Weather Forecast Using Markov Chains

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ABSTRACT. Markov Chains is a mathematical probability approach towards predicting the outcome of an event that was developed by Andrey Markov in 1906. This project would be exploring the use of Markov chains in weather forecast or weather prediction. The primary principle of Markov chains is that the future outcome is only dependent on the present outcome and independent of past outcomes. I will be exploring the trends of weather readings using the principles of Markov Chains in a sample set and attempting to predict the success of the outcomes.

1. INTRODUCTION

At a bid to prove fellow Russian Mathematician, Nekrasov wrong, Markov discovered a special model in which the Law of Large Numbers hold for dependent variables. This discovery was made by analyzing the first 20,000 characters of Eugene's Onegin poem in Markov's Onegin project. For the analysis of Onegin in the project, Markov did not include the punctuation marks and spaces, he groups the words into two states; consonant and vowels, counted pairs of the two groups and calculated the moments that is mean and variance of this data. After these, he was able to develop the stochastic matrix of the poem. A stochastic matrix is defined as a square matrix with columns or rows, of probability vectors of the unique states in Markov chains. Interestingly, in a publication by Brian Hayes, after translating the Russian poem to English language, he noticed that the quantity of vowels in the English translation was less compared to the quantity of vowels in Russian, notwithstanding the structure of the stochastic matrix obtained stayed the same.

In the case of weather forecast. Let us say we have 3 states: rainy, sunny, and cloudy. The probability of rain two times in a row would be $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \sim 11.1\%$. However, ideally, storms and hurricanes could last several days in a row and would have a much-increased probability than the 11.1% obtained. This is where Markov chains come in, to make more accurate results in probability prediction of the weather. Because the principle is only dependent on the present state to make a hypothesis on the future outcome, this makes it ideal for weather prediction.

2. METHODOLOGY

The general formula for Markov chains is listed as equation (1). This equation should only be used when the columns of the Stochastic matrix represent the probability vectors of the states in Markov matrix.

$$[x_{k+1_{mx1}}] = [S_{mxm}][x_{k_{mx1}}]$$

However, if the probability vectors are represented in the rows of the Stochastic matrix, then we use equation (2) as listed.

$$[x_{k+1_{mx1}}] = [x_{k_{mx1}}][S_{mxm}]$$

At some point of this formula, $(k+1) = s$, and at s, it can be said to be the steady state vector such that:

$$[x_{s_{mx1}}] = [S_{mxm}][x_{s_{mx1}}]$$

This means that despite being multiplied by the stochastic matrix, the probability vector of present condition is the same as future condition. This unique vector is known as Steady state vector.

3. NUMERICAL RESULTS

Let us now consider a weather stochastic matrix collated for the weather in a Region A, such that the only 2 states are "rainy" and sunny". The data is represented in Figure 1 is converted to a stochastic matrix. In Region 1, the chance of rain tomorrow is raining is at 1%, which will be represented in a probability vector. A probability vector is a non-negative vector in which all its entries sum up to 1. We can represent this probability vector as x_0 such that k = 0

Figure 1: Diagram

$S = \begin{bmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{bmatrix}, x_0 = \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix}$. Recall, $[x_{k+1_{mx1}}] = [S_{mxm}][x_{k_{mx1}}]$.

At k= 0; $x_1 = \begin{bmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{bmatrix} \begin{bmatrix} 0.99 \\ 0.01 \end{bmatrix} = \begin{bmatrix} 0.846 \\ 0.154 \end{bmatrix}$.

Therefore, the new present state probability of rain is 15.4%.

At k = 1; $x_2 = \begin{bmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{bmatrix} \begin{bmatrix} 0.846 \\ 0.154 \end{bmatrix} = \begin{bmatrix} 0.7884 \\ 0.2116 \end{bmatrix}$.

At k= 2; $x_3 = \begin{bmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{bmatrix} \begin{bmatrix} 0.7884 \\ 0.2116 \end{bmatrix} = \begin{bmatrix} 0.7884 \\ 0.2116 \end{bmatrix}$.

Comparing the resultant probability vector of $x_2$and x_3, we have the same results. This means that the new probability of rain is 21.16 % for $x_2$and x_3.Hence, indicating that the question has reached its steady state. We can therefore also say that $x_3 = x_2 = x_s$. In Figure 2 below, I made a plot of the results obtained by their respective states and the data obtained from this plot is $y = 0.0662x + 0.0474$ with $R^2 = 0.8076$.

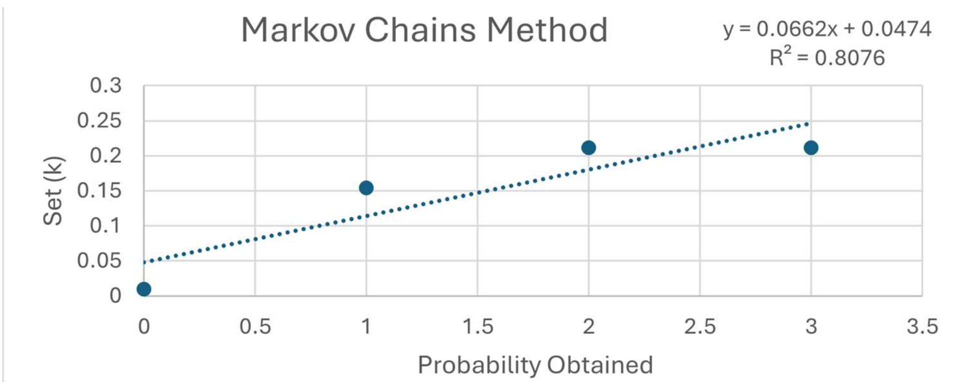

Figure 2: Plot of Markov Chains Method in Excel worksheet

On the other hand, using general probability methods:

For x_1 , The probability of rain would be 0.15 or 15%.

For x_2 , The probability of rain would be 0.15×0.15 which would result 0.0225 or 2.25%.

For x_3 , The probability of rain would be $0.15 \times 0.15 \times 0.15$ which would result in 0.00338 or 0.338%.

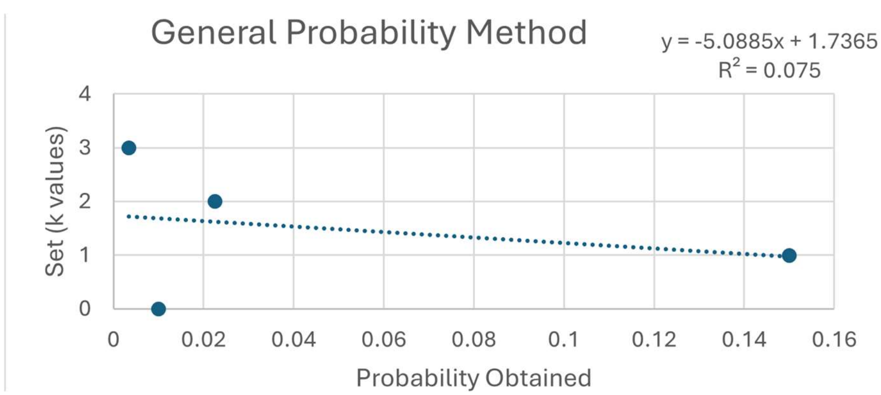

Figure 3: Plot of General Probability Method in Excel

In Figure 3 above, I made a plot of the results obtained using the general probability method to showcase their respective states and the data obtained from this plot is $y = -5.0885x + 1.7365$ with $R^2 = 0.075$.

By comparison of the two plots, we can notice the variance in the R^2 values. Markov chains has an R^2closer to 1 which indicates that the model explains all the variation in the response variable around the mean. However, the R^2 obtained for the general probability method was closer to 0 which indicated that the model does not explain any of the variation in the response variable around its mean. The latter conclusion is not correct, hence, indicating that Markov chains is a more accurate step to obtain the solution.

4.APPLICATIONS.

Markov Chains are used in various fields and industries such as:

1. Meteorology: In the prediction of weather or weather forecast as studied in this paper.
2. Economics: To predict the future worth of shares in market.
3. Search Engine Algorithms: To determine more appropriate suggestions for users.

5. CONCLUSION.

The studies of Markov Chains are an interesting one, and through the course of this paper, the primary concepts have been linked mainly with weather prediction. The general probability method has proven false through the Excel analysis, hence, supporting Markov's claims in stating that the Law of Large Numbers hold for dependent variables. Data has also proven true that by method of Markov Chains, more accurate weather predictions are made. Therefore, the deciding factors point to the conclusion of the support of using the methodology of Markov Chains to determine the weather forecast.

6. REFRENCES.

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