(5) $\beta = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_s\} \subseteq \boxed{\mathbb{F}^n}$

$(\mathbb{F}^n, \underbrace{+, \cdot}_{\text{"usuales"}})$ es un e.v. sobre el cuerpo $\mathbb{F}$.

a) $s > n \Rightarrow \boxed{\beta \text{ es l.d.}}$

Supongamos que $s = n+1$ (sin perder generalidad).

- Sea $\alpha_1, \dots, \alpha_{n+1} \in \mathbb{F}$ /

$\alpha_1 \bar{x}_1 + \alpha_2 \bar{x}_2 + \dots + \alpha_{n+1} \bar{x}_{n+1} = \bar{0}$ ¿tiene solución no trivial?

- Sea $\{\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n\}$ base canónica de $\mathbb{F}^n$.

Luego:

$$\bar{x}_i = \sum_{j=1}^{n} \underbrace{x_j^i}_{\in \mathbb{F}} \bar{e}_j \qquad \forall i = 1, \dots, s = n+1$$

$$\bar{x}_1 = x_1^1 \bar{e}_1 + x_2^1 \bar{e}_2 + \dots + x_n^1 \bar{e}_n$$

$\vdots$

$$\overline{0} = \alpha_1 \overline{x}_1 + \dots\dots + \alpha_{n+1} \overline{x}_{n+1} = \sum_{i=1}^{n+1} \alpha_i \overline{x}_i =$$

$$= \sum_{i=1}^{n+1} \alpha_i \left[\sum_{j=1}^{n} x_j^i \, \overline{e}_j \right] \overset{=}{\uparrow}$$

- Prop. distributiva
- prop. conmutativa.
- Asociativo.

$$= \sum_{j=1}^{n} \underbrace{\left[\sum_{i=1}^{n+1} \underset{\in \mathbb{K}}{\alpha_i} \, \underset{\in \mathbb{K}}{x_j^i} \right]}_{\beta_j \in \mathbb{K}} \overline{e}_j \Longrightarrow$$

$\{e_1, e_2, \dots e_n\}$ es l i

$$\Longrightarrow \sum_{i=1}^{n+1} \alpha_i x_j^i = 0 \quad \forall j = 1, \dots, n$$

$$\begin{cases} \sum_{i=1}^{n+1} \alpha_i x_1^i = 0 \\ \sum_{i=1}^{n+1} \alpha_i x_2^i = 0 \\ \vdots \\ \sum_{i=1}^{n+1} \alpha_i x_n^i = 0 \end{cases} \Longrightarrow \begin{cases} \alpha_1 x_1^1 + \dots + \alpha_n x_1^n + \alpha_{n+1} x_1^{n+1} = 0 \\ \alpha_1 x_2^1 + \dots + \alpha_n x_2^n + \alpha_{n+1} x_2^{n+1} = 0 \\ \vdots \qquad \dots \\ \alpha_1 x_n^1 + \dots + \alpha_n x_n^n + \alpha_{n+1} x_n^{n+1} = 0 \end{cases}$$

Sistema Compatible con $n+1$ incógnitas y n ecuaciones $\Longrightarrow$

$\Longrightarrow$ infinitas solucion

$\boxed{\therefore B \text{ es l.d.}}$