EECS332 Digital Image Analysis

Hough Transform

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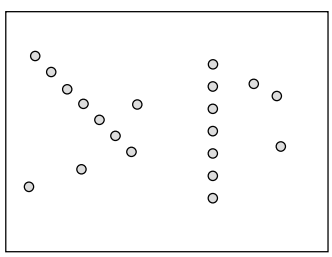
Motivations

- In our previous lecture on line fitting, we have a hidden assumption:
  - All the edge points belong to the same shape
- What if we don't have such an assumption?
  - We need to "detect" a shape
  - Let's work on the simplest case: line detection



- How many lines are there?
- Where are they?

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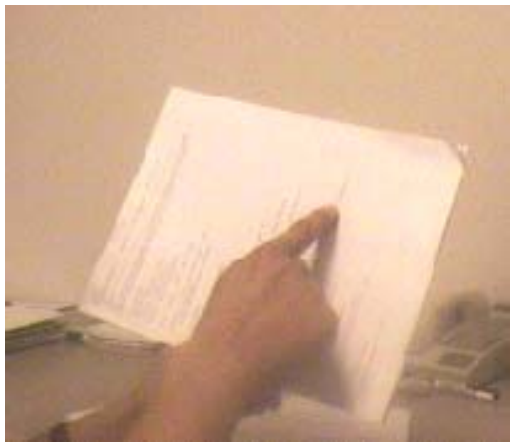
A real example



Can you find the "panel"?

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Outline

- Motivation
- Basic idea
- Implementation
- Extensions

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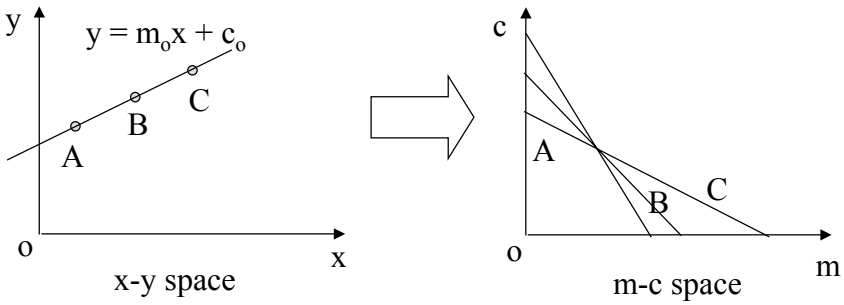
Basic Idea



$y = m_o x + c_o = f(x; m_o, c_o)$

$c = -xm + y = f(m; x, y)$

$y_A = m_o x_A + c_o$

$y_B = m_o x_B + c_o$

$y_C = m_o x_C + c_o$

$c_o = - x_A m_o + y_A$

$c_o = - x_B m_o + y_B$

$c_o = - x_C m_o + y_C$

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Basic idea

- A point (x_k, y_k) in x-y space is mapped to a line in m-c space, i.e., $c=(-x_k)m+y_k$
- A set of co-linear points in x-y space is mapped to a set of lines intersecting at a particular point (m,c)!
- We call m-c space as the "parameter space"
- This is like the voting in a parameter space

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Basic algo

- Edge detection → E
- ∀(x,y) ∈ E
  - Draw a line c=-xm+y in the m-c plane
  - i.e., if the line passes through (m,c), A[m,c] ++
- Find local maxima of A → lines in E

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Are we done?

- Of course, we need to quantize the parameter space for implementation
- m?
- c?
- Do we expect any problem?

- What about a vertical line? m=∞
- Solution?

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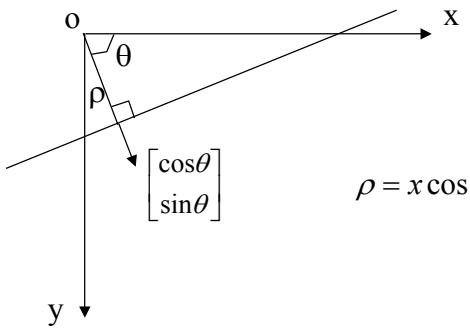
Using polar form



$$\rho = x\cos\theta + y\sin\theta = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}^T \begin{bmatrix} x \\ y\end{bmatrix}$$

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Q1: range of ρ and θ?

- Before we can quantize the parameter space, we need to know the range of the parameters

$$-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$$

$$-\sqrt{R^2 + C^2} \le \rho \le \sqrt{R^2 + C^2}$$

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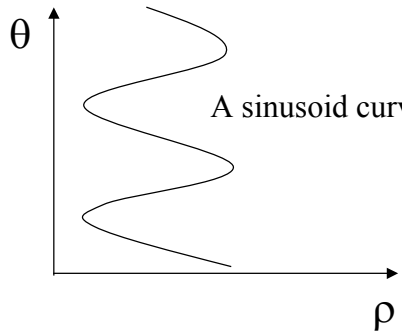
Q2: what is the curve in ρ-θ space?

- An edge point in x-y space corresponds (or vote) a curve in the ρ-θ space
- So, what does such a curve look like?



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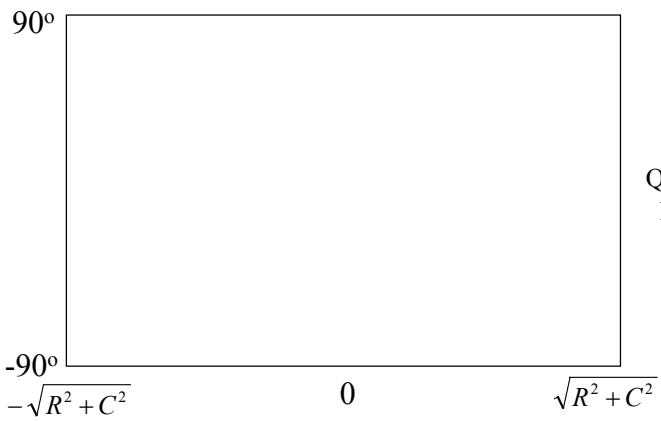
Q3: computation?

- How is the computation?



# of edge points

$K_\rho \times K_\theta \times N$

Quantization level for ρ

Quantization level for θ

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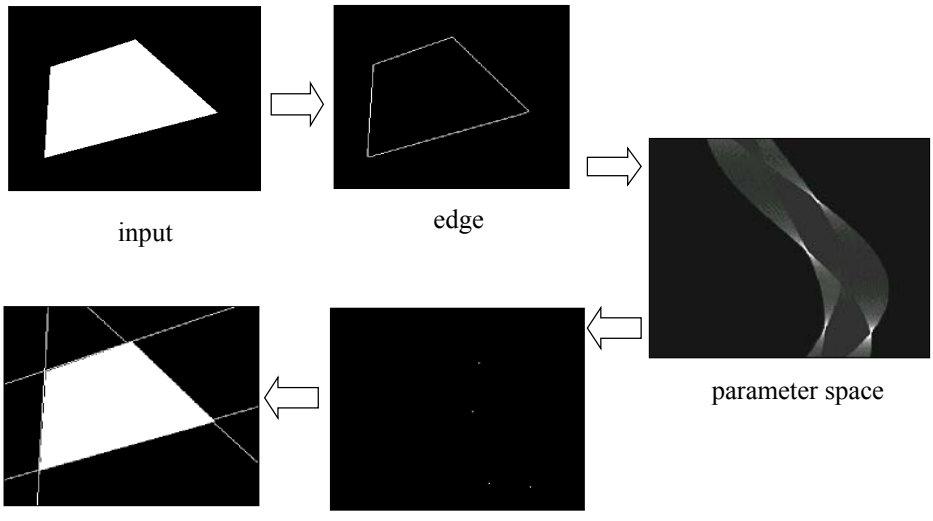
Example 1:



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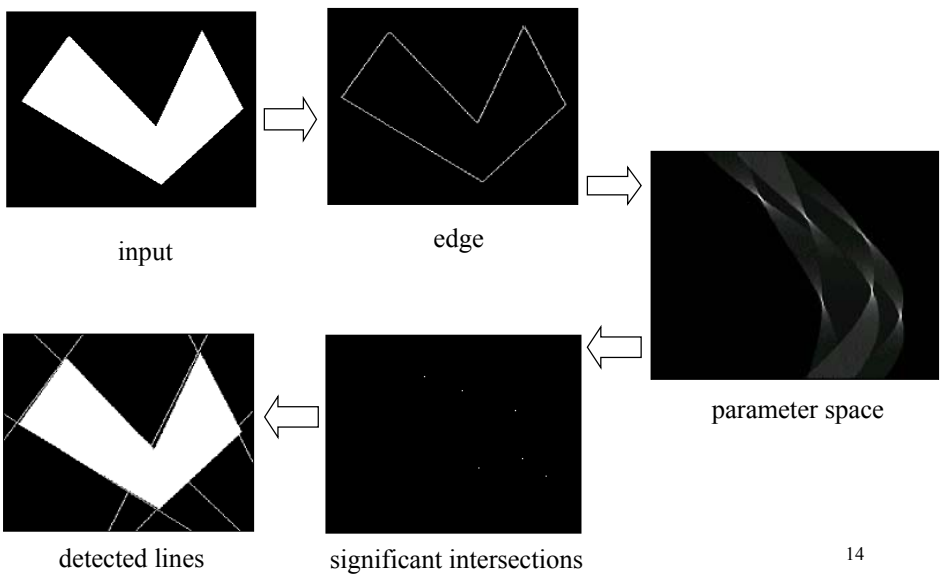
Example 2:



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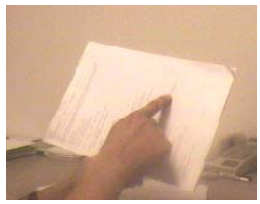
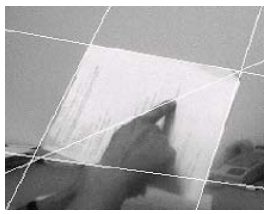
Example 3:



input

edge

parameter space



detected lines

significant intersections

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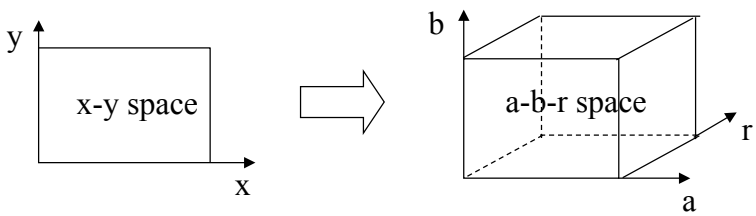
Extension

- Can we use this idea to detect a circle?

$$(x-a)^2 + (y-b)^2 = r^2$$



It is doable, but …

✓ The # of cells grows exponentially with the dim of the parameter space!

✓ Too much computation ← an inherent problem of Hough transform

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Play a trick

- But for circle detection, we can play a trick:
  - Using the direction of an edge
  - s.t., to reduce the parameter space
- Edge detection
  - We have both edge magnitude & direction!

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Trick

$$(x-a)^2+(y-b)^2=r^2$$

$$\Rightarrow \begin{cases} x=a+r\cos\theta \\ y=b+r\sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} a=x-r\cos\theta \\ b=y-r\sin\theta \end{cases}$$

$$\xrightarrow{\text{eliminate r}} \quad b=a\tan\theta-x\tan\theta+y$$

Edge detector gives us (x, y, tanθ)!

x-y- θ space ←→ a-b space

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