EECS332 Digital Image Analysis

Motion Estimation

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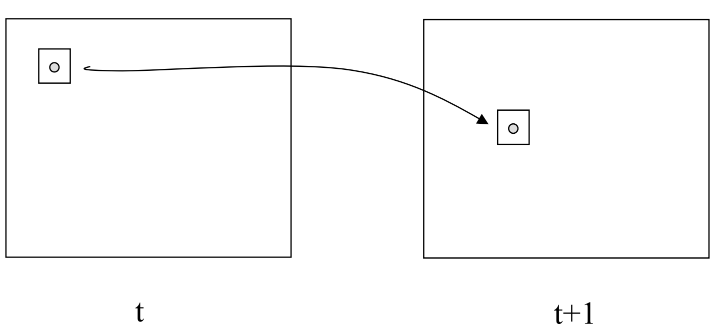
Motivations

- If something is moving in video, can you keep tracking its movements?
- The problem:



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Outline

- Motivation
- Basic questions
- Exhaustive search
- Flow-constraint equation
- Gradient-based search

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Basic questions

- Matching ← what are the criteria for matching?
- Searching ← how to find the best match?

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Matching Criterion

- SSD (sum of squared difference)

$$D = \sum_x \sum_y [I(x,y) - T(x,y)]^2$$

- Cross-correlation

$$C = \sum_x \sum_y I(x,y)T(x,y)$$

- Normalized cross-correlation

$$N = \frac{\sum_x \sum_y [I(x,y) - \bar{I}][T(x,y) - \bar{T}]}{\sqrt{[\sum_x \sum_y (I(x,y) - \bar{I})^2][\sum_x \sum_y (T(x,y) - \bar{T})^2]}}$$

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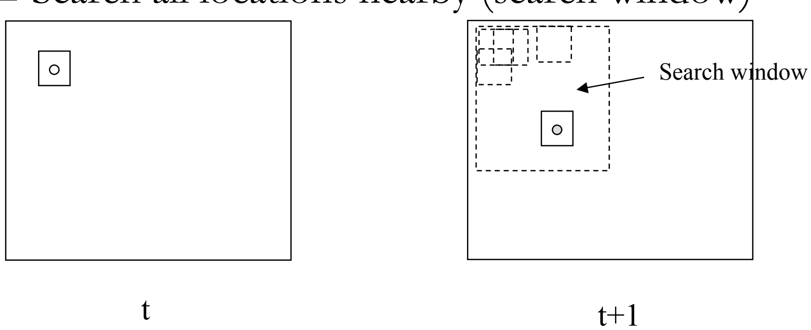
Exhaustive Search

- Search all locations nearby (search window)



Pros:

✓ easy to implement

Cons:

✓ computationally intensive

✓ can not handle rotation

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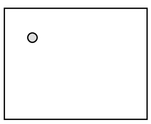
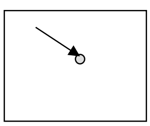
Gradient-based search

- Let I(x,y, τ) → current image
- Let I(x,y,0) → reference image or template
- For now, we assume a pure translational motion
- Constant Brightness Constraint:
  - i.e., I(x,y,0) = I(x+u, y+v, τ) $\forall (x,y) \in R$
  - where (u,v) is the displacement



I(x,y,0)



I(x+u,y+v, τ)

$$\boxed{\begin{aligned}(u^*, v^*) &= \arg\min_{(u,v)} D(u,v) \\ &= \arg\min_{(u,v)} \sum_x \sum_y [I(x+u, y+v, \tau) - I(x,y,0)]^2\end{aligned}}$$

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Flow Constraint Equation

- We perform Taylor expansion of I(x+u,y+v,t) with respect to (x,y,0):

$$I(x+u, y+v, \tau) = I(x,y,0) + \frac{\partial I(x,y,0)}{\partial x} u + \frac{\partial I(x,y,0)}{\partial y} v + \frac{\partial I(x,y,0)}{\partial t} \tau + O(t^2)$$

- denote

$$\frac{\partial I(x,y,0)}{\partial x} = I_x, \quad \frac{\partial I(x,y,0)}{\partial y} = I_y, \quad \frac{\partial I(x,y,0)}{\partial t} = I_t$$

- Since I(x+u,y+v, τ)= I(x,y,0)

$$\Rightarrow \qquad \boxed{I_x u + I_y v + I_t \tau = 0}$$

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Solution

$$\Rightarrow \quad D(u,v) = \sum_x \sum_y (I_x u + I_y v + I_t \tau)^2$$

We have
$$\nabla D(u,v) = \begin{bmatrix} \sum_x \sum_y (I_x u + I_y v + I_t \tau) I_x \\ \sum_x \sum_y (I_x u + I_y v + I_t \tau) I_y \end{bmatrix} = 0$$

Easy to see
$$\sum_x \sum_y \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\tau \sum_x \sum_y \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = -\tau \left(\sum_x \sum_y \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)^{-1} \left(\sum_x \sum_y \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix} \right)$$

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Isn't it nice?

$$\begin{bmatrix} u \\ v \end{bmatrix} = -\tau \left(\sum_x \sum_y \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)^{-1} \left(\sum_x \sum_y \begin{bmatrix} I_x I_t \\ I_y I_t \end{bmatrix} \right)$$

obtained from the template alone

I_t is image difference

- This is a closed form solution.
- One important thing → τ
  - We can not determine τ!
  - Thus, we can only solve u/τ and v/τ → velocities
  - i.e., this only provides a direction to search

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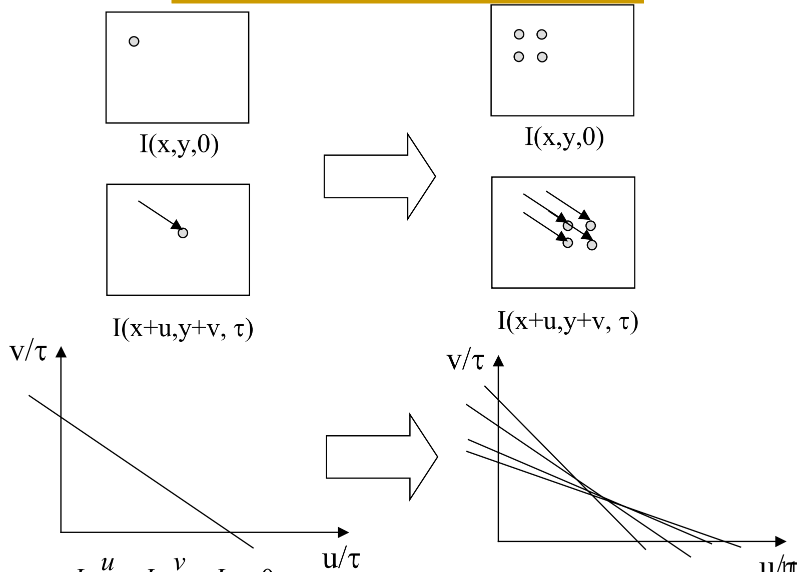
Another explanation



$$I_x \frac{u}{\tau} + I_y \frac{v}{\tau} + I_t = 0$$

Doesn't it look familiar?

A familiar solution

$$\begin{cases} I_{x1}u + I_{y1}v + I_{t1}\tau = 0 \\ I_{x2}u + I_{y2}v + I_{t2}\tau = 0 \\ \vdots \\ I_{xN}u + I_{yN}v + I_{tN}\tau = 0 \end{cases} \Rightarrow \begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \vdots \\ I_{xN} & I_{yN} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\tau \begin{bmatrix} I_{t1} \\ I_{t2} \\ \vdots \\ I_{tN} \end{bmatrix}$$

Sounds familiar?

Ax=b

You can easily figure out the solution now.

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Handling rotation?

- Assume a pure rotation

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

objective function

$$D(\theta) = \sum_x \sum_y \left[I(R(\theta)\begin{bmatrix} x \\ y \end{bmatrix}, \tau) - I(x, y, 0) \right]^2$$

Taylor expansion

$$I(R(\theta)\begin{bmatrix} x \\ y \end{bmatrix}, \tau) = I(x, y, 0) + \frac{\partial I}{\partial \theta}\theta + \frac{\partial I}{\partial t}\tau + o(t^2)$$

where $\frac{\partial I}{\partial \theta} = -\frac{\partial I}{\partial x} y + \frac{\partial I}{\partial y} x = I_\theta$

derivative

$$D(\theta) = \sum_x \sum_y (I_\theta \theta + I_t \tau)^2 \quad \Rightarrow \quad \nabla D(\theta) = \sum_x \sum_y (I_\theta \theta + I_t \tau) I_\theta$$

solution

$$\Rightarrow \quad \theta = -\tau \frac{\sum_x \sum_y I_\theta I_t}{\sum_x \sum_y I_\theta^2}$$

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