Quantum Communications

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1. Introduction

Communication has always been central to human progress, evolving through a fascinating journey that mirrors the advancement of technology and our understanding of the universe. From the simplest methods of conveying information to the complex networks we use today, each innovation in communication has fundamentally reshaped how we connect, share, and collaborate.

Iin the earliest stages of human history, communication was purely physical. Gestures, facial expressions, and simple vocalizations allowed small groups to convey basic needs and emotions. Over time, the advent of spoken language transformed these rudimentary systems into complex methods of sharing ideas and stories.

The written word marked another significant leap forward. Carvings on stone tablets and inscriptions on papyrus enabled humans to record information and transmit it across time and space. Later, the invention of the printing press democratized knowledge, allowing written material to be mass-produced and shared widely.

The industrial revolution brought the telegraph, the first technology to send information almost instantaneously over vast distances. This was followed by the telephone, radio, and television, each enabling richer, faster communication across the globe. In the late 20th century, the rise of the internet and digital communication ushered in a new era, connecting billions of people in real time through text, voice, and video.

Today, as we stand on the brink of a new technological revolution, quantum communication promises to redefine the boundaries of how we share information. Leveraging the principles of quantum mechanics (such as superposition, entanglement, and quantum state measurement) this novel form of communication offers unprecedented levels of security and potential for transmitting information.

Unlike classical methods, which rely on physical signals like electrical currents or electromagnetic waves, quantum communication encodes information in quantum bits, or qubits. These qubits exist in delicate quantum states that can represent multiple values simultaneously, enabling fundamentally different ways of transmitting and processing information.

Quantum communication not only addresses the growing need for secure data transmission but also aligns with humanity's continual pursuit of faster, more reliable, and more efficient ways to connect. By harnessing the powerful laws of quantum mechanics, we are poised to unlock a future where communication transcends the limitations of classical systems.

This introduction sets the stage for a deeper exploration of how quantum communication works, its potential applications, and its role in the ongoing evolution of human connectivity.

2. Quantum communication's basis

Before analyzing the different quantum algorithms it is necessary to understand which are the principles behind the quantum mechanics and how they can help with the quantum communication.

2.1. Quantum entanglement

The concept of entanglement has played a crucial role in the development of quantum physics and nowadays quantum correlations have come to be recognized as a novel resource that may be used to perform tasks that are either impossible or very inefficient with classical systems.

There were Einstein, Podolsky and Rosen (EPR) who first recognized this feature of quantum machinery which lies at center of interest of physics of XXI century. In their own words *This feature implies the existence* *of global states of composite system which cannot be written as a product of the states of individual subsystems.* This phenomenon, which is going to be explained in the following lines is known as "entanglement".

Let explain what is exactly the quantum entanglement. Let us focus on two quantum objects, each capable of existing in two distinct states. For example, consider the two linear polarization states $|h\rangle$ and $|v\rangle$ of photons. The basis states for the four-dimensional product space are $|hh\rangle$, $|hv\rangle$, $|vh\rangle$, and $|vv\rangle$. A general state in this product space can be expressed as:

$$|\psi\rangle = a_{hh}\,|hh\rangle + a_{hv}\,|hv\rangle + a_{vh}\,|vh\rangle + a_{vv}\,|vv\rangle\,. \tag{1}$$

In this context, we might ask whether either quantum $|h\rangle$ or $|v\rangle$ is in a well-defined state. This is not immediately evident from equation (1). If each quantum object has a specific linear polarization state, their states can be written as:

$$|\phi_1\rangle = a_{1h}\,|h\rangle + a_{1v}\,|v\rangle\,; \quad |\phi_2\rangle = a_{2h}\,|h\rangle + a_{2v}\,|v\rangle\,. \tag{2}$$

Combining these two states, the overall system can be represented as:

$$|\phi_1\phi_2\rangle = a_{1h}a_{2h}\,|hh\rangle + a_{1h}a_{2v}\,|hv\rangle + a_{1v}a_{2h}\,|vh\rangle + a_{1v}a_{2v}\,|vv\rangle\,. \tag{3}$$

Such a state is said to "factorize" because it can be written as the product of the individual states described in equation (2) and is therefore also called a *product state*. However, the state in equation (1) does not happen always, there must fulfill the following conditions:

$$a_{hh} = a_{1h}a_{2h}; \quad a_{hv} = a_{1h}a_{2v}; \quad a_{vh} = a_{1v}a_{2h}; \quad a_{vv} = a_{1v}a_{2v}. \tag{4}$$

From this, we obtain immediately the condition:

$$a_{hh}\cdot a_{vv} = a_{hv}\cdot a_{vh} \implies \text{state is product state.} \tag{5}$$

If $|\psi\rangle$ in (1) is not of the form (3), i.e., it does not factorize, one speaks of an *entangled state*:

$$a_{hh}\cdot a_{vv} \neq a_{hv}\cdot a_{vh} \implies \text{state is entangled.} \tag{6}$$

An example is the vector:

$$|\psi\rangle = \frac{|hv\rangle - |vh\rangle}{\sqrt{2}}. \tag{7}$$

Evidently, it holds that $a_{hh}\cdot a_{vv} = 0$ and $a_{hv}\cdot a_{vh} = -1$; condition (6) is fulfilled, so the state is entangled. Due to this physical phenomenom, particles involved in it share corralated states, so knowing the value of one of the qubits and the correlation that was between them we can know the values of the second state.

2.2. Quantum non-locality

In scientific experiments, outcomes are often expressed as conditional probabilities. For instance, if observer A performs an experiment labeled X, the probability of observing result a is given by $P(a|X)$.

When multiple observers conduct experiments independently, their outcomes are generally expected to be independent. For example, if observer B performs experiment Y, the marginal probability of their outcome b is given by:

$$\sum_a P(a,b|X,Y) = P(b|Y).$$

This independence reflects the principle of **locality**, which states that the results of an experiment depend only on the local state of the environment at the observer's location. Locality assumes that if two observers perform experiments at space-like separated locations (preventing any form of communication) their local outcomes can be fully described by a shared set of hidden variables λ.

However, in the quantum realm, this principle does not hold. When working with entangled quantum systems, the results of measurements at two distant locations cannot be fully explained by local hidden variables or the local state of the environment. The measurement outcome of one particle is inherently related to the state of its entangled partner, no matter how far apart they are. This lack of local explanation gives rise to **quantum non-locality**.

Non-locality is a fundamental feature of quantum mechanics and underpins applications like **quantum key distribution** (QKD) or **quantum communication**. The entangled nature of quantum systems ensures that the data generated by Alice and Bob during such protocols possesses inherent secrecy, as no shared hidden variable λ can fully describe their outcomes.

3. Quantum Communication

Quantum communication is the procedure of transferring a quantum state from one place to another. In this framework, the sender is traditionally referred to as Alice, while the receiver is called Bob. The fundamental idea is that quantum states carry quantum information (qubits) and that quantum information allows tasks to be performed that could only be achieved far less efficiently, if at all, using classical information. The best known example is quantum key distribution (QKD). In fact, there is another motivation, at least equally important to most physicists, namely the close connection between quantum communication and quantum non-locality.

There exist many different ways of achieving quantum communication. In this work some of them have been described, and used from the simplest ones to the more complex procedures. Before starting with the different procedures it is important to understand which is the basic idea under the quantum communication.

The first step to understand the quantum communication involves the quantum entanglement we have explained before. As we know, when two particles are entangled their states are correlated. Due to that, having entangled particles X and Y, the probability of measuring state y in particle B and state x in particle A is the same of measuring state y in particle B. $P(a, b|x, y) = P(b|y)$.

3.1. Quantum teleportation

Quantum teleportation forms the foundation of quantum communication. The central concept behind quantum teleportation is quantum non-locality. Instead of physically transferring a quantum object to transmit information, only the quantum state of the object is transmitted.

Let's explain how quantum teleportation works. For simplicity, we'll focus on the teleportation of a single qubit, though this process can be extended to multiple qubits. The procedure involves three main steps, as illustrated in Figure 1.

1. **Creating and distributing entangled particles:** In the first step, an entangled pair of particles is generated. These particles are then distributed between Alice and Bob. This distribution can be achieved through optical fibers for example in the case of photons. At this stage, a classical communication channel is also established between Alice and Bob, which will be used later in the process.
2. **Performing a Bell-State Measurement (BSM):** In the second step, Alice combines her entangled particle with the qubit carrying the quantum state to be teleported. She then performs a measurement known as a **Bell-State Measurement (BSM)**. This measurement outputs one of four possible **Bell states**, which encode the relationship between the two particles but not their individual quantum states. In essence, the BSM does not reveal the quantum state directly but instead relates the states of the particles. This step is technically challenging, as achieving a complete BSM is often difficult in practice.

3. **Sending the results and reconstructing the quantum state:** In the final step, Alice communicates the result of her BSM to Bob via the classical communication channel. Based on Alice's measurement result, Bob applies a corresponding set of **unitary operations** to his entangled particle. This operation transforms his particle into the original quantum state that Alice wanted to teleport. Importantly, the exact operation depends on the specific Bell state obtained in the BSM.

A key advantage of quantum teleportation is the efficiency of information transfer. In quantum communication, only **2 classical bits** are needed to transmit the results of the BSM (since there are four Bell states, and $2^2 = 4$). In contrast, classical communication would require significantly more bits to fully transmit the quantum state. This efficiency makes quantum teleportation an essential tool in quantum communication systems.

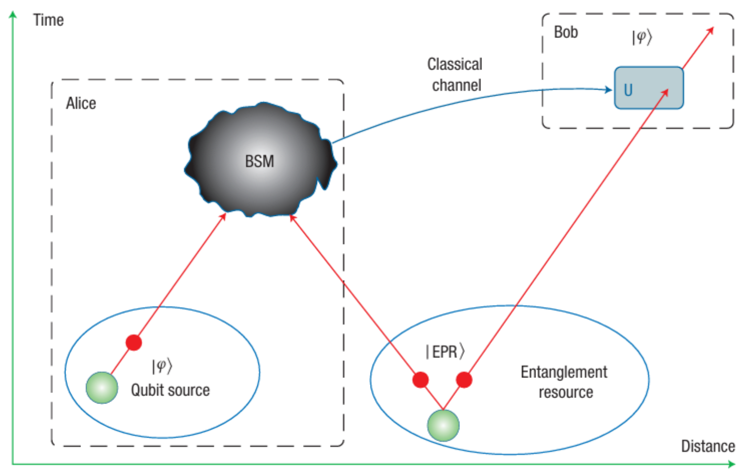


Figure 1: Quantum teleportation process.

I have implemented quantum teleportation using Qiskit in Google Colab. The algorithm follows the steps previously explained: preparation of entanglement, Bell-State Measurement (BSM), and unitary operations on Bob's qubit.

In the first step, we create entanglement between Alice's and Bob's qubits using a Hadamard gate followed by a CNOT gate. Additionally, the message qubit to be teleported is initialized in a specific quantum state ($|\psi\rangle$). In the example illustrated in Figure 2, the message qubit is prepared in the state $|\psi\rangle = 1|0\rangle + 0|1\rangle$, although other amplitudes have also been explored in the Colab implementation.

The Bell-State Measurement (BSM) is performed using a CNOT gate and a Hadamard gate. The reasoning behind this specific setup for the BSM will be discussed further in the following lines.

Finally, both the message qubit and Alice's qubit are measured, with the measurement results stored in classical registers. Based on these results, specific operations are applied to Bob's qubit. It is important to note that after entanglement, Alice's and Bob's qubits initially have the same state. If the measurement results indicate a difference, it is due to the influence of the message qubit on Alice's state during the BSM. For example:

- If the message qubit's state was $|0\rangle$, Alice's and Bob's qubits remain identical, meaning the CNOT gate does not make any change.
- If the message qubit's state was $|1\rangle$, Alice's and Bob's qubits differ, and the CNOT gate makes a X operation on Alice's qubit.

This behavior can be understood through the CNOT truth table (Table 1). The CNOT operation effectively identifies whether the two qubits are in the same or opposite states, making it instrumental in decoding the message's state.

As for the Controlled-Z (CZ) gate, it serves to adjust the phase of Bob's qubit based on the amplitude's sign in the message qubit. If the amplitudes of the message qubit are negative, this negativity must also be reflected in Bob's qubit. However, since the final measurement of Bob's qubit collapses the quantum state, the negative sign of the amplitudes does not affect the classical measurement outcomes.

Control (Input)	**Target (Input)**	**Target (Output)**
0	0	0
0	1	1
1	0	1
1	1	0

Table 1: Truth table of CNOT Gate

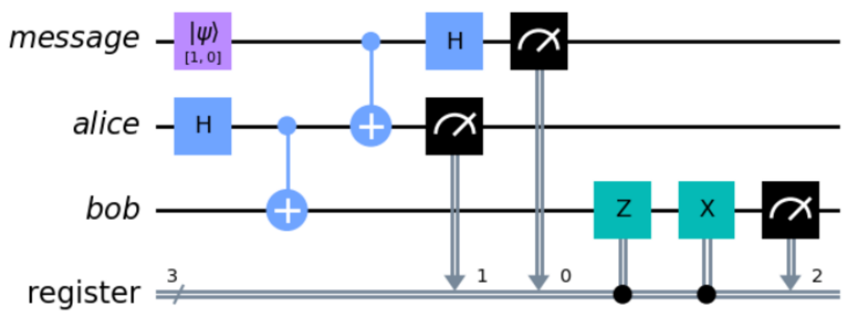


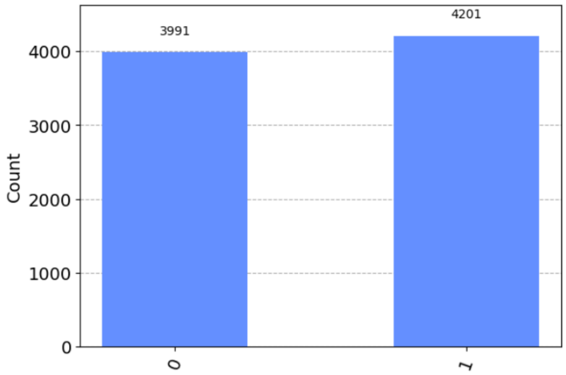
Figure 2: Algorithm of the quantum teleportation

I executed the algorithm on both ideal (noise-free) and noisy quantum devices. While there are too many results to present in detail, the key findings can be summarized as follows:
When executed on an ideal device, the probability distribution of the measured states closely matches the expected theoretical values (see Figure 3a). However, when the communication is performed on a noisy device, the proportions deviate from the ideal case. For instance, as shown in Figure 3b, some measurements incorrectly detect the state $|0\rangle$, even though its probability should theoretically be zero. This discrepancy is indicative of noise in the system and directly impacts the fidelity of the communication.
On the noisy device, 8192 measurements were performed, of which 947 resulted in errors. This corresponds to a fidelity of 0.88, meaning that 88% of the transmitted messages were successfully communicated without contamination. Fidelity is a critical metric in quantum communication as it quantifies the likelihood of accurately receiving an uncorrupted message.
As noise and perturbed information are significant challenges in quantum communication, the following sections will explore techniques to mitigate these issues and improve fidelity.

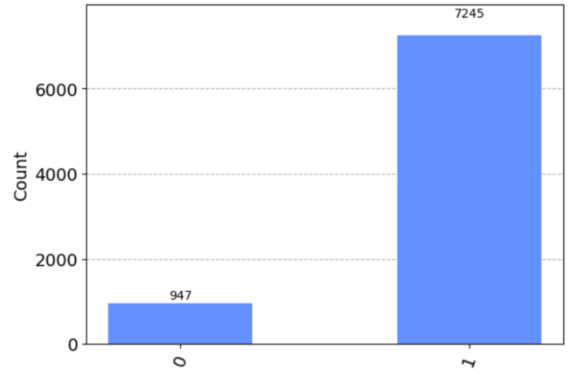
4. Quantum communications for long distances

Before implementing quantum teleportation, it is essential to first establish shared entangled qubits. While this is generally not a problem (since entangled particles, such as photons, can travel at the speed of light) it becomes challenging when dealing with very large distances.

For instance, consider the scenario of communicating between Earth and Jupiter, which are separated by a distance of approximately 636 million kilometers. Transmitting a particle to Jupiter would take nearly 89



(a) State of Bob's qubit for a message in state $\frac{1}{\sqrt(2)}$—0¿ + $\frac{1}{\sqrt(2)}$—1¿. The device is ideal (no noise)



(b) State of Bob's qubit for a message in state 0—0¿ + 1—1¿. The device has noise.

Figure 3: State of Bob's qubitafter teleportation

days, representing the time required to establish a direct communication channel between the two planets. Now, suppose we also have inhabitants on Mars, located at a distance of 225 million kilometers from Earth. In this case, establishing a communication channel between Earth and Mars would take about 31 days, while setting up a channel between Mars and Jupiter would require 58 days.

To achieve communication between Earth and Jupiter in this scenario, we could use Mars as an intermediate node. By entangling the qubits communicating between Earth and Mars, and separately entangling the qubits communicating between Mars and Jupiter, we can combine these two links. This process, which involves entangling two qubits with no shared history, is called **entanglement swapping**. The key idea is to first create entanglement between relatively close nodes and then extend the entanglement step by step. This entire process, which facilitates long-distance quantum teleportation via entanglement swapping, is known as a **relay**.

4.1. Quantum relays

Quantum relays have emerged as a promising method for enabling communication between distant locations. While they represent a significant advancement, the distances achievable through quantum relays are still inherently limited. This limitation arises from the requirement that the entanglement of pairs A–B and B–C must first be successfully established before the entanglement can be swapped to connect A–C.

Another challenge associated with quantum relays is the probability of photon loss or alteration. The introduction of relays does not change the overall probability of successful photon propagation. Specifically, the likelihood of photons successfully traveling between A and B, and between B and C, remains equivalent to the probability of a photon propagating directly from A to C. This presents a practical issue, as photons may be lost during transmission through optical fibers.

Despite these challenges, quantum relays offer a solution to a key problem in quantum communication: the phenomenon of **dark counts**. Dark counts occur when false-positive measurements or noise in the system lead to erroneous detections. Unlike traditional systems, quantum relays are inherently resilient to dark counts, making them a valuable tool in enhancing the reliability of quantum communication systems.

In the following figure (4), we present the circuit for entanglement swapping. Initially, we have four qubits: the first two belong to Alice, while the third and fourth belong to Bob. Before the red line (representing the start of entanglement swapping), Alice's qubits are entangled with each other (her first qubit is entangled with her second qubit), and the same applies to Bob's.

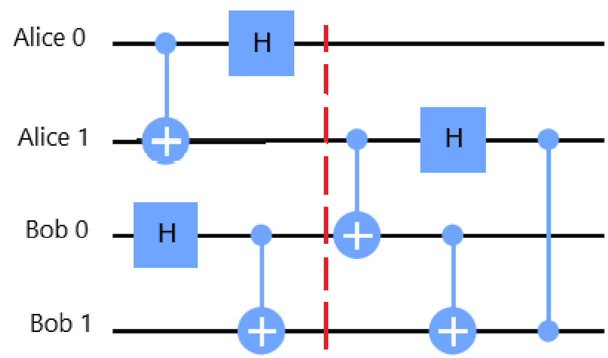


Figure 4: Representation of entanglement swapping.

At this stage (after the red line), both Alice and Bob send one of their qubits to Mary (not explicitly shown in the figure). Mary performs the entanglement swapping operation. This part of the circuit mirrors the process of quantum teleportation: after a Bell-state measurement (BSM) and some correction gates, Alice's first qubit is effectively transmitted to Bob's second qubit. The critical outcome here is that Alice's and Bob's qubits become entangled. This entanglement arises because their qubits share the same quantum state, thereby forming an entangled pair in state $\Phi+$.

An interesting phenomenon occurs during this process: although Mary plays a crucial role in enabling the entanglement swapping, she loses her entanglement with Alice and Bob. Thus, while Mary is essential in establishing the connection, she retains no information about the final entangled state.

With the circuit for entanglement swapping established, we can now extend this to create a quantum relay (Figure 5). To do this, two additional elements must be incorporated. First, we explicitly represent Mary in the diagram. Second, we apply corrections to address errors that might arise during the entanglement swapping process.

As it can be seen in this quantum relay representation, Mary is shown swapping her second qubit with Alice's qubit. The diagram also includes a black box representing the entanglement swapping operation discussed earlier. Additionally, rotation gates ($R_x(\pi/2)$ and $R_x(-\pi/2)$) are applied as part of Deutsch's correction.

Deutsch's correction is used to address specific issues that can occur during the protocol. When the intermediate qubits are measured (typically in the Bell basis) during entanglement swapping, random phase rotations may be introduced into the final entangled state shared by Alice and Bob. These phase errors mean that even if the swapping process succeeds, Alice and Bob might end up with a Bell state rotated in phase. The rotation gates ($R_x(\pi/2)$ and $R_x(-\pi/2)$) are applied to correct these phase errors, ensuring that Alice and Bob obtain the intended entangled state.

Finally, a CNOT gate and a Hadamard gate can be seen in Alice's first qubit and in Bob's second qubit. This is not compulsory in the algorithm but it can be used to check which of the Bell states do those qubits represent. In table 2 can be seen that after those two gates are applied the Bell state that is being represented can be seen. This is very interesting as we can see how the communication has been done.

In the accompanying Colab notebook provided with this work, several experiments have been conducted. Specifically, a quantum relay has been executed on both noisy and ideal quantum computers. Since ideal quantum computers consistently produce perfect results and do not yet exist in practice, we will focus on the results obtained from the noisy device.

Using the noisy device, we achieved promising results, with the entanglement being successfully established

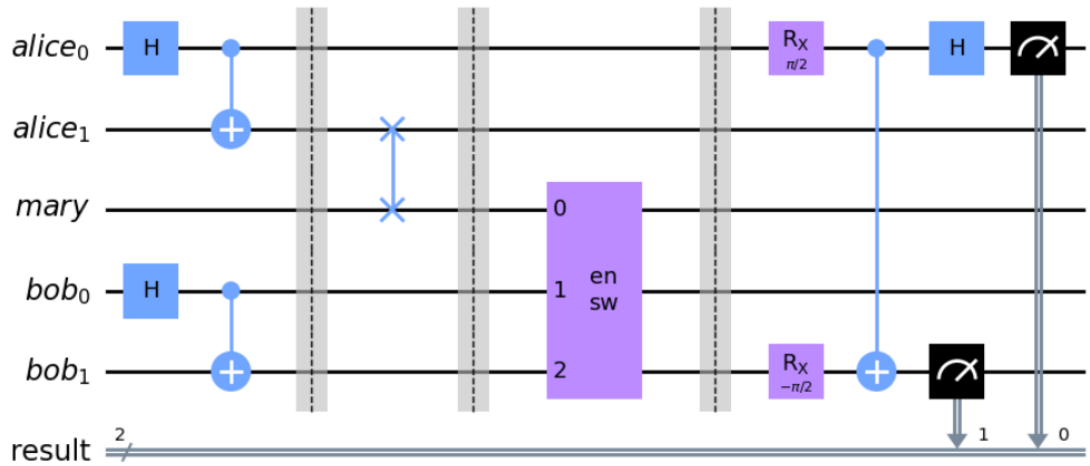


Figure 5: Representation of a quantum relay.

Input	**After CNOT**	**After Hadamard**
(—00⟩ + —11⟩)/$\sqrt{2}$	(—00⟩ + —11⟩)/$\sqrt{2}$	—00⟩
(—00⟩ - —11⟩)/$\sqrt{2}$	(—00⟩ - —11⟩)/$\sqrt{2}$	—10⟩
(—01⟩ + —10⟩)/$\sqrt{2}$	(—01⟩ + —10⟩)/$\sqrt{2}$	—01⟩
(—01⟩ - —10⟩)/$\sqrt{2}$	(—01⟩ - —10⟩)/$\sqrt{2}$	—11⟩

Table 2: Identification of Bell states.

with a fidelity of 0.86. This fidelity is slightly lower than the 0.88 observed in the simple teleportation experiment. As discussed earlier, increasing the distance that particles must travel leads to a higher likelihood of interference and errors. This naturally affects the fidelity of the system.

While errors are inevitable in long-distance quantum communications, this raises an important question: can we improve the performance of these systems despite these challenges? The following sections will explore potential strategies and solutions to address these issues.

4.2. Quantum repeaters

To address the issue of distance limitations in quantum communication, a groundbreaking technology known as quantum repeaters is required. These devices leverage the principles of quantum relays and quantum memories to extend the reach of quantum networks.

The fundamental concept behind quantum repeaters is as follows: if entanglement distribution succeeds between nodes A and B but fails between nodes B and C, the A–B entanglement can be temporarily stored in a quantum memory. This allows the B–C entanglement distribution process to be restarted without losing the previously established entanglement between A and B.

Additionally, quantum memories offer another significant advantage: they can be employed in multi-qubit gates for purification techniques. These techniques are crucial for verifying the success of the entanglement process and improving its fidelity. In this approach, we will focus on the latter application of quantum memories, utilizing them to enhance the reliability and accuracy of the communication process.

4.3. Error correction

To perform error correction in the quantum circuit, we will use Bennett's protocol. Noise in quantum systems can randomize quantum states in various ways, but this protocol is designed to identify and retain only the cases where noise affects both pairs similarly. The underlying assumption of Bennett's protocol is that if noise disrupts one pair of entangled qubits, the remaining entangled pairs are not necessarily affected. This assumption enables selective retention of high-fidelity entangled pairs.

Bennett's protocol involves applying a CNOT operation between the qubits to be entangled and their corresponding auxiliary (correction) qubits. Each auxiliary qubit is then compared to its counterpart. If the values of the auxiliary qubits match, it can be inferred that the entanglement has been successfully established for the corresponding pair. However, if the values differ, the affected qubits are discarded, as it is assumed that the entanglement was disrupted by noise.

Let us consider some illustrative examples (Tables 3, 4, 5, and 6). In the first example, all qubits exhibit matching values, indicating successful entanglement, so no restrictions are applied. In the second case, one of Alice's auxiliary qubits does not match its counterpart in Bob's system. Despite the primary qubits being correctly entangled, the protocol discards the communication to maintain high fidelity. In the third example, errors occur in two qubits, but the presence of a second auxiliary qubit allows for successful identification and rejection of the faulty entanglement. Finally, in Table 6, all auxiliary qubits of either Alice or Bob exhibit errors. Even though the primary qubits are not well entangled, the protocol approves the communication due to the assumption that auxiliary qubit errors do not necessarily reflect entanglement failure.

Before Bennett's		**After Bennett's**	
Alice	Bob	Alice	Bob
0	0	0	0
0	0	0	0
0	0	0	0

Table 3: Comparison of states before and after Bennett's protocol (all the states are identical).

Before Bennett's		**After Bennett's**	
Alice	Bob	Alice	Bob
0	0	0	0
1	0	1	0
0	0	0	0

Table 4: Comparison of states before and after Bennett's protocol (one of the correction qubits has failed)

Before Bennett's		**After Bennett's**	
Alice	Bob	Alice	Bob
0	1	0	1
0	1	0	0
0	0	0	1

Table 5: Comparison of states before and after Bennett's protocol (one of the correction and one from the entangling qubits has failed)

When evaluating the performance of our system after applying corrections, two metrics must be considered. First, the **yield**, which measures the proportion of accepted communications, and second, the **fidelity**, which assesses the accuracy of the accepted communications by indicating how many erroneous communications were mistakenly approved. It is evident that as the number of correction qubits increases, the yield decreases

Before Bennett's		**After Bennett's**	
Alice	Bob	Alice	Bob
1	0	0	0
1	0	0	0
1	0	0	0

Table 6: Comparison of states before and after Bennett's protocol (all Alice's or Bob's qubits have failed)

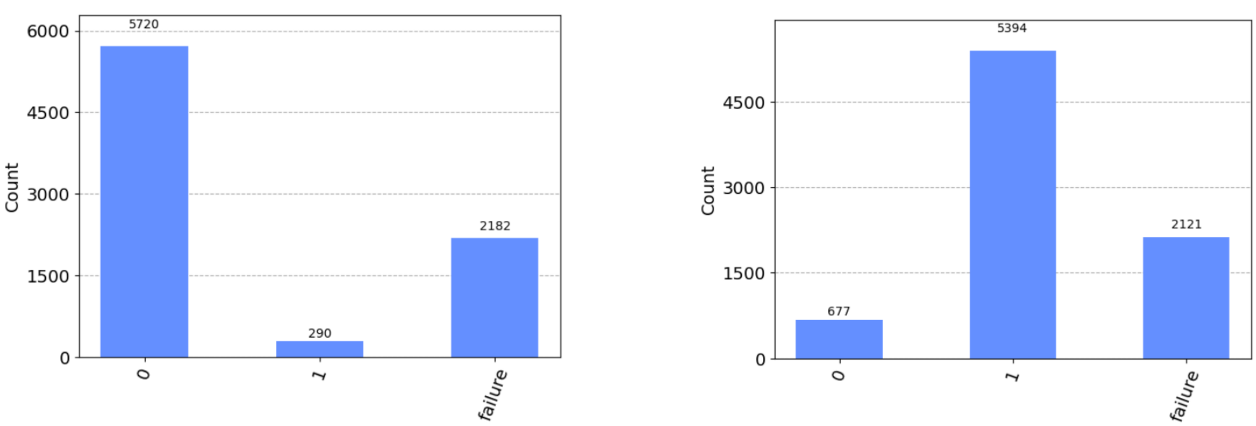
while the fidelity improves, as more strong conditions are applied to accept communications.

In the experiments I made, I used 1 and 2 correction qubits. The results showed yields of 0.84 and 0.68, respectively, and fidelities of 0.85 and 0.90, respectively. I chose not to experiment with more correction qubits because the waiting time increases significantly as the number of correction qubits rises.

4.4. Communication using quantum repeaters

Once the quantum repeaters have been established, sending messages becomes a straightforward process. The procedure is quite similar to quantum teleportation, as it involves performing a Bell-State Measurement (BSM) on Alice's qubit and the message qubit. After measuring the result, the corresponding CNOT and CZ gates must be applied in the same manner as in the teleportation protocol. An important detail to note is that the BSM applied to Alice's and Bob's qubits—used to determine the Bell state—must be accounted for and effectively reversed.

I executed the algorithm on both noisy and ideal devices. Since the results from ideal devices are predictable, I will focus on the outcomes from noisy devices. In Figure 6, two representative results are displayed. It can be observed that, in some instances, the algorithm identifies a failure, and those results are disregarded. This highlights the importance of accounting for errors when working with noisy quantum devices.



(a) State of Bob's qubit for a message in state —0¿

(b) State of Bob's qubit for a message in state —1¿

Figure 6: State of Bob's qubit for different messages

5. Conclusion

In this work, we have demonstrated that quantum communication is a feasible approach to transmitting information over various distances, leveraging the principles of quantum entanglement and teleportation. Through the use of techniques such as quantum relays, quantum repeaters, and error-correction protocols like Bennett's protocol, we have shown that it is possible to address and mitigate many of the errors and challenges inherent in quantum systems. While noise and interference remain significant obstacles, these methods provide practical solutions to enhance the fidelity and reliability of quantum communication networks.

Although our experiments have focused on transmitting a single qubit, the principles and protocols described can be scaled by repeating the process to transmit multiple qubits, enabling the communication of complex and extensive messages. This scalability underscores the potential of quantum communication for real-world applications, such as secure data transmission and long-distance networking.

Looking forward, advancements in technologies like quantum memories and fault-tolerant quantum gates will play a critical role in overcoming current limitations, such as distance constraints and noise-related errors. These developments are paving the way for the realization of robust, large-scale quantum communication networks that could revolutionize the way information is transmitted and shared across the globe.

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