Shape From Shading

Introduction to Computational Photography:

EECS 395/495

Northwestern University

Shape From Shading

Method for recovering 3D shape information from a single image using shading.

Topics:

(1) Human Perception of Shape

(2) Stereographic Projection

(3) Shape From Shading Constraints

(4) Numerical Shape from Shading

Review: Shape from a Single Image?

Given Image I, Source Direction $\mathbf{s}$ and Surface Reflectance…

$$\underbrace{\text{Source Direction } \mathbf{s} \text{ and Surface Reflectance}}_{\text{Reflectance Map } R(p,q)}$$

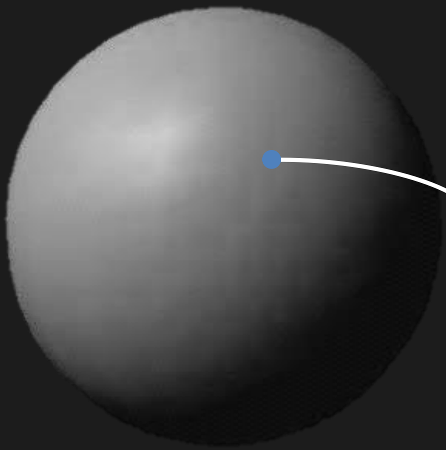
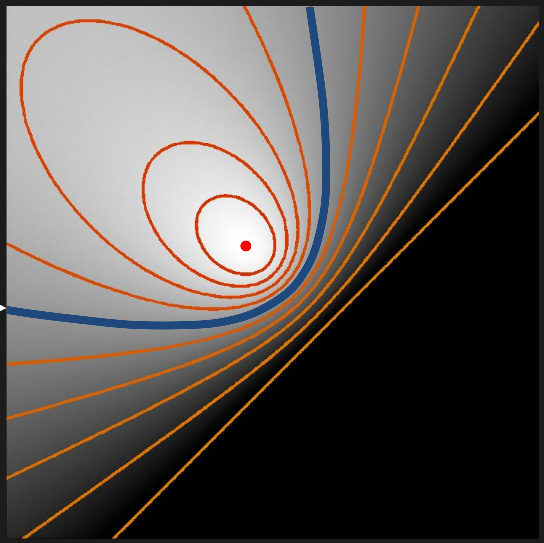


Image I



Reflectance Map $R(p,q)$

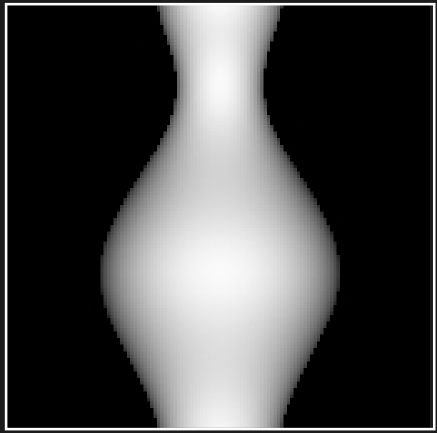
…Can we find Surface Gradients (p,q) at a pixel from its Brightness?

NO

Brightness of a pixel maps to infinite (p,q) values along a corresponding iso-brightness contour on the reflectance map.

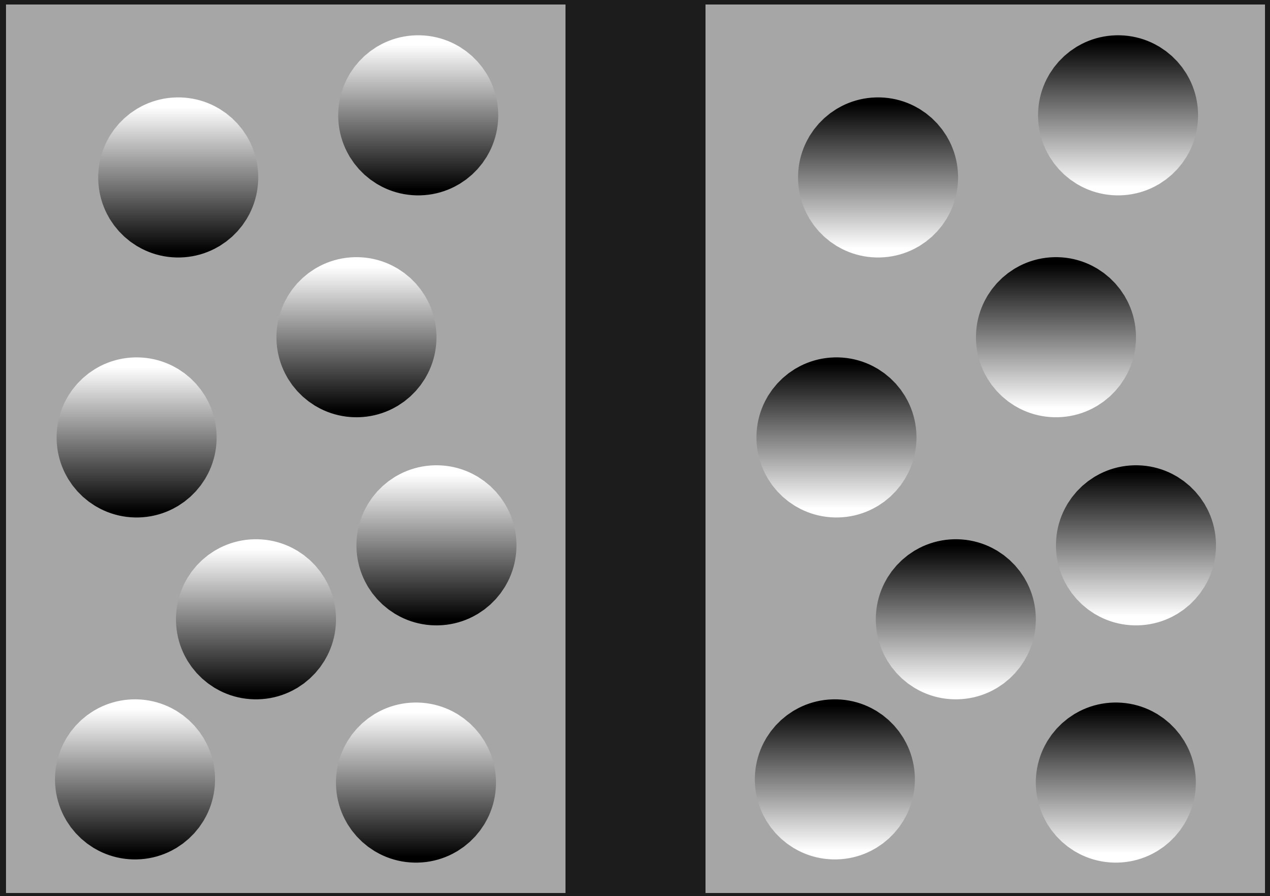
Shape From Shading in Humans

We effortlessly perceive shape from a single image.



We make many assumptions in doing so.

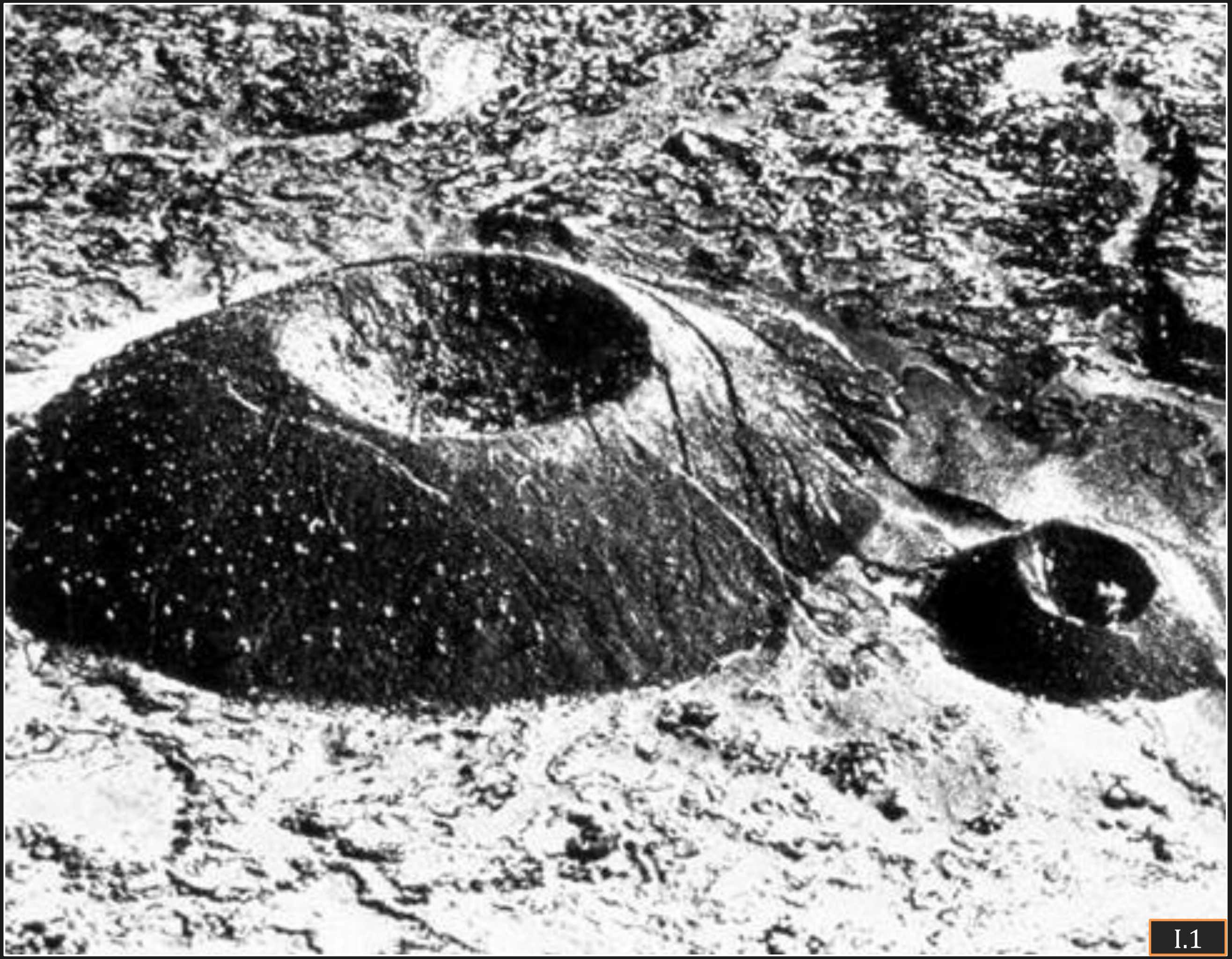
We Assume Light Source is Above Us!



The shaded objects in the left panel are usually seen as convex, whereas those in the right panel are usually seen as concave.

[Ramachandran 1990]

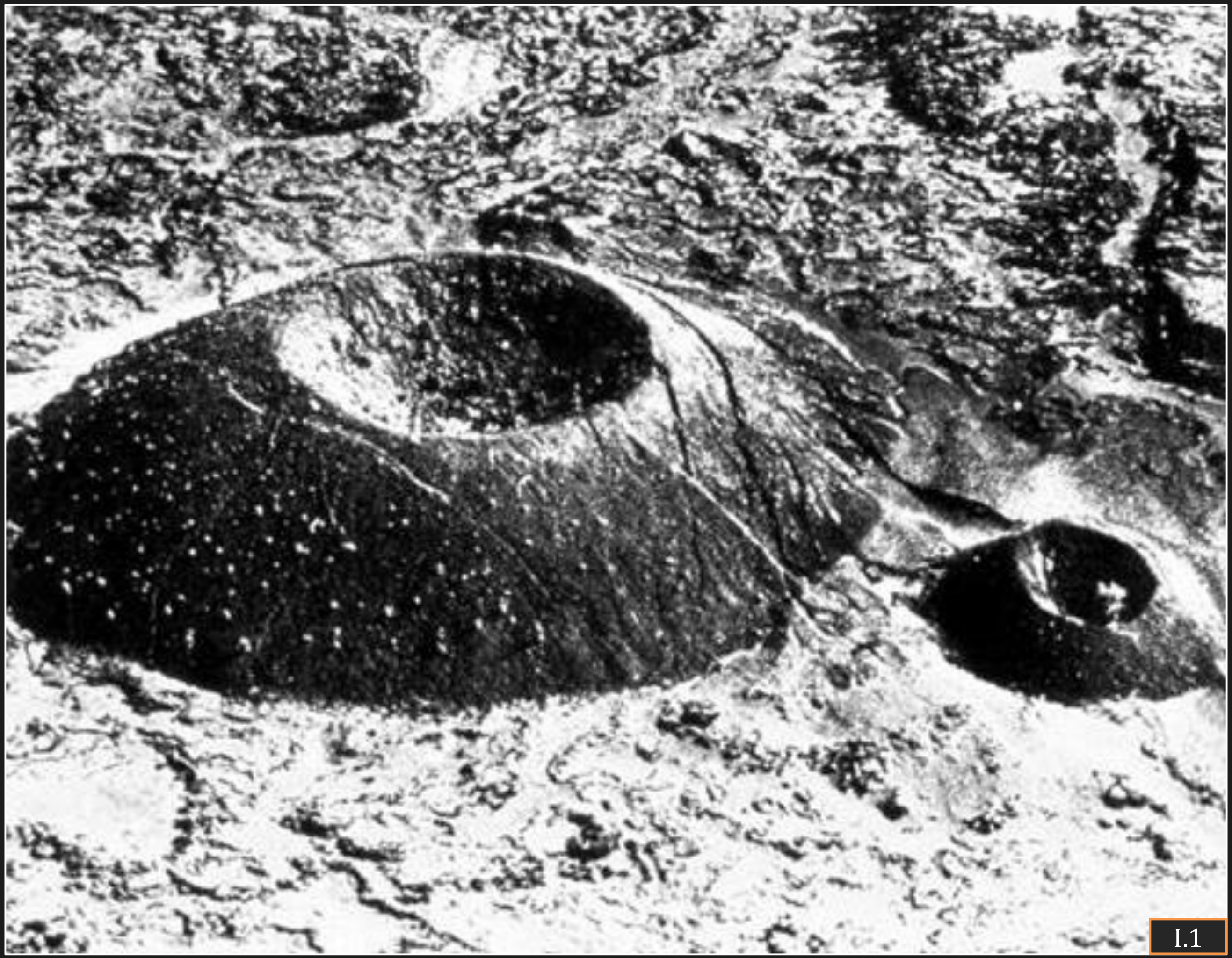
We Assume Light Source is Above Us!



Crater on a Mound

[Rittenhouse 1786]

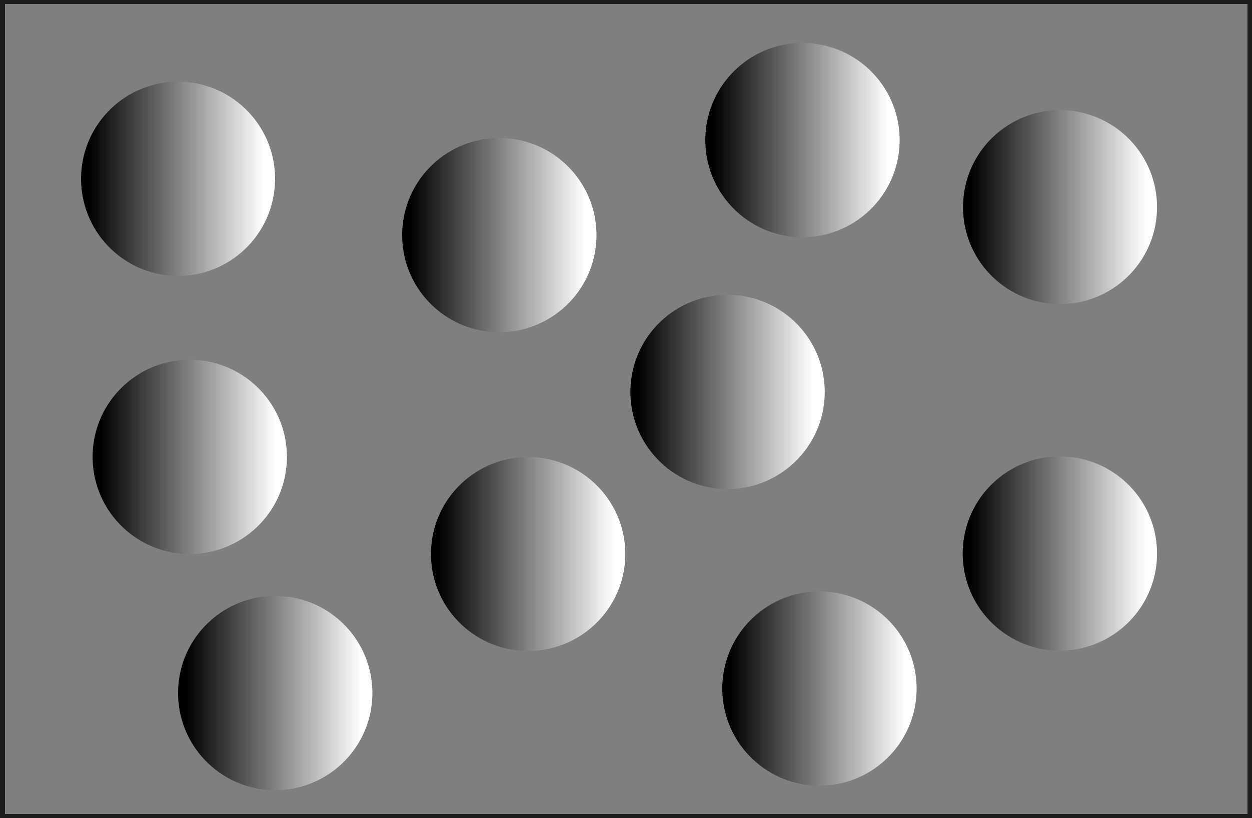
We Assume Light Source is Above Us!



Mound in a Crater

[Rittenhouse 1786]

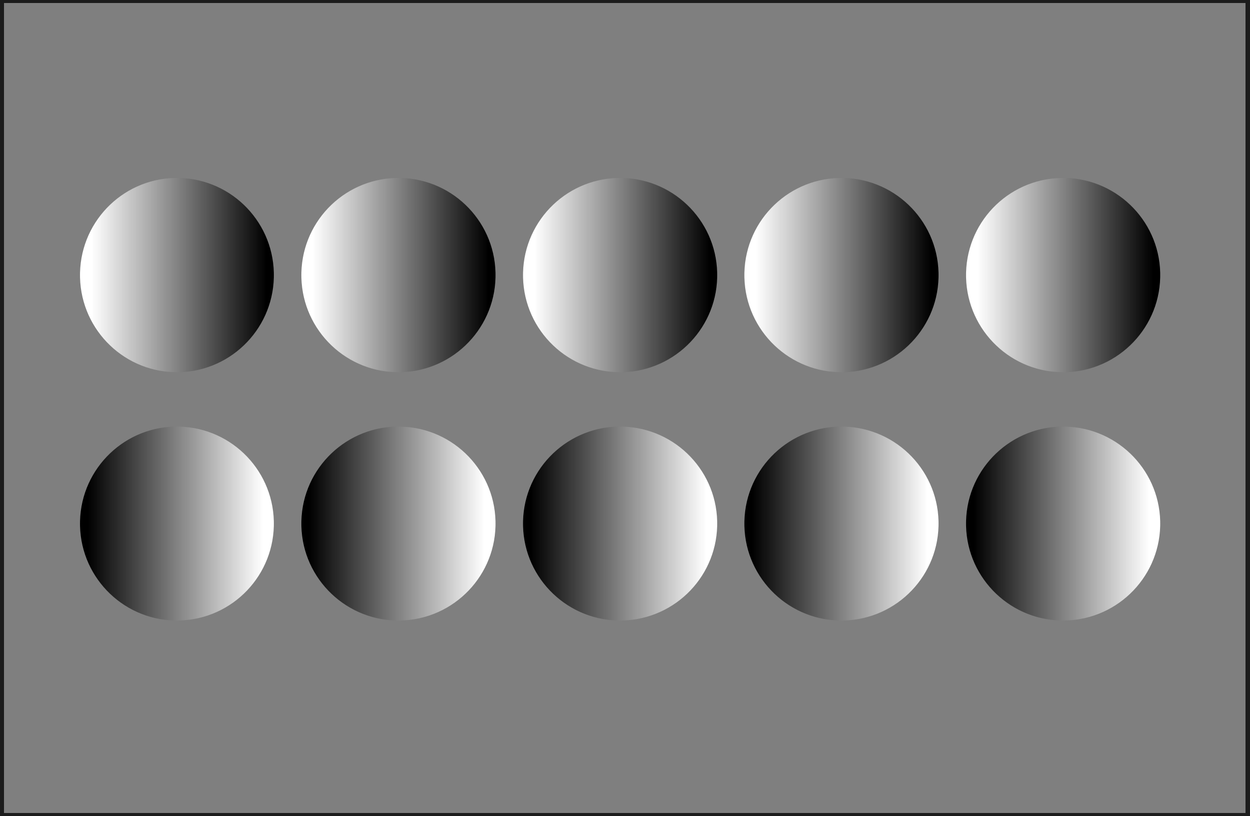
What If Illumination is Sideways?



Spheres or Cavities? It depends on where you think the light source is. You can reverse the depth of the objects by mentally shifting the light source from left to right.

[Ramachandran 1990]

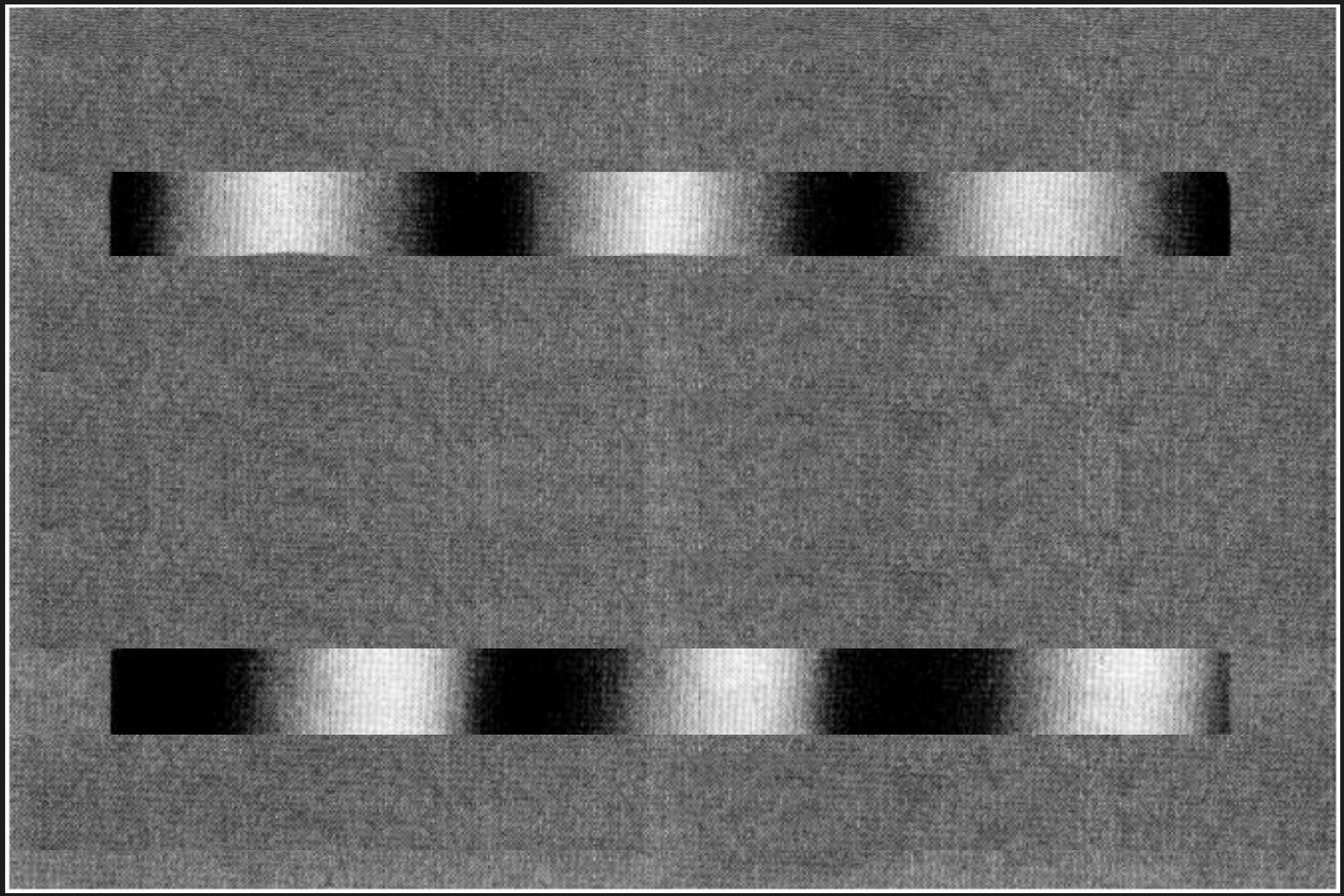
We Assume Uniform Global Illumination



Objects in one row can be seen as either convex or concave if the other row is excluded; but when both rows are viewed simultaneously, seeing one row as convex forces the other to be perceived as concave.

[Ramachandran 1990]

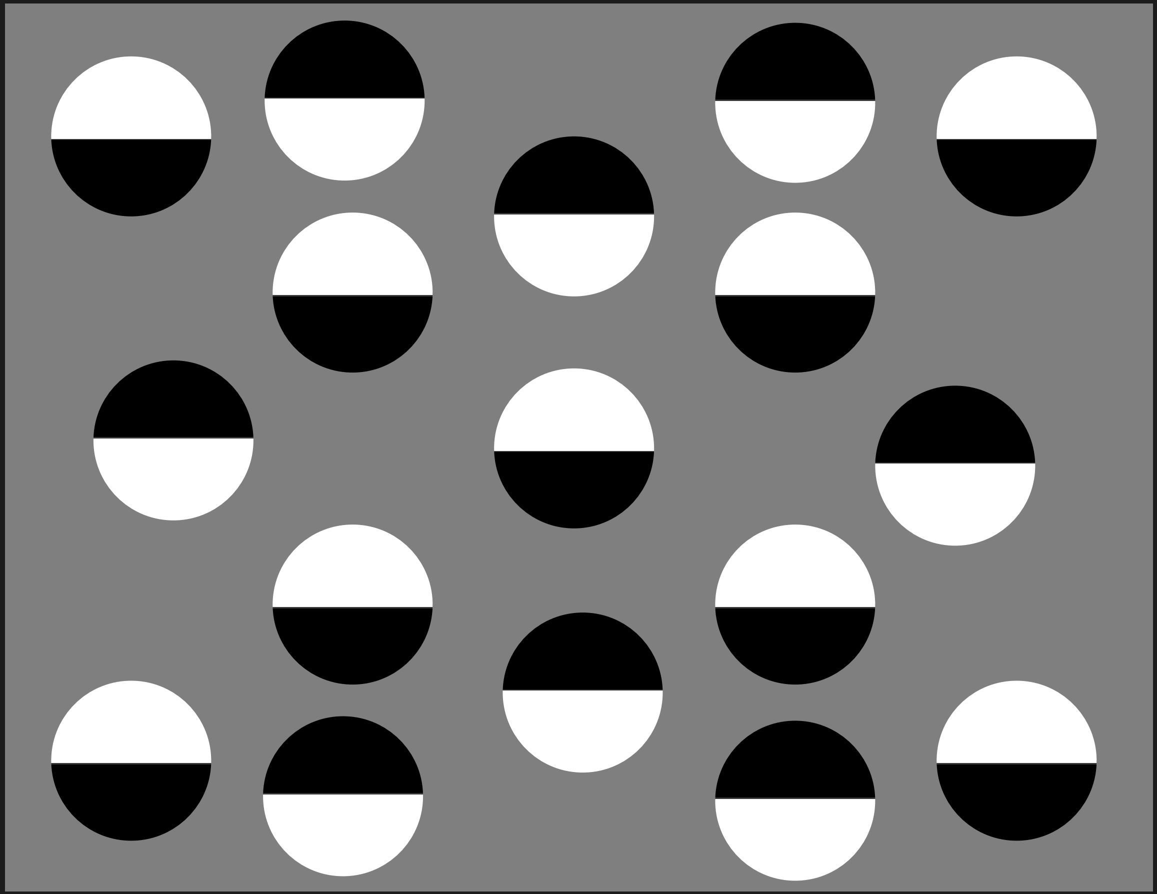
Boundaries Influence Perceived Shape



Both images have the same shading variation but the top image suggests three cylinders lit vertically and the bottom one a corrugated sheet lit from far right.

[Ramachandran 1990]

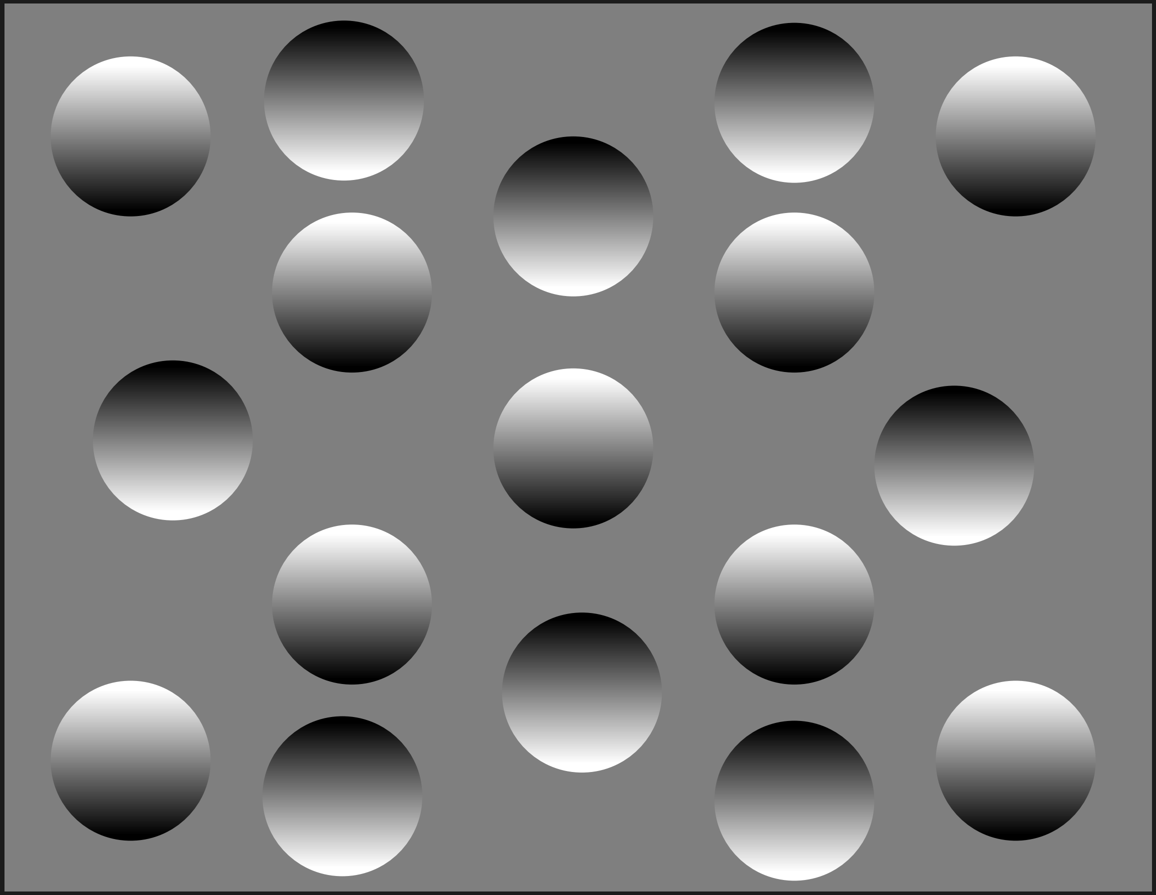
Perceptual Grouping



Perceptual grouping of objects with the identical appearance is difficult to achieve without shading.

[Ramachandran 1990]

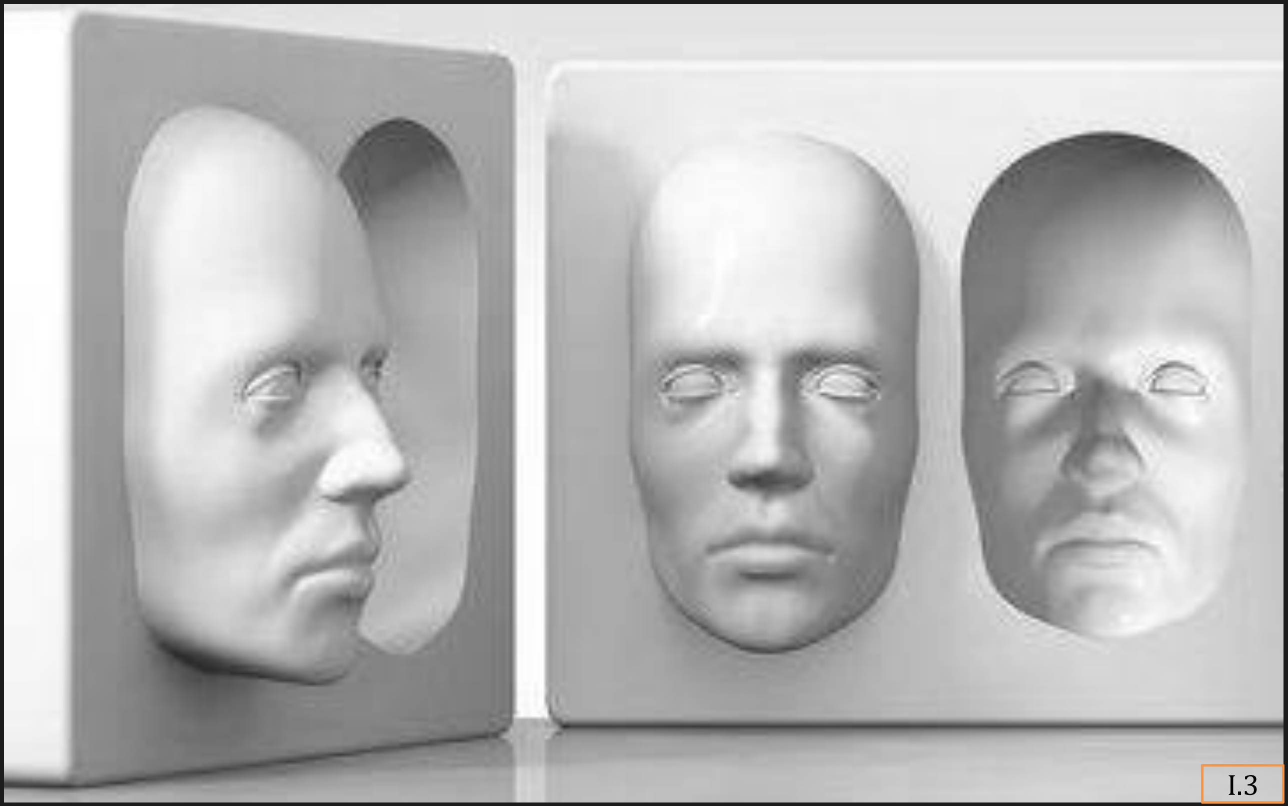
We Use Shading for Perceptual Grouping



Objects that are light on top are usually perceived as convex objects that can be mentally grouped and segregated (to form an X pattern) from the background of concave objects.

[Ramachandran 1990]

We Tend to “See” the Familiar



Hollow-Mask interiors lit from above produce an eerie impression of protruding face lit from below. In interpreting shaded images the brain usually assumes light shining from above but here it rejects the assumptions in order to interpret the images as normal, convex objects.

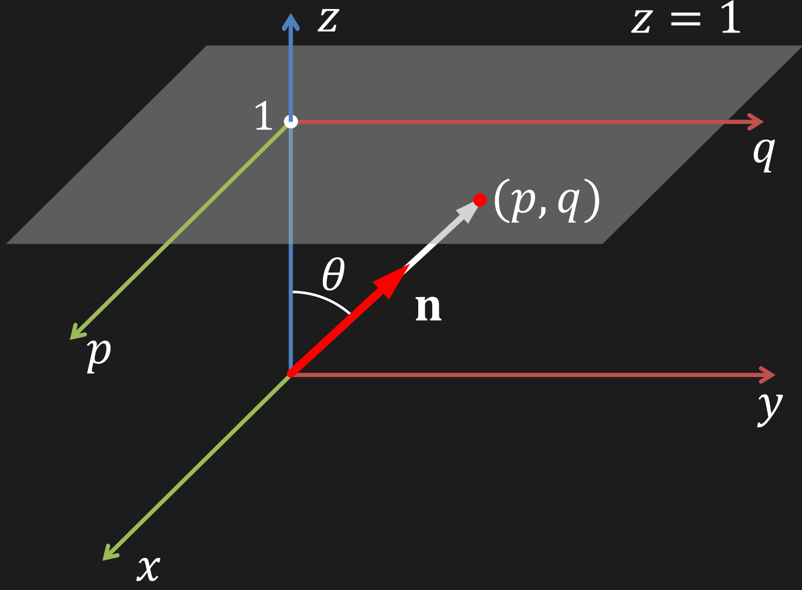
[Ramachandran 1990]

Shape From Shading

Solutions: Use assumptions and constraints

Stereographic Projection: fg Space

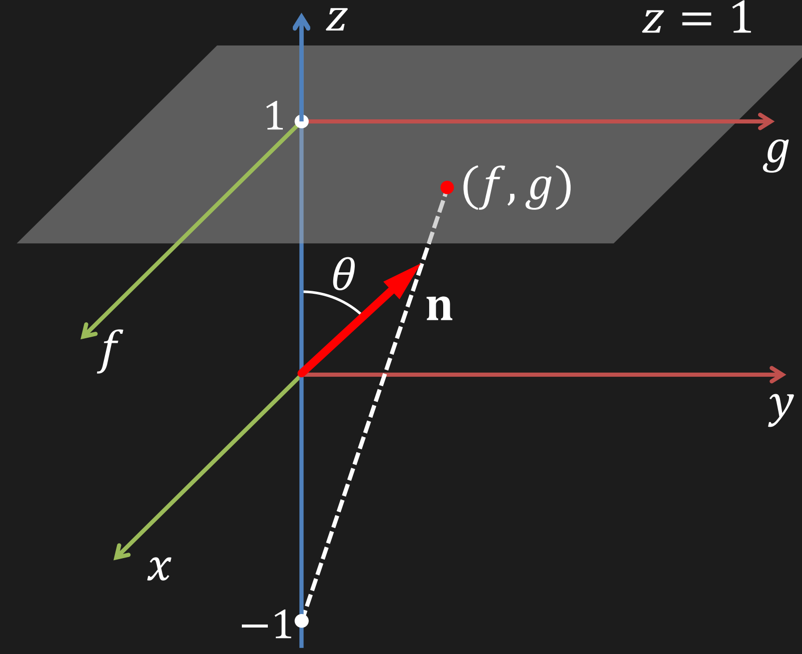
pq Space



Problem:

p or q is infinite when $\theta = 90°$.

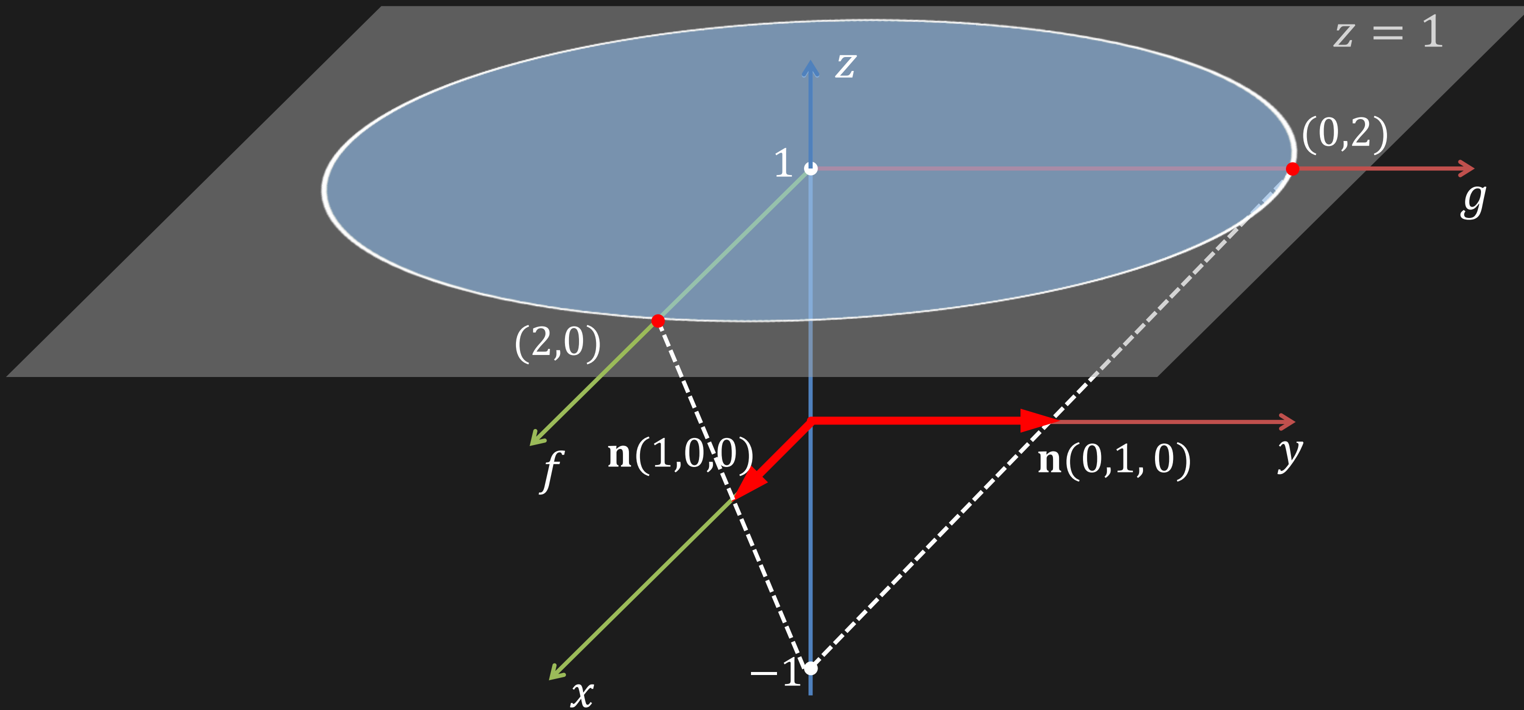
fg Space



$$f = \frac{2p}{1 + \sqrt{p^2 + q^2 + 1}}$$

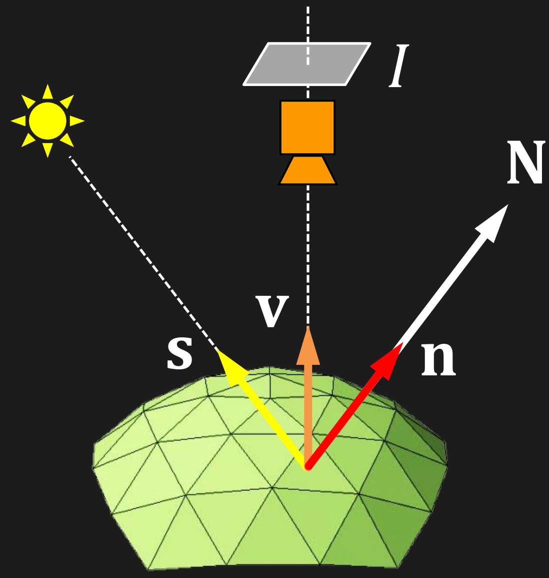
$$g = \frac{2q}{1 + \sqrt{p^2 + q^2 + 1}}$$

Stereographic Projection: fg Space



All possible values of surface gradients
lie within a circle of radius 2 on the fg Plane.

Reflectance Map $R_s(f,g)$



$\mathbf{v} = (0,0,1)$

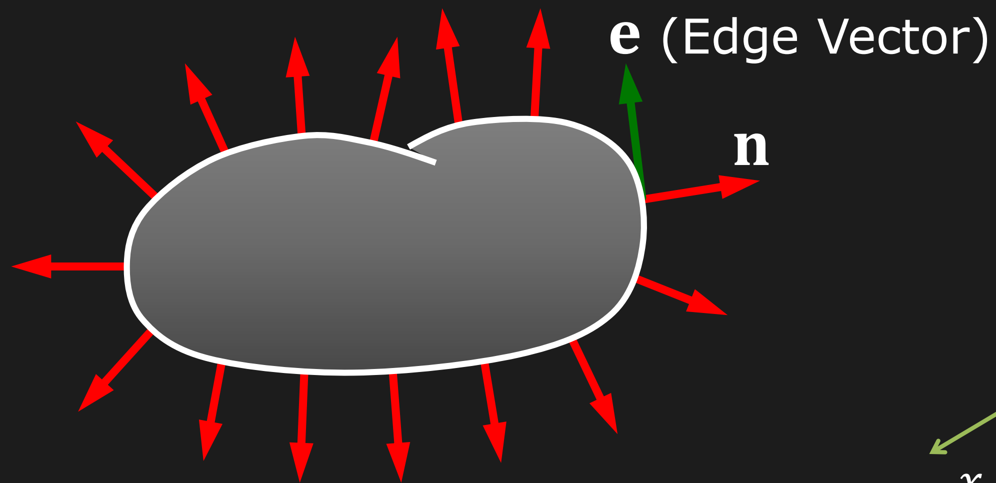
Surface Normal:

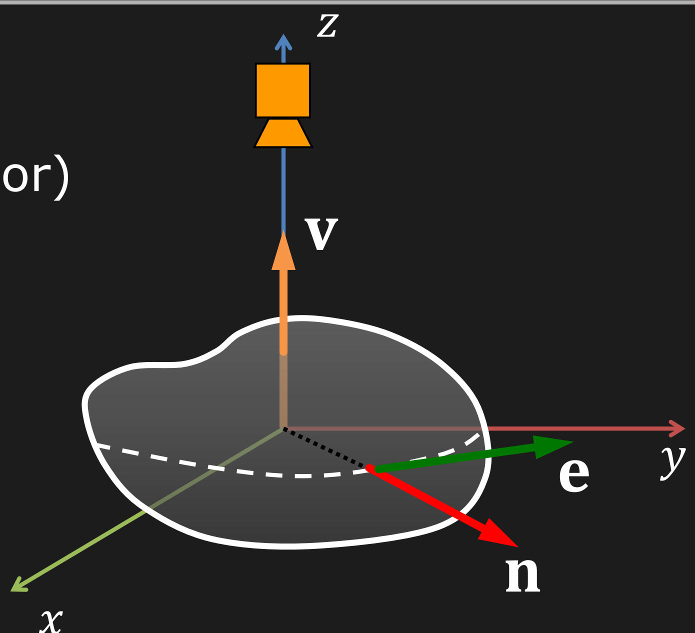
$\mathbf{n} \equiv (p,q) \equiv (f,g)$

For a given Source Direction $\boldsymbol{s}$ and Surface Reflectance, Reflectance Map relates Image Intensity to its Surface Gradients:

$$\boxed{I = R_s(f,g)}$$

Occluding Boundaries





$\mathbf{n} \perp \mathbf{e}, \ \mathbf{e} \perp \mathbf{v}$

($\mathbf{e}$ and $\mathbf{v}$ are known)

$$\boxed{\mathbf{n} = \mathbf{e} \times \mathbf{v}}$$

Surface normals on the occluding boundary can be used Boundary Conditions for Shape from Shading.

Image Irradiance Constraint

Assumption: Image irradiance should follow the reflectance map. That is, $I(x, y) = R_s(f, g)$

Minimize:

$$e_r = \iint (I(x, y) - R_s(f, g))^2 \, dx \, dy$$

Aim: Penalize errors in image irradiance and the reflectance map.

Smoothness Constraint

Assumption: Object surface is Smooth.

That is, the gradient values (f, g) vary "smoothly."

Minimize:

$$e_s = \iint \left(f_x^2 + f_y^2\right) + \left(g_x^2 + g_y^2\right) dx\, dy$$

where: $f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$, $g_x = \frac{\partial g}{\partial x}$ and $g_y = \frac{\partial g}{\partial y}$

Aim: Penalize rapid changes in f and g during surface estimation.

Shape From Shading

Find surface gradients (f, g) at all image points that minimize the function:

$$\boxed{e = e_s + \lambda e_r}$$

where:

e_s: Smoothness Constraint

e_r: Image Irradiance Error

λ: Weight (Use large value when brightness measurement is accurate; small otherwise)

Numerical Shape from Shading

Smoothness Error at point (i, j)

$$e_{s_{i,j}} = \frac{1}{4}\left(\left(f_{i+1,j} - f_{i,j}\right)^2 + \left(f_{i,j+1} - f_{i,j}\right)^2\right.$$

Image Irradiance Error at point (i, j)

$$e_{r_{i,j}} = \left(I_{i,j} - R_s\left(f_{i,j}, g_{i,j}\right)\right)^2$$

Find $\left(f_{i,j}, g_{i,j}\right)$ for all (i, j) that minimizes:

$$\boxed{e = \sum_i \sum_j \left(e_{s_{i,j}} + \lambda\, e_{r_{i,j}}\right)}$$

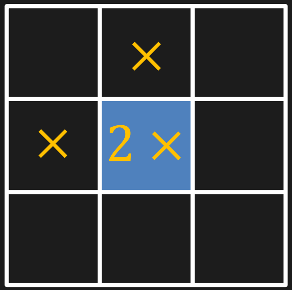
[Ikeuchi 1981]

Numerical Shape from Shading

If $\left(f_{k,l}, g_{k,l}\right)$ minimizes e, then $\dfrac{\partial e}{\partial f_{k,l}} = 0$ and $\dfrac{\partial e}{\partial g_{k,l}} = 0$

Given an image of size $N \times N$, there are $2N^2$ unknowns.
(N^2 $f_{i,j}$'s and N^2 $g_{k,l}$'s)

However, note that each $f_{i,j}$ and $g_{k,l}$ appears in 4 equations respectively.



[Ikeuchi 1981]

Numerical Shape from Shading

If $(f_{k,l}, g_{k,l})$ minimizes e, then $\dfrac{\partial e}{\partial f_{k,l}} = 0$ and $\dfrac{\partial e}{\partial g_{k,l}} = 0$

Therefore:

Eq 1: $$\frac{\partial e}{\partial f_{k,l}} = 2\left(f_{k,l} - \bar{f}_{k,l}\right) - 2\lambda\left(I_{k,l} - R_s\left(f_{k,l}, g_{k,l}\right)\right)\frac{\partial R}{\partial f}\bigg|_{f_{k,l}} = 0$$

Eq 2: $$\frac{\partial e}{\partial g_{k,l}} = 2\left(g_{k,l} - \bar{g}_{k,l}\right) - 2\lambda\left(I_{k,l} - R_s\left(f_{k,l}, g_{k,l}\right)\right)\frac{\partial R}{\partial g}\bigg|_{g_{k,l}} = 0$$

where $\bar{f}_{i,j}$ and $\bar{g}_{k,l}$ are local averages.

$$\bar{f}_{i,j} = \tfrac{1}{4}\left(f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}\right)$$

$$\bar{g}_{k,l} = \tfrac{1}{4}\left(g_{i+1,j} + g_{i-1,j} + g_{i,j+1} + g_{i,j-1}\right)$$

[Ikeuchi 1981]

Numerical Shape from Shading

If $\left(f_{k,l}, g_{k,l}\right)$ minimizes e, then $\dfrac{\partial e}{\partial f_{k,l}} = 0$ and $\dfrac{\partial e}{\partial g_{k,l}} = 0$

Therefore:

Eq 1: $$\frac{\partial e}{\partial f_{k,l}} = 2\left(f_{k,l} - \bar{f}_{k,l}\right) - 2\lambda\left(I_{k,l} - R_s\left(f_{k,l}, g_{k,l}\right)\right)\left.\frac{\partial R}{\partial f}\right|_{f_{k,l}} = 0$$

Eq 2: $$\frac{\partial e}{\partial g_{k,l}} = 2\left(g_{k,l} - \bar{g}_{k,l}\right) - 2\lambda\left(I_{k,l} - R_s\left(f_{k,l}, g_{k,l}\right)\right)\left.\frac{\partial R}{\partial g}\right|_{g_{k,l}} = 0$$

Moving all $f_{k,l}$'s and $g_{k,l}$'s to one side, we get…

[Ikeuchi 1981]

Iterative Solution

Update Rule:

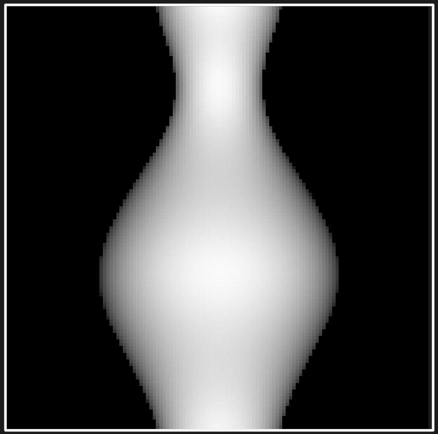
$$f_{k,l}^{(n+1)} = \bar{f}_{k,l}^{(n)} + \lambda \left(I_{k,l} - R\left(f_{k,l}^{(n)}, f_{k,l}^{(n)} \right) \right) \left. \frac{\partial R}{\partial f} \right|_{f_{k,l}^{(n)}}$$

$$g_{k,l}^{(n+1)} = \bar{g}_{k,l}^{(n)} + \lambda \left(I_{k,l} - R\left(f_{k,l}^{(n)}, g_{k,l}^{(n)} \right) \right) \left. \frac{\partial R}{\partial g} \right|_{g_{k,l}^{(n)}}$$

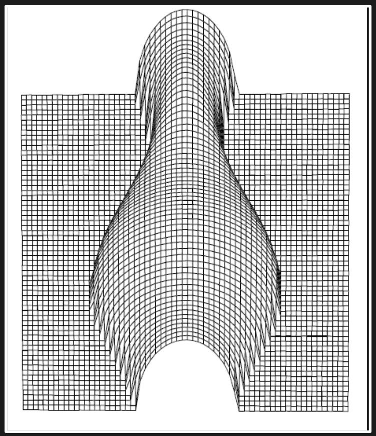
- Use known (f, g) values on the occluding boundary to constrain the solution.
- Iteratively compute (f, g) until the solution has converged.

[Ikeuchi 1981]

Results: Synthetic Objects

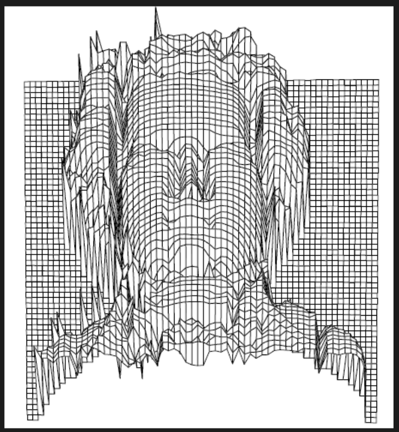


Scene



Recovered Shape

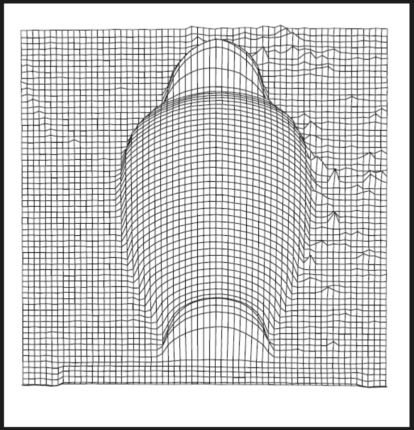
Scene



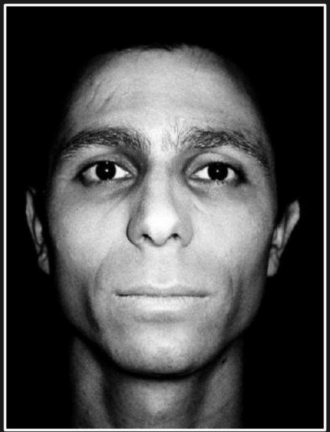
Recovered Shape

Results: Real Objects

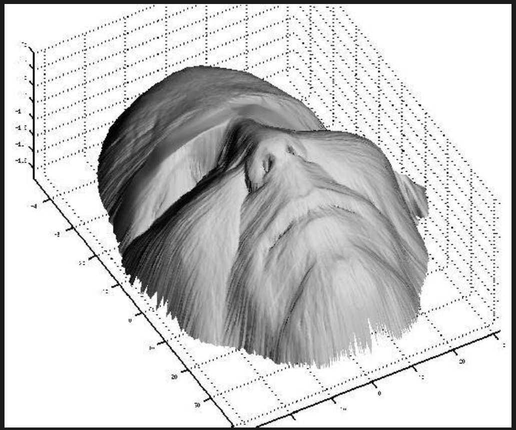
Scene



Recovered Shape



Scene



Recovered Shape

References

Textbooks:

Robot Vision (Chapter 11)
Horn, B. K. P., MIT Press

Articles and Papers:

[Ikeuchi 1981] K. Ikeuchi and B. K. P. Horn. “Numerical Shape from Shading and Occlusing Boundaries.” Artificial Intelligence, 1981.

[Ramachandran 1990] V. S. Ramachandran. “Perceiving shape from shading.” Scientific American magazine. 1990.

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I.2 Adapted from V. S. Ramachandran, “Perceiving shape from shading.” Scientific American magazine. 1990.

I.3 http://www.sciencephoto.com/media/351214/enlarge

I.4 Adapted from R. Zhang, P. Tsai, J. Cryer, M. Shah. “Shape-from-Shading: A Survey,” PAMI, 1999.