Class	AE 426 Project 1
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Objective

This report is trying to obtain results for the differential equation in Question 1, and the Pendulum equation making use of the ode45 function in MATLAB.

Methodology

For Question 1.

Making use of the state vectors, we defined:

$$\vec{r} = \begin{bmatrix} x \\ \dot{x} \\ y \end{bmatrix}, \qquad \dot{\vec{r}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \end{bmatrix}$$

$$\dot{\vec{r}} = \begin{bmatrix} \dot{x} \\ \dot{x} - 2y \\ 2y + x \end{bmatrix} = \begin{bmatrix} \vec{r}(2) \\ \vec{r}(2) - \vec{r}(3) \\ 2\vec{r}(3) + \vec{r}(1) \end{bmatrix}$$

For Question 2.

When we look at the rod as a whole in circular motion, the tension cancels out. The motion along the $\widehat{e_r}$ direction is not taken into account. Therefore, we derive the motion of the pendulum as:

$$\sum \tau = I\alpha$$

$$-kl\dot{\theta}\widehat{e_\theta} - \mathrm{mg}l \sin\theta\, \widehat{e_\theta} = ml^2\, \ddot{\theta}\widehat{e_\theta}$$

$$\ddot{\theta} = \frac{-k\dot{\theta}}{ml} - \frac{g \sin\theta}{l}$$

$$\ddot{\theta} = -\,\frac{k\dot{\theta}}{2} - \frac{9.81 \sin\theta}{2}$$

We then obtained the state vectors:

$$\vec{r} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}, \quad \dot{\vec{r}} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \vec{r}(2) \\ -\vec{r}(2)(\frac{k}{2}) - \frac{9.81 \sin \vec{r}(1)}{2} \end{bmatrix}$$

Results & Discussion

For Question 1.

Using MATLAB function ode45, I selected a maximum time of 8 and minimum time of 1, I obtained the following graphical results shown in the figures of the next page.

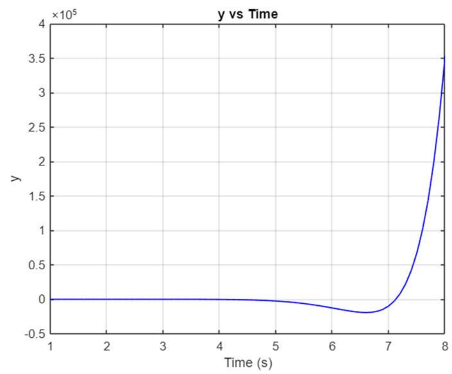

Figure1: Plot of y vs Time for a timespan of (1:8)

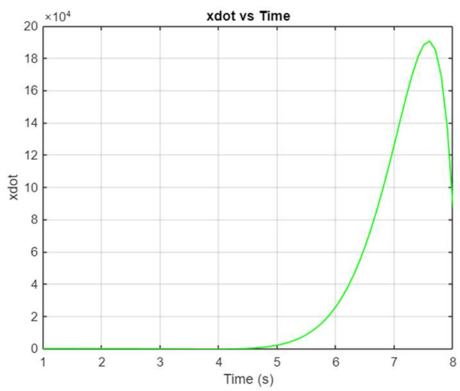

Figure 2: Plot of xdot vs Time for a time span of (1:8)

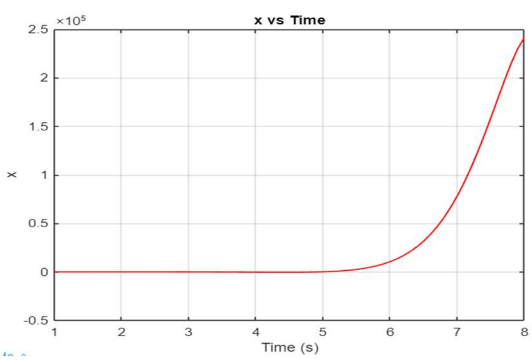

Figure 3: Plot of x vs Time for timespan of (1:8)

To observe how these solutions behave with time with each other, I plotted figure 4 below. It can be seen that all though the xdot function seemed to have the most increase from $t \leq 7.3$, xdot ended up having the most decrease.

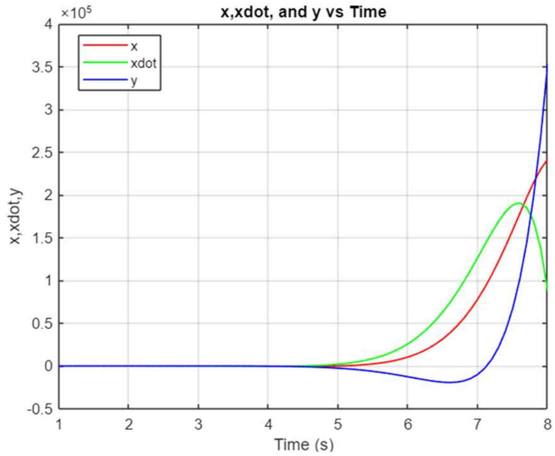

Figure 4: Combined plot of x, xdot, and y.

To further make a conclusion on the results (and to test the end of solutions for the equation). I increased the time span to a value of 1000 with a 1 step increase. My results are plotted in Figure 5 below. Therefore, it can be concluded that after a time range of 405s there is no solution.

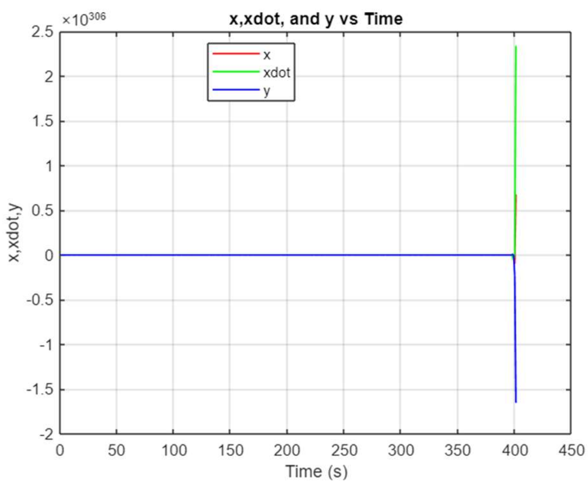

Figure 5: Plot of x, xdot, and y for a timespan of (1:1000)

<u>For Question 2.</u>

To obtain a dampening coefficient, the formula obtained was $c = 2d\sqrt{(mgl)}$, and the dampening coefficient obtained was 0.2215. However, after running the code, I discovered that the amplitude difference in the plot of Angular velocity vs. Time produces inconsistent dampening percentages amongst the different peaks. Using MATLAB function “findpeaks”, I visually (and numerically) iterated the dampening coefficients. I calculated dampening coefficient with another formula shown in the MATLAB code and got 0.0722, it was from this value I then iterated and concluded the project with a dampening coefficient of 0.0749.

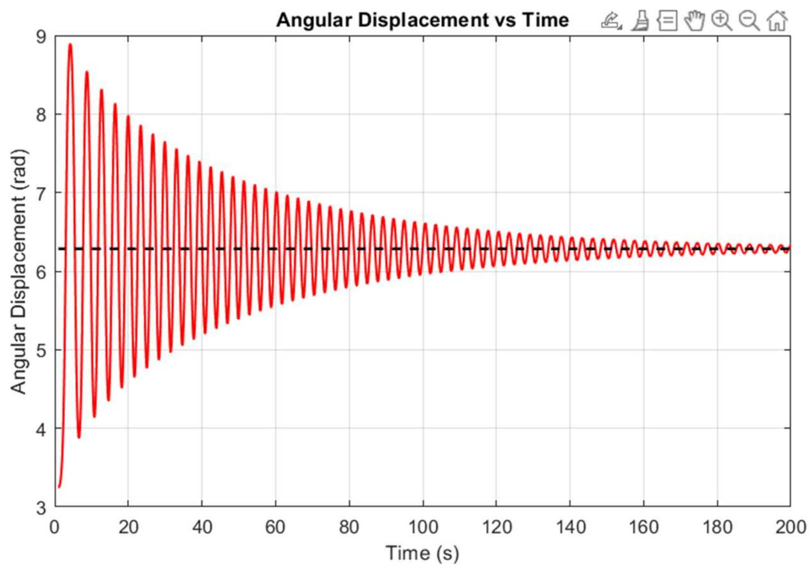

Figure 6: Plot of Angular Displacement with Time

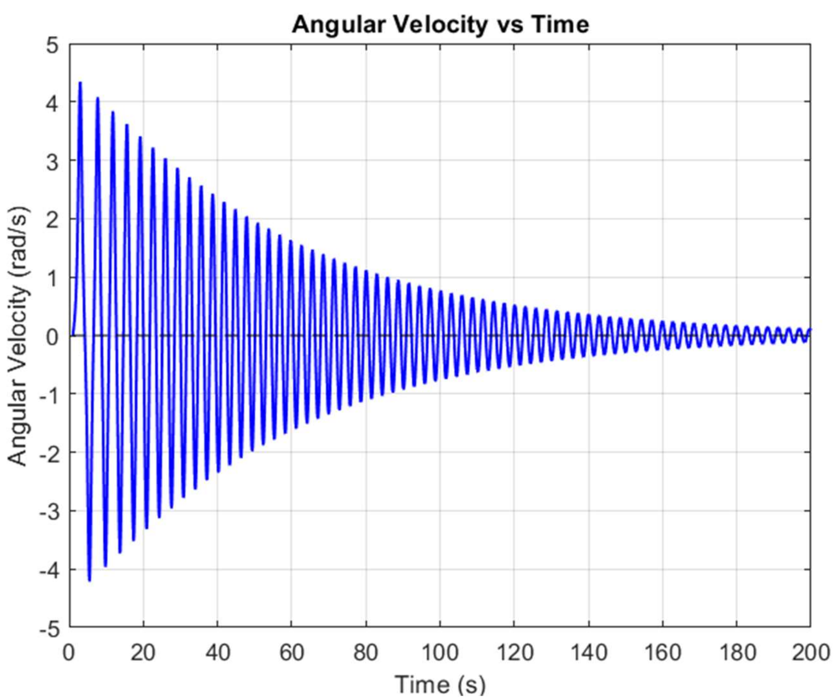

Figure 7: Plot of Angular Velocity with Time

The pendulum point at $\theta = \pi$ can be said to be unstable equilibrium, because once it was perturbated. From the plot of Angular displacement vs. Time as shown in Figure 2, the pendulum can be seen moving through 2π. This is a characteristic of unstable equilibrium and signifies that although the pendulum has enough energy to oscillate, the oscillations will not stabilize at the unstable equilibrium but will instead decay towards a stable state, resting at the bottom position due to damping (~5%).