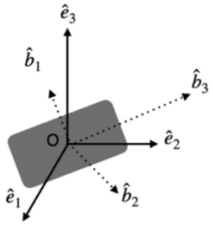
Project 3 (due 5th December) 8%

Consider a spacecraft B moving with respect to frame E. B possesses the following inertia characteristics:



$I_1 = 400\ \text{kgm}^2$
$I_2 = 400\ \text{kgm}^2$
$I_3 = 100\ \text{kgm}^2$

Assume that B is subject to a constant torque T in the direction of b1:

(a) Write the kinematic equations of motion assuming quaternions. Derive the dynamic equations of motion. In which basis is the angular velocity incorporated?
(b) Assume that at t = 0 s, $\hat{b}_i = \hat{e}_i$, $\omega_1(0) = 1\ \text{rad/s}$, $\omega_2(0) = -1\ \text{rad/s}$, $\omega_3(0) = 2\ \text{rad/s}$, $T = 80\ \text{Nm}$. Solve analytically the time history of the angular velocities with respect to time and plot all the three curves together for 3 cycles of motion.
(c) What is the value of the quaternions for t=0 s? Numerically integrate the dynamic and kinematic differential equations simultaneously for at least 3 cycles of the motion and plot the three curves together in one plot. What angle sequence are you using to represent the three rotations?
(d) Modify the script such that the output information includes the precession and the nutation. Produce a list every 1 second interval until t = 12 s for the time, each component of angular velocity, C12, C22, C33, precession, nutation, and quaternions. Express angles in degrees. Explain how you determine the correct quadrant for the angles.
(e) Plot the precession and nutation angles separately as functions of time. Initially, consider that the nutation is equal to 0. Does it ever return to zero? Why?
(f) Repeat the analysis without any torque but same initial conditions and pinpoint the differences; How do the angles discussed in class for torque-free motion behave? Do they behave in the same way as the numerical simulation? If not, why? How are they related? Given the simulation results for precession and nutation, is it possible to compute the analytical torque-free precession and nutation angles from the space and body cones?
(g) Plot C_{31} as a function of C_{21} (make sure the plot is scaled such that both values stay between 1 and -1). This plot results in a view down the $\hat{e_1}$ axis. The positive direction is into or out of the page? How is the curve? On the plot, mark the time t = 0.2 s. Sketch the precession angle at such time. If you measure the angle, does it match the value for the precession that you computed in the simulation?

You have to present the project in a 4 pages report maximum including all the process for the process, and also references and figures. Put your MATLAB code in appendix (doesn't count towards page limit).