Photometric Stereo

Introduction to Computational Photography:

EECS 395/495

Northwestern University

Photometric Stereo

Method for recovering 3D shape information from image intensity (brightness).

Topics:

(1) Gradient Space

(2) Reflectance Map

(3) Photometric Stereo

(4) Calibration-Based Photometric Stereo

(5) Shape from Surface Normal

Image Intensity

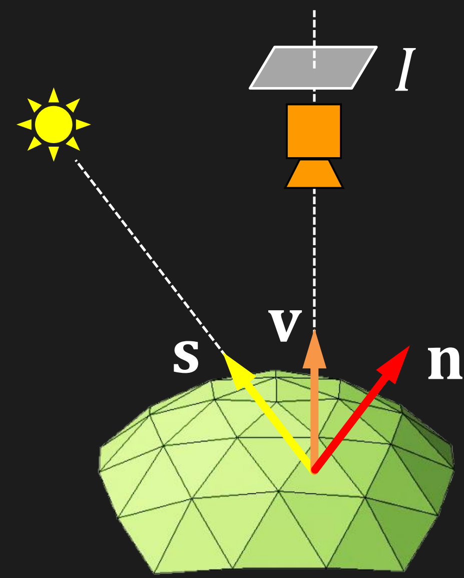


Image Intensity $I = \mathcal{F}$(Source Direction $\mathbf{s}$,
Surface Normal $\mathbf{n}$,
Surface Reflectance)

Photometric Stereo

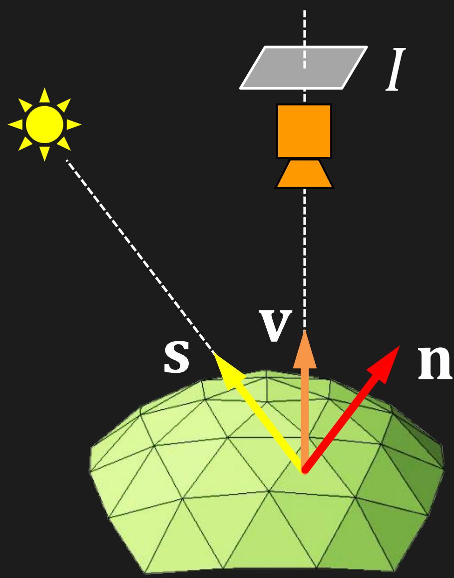


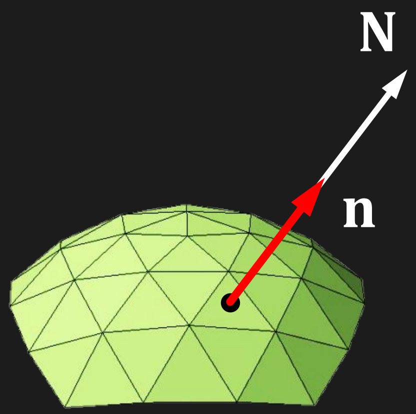
Image Intensity $I = \mathcal{F}$(Source Direction $\mathbf{s}$, GIVEN

GIVEN Surface Normal $\mathbf{n}$, ?

Surface Reflectance) GIVEN

Surface Gradient and Normal

Let $z = f(x, y)$ represent a 3D surface.



Surface Gradient:

$$\left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}\right) = (p, q)$$

In Horn $\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = (p, q)$

Surface Normal:

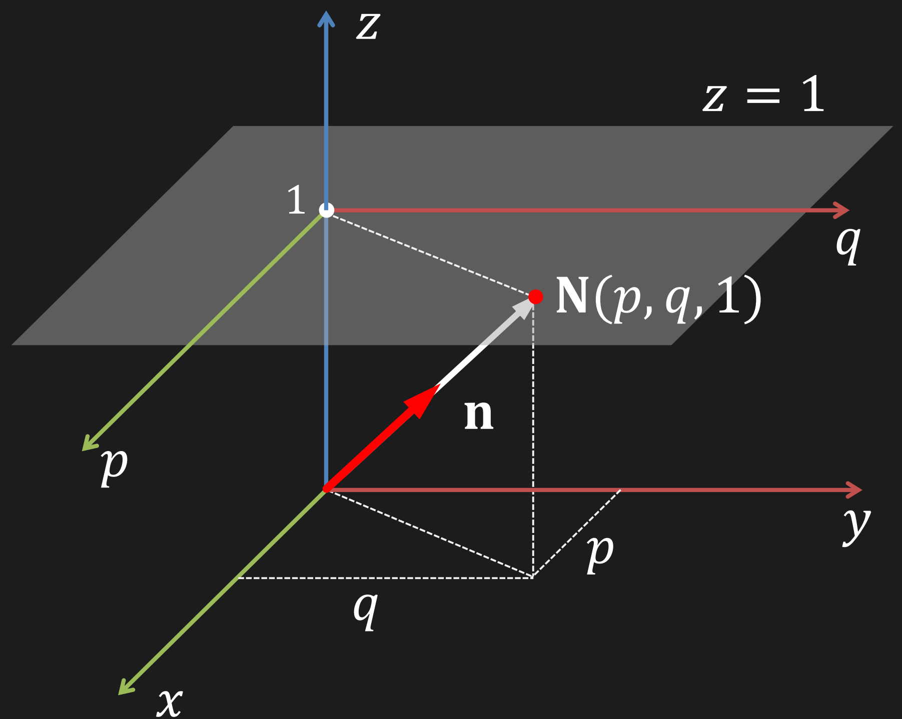
$$\mathbf{N} = \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right) = (p, q, 1)$$

Unit Surface Normal: $\boxed{\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}}$

Surface Normal represented with only two parameters (p, q).

Gradient Space

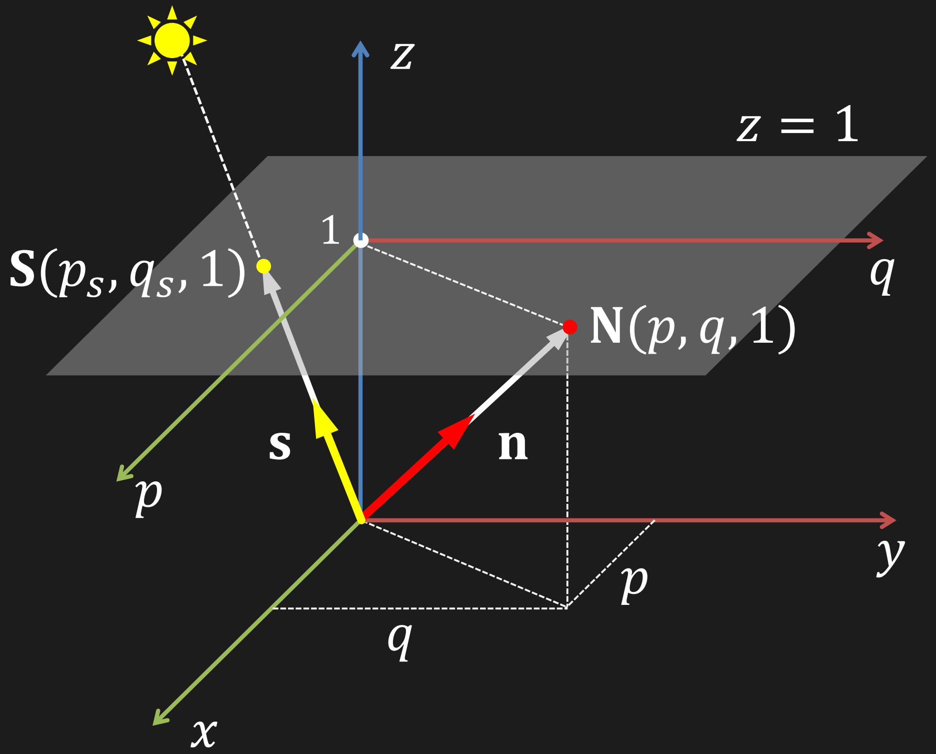
Plane $z = 1$ is called the Gradient Space or pq Plane



Every point (p, q) in the Gradient Space corresponds to a unique orientation.

Gradient Space

Plane $z = 1$ is called the Gradient Space or pq Plane



Surface Normal:

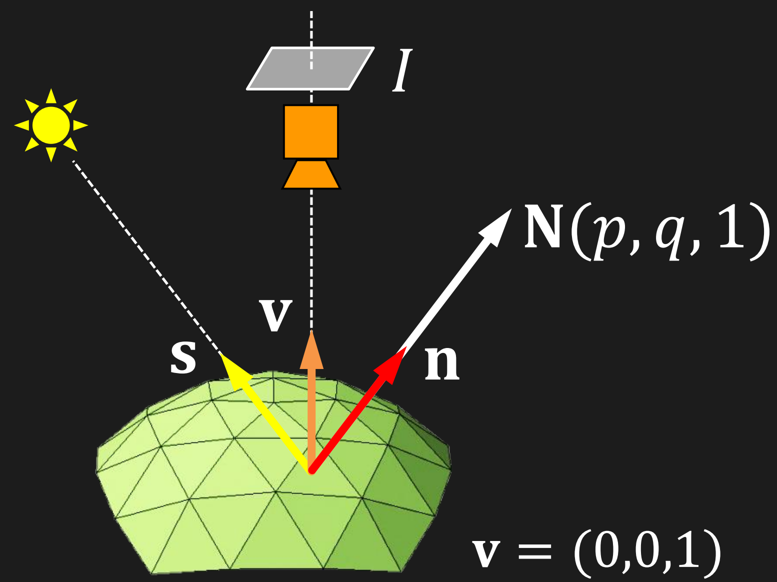
$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source Direction:

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_s, q_s, 1)}{\sqrt{{p_s}^2 + {q_s}^2 + 1}}$$

Every point (p, q) in the Gradient Space corresponds to a unique orientation.

Reflectance Map $R(p, q)$



For a given Source Direction $\boldsymbol{s}$ and Surface Reflectance, Image Intensity at a point (x,y):

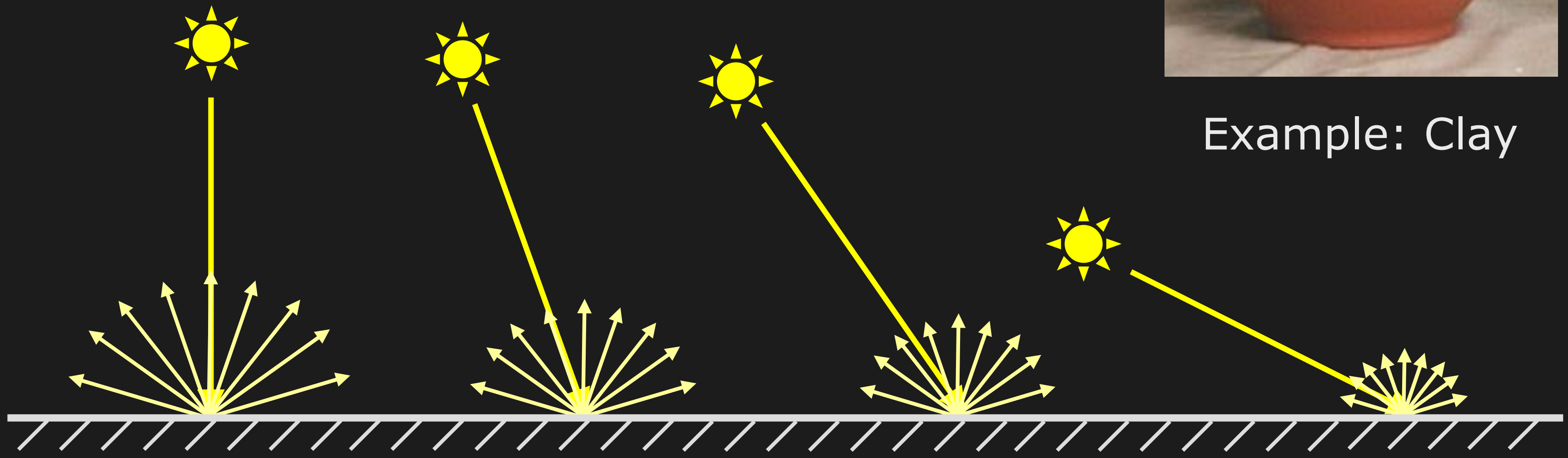
$$\boxed{I = R(p, q)}$$

Reflectance Map

Review: Lambertian Surface

Image irradiance I is independent of viewing direction.

Example: Clay

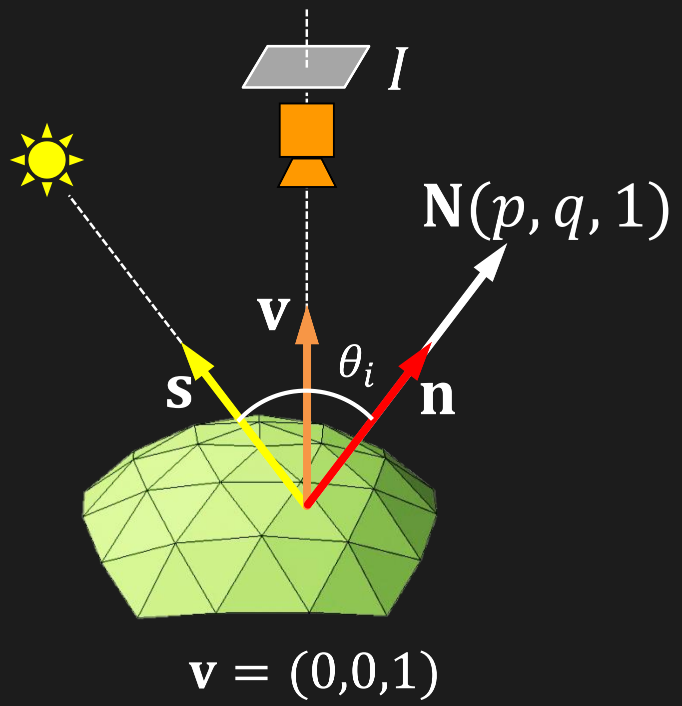


Lambertian (Diffuse or Matte) Surface

Reflectance Map: Lambertian Surface

Image Irradiance:

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \frac{\rho}{\pi} kc(\mathbf{n} \cdot \mathbf{s})$$



where ρ: Surface Albedo (Reflectance)

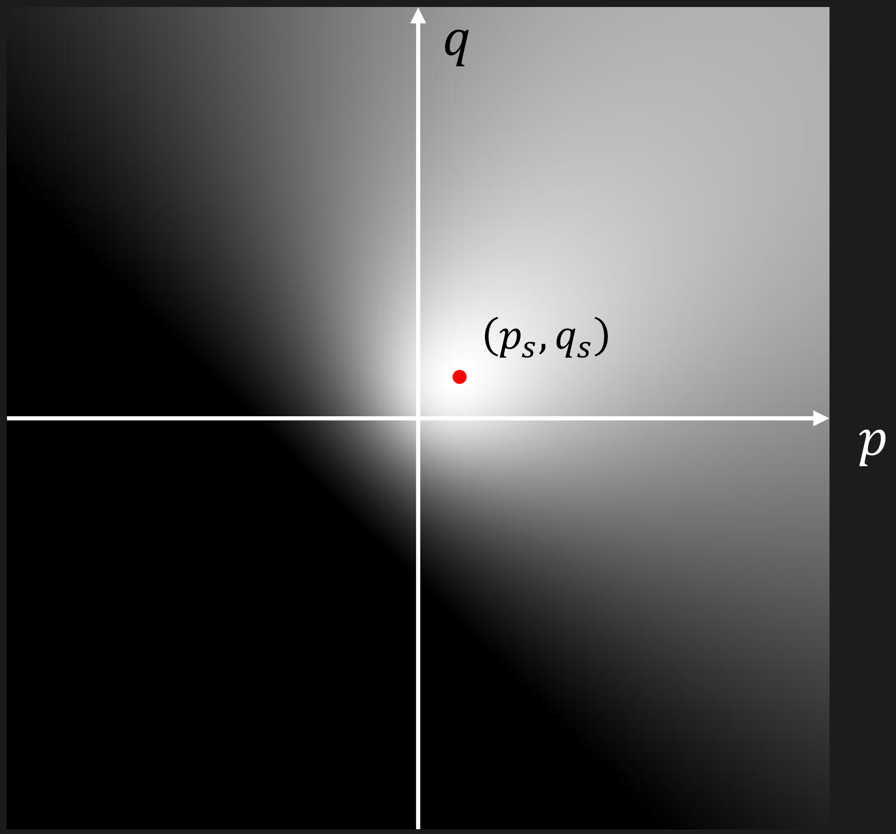
k: Source Brightness

c: Constant (Optical System)

Let $\frac{\rho}{\pi} kc = 1$ then, $\boxed{I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s}}$

Reflectance Map: Lambertian Surface

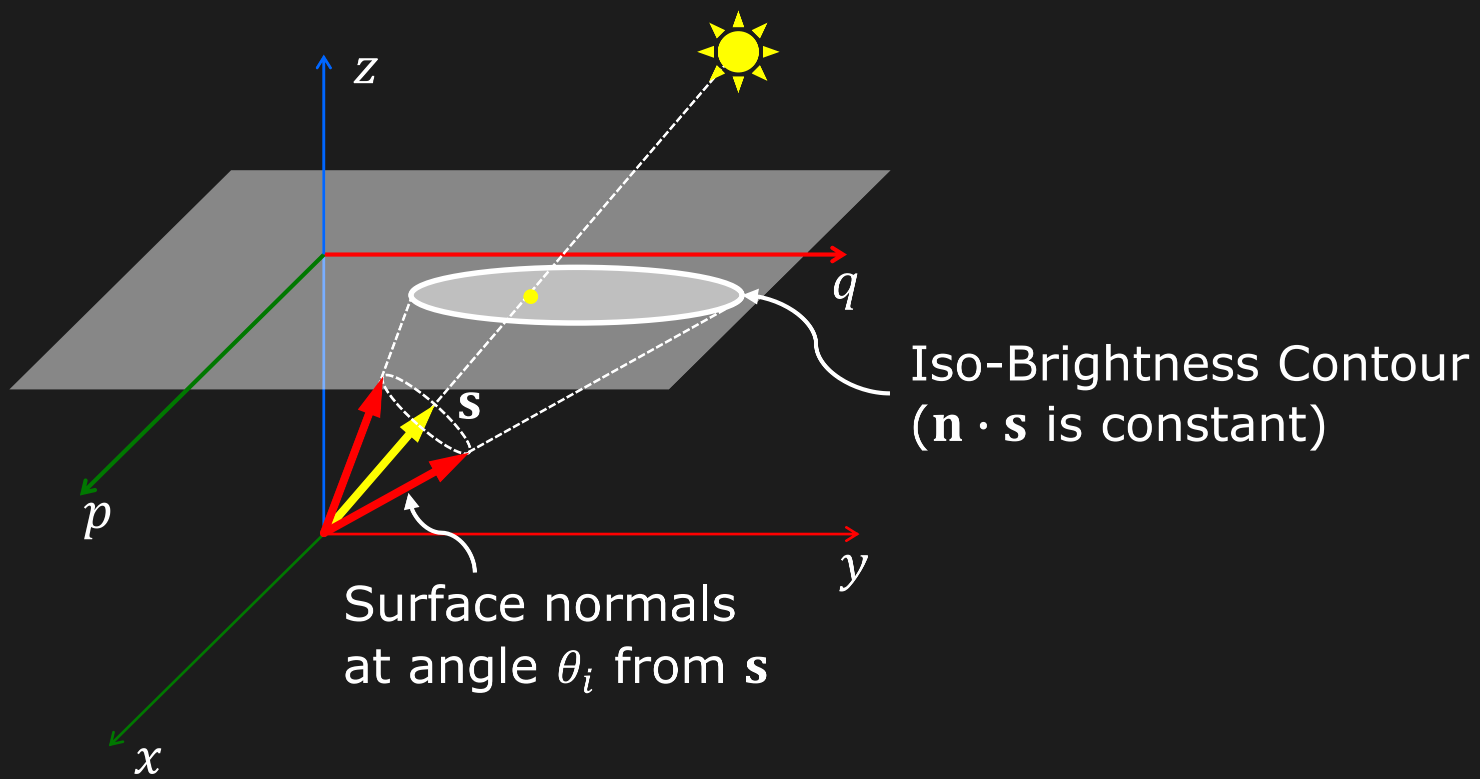
$$I = \mathbf{n} \cdot \mathbf{s} = \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{{p_s}^2 + {q_s}^2 + 1}} = R(p, q)$$



Reflectance Map

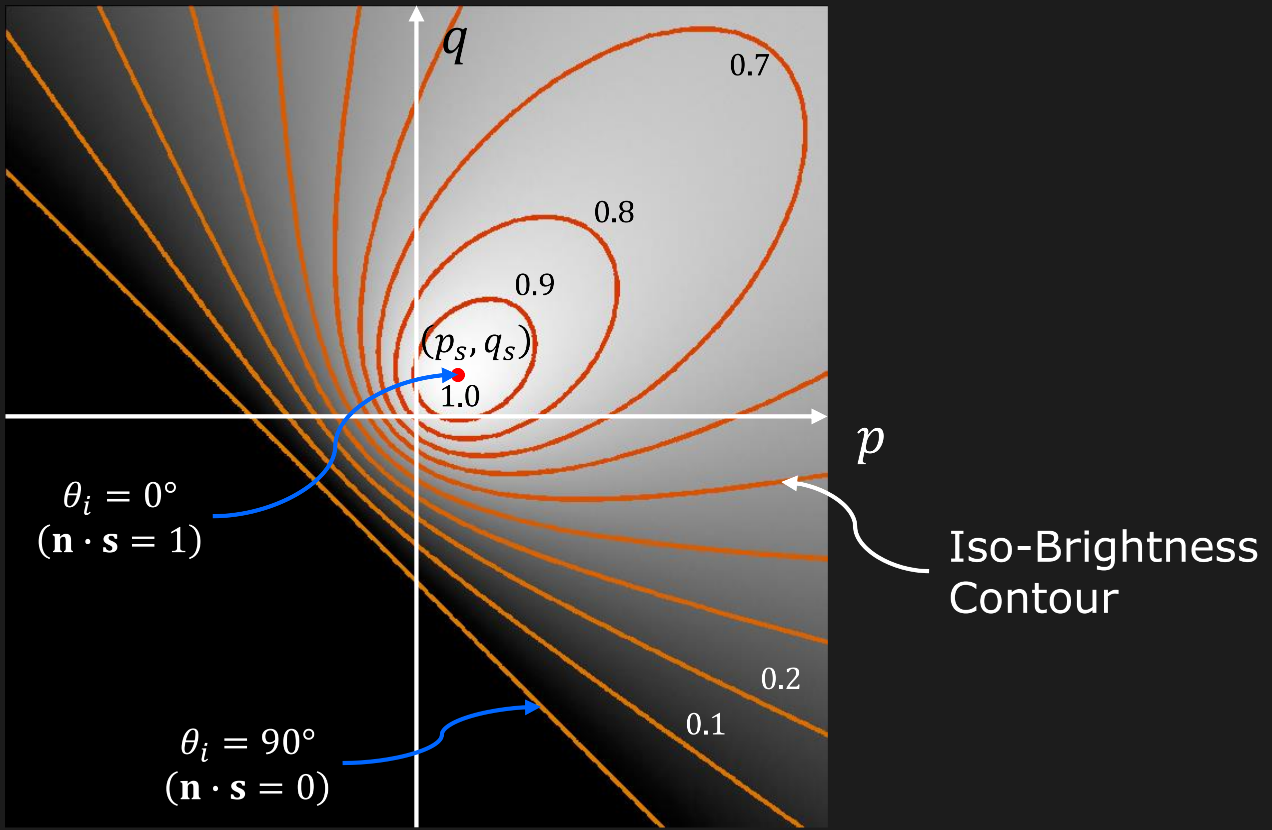
Reflectance Map: Iso-Brightness Contours

$$I = \mathbf{n} \cdot \mathbf{s} = \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{p_s{}^2 + q_s{}^2 + 1}} = R(p, q)$$



Reflectance Map: Lambertian Surface

$$I = \mathbf{n} \cdot \mathbf{s} = \frac{p p_s + q q_s + 1}{\sqrt{p^2 + q^2 + 1}\sqrt{{p_s}^2 + {q_s}^2 + 1}} = R(p, q)$$



Reflectance Map

Shape from a Single Image?

Given Image I, $\underbrace{\text{Source Direction } \mathbf{s} \text{ and Surface Reflectance}}_{\text{Reflectance Map } R(p,q)}$

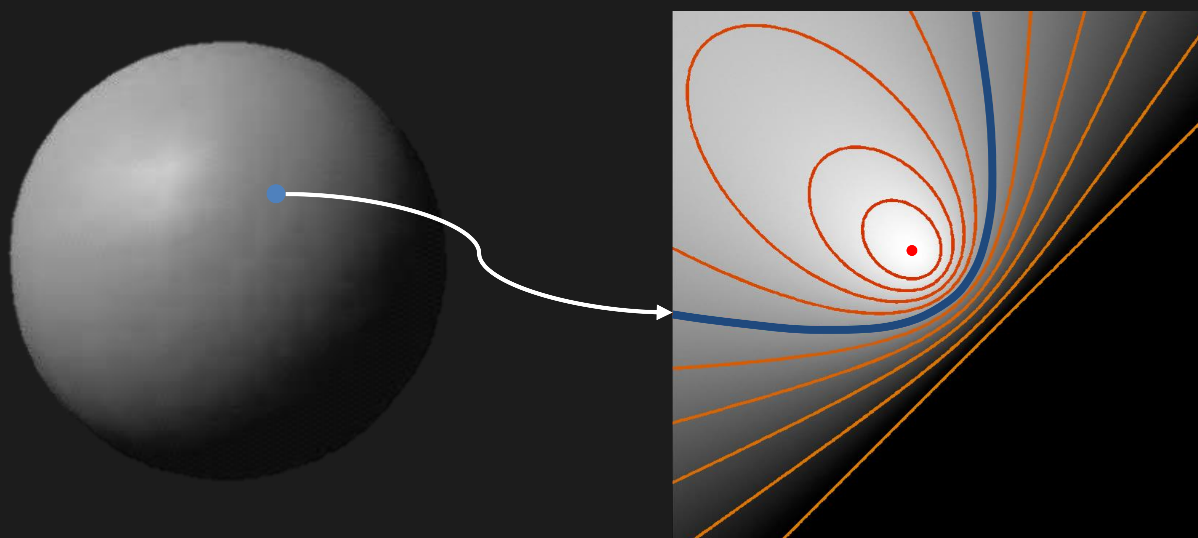


Image I Reflectance Map $R(p,q)$

Can we estimate Surface Gradients (p,q) at each pixel? $\boxed{\text{NO}}$

Intensity at each pixel maps to infinite (p,q) values along the corresponding iso-brightness contour.

Photometric Stereo

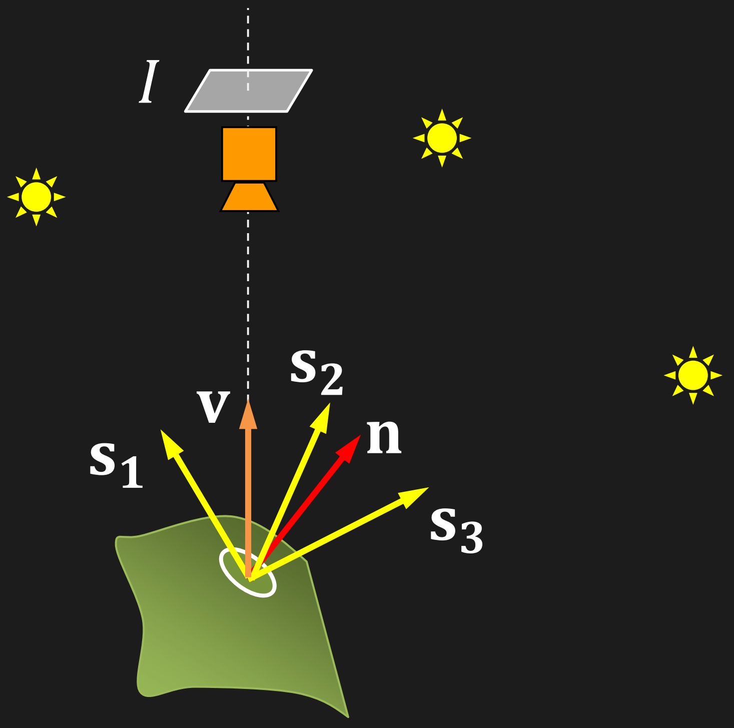
Idea: Use multiple images under different lighting to resolve the ambiguity in surface orientation.

Notation:

Direction of Source i: $\mathbf{s_i} \equiv \left(p_{s_i}, q_{s_i}\right)$

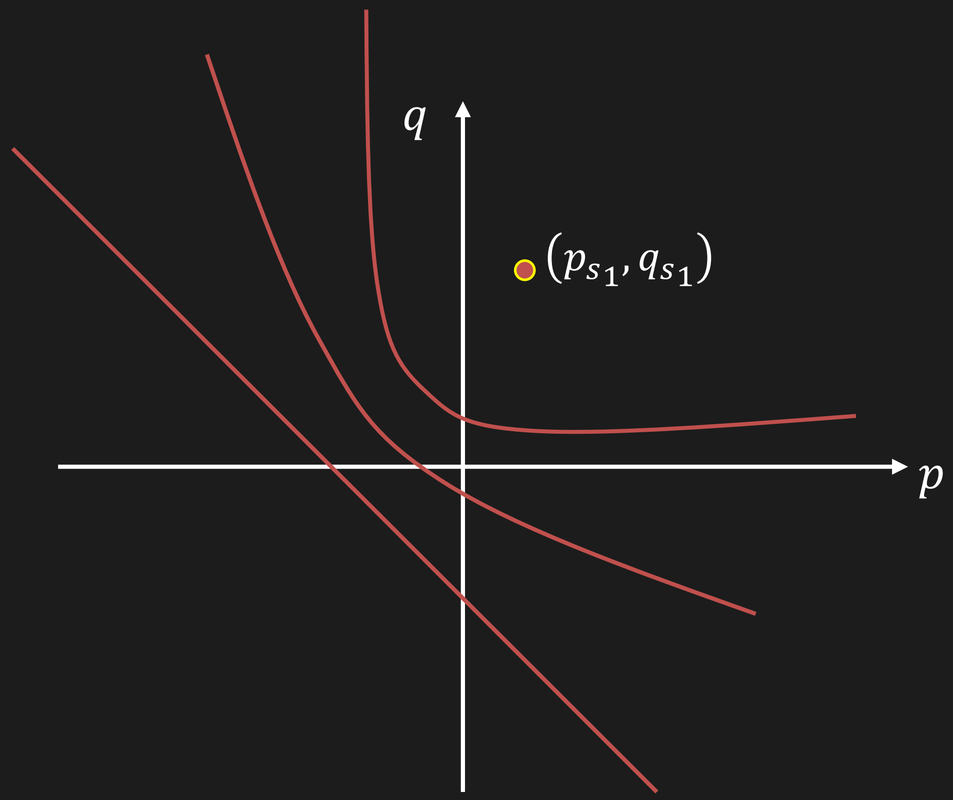
Reflectance Map for Source i: $R_i(p, q)$

Image intensity produced by Source i: $I_{\boldsymbol{i}}(\boldsymbol{x}, \boldsymbol{y})$



Photometric Stereo: Basic Idea

Capture Image I_1 under light source $\mathbf{s}_1(p_{s_1}, q_{s_1})$.



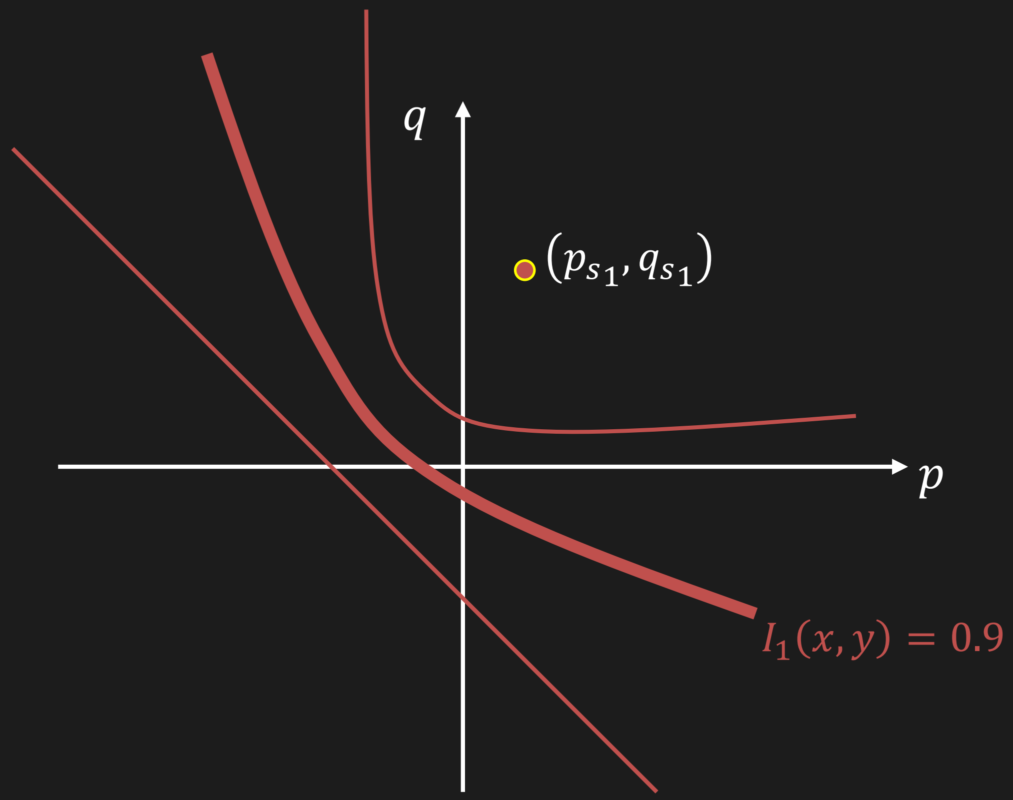
Reflectance Map $R_1(p, q)$

Photometric Stereo: Basic Idea

Capture Image I_1 under light source $\mathbf{s}_1(p_{s_1}, q_{s_1})$.

For Example:

Let $I_1(x, y) = 0.9$



Infinite Solutions Exist for (p, q)

Reflectance Map $R_1(p, q)$

Photometric Stereo: Basic Idea

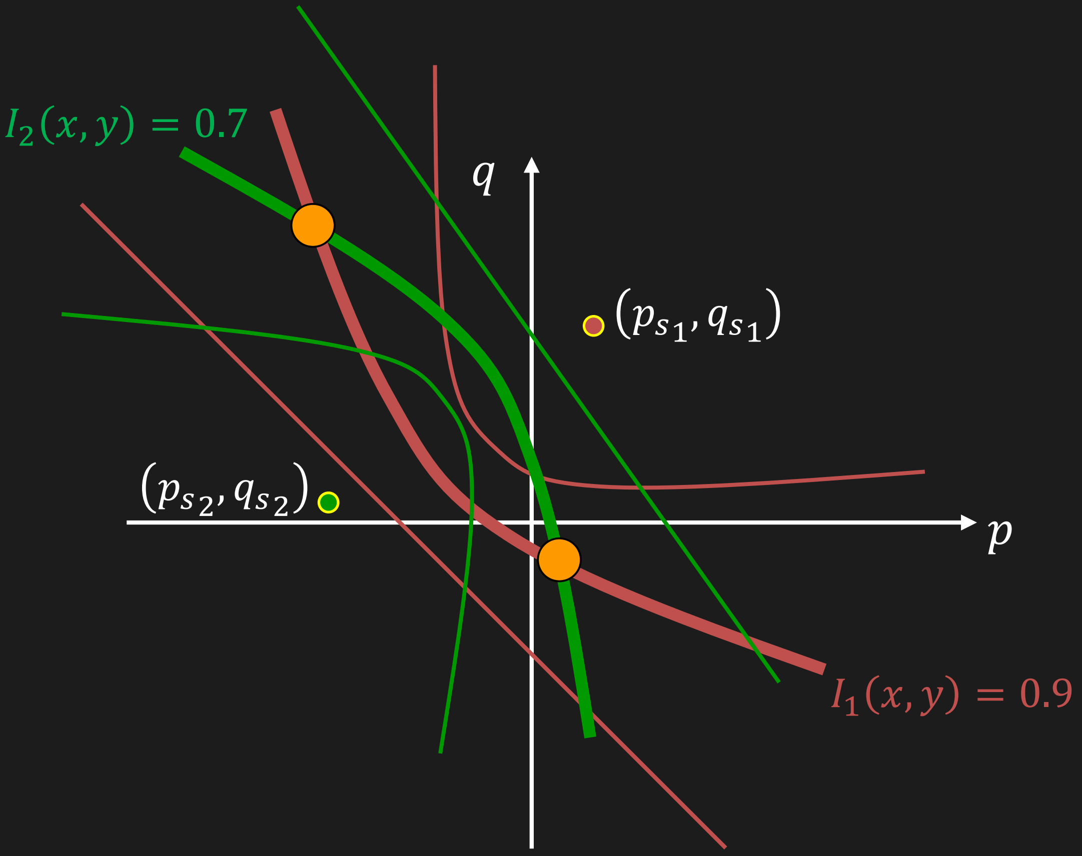
Capture Image I_2 under light source $\mathbf{s}_2(p_{s_2}, q_{s_2})$.

For Example:

Let $I_1(x, y) = 0.9$

$I_2(x, y) = 0.7$

Two Solutions Exist for (p, q)



Reflectance Maps $R_1(p, q)$, $R_2(p, q)$

Photometric Stereo: Basic Idea

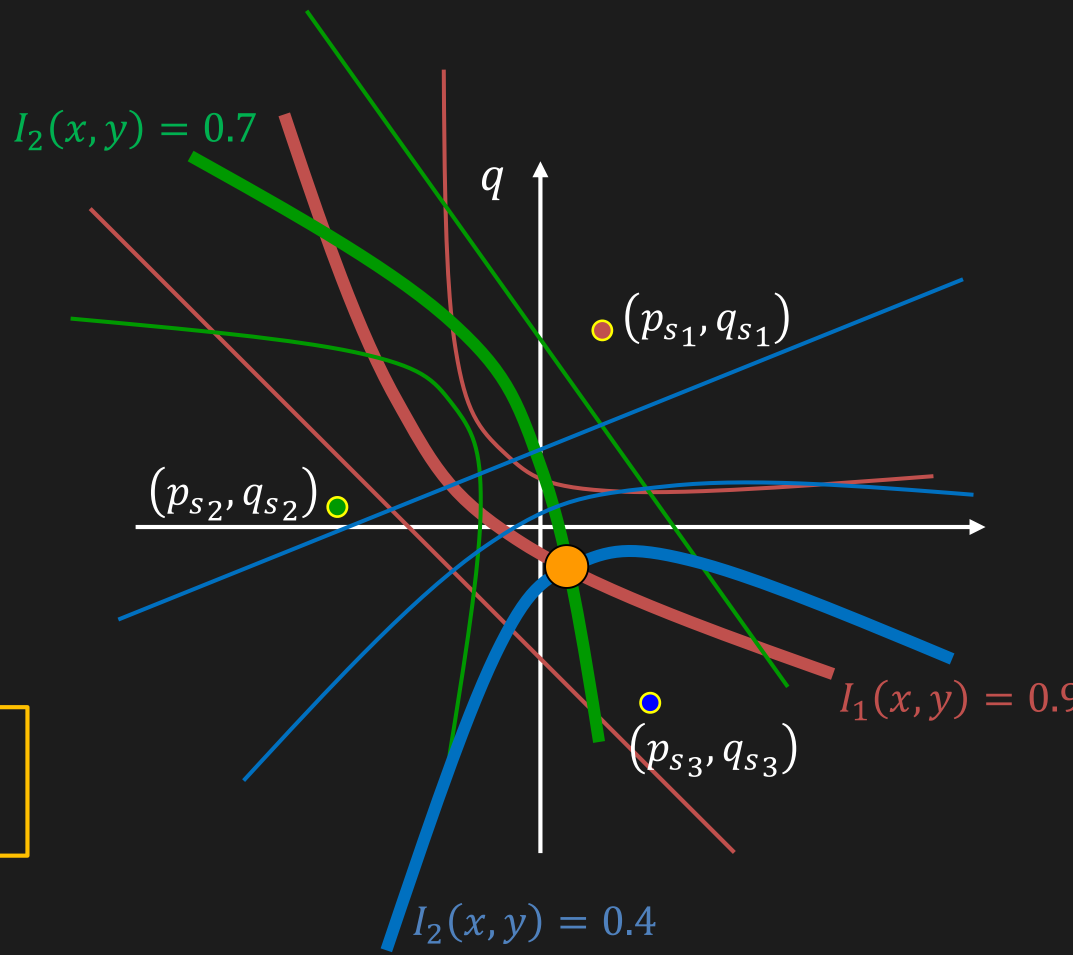
Capture Image I_3 under light source $\mathbf{s}_3(p_{s_3}, q_{s_3})$.

For Example:

Let $I_1(x, y) = 0.9$

$I_2(x, y) = 0.7$

$I_3(x, y) = 0.4$



Unique Solution for Surface Orientation: (p', q')

Reflectance Maps $R_1(p, q)$, $R_2(p, q)$, $R_3(p, q)$

Photometric Stereo: Basic Idea

Step 1: Take K images with K known light sources.

Step 2: Using known source direction and BRDF,
construct reflectance map for each source direction.

Step 3: For each pixel location (x, y), find (p, q) as
the intersection of K reflectance map curves. This is the surface
normal at pixel (x, y).

Smallest K needed depends on the material properties.

Example: $K = 3$ for Lambertian Surfaces.

Photometric Stereo: Lambertian Case

Image Irradiance measured at point (x, y) under each of the three light sources:

$$I_1 = \frac{\rho}{\pi}\mathbf{n}\cdot\mathbf{s_1} \qquad I_2 = \frac{\rho}{\pi}\mathbf{n}\cdot\mathbf{s_2} \qquad I_3 = \frac{\rho}{\pi}\mathbf{n}\cdot\mathbf{s_3}$$

where: $\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ and $\mathbf{s}_i = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$

We can write this in matrix format.

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\text{Measured}} = \frac{\rho}{\pi} \underbrace{\begin{bmatrix} s_{x_1} & s_{y_1} & s_{z_1} \\ s_{x_2} & s_{y_2} & s_{z_2} \\ s_{x_3} & s_{y_3} & s_{z_3} \end{bmatrix}}_{S_{3\times 3}\ \text{(Known)}} \mathbf{n}$$

Photometric Stereo: Lambertian Case

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{\rho}{\pi} \underbrace{\begin{bmatrix} s_{x_1} & s_{y_1} & s_{z_1} \\ s_{x_2} & s_{y_2} & s_{z_2} \\ s_{x_3} & s_{y_3} & s_{z_3} \end{bmatrix}}_{S_{3\times3}\ \text{(Known)}} \mathbf{n} \Rightarrow \left\{ \begin{array}{l} I = S\mathbf{M} \\ \text{where: } \mathbf{M} = \frac{\rho}{\pi}\mathbf{n} \end{array} \right.$$

Solution: $\mathbf{M} = (S)^{-1} I$

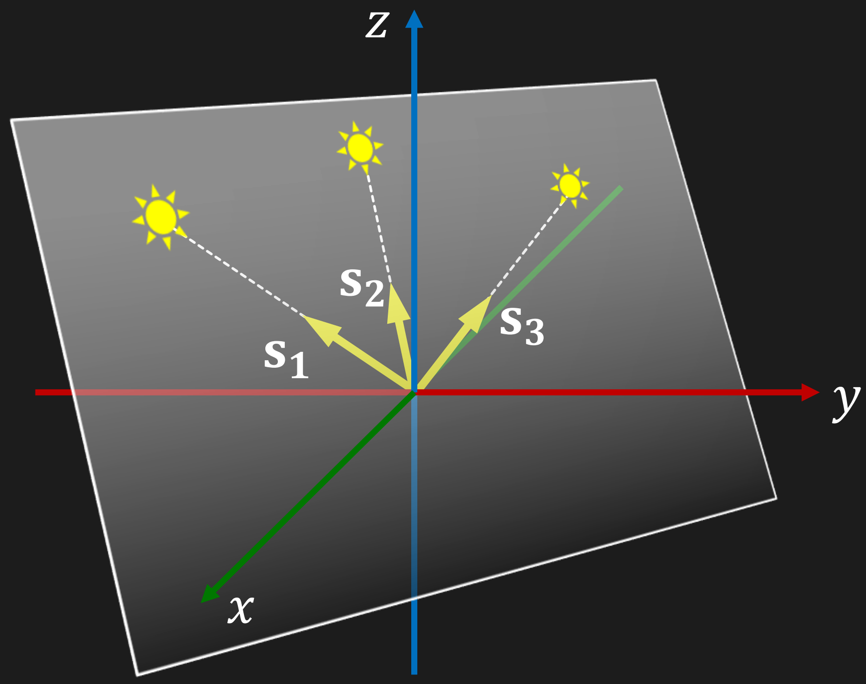
Surface Normal: $\mathbf{n} = \frac{\mathbf{M}}{|\mathbf{M}|}$ Albedo: $\frac{\rho}{\pi} = |\mathbf{M}|$

When Does It Not Work?

When $S_{3\times 3}$ is not invertible.

That is, when one source direction can be represented as a linear combination of the other two.

$$\mathbf{s_3} = \alpha \mathbf{s_1} + \beta \mathbf{s_2}$$



All sources and the origin lie on a plane

Photometric Stereo: Lambertian Case

Get better results by using more $(K > 3)$ light sources

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_K \end{bmatrix}}_{I_{K\times 1}} = \frac{\rho}{\pi} \underbrace{\begin{bmatrix} s_{x_1} & s_{y_1} & s_{z_1} \\ s_{x_2} & s_{y_2} & s_{z_2} \\ \vdots & \vdots & \vdots \\ s_{x_K} & s_{y_K} & s_{z_K} \end{bmatrix}}_{S_{K\times 3}} \mathbf{n} \Rightarrow \begin{cases} I = S\mathbf{M} \\ \text{where: } \mathbf{M} = \dfrac{\rho}{\pi}\mathbf{n} \end{cases}$$

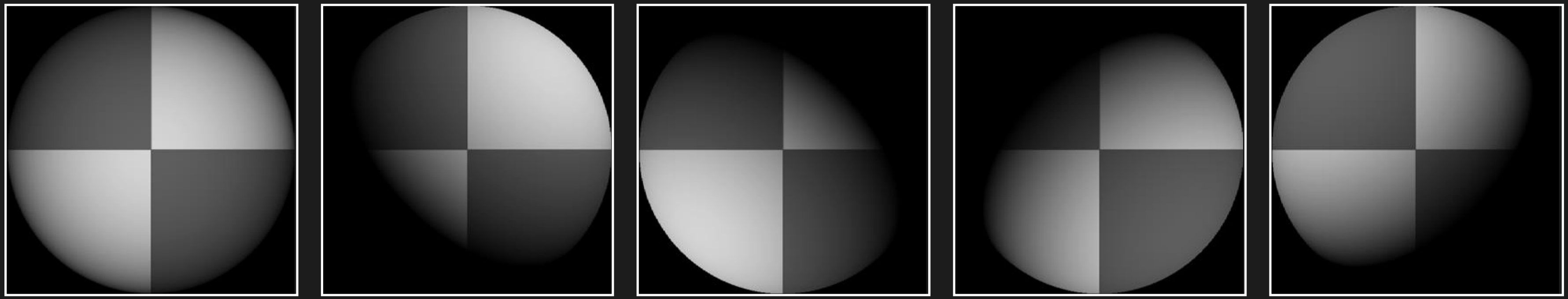
$S_{K\times 3}$ is not a square matrix and hence not invertible.

Solution: Use Least Squares Estimation

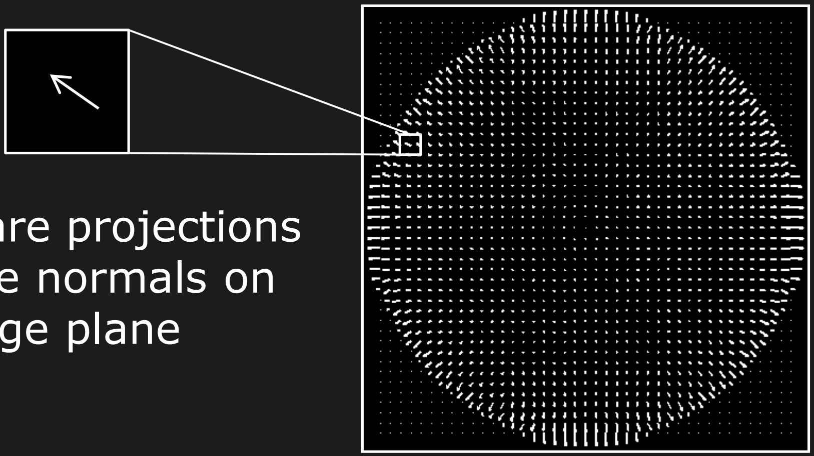
$$S^T I = S^T S \mathbf{M}$$

$$\mathbf{M} = \underbrace{(S^T S)^{-1}}_{3\times 3} S^T I \quad \text{(Pseudo-inverse)}$$

Results: Lambertian Sphere

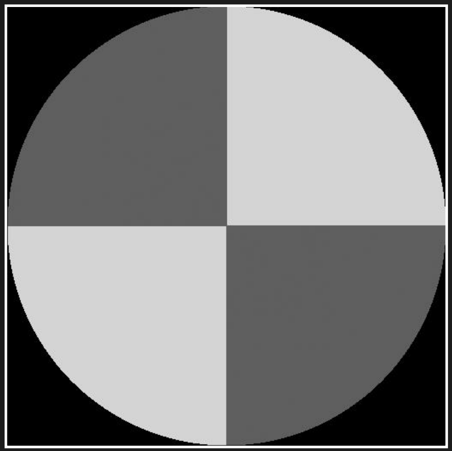


Input Images



Needles are projections of surface normals on image plane

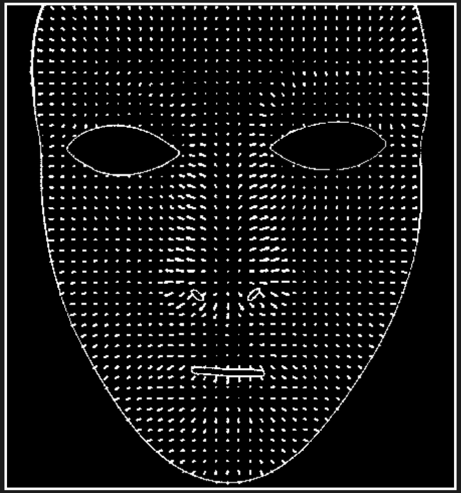
Estimated Surface Normals



Estimated Albedo

Results: Lambertian Mask

Input Images



Estimated Surface Normals

Estimated Albedo

Results: Lambertian Toy

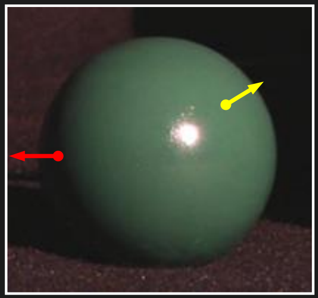
Input Images

Estimated Surface Normals

Estimated Albedo

Calibration-based Photometric Stereo

Use a Calibration Object (ex: Sphere) of Known Size, Shape and Same Reflectance as the scene objects.



Calibration Sphere

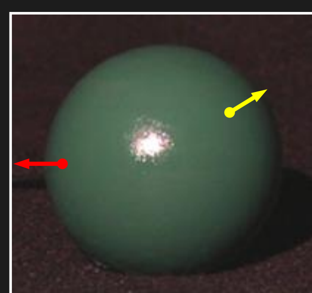
Scene

Orientation Consistency: Points with the same surface normal produce the same set of intensities under different lighting.

[Silver 1980]

Calibration-based Photometric Stereo

Use a Calibration Object (ex: Sphere) of Known Size, Shape and Same Reflectance as the scene objects.



Calibration Sphere

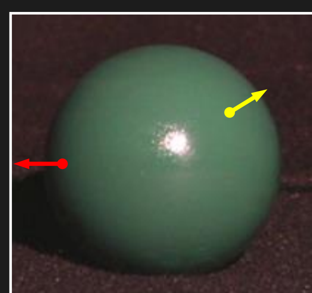
Scene

Orientation Consistency: Points with the same surface normal produce the same set of intensities under different lighting.

[Silver 1980]

Calibration-based Photometric Stereo

Use a Calibration Object (ex: Sphere) of Known Size, Shape and Same Reflectance as the scene objects.



Calibration Sphere

Scene

Orientation Consistency: Points with the same surface normal produce the same set of intensities under different lighting.

[Silver 1980]

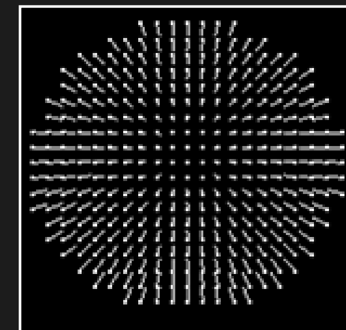
Calibration Procedure

Image 1

Image 2

…

Image K



Surface Normals
$(p, q, 1)$

Step 1: Capture $K \geq 3$ images under K different light sources.

Each point on the sphere produces K image intensities $(I_1, I_2, \dots, I_K)$ corresponding to the K light sources.

Step 2: Using the known size of the sphere, estimate the surface normal $(p, q, 1)$ for every point on the sphere.

[Silver 1980]

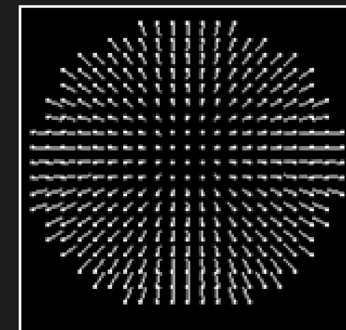
Calibration Procedure

Image 1

Image 2

…

Image K



Surface Normals $(p, q, 1)$

Step 3: Create a lookup table for the K-tuple: $(I_1, I_2, \dots, I_K) \rightarrow (p, q)$

Populate the lookup table with $(I_1, I_2, \dots, I_K)$ and (p, q) for each pixel on the sphere.

I_1	I_2	…	I_K	p	q

[Silver 1980]

Looking Up Surface Normal

Step 4: Capture K images of the scene object under the same K light sources.

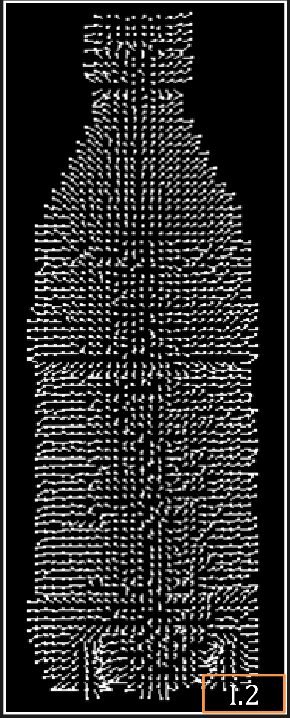
Step 5: For each pixel in the scene, use Lookup Table to map $(I_1, I_2, \ldots, I_K) \rightarrow (p, q)$

Image 1

Image 2

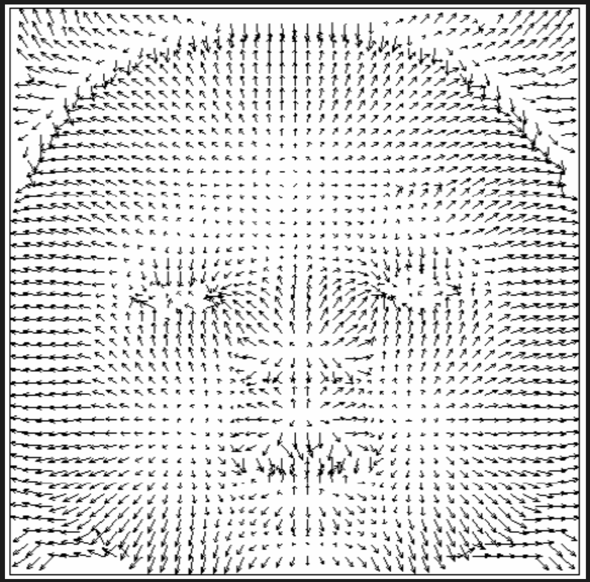
...

Image K



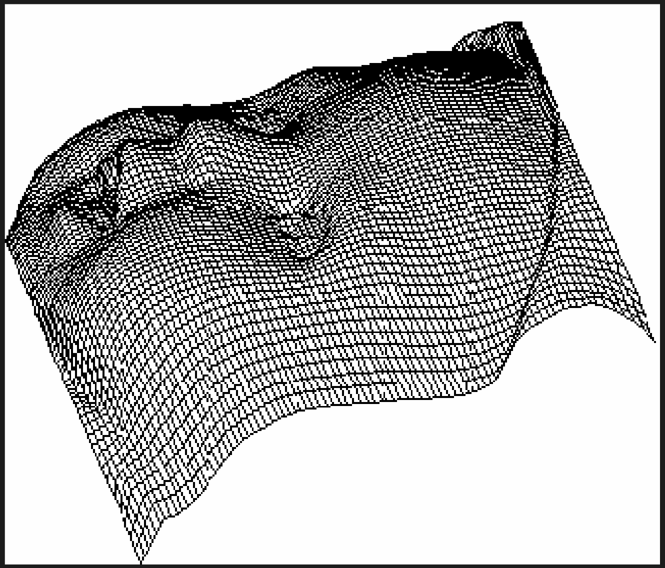
Estimated Surface Normals

Shape From Surface Normals



Differentiation

Integration



Gradient/Normal Map
$[p(x,y), q(x,y), 1]$

Shape or
Depth Map (z)

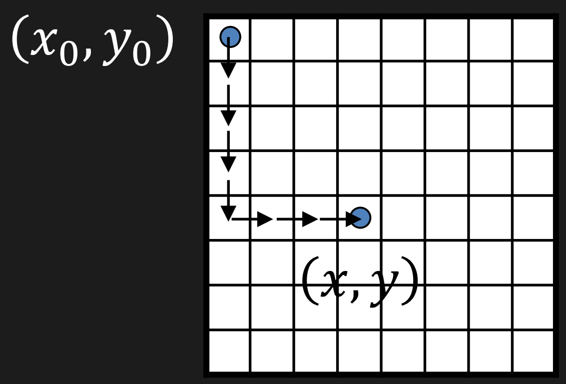
Shape From Surface Normals

Estimate surface by integrating surface gradient

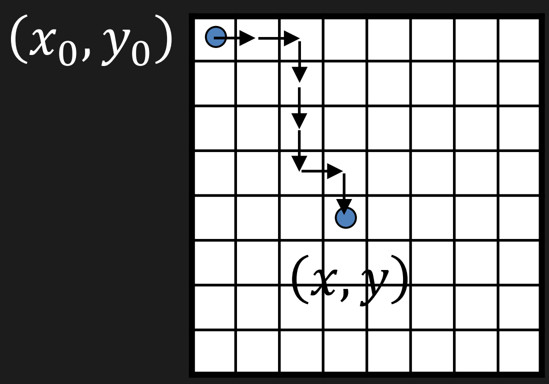
$$\boxed{z(x,y) = z(x_0,y_0) + \oint_{(x_0,y_0)}^{(x,y)} -(p\,dx + q\,dy)}$$

where (x_0, y_0) is a any reference point and $z(x_0, y_0) = 0$.

$z(x, y)$ obtained by integration along any path from (x_0, y_0).



or



Naïve Algorithm for Estimating Shape

1. Initialize reference depth

   $z(0,0) = 0$

2. Compute depth for first column

   for $y = 1$ to $(H-1)$

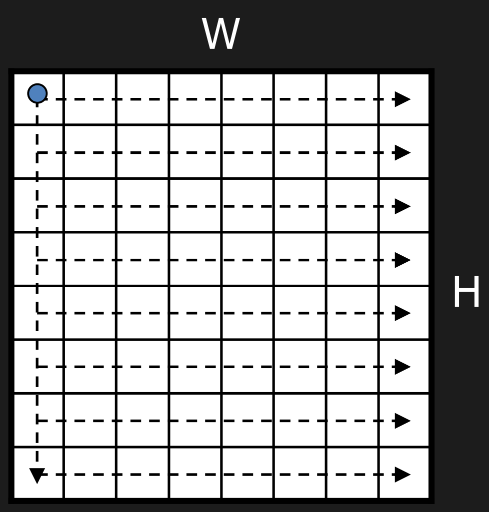
   $z(0,y) = z(0,y-1) - q(0,y)$

3. Compute depth for each row

   for $y = 0$ to $(H-1)$

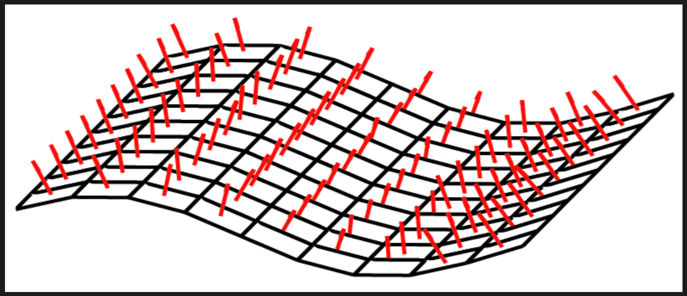
   for $x = 1$ to $(W-1)$

   $z(x,y) = z(x-1,y) - p(x,y)$

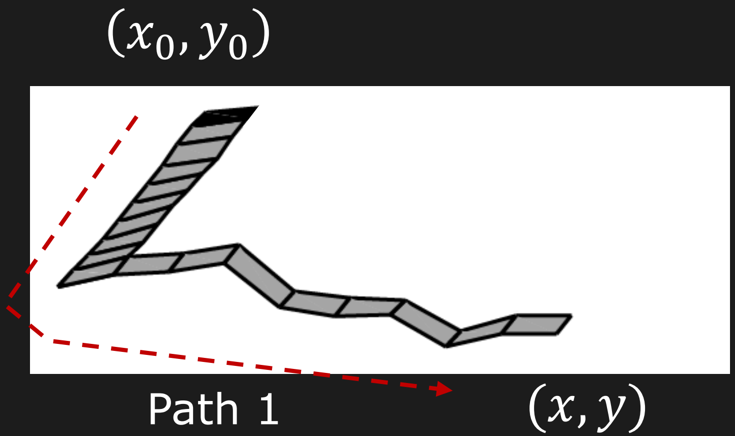


Computed Depth Map

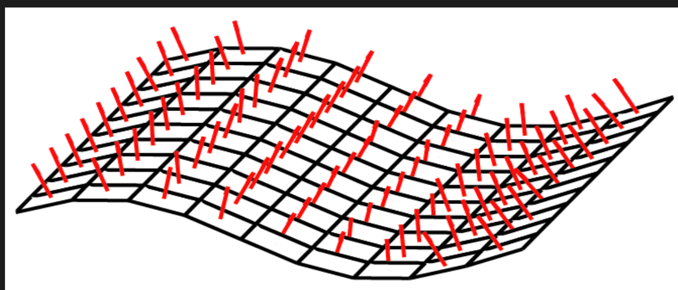
Noise Sensitivity of Computed Shape



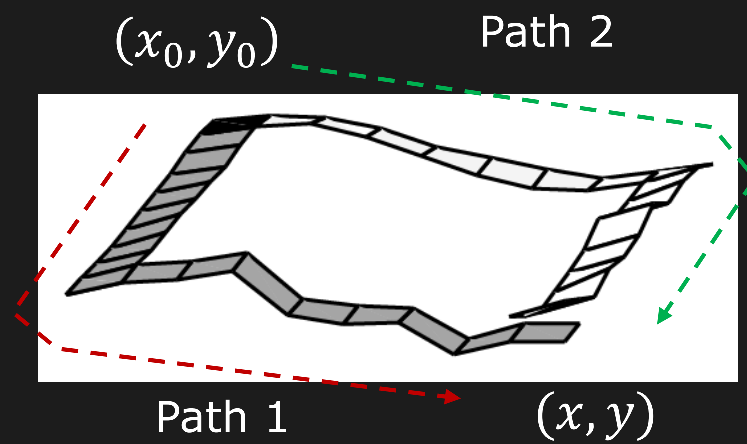
Actual Surface shown with noisy estimates of surface gradient



Noise Sensitivity of Computed Shape



Actual Surface shown with noisy estimates of surface gradient



Depth computed from noisy gradients depends on the integration path.

One Solution: Compute depth maps using different paths. Then, find average of computed depth maps to reduce error.

Estimating Shape Using Least Squares

Minimize the errors between measured surface gradients (p, q) and surface gradients of estimated surface $z(x, y)$.

Error Measure:

$$D = \iint_{Image} \left(\frac{\partial z}{\partial x} + p\right)^2 + \left(\frac{\partial z}{\partial y} + q\right)^2 dxdy$$

where $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ are gradients of the estimated surface.

We need to find $z(x, y)$ that minimizes D

[Horn 1990]

Frankot-Chellappa Algorithm

Minimize objective function D in Fourier Domain.

Let $Z(u,v)$, $P(u,v)$ and $Q(u,v)$ be the Fourier Transforms of $z(x,y)$, $p(x,y)$ and $q(x,y)$, respectively. Then:

$$z(x,y) = \iint_{-\infty}^{\infty} Z(u,v) e^{i2\pi(ux+vy)} du dv$$

$$p(x,y) = \iint_{-\infty}^{\infty} P(u,v) e^{i2\pi(ux+vy)} du dv$$

$$q(x,y) = \iint_{-\infty}^{\infty} Q(u,v) e^{i2\pi(ux+vy)} du dv$$

Substitute for $z(x,y)$, $p(x,y)$ and $q(x,y)$ in equation for D.

[Frankot 1988]

Frankot-Chellappa Algorithm

Find $Z(u,v)$ that minimizes D using $\frac{\partial D}{\partial Z} = 0$.

Solution: $$\tilde{Z}(u,v) = \frac{iuP(u,v) + ivQ(u,v)}{u^2 + v^2}$$

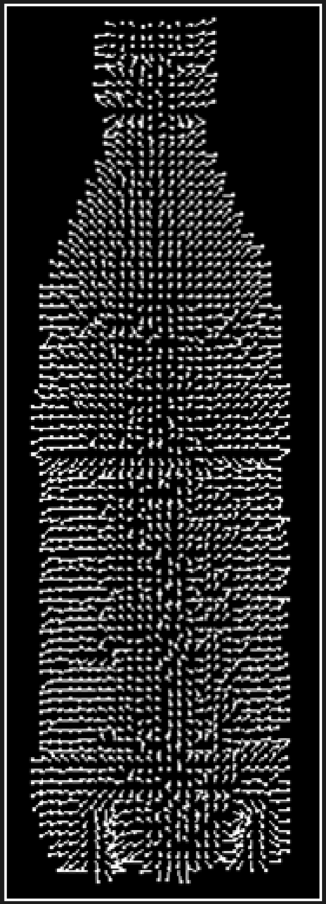
This is the Fourier Transform of the best fit surface.

Compute Inverse Fourier Transform to obtain $\tilde{z}(x,y)$.

$$\tilde{Z}(u,v) \longrightarrow \boxed{\text{IFT}} \longrightarrow \tilde{z}(x,y)$$

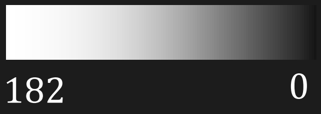
[Frankot 1988]

Results: 3D Surface Reconstruction



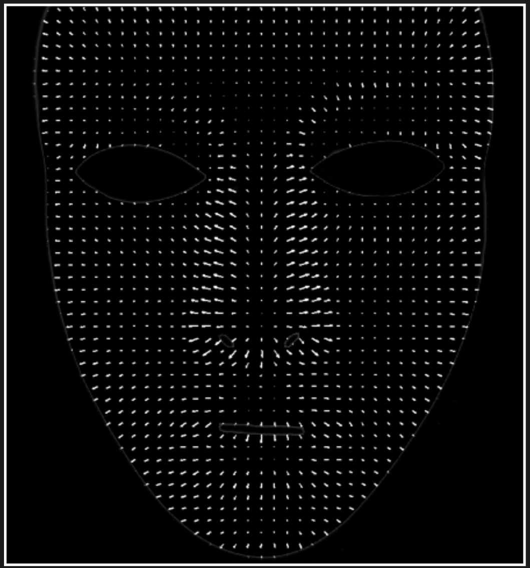
Surface Normals

Estimated Depth Map
$z = f(x, y)$



Estimated Surface
(Rendered)

Results: 3D Surface Reconstruction

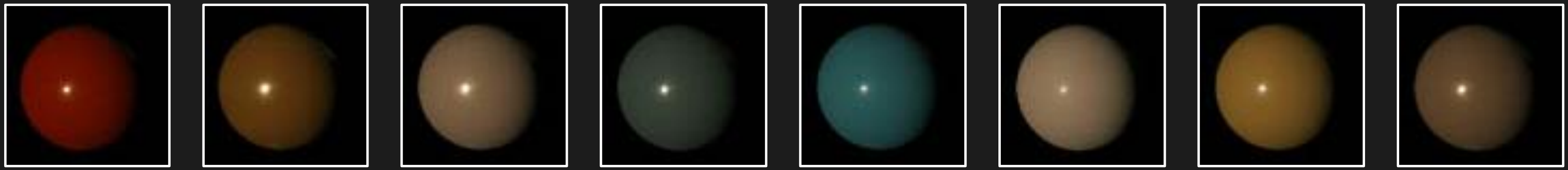


Surface Normals

Estimated Depth Map
$z = f(x, y)$

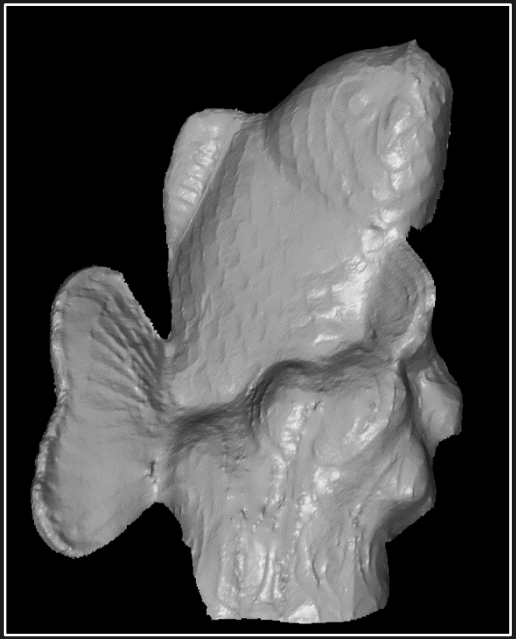
Estimated Surface
(Rendered)

Results: Fish Toy



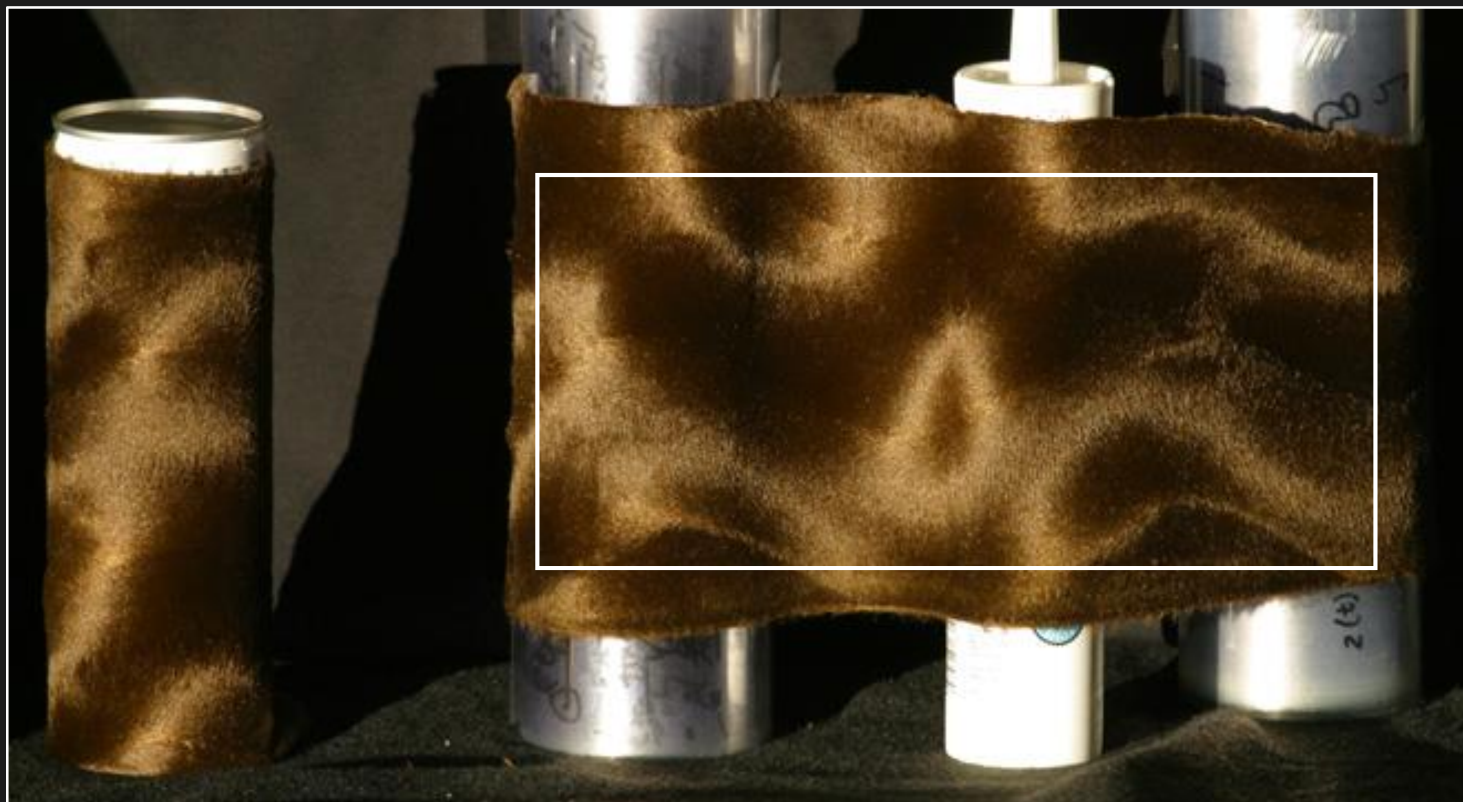
Calibration Spheres

Scene



Estimated Surface
(Rendered)

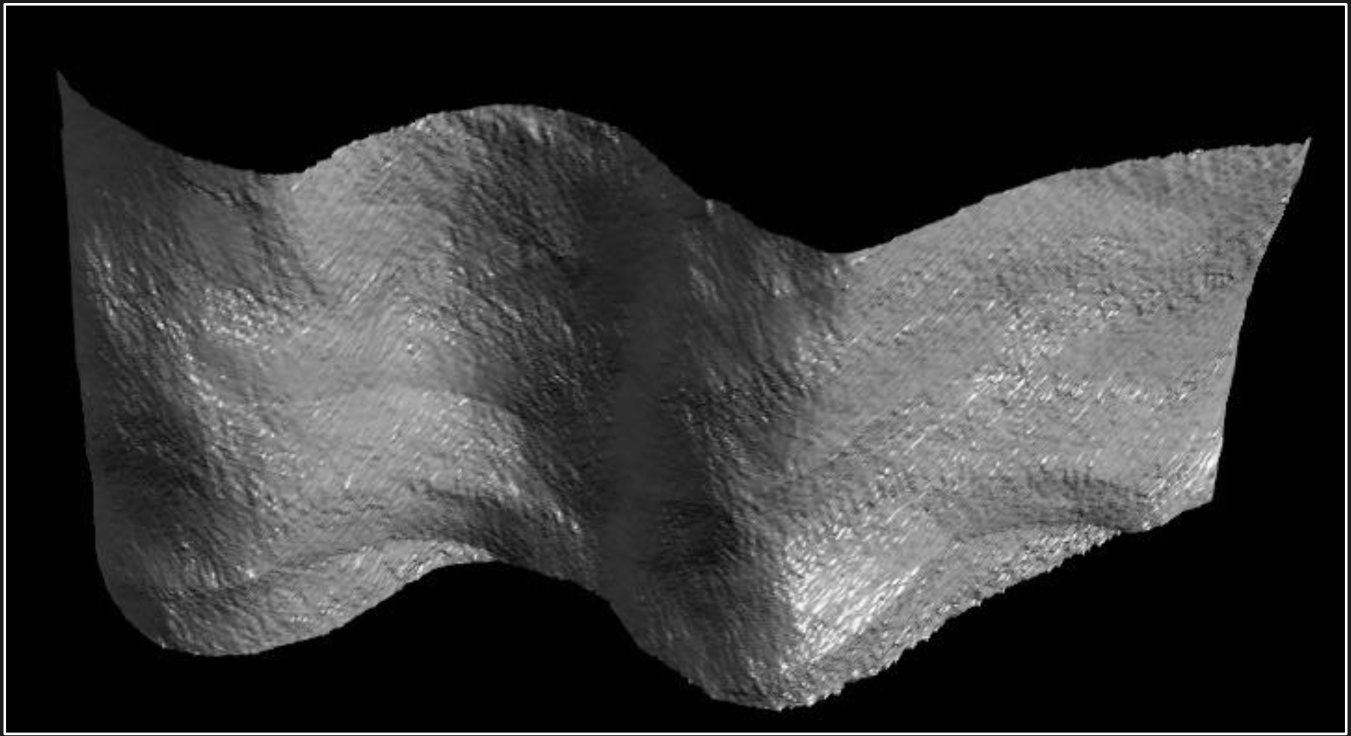
Results: Brushed Fur



Calibration Cylinder

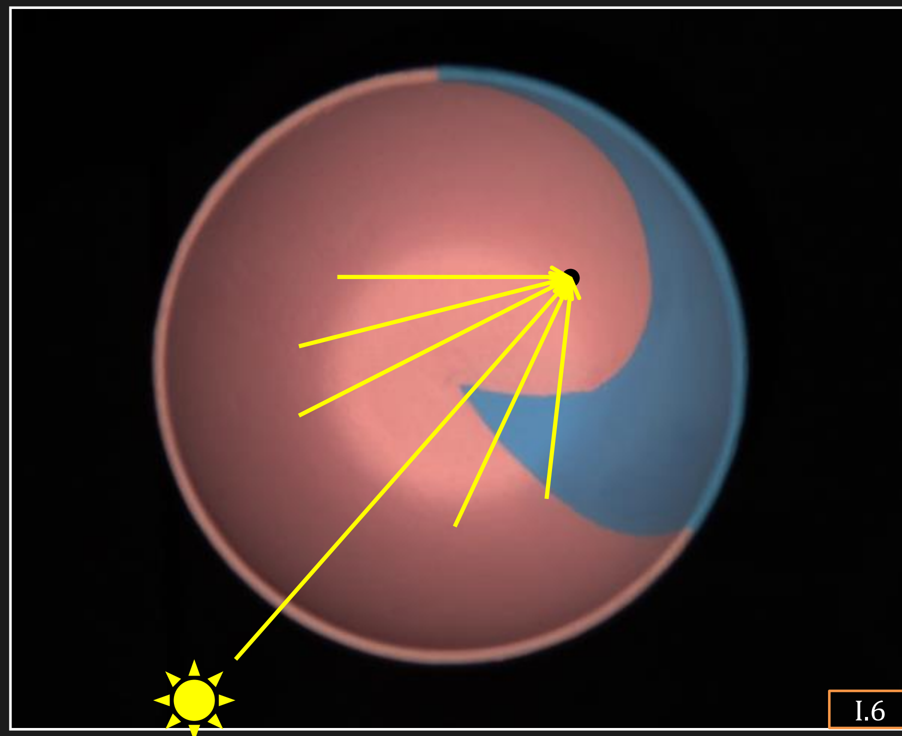
Scene

Photometric Stereo Results: Brushed Fur



Estimated Surface
(Rendered)

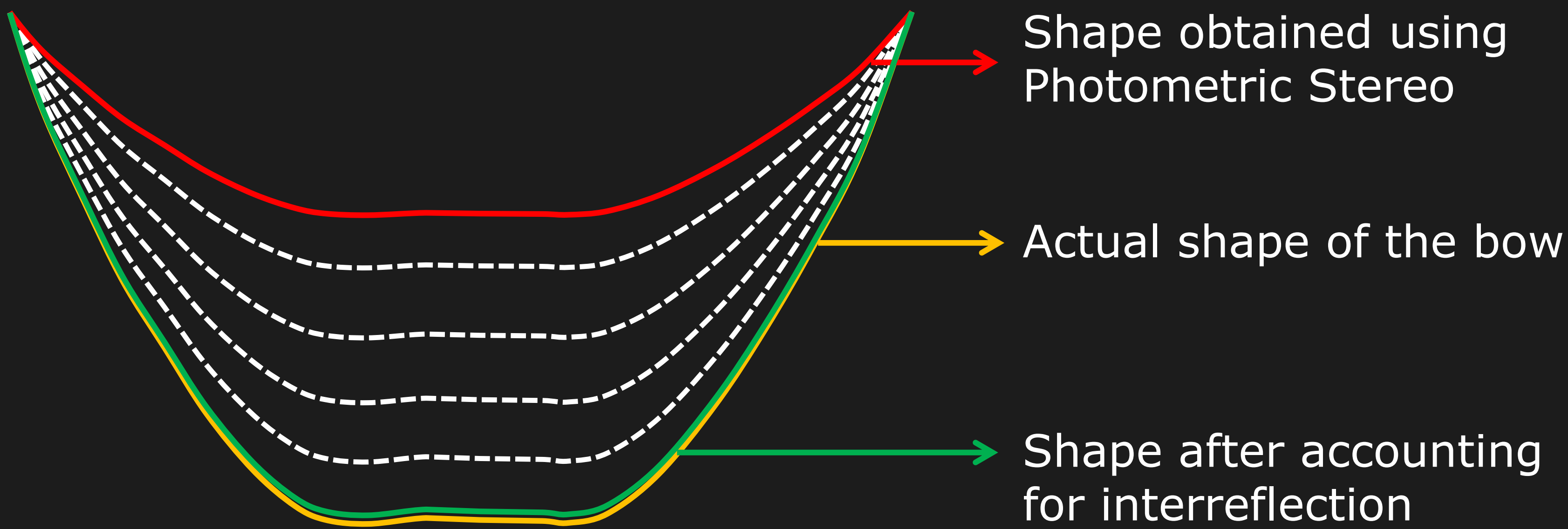
Interreflection Problem



Brightness of a scene point is not only due to the light source but also due to light reflected by other scene points.

Photometric Stereo provides incorrect shape!

Shape from Interreflection



Iterative Estimation of Bowl Depth

[Nayar 1991]

References: Textbooks

Textbooks:

Robot Vision (Chapter 10)
Horn, B. K. P., MIT Press

Computer Vision: A Modern Approach (Chapter 5)
Forsyth, D and Ponce, J., Prentice Hall

References: Papers

[Frankot 1988] R. Frankot and R. Chellappa. “A Method for Enforcing Integrability in Shape from Shading Algorithm.” IEEE PAMI, 1988.

[Hertzmann 2005] A. Hertzmann, S. M. Seitz. “Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs.” IEEE PAMI, 2005.

[Horn 1990] Horn, B.K.P. “Height and Gradient from Shading,” IJCV, 1990.

[Nayar 1991] S.K. Nayar, K. Ikeuchi and T. Kanade. "Shape from Interreflections.“ IJCV, 1991.

[Silver 1980] W.M. Silver. “Determining Shape and Reflectance Using Multiple Images.” Master’s thesis, MIT, Cambridge, Mass., 1980.

[Woodham 1980] R. Woodham, “Photometric Method for Determining Surface Orientation from Multiple Images.” Optical Engineering, 1980.

Image Credits

I.1 Adapted from Forsyth, D and Ponce, J., Computer Vision: A Modern Approach (Chapter 5), Prentice Hall.

I.2 http://www.cs.washington.edu/education/courses/cse455/04wi/projects/project3/project3.htm

I.3 http://grail.cs.washington.edu/projects/sam/

I.4 http://grail.cs.washington.edu/projects/sam/

I.5 http://grail.cs.washington.edu/projects/sam/

I.6 http://www1.cs.columbia.edu/CAVE/projects/separation/photometric_stereo_gallery.php