Class	AE 426 Project 2
Author(s)	Serena Ikemhe Elijah
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Instructor	Professor David Garcia

Objectives

This report is to obtain results for the for the Project 2 problem of a spinning top.

Assumptions

The inertia of the total body is assumed to be a cone. Although the object is a half sphere and cone, it can be assumed that the inertia of the half sphere is negligible. We also assumed the value of friction , M_f = -0.095

Methodology

For a 3-1-3 rotation matrix, the DCM is :

Taking:

c1 = $\cos\emptyset$; s1 = $\sin\emptyset$; c2 = $\cos\theta$; s2 = $\sin\theta$; c3 = $\cos\varphi$; s3 = $\sin\varphi$

$$\begin{matrix} \cos\emptyset & -\sin\emptyset & 0 \\ \sin\emptyset & \cos\emptyset & 0 \\ 0 & 0 & 1 \end{matrix} * \begin{matrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{matrix} * \begin{matrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\text{Final DCM} = \begin{matrix} c1c3 - c2s1s3 & -c1s3 - c2c3s1 & s1s2 \\ c3s1 + c1c2s3 & c1c2c3 - s1s3 & -c1s2 \\ s2s3 & c3s2 & c2 \end{matrix}$$

Inertia calculations: Since we are assuming that the whole body is a cylinder:

$$I_x = I_y = \frac{3}{10}M(r^2 + 4h^2) = 0.00081 ; \quad I_z = \frac{3}{10}Mr^2 = 0.0098$$

Angular velocity calculations:

$$\bar{w} = w_n\,\hat{\imath} + w_p\,\hat{K} + w_s\hat{k}$$

$$\bar{w} = w_n\,\hat{\imath} + w_p(\sin\theta\,\hat{\jmath} + \cos\theta\hat{k}) + w_s\hat{k}$$

$w_x = w_n$; $w_y = w_p \sin\theta\,\hat{\jmath}$; $w_z = w_p\cos\theta\hat{k} + w_s\hat{k}$;

$$\dot{w}_x = \dot{w}_n ; \dot{w}_y = \dot{w}_p\sin\theta + w_p w_n\cos\theta \;\; ; \; \dot{w}_z = \dot{w}_s + \dot{w}_p\cos\theta - w_p w_n\sin\theta$$

$$M_{tot} = mgd\sin\theta\,\hat{\imath} + M_f\hat{k}$$

$$M_{tot} = A\dot{w_n}\hat{\imath} + A(\dot{w}_p\sin\theta + w_p w_n\cos\theta)\hat{\jmath} + C(\dot{w}_s + \dot{w}_p\cos\theta - w_p w_n\sin\theta)\hat{k} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ w_n & w_n\sin\theta & w_p\cos\theta \\ Aw_n & Aw_p\sin\theta & C(w_s + w_p\cos\theta) \end{vmatrix}$$

Equation of Motion:

$$mgd\sin\theta\,\hat{\imath} + M_f\hat{k} = A\dot{w_n}\hat{\imath} + A(\dot{w}_p\sin\theta + w_p w_n\cos\theta)\hat{\jmath} + C(\dot{w}_s + \dot{w}_p - w_p w_n\sin\theta)\hat{k} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ w_n & w_n\sin\theta & w_p\cos\theta \\ Aw_n & Aw_p\sin\theta & C(w_s + w_p\cos\theta) \end{vmatrix}$$

Equating components: $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ we are able to get the equations of motion considering the spin, nutation, and precession rates.

$$\dot{w}_n = \frac{mgd\sin\theta - Cw_p w_s\sin\theta - (C - A)w_p{}^2\sin\theta\cos\theta}{A}$$

$$\dot{w}_p = \frac{-Aw_p w_n\cos\theta - Cw_n w_s - (C - A)w_p w_n\cos\theta}{A\sin\theta}$$

$$\dot{w}_s = -\frac{M_f}{C} + w_p w_n\sin\theta - \dot{w}_p\cos\theta$$

Results & Discussion

For this project, we ran two different initial θ values. For $\theta 1 = 0.01$, I ran the iteration of t = 0:0.1:10, an iteration of t = 0:10:100, and a final iteration of

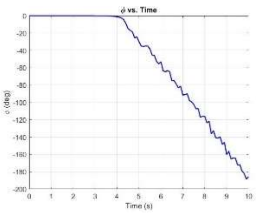

Figure 1: Graph of Phi vs. time,10s.

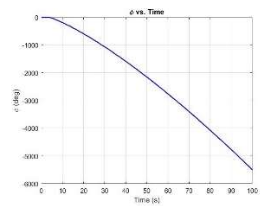

Figure 2 : Graph of Phi vs. time, 100s.

On a closer obervance of figure 1, sometime after t = 4 but before t = 5, we can observe a change in the system. The change at this point in the system can be observed in all the graphs that have t = 10. At this point, we can deduce or make an intelligent guess that the system touches the floor.

As the behaviour of Phi is observed further we notice that the motion smoothens out. This indicates that over time the system either stabilizes or the effects of short term flunctuations diminish. If we stick by our intelligent guess that the surface of the top touches the floor between t = 4, and t= 5. Then we can say that the motion of the body indeed smoothes out.

The variation of the change in Phi, is shown below in figures 3 and 4 below.

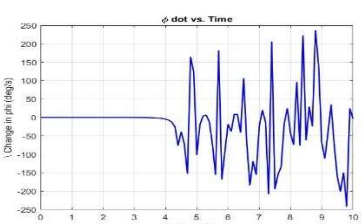

Figure 3 : Graph of Phi_dot vs. time, 10s.

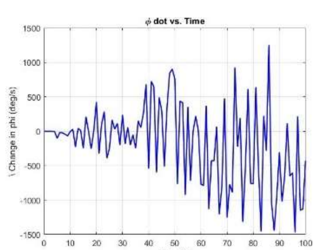

Figure 4 : Graph of Phi_dot vs. time, 100s.

The trend of Psi with time, varies differently from phi. Phi was decreasing with time, while Phi increased in time. However like phi, once trended against a time, 100s, It's motion smoothened out. Psi also changes in motion somewhere between t = 4 , and t = 5. This further strengthens my hypothesis of the motion top touching the floor at that point.

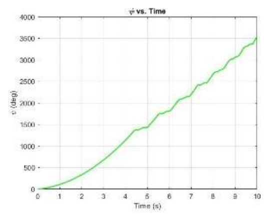

Figure 5: : Graph of Psi vs. time, 10s

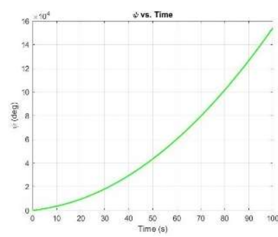

Figure 6: Graph of Psi vs. time, 100s.

The graph of Psi_dot changing with time is observed in figures 7 & 8. The values trend negative, which is opposite of the trend for just psi, but we are able to deduce that this indicates that the body is experiencing instability due to noise (disturbances) resulting from the torque.

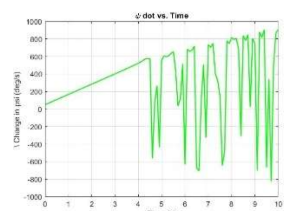

Figure 7: Graph of Psi_dot vs. time, 10s

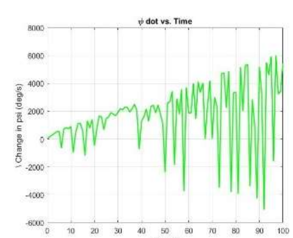

Figure 8: Graph of Psi_dot vs. time, 100s.

The plot of theta verses time is shown in the figure below.

We begin the initial condition with $\theta = 0.01$. From the graph, we can observe that the change in theta vs. time, the changing increasing oscillations may signal the effects of friction.

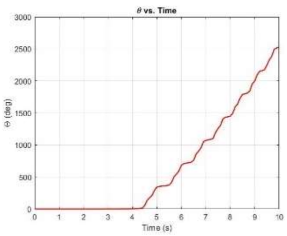

Figure 9: Graph of theta vs. time, 10s.

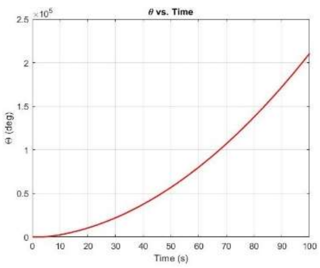

Figure 10: Graph of theta vs. time, 100s.

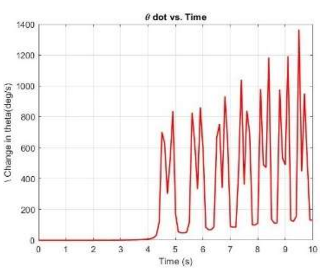

Figure 11: Graph of theta_dot vs. time, 10s.

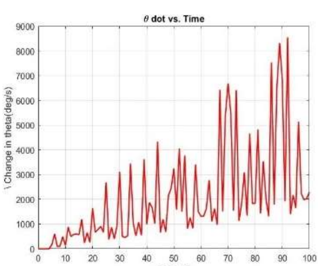

Figure 12: Graph of thetadot vs. time, 100s.

The combined graph to observe the changes visually for angles are shown in Figure 13, while the combined graph to observe the changes for the rates can be shown in Figure 14.

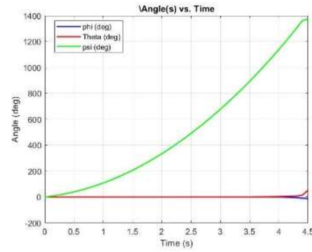

Figure 13: Graph of phi, theta, psi vs. time, 4.5s.

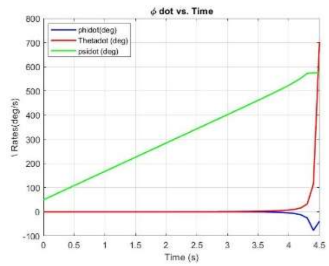

Figure 14: Graph of phidot, thetadot, psidot vs. time, 4.5s.

Taking more analysis and changing the the value to $\theta = 0.051$. We can observe that it produces more oscillations and has an increasing psi.

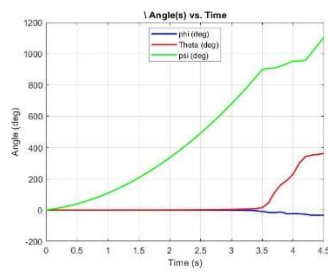

Figure 15: Graph of phi, theta, psi vs. time, 4.5s.

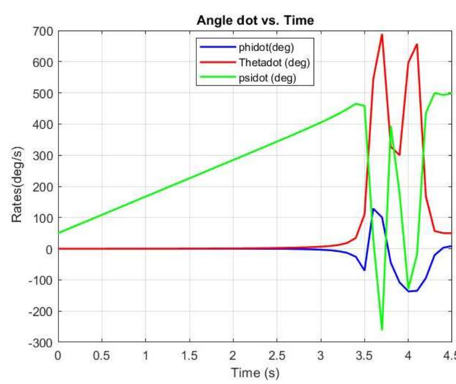

Figure 16: Graph of phidot, thetadot, psidot vs. time, 4.5s.

Appendix

```
clc
clear
close all
%Serena Elijah
%AE 426 Project 2
syms phi theta psi wp_dot ws_dot wn_dot wp_ddot ws_ddot wn_ddot A B C ws wn wp m g d
%phi is the first angle of rotation
%theta is the second angle of rotation
%psi is the third angle of rotation
m = 3 ; %kg
rl = 3/100 ; %m
g = 9.81 ; %m/s^2
d = 5/100 ; %m d=h
%phi = 0; theta = 0.01; psi = 0 ; wp = 0 ;  wn = 0; ws= 50 ;
%r0 = [phi theta psi wp wn ws] ; %initial conditions
r0 = [ 0 0.051 0 0 0 50] ; %Initial condition
tspan = 0:0.1:4.5 ; %time span
%INERTIA
A  = (3/10)*m*(rl^2)
C = (3/10)*m*(rl^2 + 4*d^2)
%Ode45 to solve differential equation using initial conditions
options = odeset('RelTol', 1e-9, 'AbsTol', 1e-9) ;
[t,r] = ode45(@(t,r) eom(t, r), tspan, r0, options) ;
Plots
%% Plotting Results
% Plot for r(1) vs t (phi)
figure;
plot(t, r(:, 1), 'b', 'LineWidth', 1.5);
hold on;
plot(t, r(:, 2), 'r', 'LineWidth', 1.5)
hold on;
plot(t, r(:, 3), 'g', 'LineWidth', 1.5);
```

```
legend('phi (deg)', 'Theta (deg)', 'psi (deg)', 'Location', 'best')

title(' Angle(s) vs. Time');

xlabel('Time (s)');

ylabel('Angle (deg)');

grid on;

figure;

plot(t, r(:, 4), 'b', 'LineWidth', 1.5);

hold on;

plot(t, r(:, 5), 'r', 'LineWidth', 1.5);

hold on;

plot(t, r(:, 6), 'g', 'LineWidth', 1.5);

title('Angle dot vs. Time');

xlabel('Time (s)');

ylabel(' Rates(deg/s)');

legend('phidot(deg)', 'Thetadot (deg)', 'psidot (deg)', 'Location', 'best')


grid on;

Functions eom

function drdt = eom(t,r)

m = 3 ; %kg

rl = 3/100 ; %m

g = 9.81 ; %m/s^2

d = 5/100 ; %m d=h

C  = (3/10)*m*(rl^2) ;

A = (3/10)*m*(rl^2 + 4*d^2) ;

Mf = -0.095;

%drdt = [wp wn ws wp_dot wn_dot ws_dot]

drdt(1) = r(4) ;

drdt(2) = r(5)  ;

drdt(3) = r(6) ;

drdt(4) = (1/A*sind(r(2)))*( (-A*r(4)*r(5)*cosd(r(2))) + (-C*r(6)*r(5)) + ((A-C)*r(4)*r(5)*cosd(r(2))) ) ;

drdt(5) = (1/A)*((m*g*d*sind(r(2))) + (-C*r(4)*r(6) *sind(r(2))) + (-(C-A)*r(4)^2*cosd(r(2))*sind(r(2)))); %wn_dot

drdt(6) = -(Mf/C)  -(drdt(4)*cosd(r(2))) + r(4)*r(5)*sind(r(2)); %ws_dot

drdt = drdt' ;

end
```