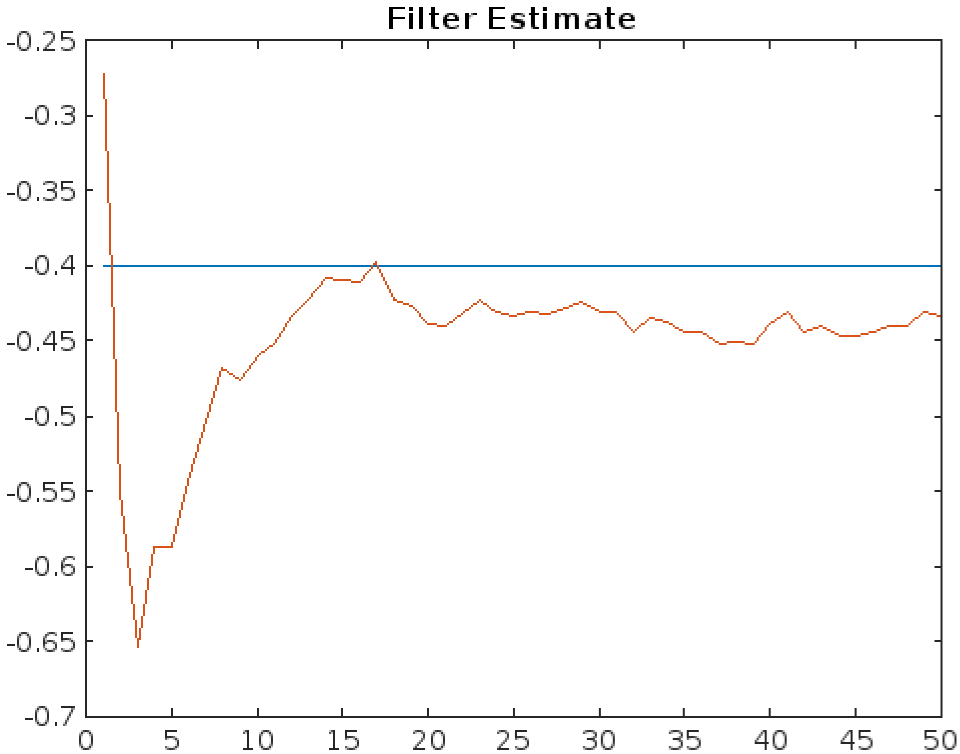
```
%
% Title: Kalman Filter
% Author: Karthik
% Date: 12/3/2023
%

A = 1;
B = 0;
H = 1;
Q = 1e-5;
R = 0.2^2;
x_true = -0.4;

% Assumptions
x_hat_0 = 0;
P_0 = 1;

% Noisy measurements
no_iterations = 50;
noise = randn(no_iterations, 1) * sqrt(R);
measurements = x_true * ones(no_iterations, 1) + noise;

x_hat = zeros(no_iterations, 1);
P = zeros(no_iterations, 1);

waveforms = zeros(no_iterations, 3);
for k = 1:no_iterations
  if k == 1
    x_hat_pred = x_hat_0;
    P_pred = P_0;
  else
    x_hat_pred = A * x_hat(k-1);
    P_pred = A * P(k-1) * A' + Q;
  end

  K = P_pred * H' / (H * P_pred * H' + R);
  x_hat(k) = x_hat_pred + K * (measurements(k) - H * x_hat_pred);
  P(k) = (eye(size(P_pred)) - K * H) * P_pred;

  waveforms(k, 1) = x_true;
  waveforms(k, 2) = measurements(k);
  waveforms(k, 3) = P(k);
  waveforms(k, 4) = x_hat(k);
end

figure;
plot(waveforms(:, 1));
title('True Value');

figure;
plot(waveforms(:, 2));
title('Noisy Measurements');
```

```
figure;
plot(waveforms(:, 3));
title('State Covariance');

figure;
plot(waveforms(:, 1));
hold on;
plot(waveforms(:, 4));
title('Filter Estimate');
```




Noisy Measurements
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
0
10
20
30
40
50


State Covariance
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
10
20
30
40
50



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Results & Analyzation

The code generates plots of the true voltage, noisy measurements, and filtered estimate. The true voltage graph as shown below represents the actual voltage of the system being measured. It is a smooth and consistent waveform of value -0.4V.

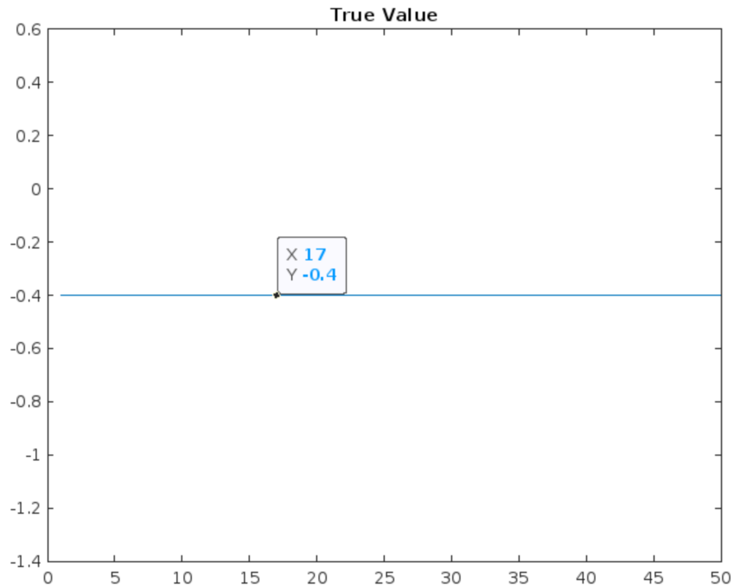

Figure 1 True value

The noisy measurements graph represents the readings obtained from the measurement sensors. These readings are corrupted by noise, making it difficult to determine the true voltage from the measurements directly. The noise is evident in the fluctuations and irregularities in the waveform.

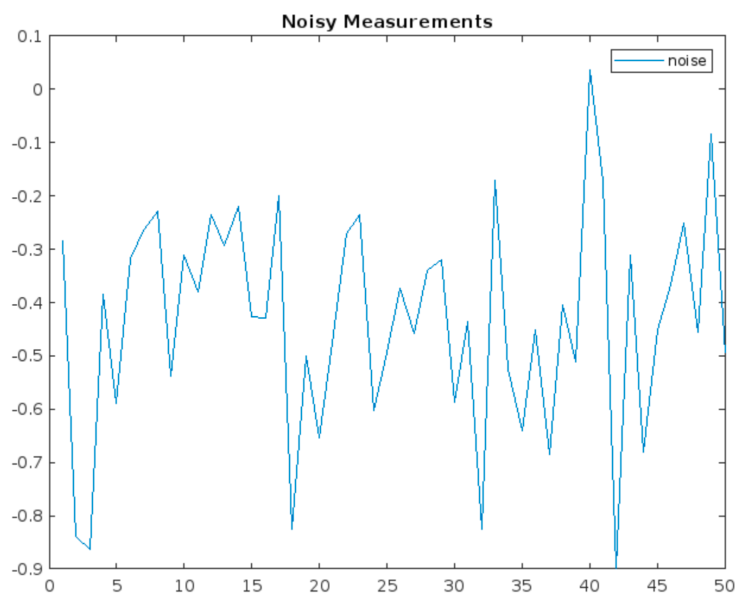

Figure 2 Noise measurements plot

The state covariance graph, also known as the uncertainty ellipse or sigma ellipse, represents the uncertainty associated with the state estimate provided by the Kalman filter. It provides valuable insights into the effectiveness of the Kalman filter and the reliability of the state estimate.

A shrinking ellipse indicates that the filter is reducing the uncertainty and improving the accuracy of the state estimate. Conversely, a growing ellipse suggests that the filter is encountering increased noise or disturbances, leading to higher uncertainty.

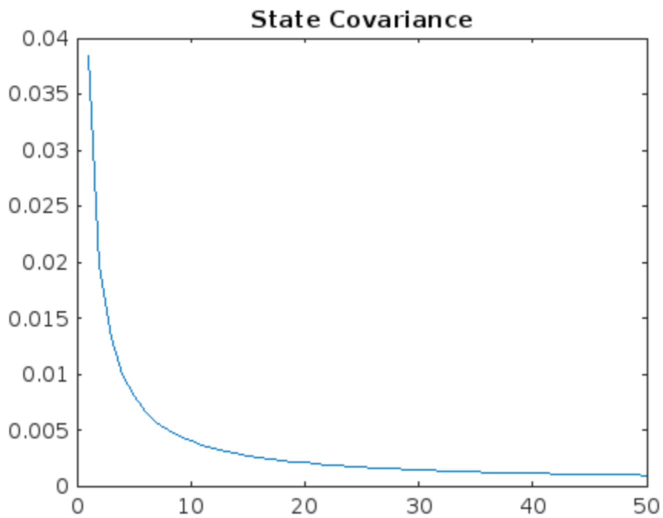

Figure 3 State covariance P(k)

The filtered estimate graph represents the voltage estimated by the Kalman filter. The Kalman filter combines the system model with the noisy measurements to produce an estimate of the true voltage.

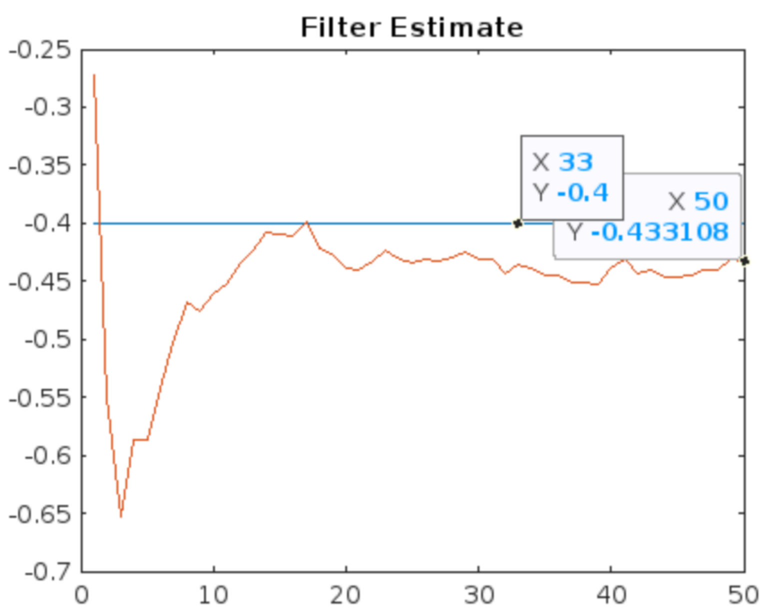

Figure 4 The filter estimate (orange) reaching the true value (blue)

The filtered estimate (-0.433V, orange signal) closely approaches the true voltage waveform (-0.4V), indicating that the Kalman filter effectively removes the noise from the measurements.

References

- Maybeck, P. S. (1982). Stochastic models, estimation, and control, Volume 1. Academic Press.
- Gelb, A. (1974). Applied optimal estimation. MIT Press.

Conclusion

The implementation of the Kalman filter successfully estimated the true state of the system from noisy measurements. The filtered estimate closely tracked the true state, even in the presence of significant noise. This demonstrates the effectiveness of the Kalman filter in state estimation problems and its ability to handle uncertainty in measurements. The Kalman filter's recursive approach allows it to adapt to changing system conditions and maintain accurate state estimates over time. This makes it a valuable tool for real-time state tracking and control applications.