(14) $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ base de $\mathbb{F}^3$.

Veamos que $\beta = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ También es una base.

$\bar{u}_1 = \bar{v}_1$, $\bar{u}_2 = \bar{v}_1 + \bar{v}_2$, $\bar{u}_3 = \bar{v}_1 + \bar{v}_2 + \bar{v}_3$

- Observación: $\beta \subseteq \mathbb{F}^3$ pues $\bar{u}_i \in \beta$ $\forall i = 1, 2, 3$ (por Axioma de Clausura para +)

- β es base $\iff$ β es li y β genera $\mathbb{F}^3$.

- Veamos que β es l.i:

Sea $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{F}$ / $\alpha_1 \bar{u}_1 + \alpha_2 \bar{u}_2 + \alpha_3 \bar{u}_3 = \bar{0}$

$$\bar{0} = \alpha_1 \bar{u}_1 + \alpha_2 \bar{u}_2 + \alpha_3 \bar{u}_3 =$$

$$= \alpha_1 \bar{v}_1 + \alpha_2 (\bar{v}_1 + \bar{v}_2) + \alpha_3 (\bar{v}_1 + \bar{v}_2 + \bar{v}_3) =$$

$$= \alpha_1 \bar{v}_1 + \alpha_2 \bar{v}_1 + \alpha_2 \bar{v}_2 + \alpha_3 \bar{v}_1 + \alpha_3 \bar{v}_2 + \alpha_3 \bar{v}_3 =$$

$$= \bar{v}_1 (\alpha_1 + \alpha_2 + \alpha_3) + \bar{v}_2 (\alpha_2 + \alpha_3) + \alpha_3 \bar{v}_3 \Longrightarrow$$

$\xrightarrow{\{\bar{v}_1, \bar{v}_2, \bar{v}_3\} \text{ es li}}$ $\begin{cases} \alpha_1 + \alpha_2 + \alpha_3 = 0 & \Longrightarrow \boxed{\alpha_3 = 0} \\ \alpha_2 + \alpha_3 = 0 & \longrightarrow \boxed{\alpha_2 = 0} \\ \boxed{\alpha_3 = 0} \end{cases}$

$\boxed{\therefore \beta \text{ es li}}$ (I)

Escaneado con Cam

Como:

$$\left.\begin{array}{l} \beta \text{ es l.i} \\ \beta \subseteq \mathbb{F}^3, \ |\beta| = 3 \\ \dim{}^{3} = 3 \end{array}\right\} \underset{(\text{Ej } 5,c)}{\Longrightarrow} \beta \text{ genera } \mathbb{F}^3 \quad \text{(II)}$$

De (I) y (II) β es base de $\mathbb{F}^3$.