classification_intro_professor

September 17, 2024

1 Introduction to Classification.

```
Notebook version: 2.4 (Sep 17, 2024)

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Changes: v.1.0 - First version. Extracted from a former notebook on K-NN
         v.2.0 - Adapted to Python 3.0 (backcompatible with Python 2.7)
         v.2.1 - Minor corrections affecting the notation and assumptions
         v.2.2 - Updated index notation
         v.2.3 - Adaptation to slides conversion
         v.2.4 - (JCS) Use of pandas. Updated python code.
```

[1]:
```
%matplotlib inline

# Import some libraries that will be necessary for working with data and␣
  ↪displaying plots
import matplotlib.pyplot as plt
import pandas as pd

from sklearn.model_selection import train_test_split
from sklearn.preprocessing import LabelEncoder
```

1.1 1. The Classification problem

In a generic classification problem, we are given an **observation vector** $\mathbf{x} \in \mathbb{R}^N$ which is known to belong to one and only one **category** or **class**, y, from the set $\mathcal{Y} = \{0, 1, \ldots, M-1\}$.

The goal of a classifier system is to **predict** y based on $\mathbf{x}$.

To design the classifier, we are given a collection of labelled observations $\mathcal{D} = \{(\mathbf{x}_k, y_k)\}_{k=0}^{K-1}$ where, for each observation $\mathbf{x}_k$, the value of its true category, y_k, is known.

1.1.1 1.1. Binary Classification

We will focus mostly in binary classification problems, where the label set is binary, $\mathcal{Y} = \{0, 1\}$.

Despite its simplicity, this is the most frequent case. Many multi-class classification problems are usually solved by decomposing them into a collection of binary problems.

1.1.2 1.2. The independence and identical distribution (i.i.d.) assumption.

The classification algorithms, as many other machine learning algorithms, are based on two major underlying hypothesis:

- **Independence**: All samples are statistically independent.
- **Identical distribution**: All samples in dataset $\mathcal{D}$ have been generated by the same distribution $p_{\mathbf{X},Y}(\mathbf{x}, y)$.

The i.i.d. assumption is essential to guarantee that a classifier based on $\mathcal{D}$ has a good perfomance when applied to new input samples.

The **underlying distribution is unknown** (if we knew it, we could apply classic decision theory to make optimal predictions). This is why we need the data in $\mathcal{D}$ to design the classifier.

1.2 2. A simple classification problem: the Iris dataset

(Iris dataset presentation is based on this Tutorial by Jason Brownlee)

As an illustration, consider the Iris dataset , taken from the UCI Machine Learning repository . Quoted from the dataset description:

> *This is perhaps the best known database to be found in the pattern recognition literature. The data set contains 3 classes of 50 instances each, where each class refers to a type of iris plant. [...] One class is linearly separable from the other 2; the latter are NOT linearly separable from each other.*

The *class* is the species, which is one of *setosa*, *versicolor* or *virginica*. Each instance contains 4 measurements of given flowers: sepal length, sepal width, petal length and petal width, all in centimeters. The last column shows the class of each instance.

[2]:
```
df_data = pd.read_csv('datasets/iris.data', header=None)
df_data.head(10)
```

[2]:

	0	1	2	3	4
0	5.1	3.5	1.4	0.2	Iris-setosa
1	4.9	3.0	1.4	0.2	Iris-setosa
2	4.7	3.2	1.3	0.2	Iris-setosa
3	4.6	3.1	1.5	0.2	Iris-setosa
4	5.0	3.6	1.4	0.2	Iris-setosa
5	5.4	3.9	1.7	0.4	Iris-setosa
6	4.6	3.4	1.4	0.3	Iris-setosa
7	5.0	3.4	1.5	0.2	Iris-setosa
8	4.4	2.9	1.4	0.2	Iris-setosa
9	4.9	3.1	1.5	0.1	Iris-setosa

We can inspect some basic statistics of the data features using the `describe` method from the `pandas` package.

[3]:
```
df_data.describe()
```

[3]:

	0	1	2	3
count	150.000000	150.000000	150.000000	150.000000
mean	5.843333	3.054000	3.758667	1.198667
std	0.828066	0.433594	1.764420	0.763161
min	4.300000	2.000000	1.000000	0.100000
25%	5.100000	2.800000	1.600000	0.300000
50%	5.800000	3.000000	4.350000	1.300000
75%	6.400000	3.300000	5.100000	1.800000
max	7.900000	4.400000	6.900000	2.500000

We will separate the features and the target categories into a feature matrix and a list of class labels. Labels will be mapped to integers to facilitate the treatment.

[4]:
```
# Extract the features and the classes from the dataframe
X = df_data.iloc[:, 0:4].values
C = df_data.iloc[:, 4].values

# Map the classes to integers
Y = LabelEncoder().fit_transform(C)
# Get a dictionary mapping from integers to classes
int_to_class = dict(zip(Y, C))
# Get a dictionary mapping from classes to integers
class_to_int = dict(zip(C, Y))

n_clases = len(int_to_class)
n_samples, n_features = X.shape

# Show the data dimensions:
print(f"-- The feature matrix contains {n_samples} samples and {n_features}␣
 ↪features.")

# Pretty print the mapping
print("-- The following mapping from integers to classes will be used:")
for k, v in int_to_class.items():
    print(f"     {k} -> {v}")
```

```
-- The feature matrix contains 150 samples and 4 features.
-- The following mapping from integers to classes will be used:
     0 -> Iris-setosa
     1 -> Iris-versicolor
     2 -> Iris-virginica
```

1.2.1 2.1. Training and test

Next, we will split the data into two sets:

- **Training set**, that will be used to learn the classification model
- **Test set**, that will be used to evaluate the classification performance

To make the partition, we can use the `train_test_split` method from `sklearn`.

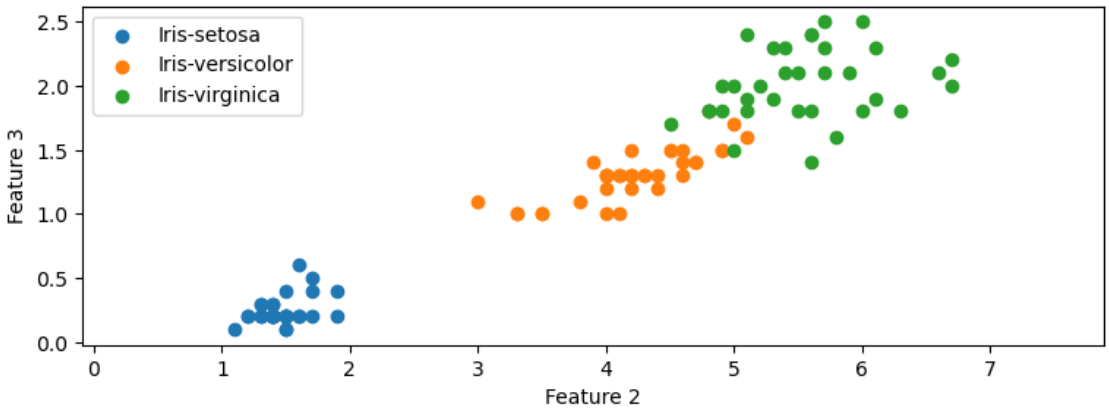
[5]:
```
# Split the dataset into training and testing sets
X_trn_all, X_tst_all, Y_trn_all, Y_tst_all = train_test_split(
    X, Y, test_size=0.33, random_state=42)

# Dataset sizes
n_trn = len(Y_trn_all)
n_tst = len(Y_tst_all)

# Show the data dimensions:
print(f"-- {n_trn} training samples.")
print(f"-- {n_tst} test samples.")
```

```
-- 100 training samples.
-- 50 test samples.
```

1.2.2 2.2. Scatter plots

To get some intuition about this four dimensional dataset we can plot 2-dimensional projections taking only two variables each time.

[6]:
```
fh = 2  # First feature to plot
fv = 3  # Second feature to plot

plt.figure(figsize=(9, 3))
for i in range(n_clases):
    # Plot the samples of class i
    plt.scatter(X_trn_all[Y_trn_all == i, fh], X_trn_all[Y_trn_all == i, fv],
                label=int_to_class[i])
plt.axis('equal')
plt.xlabel(f'Feature {fh}')
plt.ylabel(f'Feature {fv}')
plt.legend()
plt.show()
```



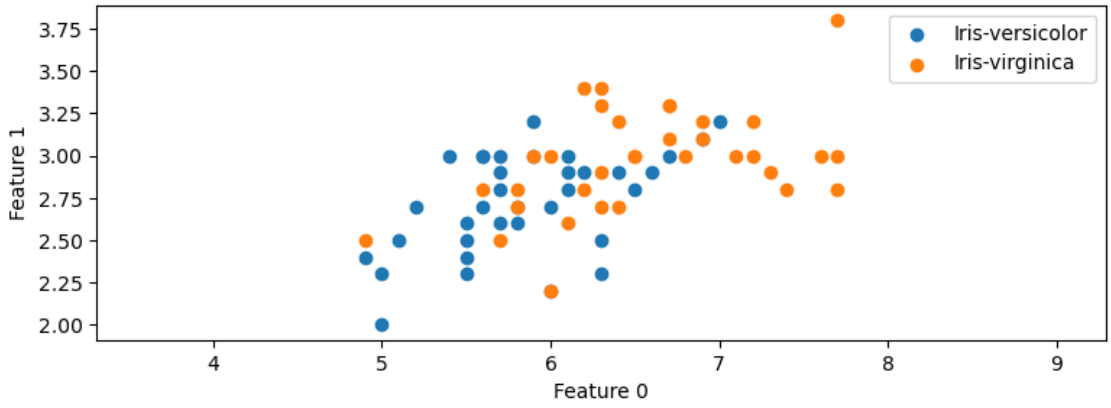
In the following, we will design a classifier to separate classes "Versicolor" and "Virginica" using x_0 and x_1 only. To do so, we build a training set with samples from these categories, and a binary label $y^{(k)} = 1$ for samples in class "Virginica", and 0 for "Versicolor" data.

[7]:
```
# Select two classes
ignored_class = 'Iris-setosa'
c0 = 'Iris-versicolor'
c1 = 'Iris-virginica'
# Select two features
ind = [0, 1]

# New map indices to classes
int_to_class_2 = {0: c0, 1: c1}
class_to_int_2 = {c0: 0, c1: 1}

# Select samples
X_trn = X_trn_all[Y_trn_all != class_to_int[ignored_class], ]
X_tst = X_tst_all[Y_tst_all != class_to_int[ignored_class], :]
# Select features
X_trn = X_trn[:, ind]
X_tst = X_tst[:, ind]

# Select labels for the selected features
Y_trn = Y_trn_all[Y_trn_all != class_to_int[ignored_class]]
Y_tst = Y_tst_all[Y_tst_all != class_to_int[ignored_class]]

# Map to new class indices
Y_trn = Y_trn == class_to_int[c1]
Y_tst = Y_tst == class_to_int[c1]

# Dataset sizes
n_trn = len(Y_trn)
n_tst = len(Y_tst)
# Show the data dimensions:
print(f"-- {n_trn} training samples.")
print(f"-- {n_tst} test samples.")
```

```
-- 69 training samples.
-- 31 test samples.
```

[8]:
```
plt.figure(figsize=(9, 3))
for i in [0, 1]:
    # Plot the samples of class i
    plt.scatter(X_trn[Y_trn == i, 0], X_trn[Y_trn == i, 1],
                label=int_to_class_2[i])
```

```
plt.axis('equal')
plt.xlabel(f'Feature {ind[0]}')
plt.ylabel(f'Feature {ind[1]}')
plt.legend()
plt.show()
```



1.3 3. A Baseline Classifier: Maximum A Priori.

For the selected data set, we have two clases and a dataset with the following class proportions:

[9]:
```
print(f'Class 0 ({c0}): {n_trn - sum(Y_trn)} samples')
print(f'Class 1 ({c1}): {sum(Y_trn)} samples')
```

```
Class 0 (Iris-versicolor): 35 samples
Class 1 (Iris-virginica): 34 samples
```

The maximum a priori classifier assigns any sample $\mathbf{x}$ to the most frequent class in the training set. Therefore, the class prediction y for any sample $\mathbf{x}$ is

[10]:
```
y = int(2*sum(Y_trn) > n_trn)
print(f'y = {y} ({c1 if y==1 else c0})')
```

```
y = 0 (Iris-versicolor)
```

The error rate for this baseline classifier is:

[11]:
```
# Training and test error arrays
E_trn = (Y_trn != y)
E_tst = (Y_tst != y)

# Error rates
pe_trn = float(sum(E_trn)) / n_trn
pe_tst = float(sum(E_tst)) / n_tst
print('Pe(train):', str(pe_trn))
```

```
print('Pe(test): ', str(pe_tst))
```

```
Pe(train): 0.4927536231884058
Pe(test):  0.5161290322580645
```

The error rate of the baseline classifier is a simple benchmark for classification. Since the maximum a priori decision is independent on the observation, $\mathbf{x}$, any classifier based on $\mathbf{x}$ should have a better (or, at least, not worse) performance than the baseline classifier.

1.4 3. Parametric vs non-parametric classification.

Most classification algorithms can be fitted to one of two categories:

1. **Parametric classifiers**: to classify any input sample $\mathbf{x}$, the classifier applies some function $f_{\mathbf{w}}(\mathbf{x})$ which depends on some parameters $\mathbf{w}$. The training dataset is used to estimate $\mathbf{w}$. Once the parameter has been estimated, the training data is no longer needed to classify new inputs.
2. **Non-parametric classifiers**: the classifier decision for any input $\mathbf{x}$ depend on the training data in a direct manner. The training data must be preserved to classify new data.