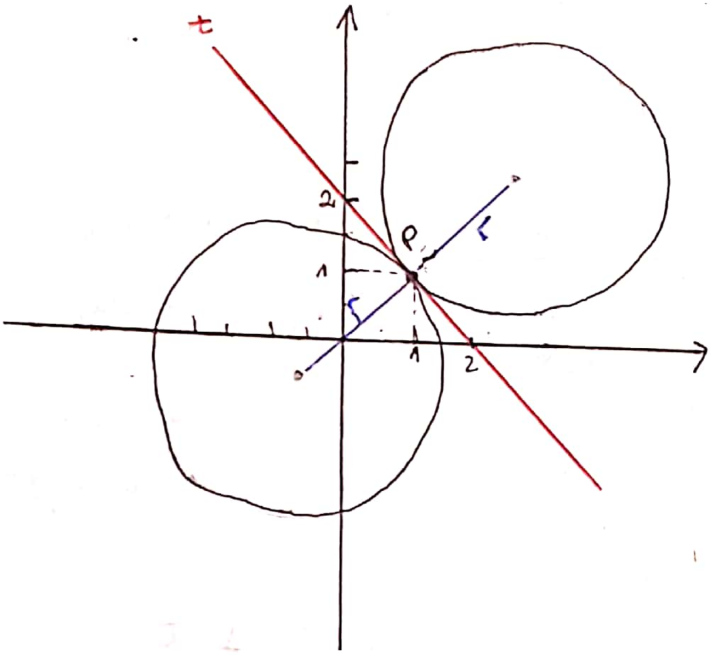
6 c) $\mathcal{C}(C(x_0,y_0),2)$ / t) $x+y-2=0$ es tangente a $\mathcal{C}$ en $P(1,1)$



La ecuación de $\mathcal{C}(C(x_0,y_0),2)$ es:

$$(x-x_0)^2 + (y-y_0)^2 = 2^2$$

Debemos hallar las coordenadas del centro $C(x_0,y_0)$

$$P(1,1) \in \mathcal{C} \Rightarrow (1-x_0)^2 + (1-y_0)^2 = 2^2$$

$$1 - 2x_0 + x_0^2 + 1 - 2y_0 + y_0^2 = 4$$

$$x_0^2 + y_0^2 - 2x_0 - 2y_0 + 2 = 4$$

$$\boxed{x_0^2 + y_0^2 - 2x_0 - 2y_0 = 2 \qquad (1)}$$

Por otro lado, la distancia de la recta tangente t) al centro $C(x_0,y_0)$ de $\mathcal{C}$ es 2, radio de $\mathcal{C}$.

Luego:

$$d(t, C) = 2$$

$\{ t)\ x + y - 2 = 0$ donde $\bar{n} = (1,1) \perp t$

$$\downarrow$$

$$\frac{|x_0 + y_0 - 2|}{\sqrt{1^2 + 1^2}} = 2$$

$$|x_0 + y_0 - 2| = 2\sqrt{2} \Rightarrow x_0 + y_0 - 2 = 2\sqrt{2} \quad \text{ó} \quad x_0 + y_0 - 2 = -2\sqrt{2}$$

Caso 1: $x_0 + y_0 - 2 = 2\sqrt{2}$ y consideramos (1)

$$\begin{cases} y_0 = 2\sqrt{2} + 2 - x_0 \\ x_0^2 + y_0^2 - 2x_0 - 2y_0 = 2 \end{cases}$$

Reemplazamos y_0 en la segunda ecuación:

$$x_0^2 + (2\sqrt{2} + 2 - x_0)^2 - 2x_0 - 2(2\sqrt{2} + 2 - x_0) = 2$$

$$x_0^2 + (2\sqrt{2}+2)^2 - 2(2\sqrt{2}+2)x_0 + x_0^2 - \cancel{2x_0} - 4\sqrt{2} - 4 + \cancel{2x_0} = 2$$

$$2x_0^2 + (-4\sqrt{2} - 4)x_0 + (4\sqrt{2} + 6) = 0$$

Aplicando la resolvente obtenemos:

$$x_0 = 1 + \sqrt{2}$$

$$\therefore y_0 = 2\sqrt{2} + 2 - (1 + \sqrt{2}) = \sqrt{2} + 1$$

$\therefore$ El centro es $C(1+\sqrt{2}, 1+\sqrt{2})$

Caso 2 $\quad x_0+y_0-2=-2\sqrt{2}$ y consideramos (1):

$$\begin{cases} y_0 = -2\sqrt{2}+2-x_0 \\ x_0^2+y_0^2-2x_0-2y_0=2 \end{cases}$$

Reemplazamos y_0 en la segunda ecuación:

$$x_0^2+(-2\sqrt{2}+2-x_0)^2-2x_0-2(-2\sqrt{2}+2-x_0)=2$$

$$x_0^2+(-2\sqrt{2}+2)^2+2(-2\sqrt{2}+2)(-x_0)+x_0^2-2x_0+4\sqrt{2}-4+2x_0=2$$

$$2x_0^2+(4\sqrt{2}+4)x_0+(-4\sqrt{2}+6)=0$$

Aplicando la resolvente: $x_0=1-\sqrt{2}$

$$\therefore y_0=-2\sqrt{2}+2-(1-\sqrt{2})=1-\sqrt{2}$$

$\therefore$ El centro, en este caso, es $C(1-\sqrt{2},1-\sqrt{2})$

$\therefore$ Las ecuaciones de las circunferencias, de radio 2, tangentes a t en $P(1,1)$ son:

$$\mathcal{C}_1: \left(x-(1+\sqrt{2})\right)^2+\left(y-(1+\sqrt{2})\right)^2=4$$

$$\mathcal{C}_2: \left(x-(1-\sqrt{2})\right)^2+\left(y-(1+\sqrt{2})\right)^2=4$$