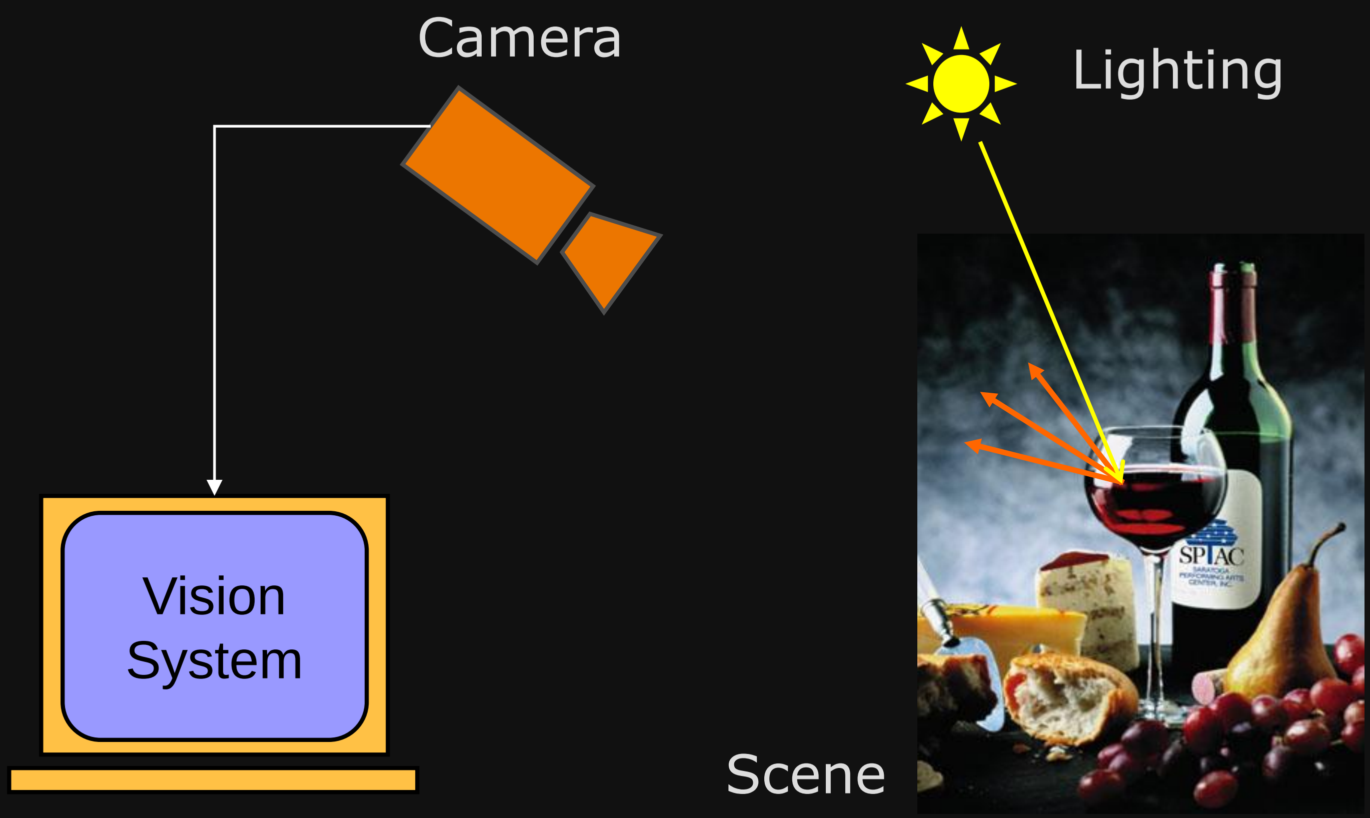
Radiometry and Reflectance

Introduction to Computational Photography:

EECS 395/495

Northwestern University

From 2D to 3D



We need to understand the relation between lighting, surface reflectance and image intensity.

Radiometry and Reflectance

To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.

Topics:

(1) Image Intensity

(2) Radiometric Concepts

(3) Image Formation

(4) Reflectance Models

Image Intensity

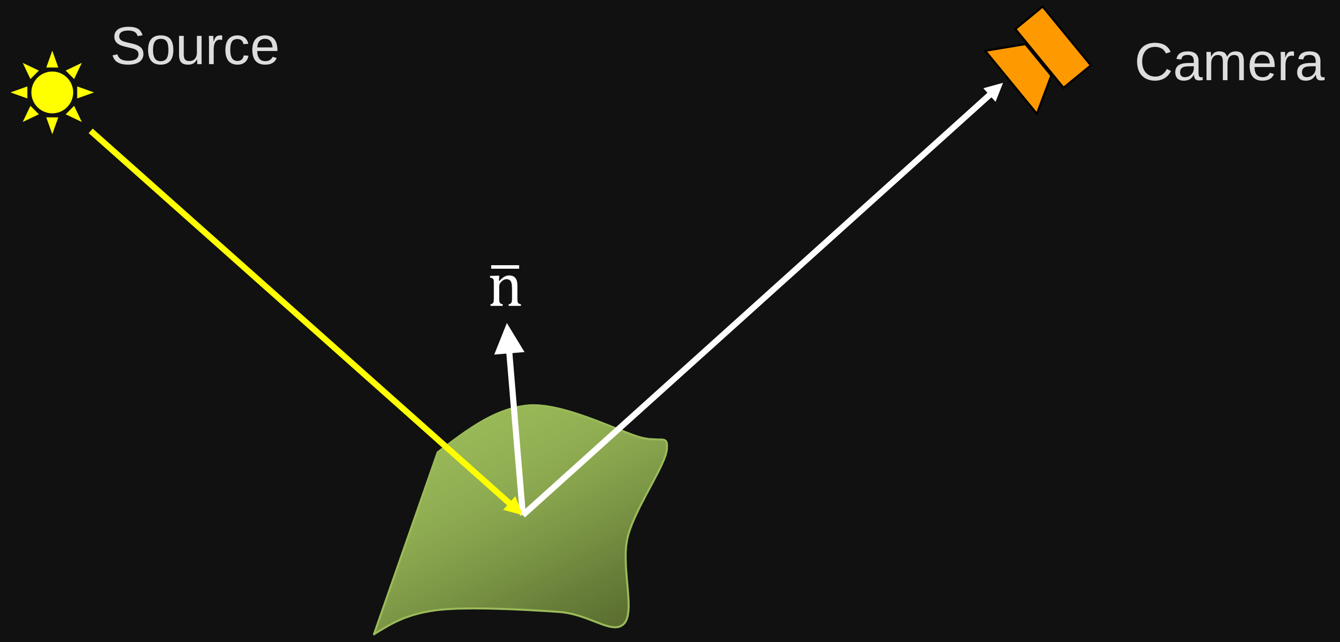


Image Intensity = f (Illumination,

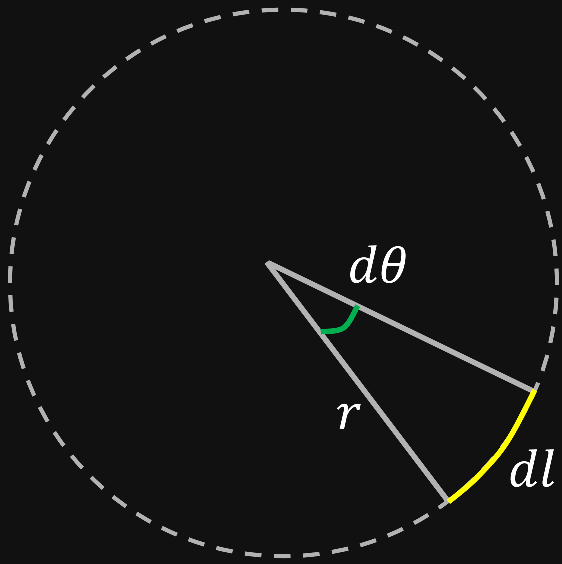
Surface Orientation,

Surface Reflectance)

Image intensity understanding is under-constrained!

Concept: Angle (2D)

$$\boxed{d\theta = \frac{dl}{r}}$$



Unit: radian (rad)

$d\theta$ is dimensionless

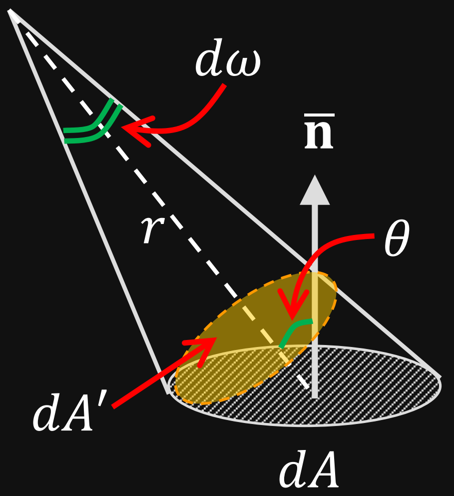
However, rad is used as its unit to distinguish from other dimensionless quantities.

Concept: Solid Angle (3D)

$$\boxed{d\omega = \frac{dA'}{r^2} = \frac{dA\ \cos\theta}{r^2}}$$

Unit: steradian (sr)

$d\omega$ is dimensionless



dA': Foreshortened Area

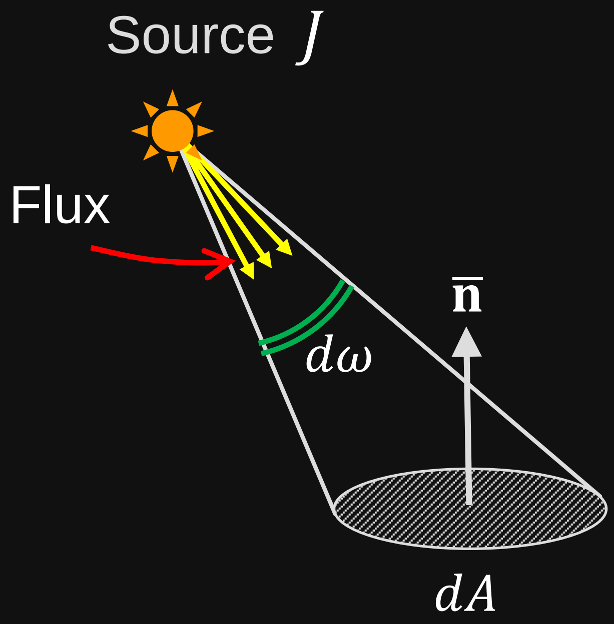
What is the solid angle subtended by a hemisphere? 2π

Concept: Light Flux

Power emitted within a solid angle

$$\boxed{d\Phi}$$

Unit: watts (W)

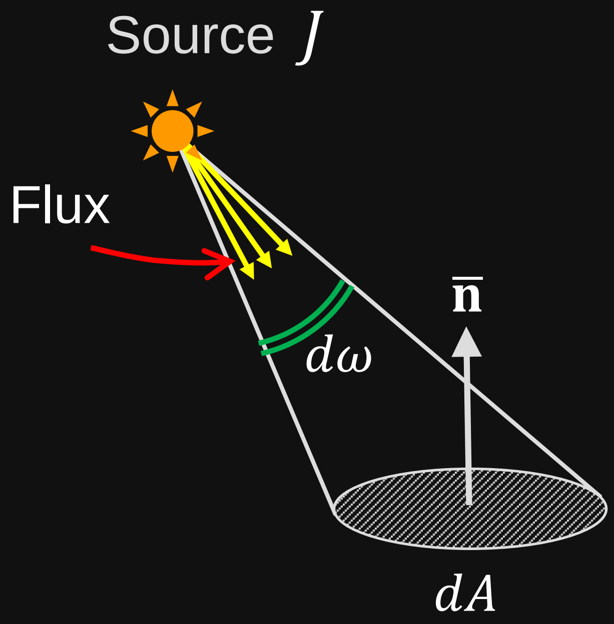


Concept: Radiant Intensity

Light flux emitted per unit solid angle

$$\boxed{J = \frac{d\Phi}{d\omega}}$$

Unit: Wsr^{-1}

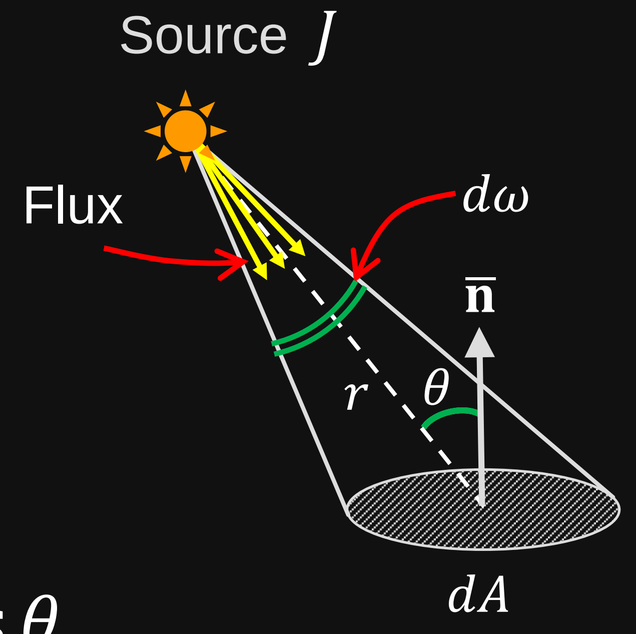


Concept: Surface Irradiance

Light flux incident per unit surface area

$$\boxed{E = \frac{d\Phi}{dA}}$$

Unit: Wm^{-2}



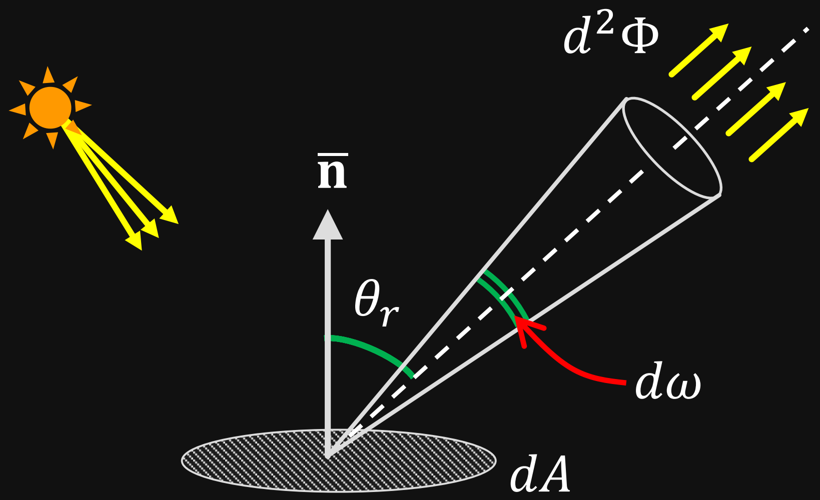
$$E = \frac{J.d\omega}{dA} = \frac{J.\dfrac{dA.\cos\theta}{r^2}}{dA} = \frac{J\ \cos\theta}{r^2}$$

Concept: Surface Radiance

Light flux emitted per unit foreshortened area per unit solid angle

$$L = \boxed{\frac{d^2\Phi}{(dA\cos\theta_r)\,d\omega}}$$

Unit: $Wm^{-2}sr^{-1}$



$dA\cos\theta_r$: Foreshortened Area

Note:

- L depends on direction θ_r : $L(\theta_r)$
- Surface can radiate into the whole hemisphere.
- L depends on reflectance properties of surface

Scene Radiance and Image Brightness

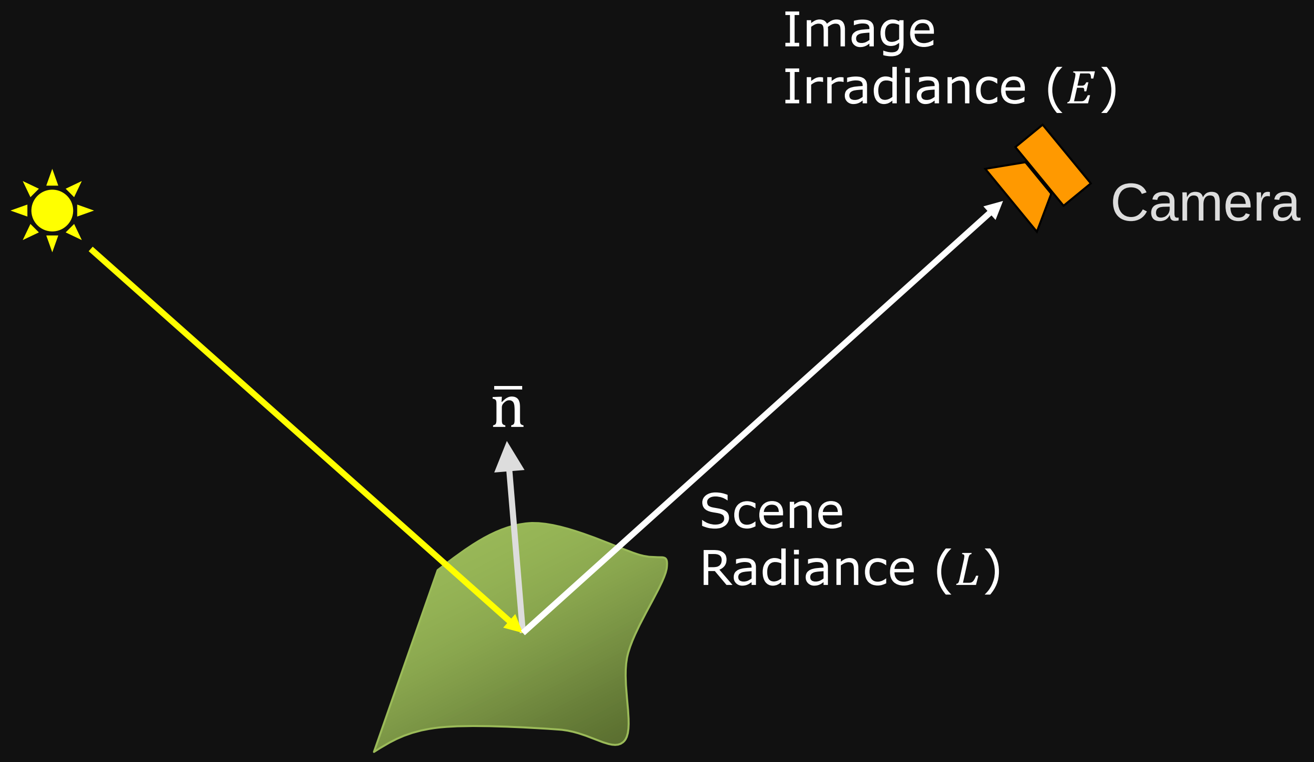
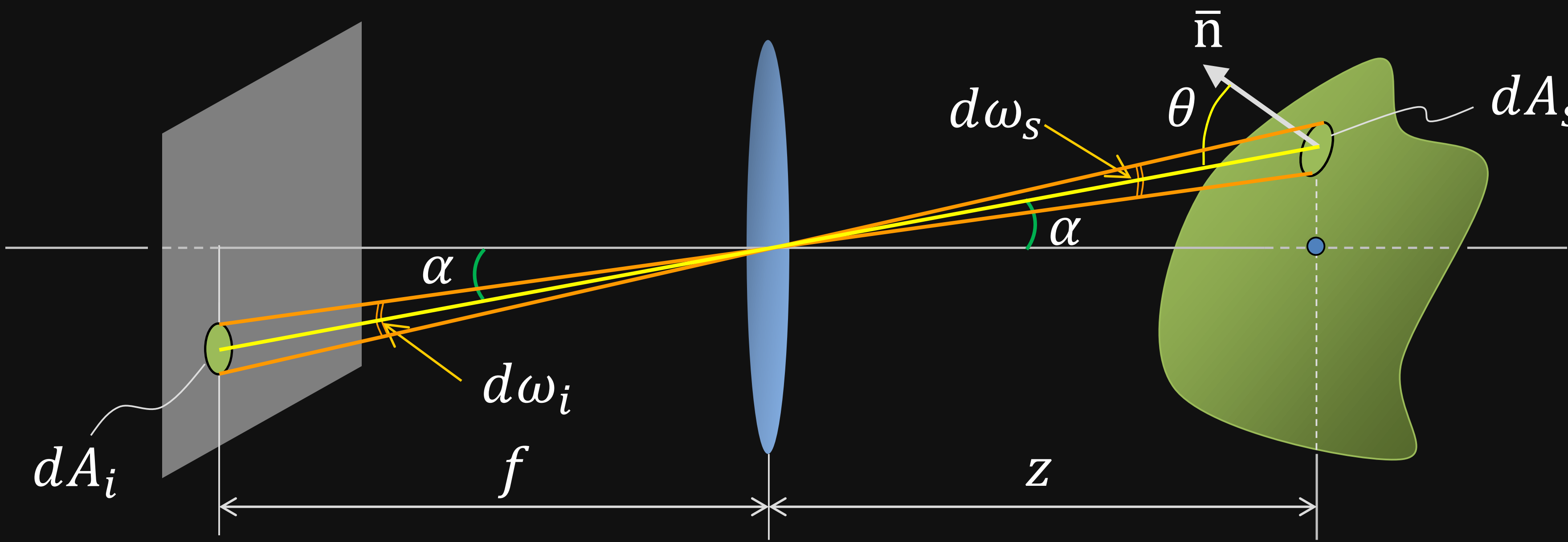


Image Irradiance and Scene Radiance

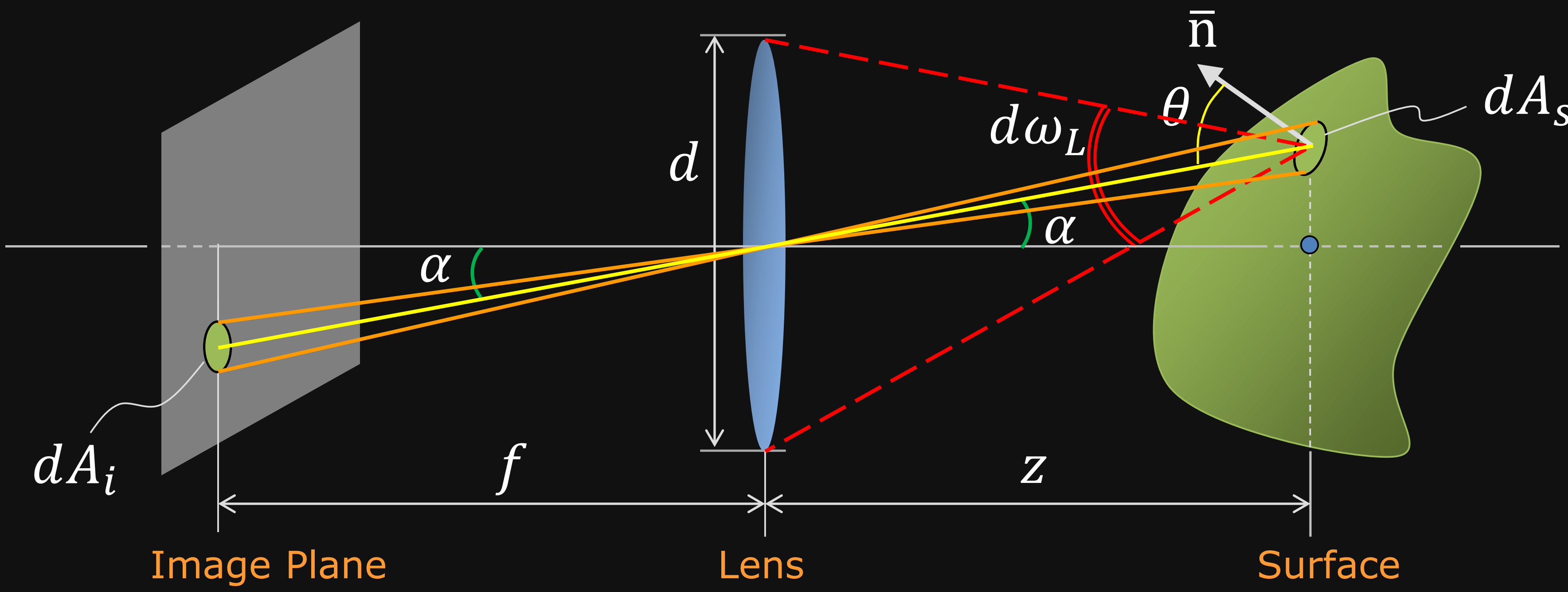


Solid Angles: $d\omega_i = d\omega_s$

$$\frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2} \quad \Rightarrow \quad \boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2}$$

Equation (1)

Image Irradiance and Scene Radiance

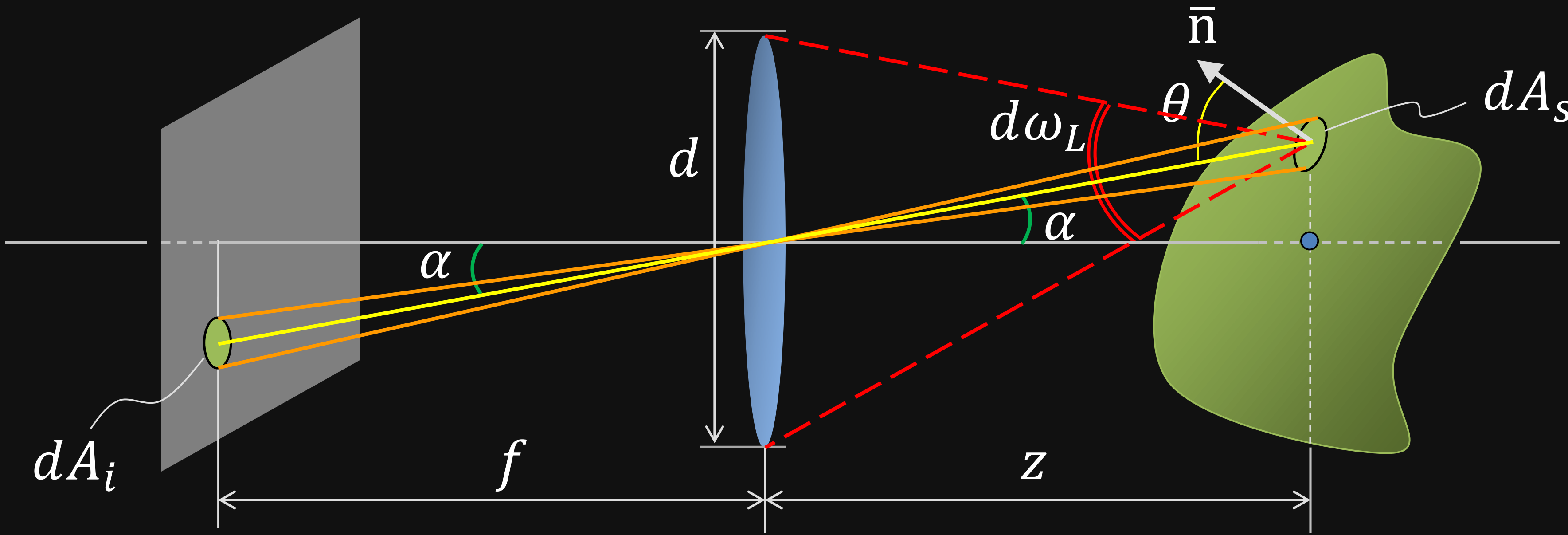


Solid Angle subtended by the lens:

$$\boxed{d\omega_L = \frac{\pi d^2}{4} \frac{\cos\alpha}{(z/\cos\alpha)^2}}$$

Equation (2)

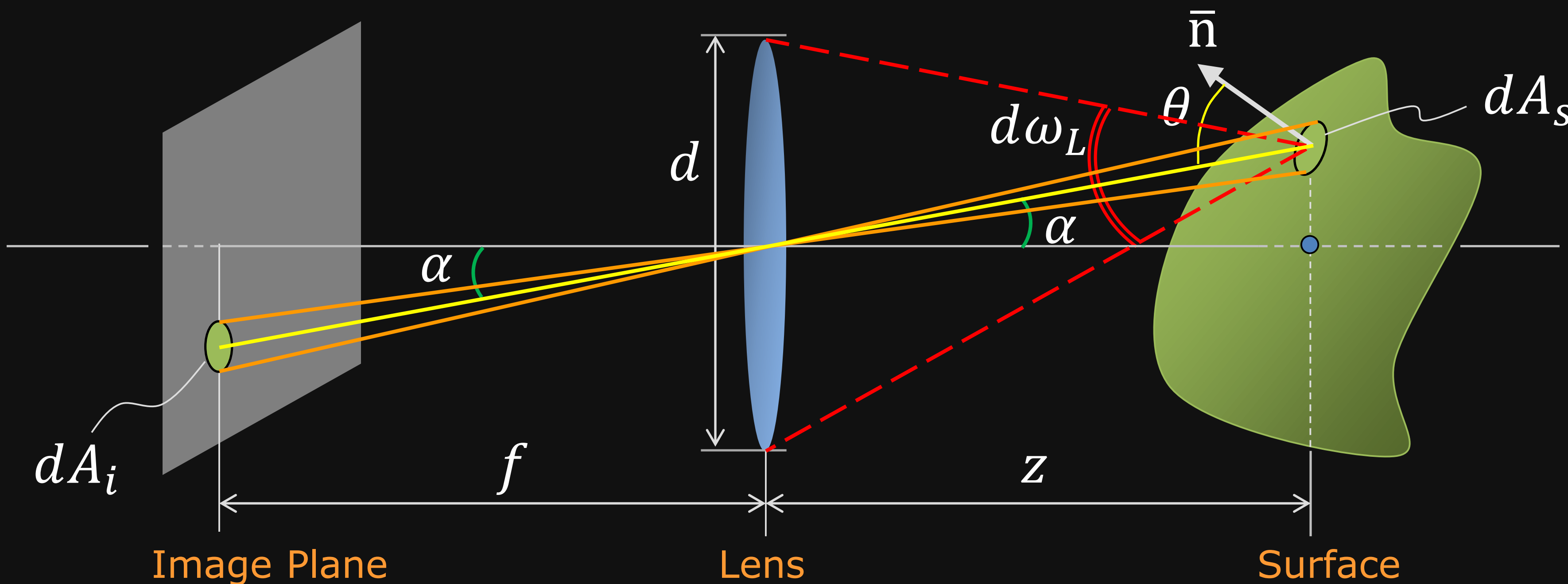
Image Irradiance and Scene Radiance



Energy Conservation:

Flux received by lens from dA_s = Flux projected onto dA_i

Image Irradiance and Scene Radiance



Scene Radiance:

Flux received by lens from dA_s

$$L = \frac{d^2\Phi}{(dA_s \cos\theta)d\omega_L}$$

$$\boxed{d^2\Phi = L \cdot (dA_s \cos\theta)d\omega_L}$$

Equation (3)

Image Irradiance and Scene Radiance

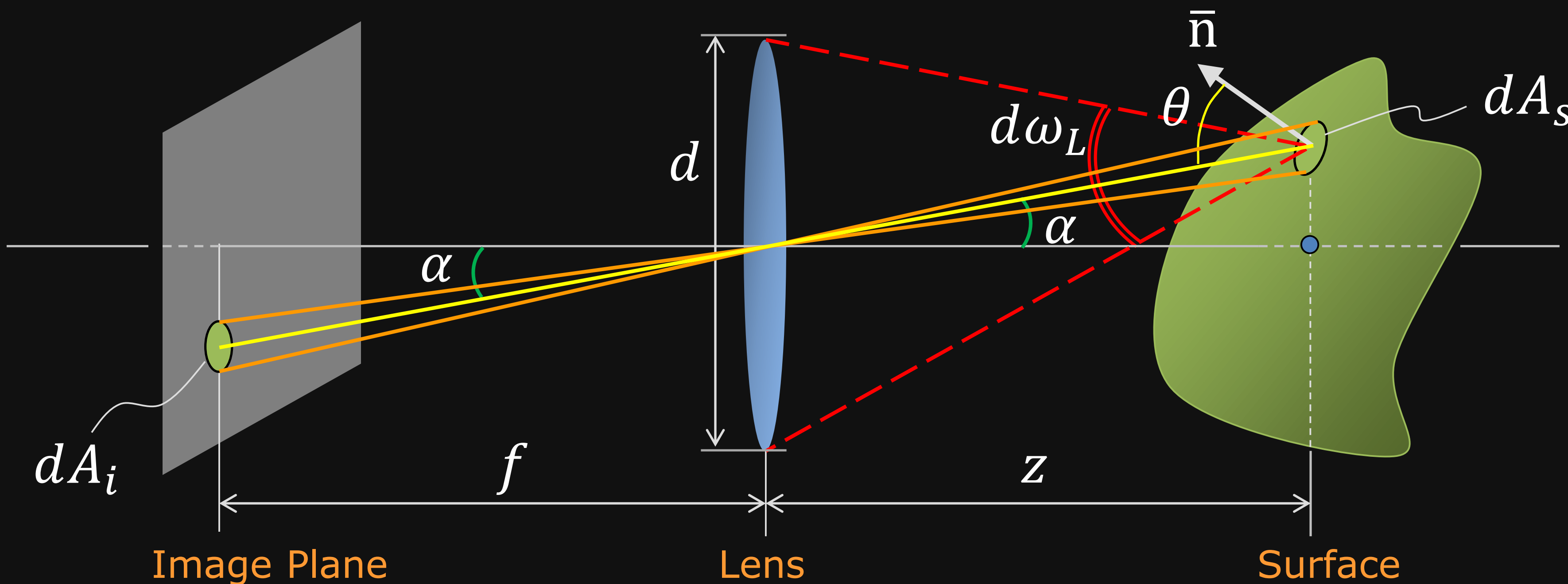


Image Irradiance: Flux projected onto dA_i

$$E = \frac{d\Phi}{dA_i} \Rightarrow \boxed{d\Phi = E \cdot dA_i}$$

Equation (4)

Image Irradiance and Scene Radiance

Equation (1)

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

Equation (2)

$$d\omega_L = \frac{\pi d^2}{4}\frac{\cos\alpha}{(z/\cos\alpha)^2}$$

Equation (3)

$$d^2\Phi = L \cdot (dA_s \cos\theta) d\omega_L$$

Equation (4)

$$d\Phi = E \cdot dA_i$$

$$\boxed{E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha}$$

Image Irradiance and Scene Radiance

$$\boxed{E = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha}$$

Image Irradiance

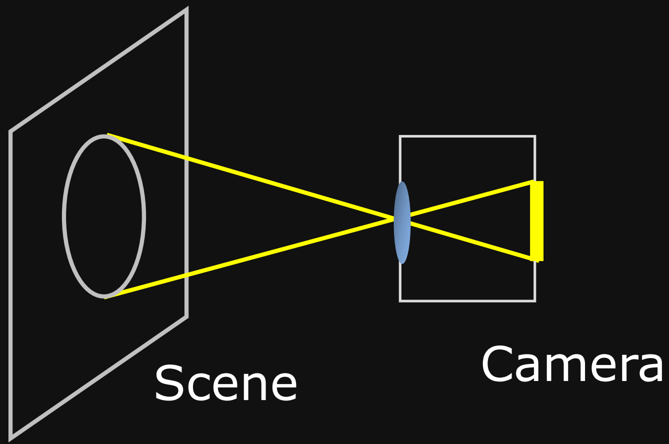
Scene Radiance

1/(F-number)

- Image Irradiance is proportional to Scene Radiance
- Image brightness falls off from the image center due to the $\cos^4 \alpha$ term.
- For small fields of view, effects of $\cos^4 \alpha$ are small

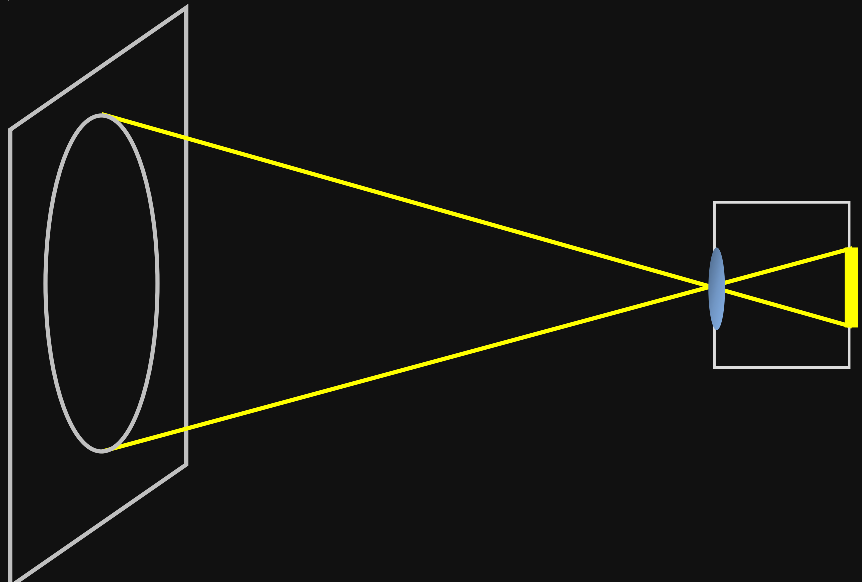
Image Irradiance and Scene Radiance

Does image brightness vary with scene depth?



$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

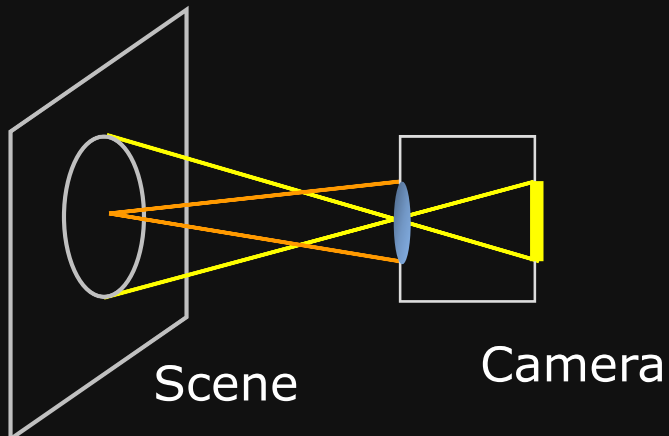
NO.



Larger the scene depth, larger the area of light accumulation.

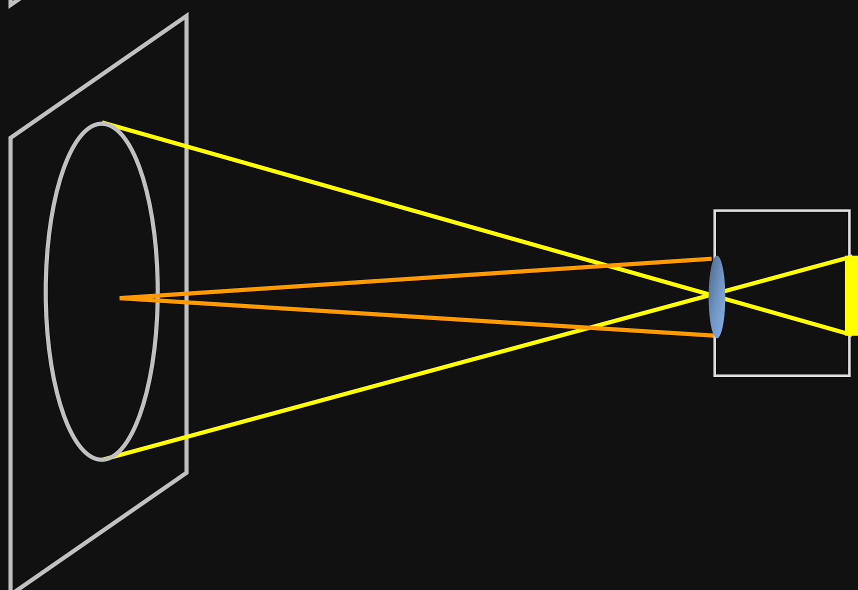
Image Irradiance and Scene Radiance

Does image brightness vary with scene depth?



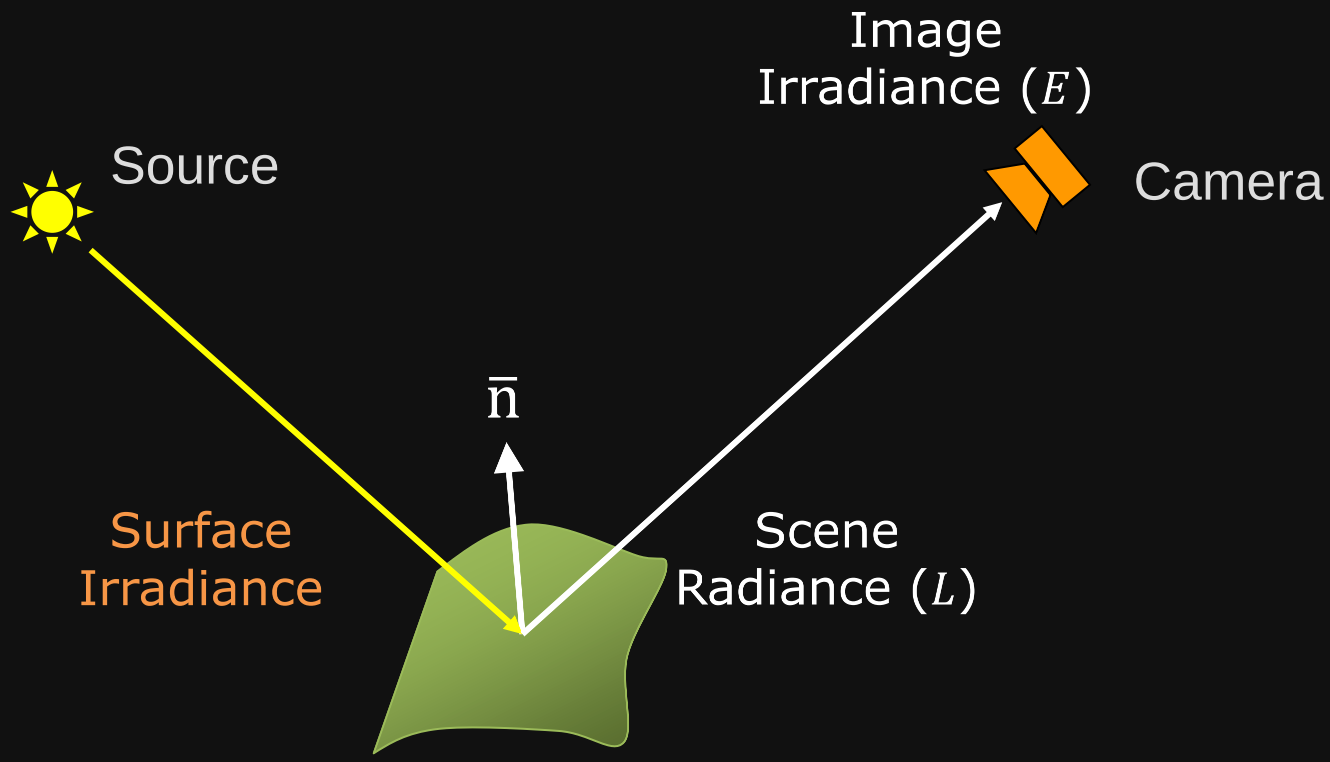
$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

NO.



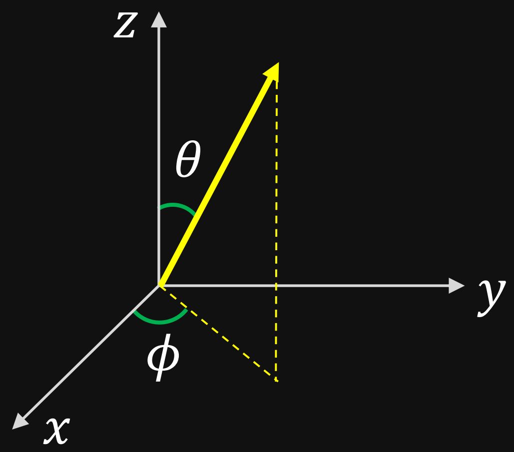
Larger the scene depth, smaller the solid angle subtended by each point onto the lens, and hence less light from each point.

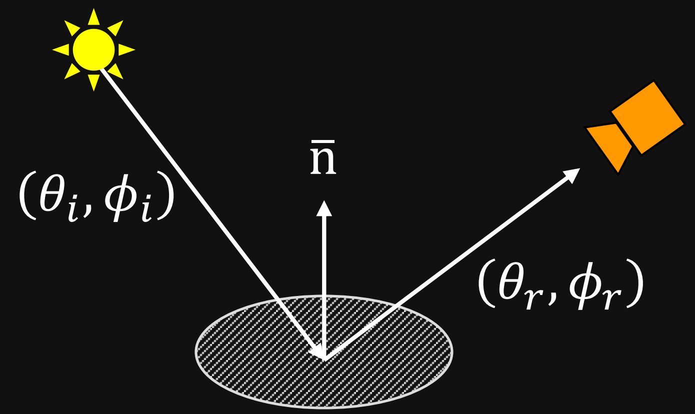
Surface Appearance



Surface reflection depends on both the viewing and illumination directions.

Bidirectional Reflectance Distribution Function (BRDF)





$E(\theta_i, \phi_i)$: Irradiance due to source in direction (θ_i, ϕ_i)

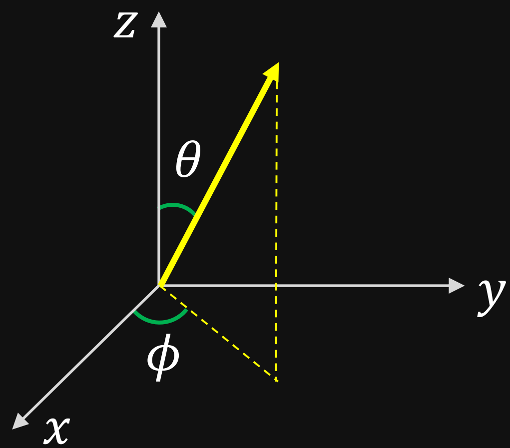
$L(\theta_r, \phi_r)$: Radiance of surface in direction (θ_r, ϕ_r)

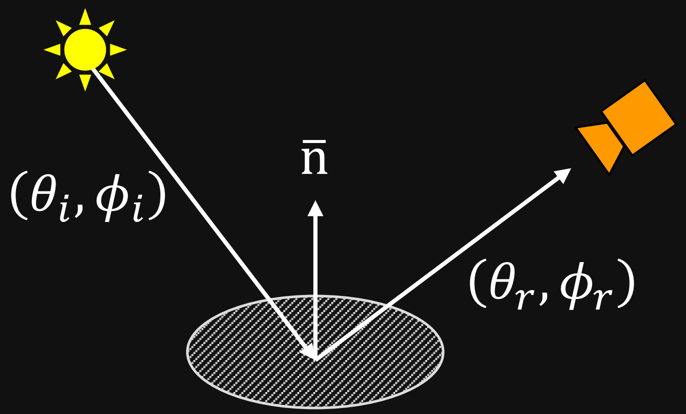
BRDF: $f(\theta_i, \phi_i, \theta_r, \phi_r) = \dfrac{L(\theta_r, \phi_r)}{E(\theta_i, \phi_i)}$

Unit: 1/sr

[Nicodemus 1977]

Properties of BRDF





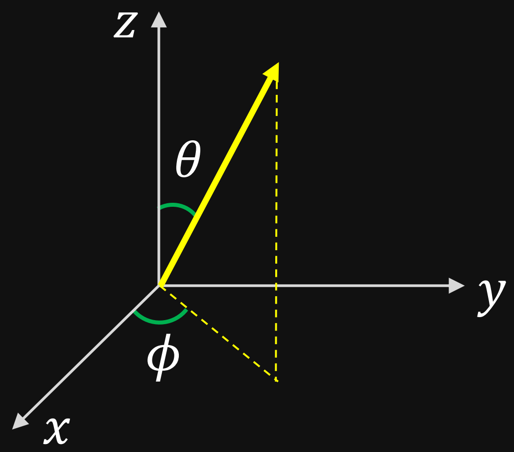
Non-Negative:

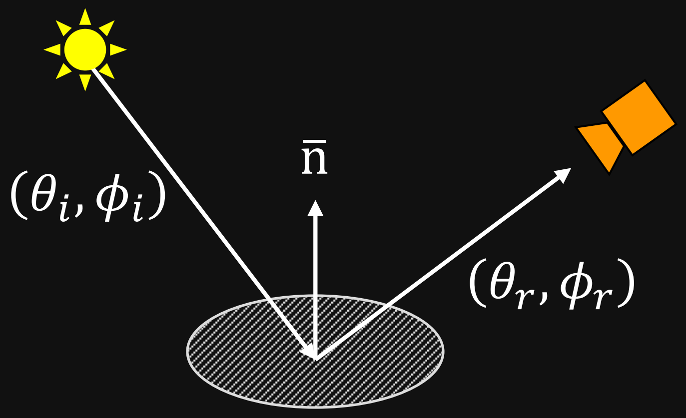
$$f(\theta_i, \phi_i, \theta_r, \phi_r) > 0$$

Helmholtz Reciprocity:

$$f(\theta_i, \phi_i, \theta_r, \phi_r) = f(\theta_r, \phi_r, \theta_i, \phi_i)$$

BRDF for Isotropic Surfaces



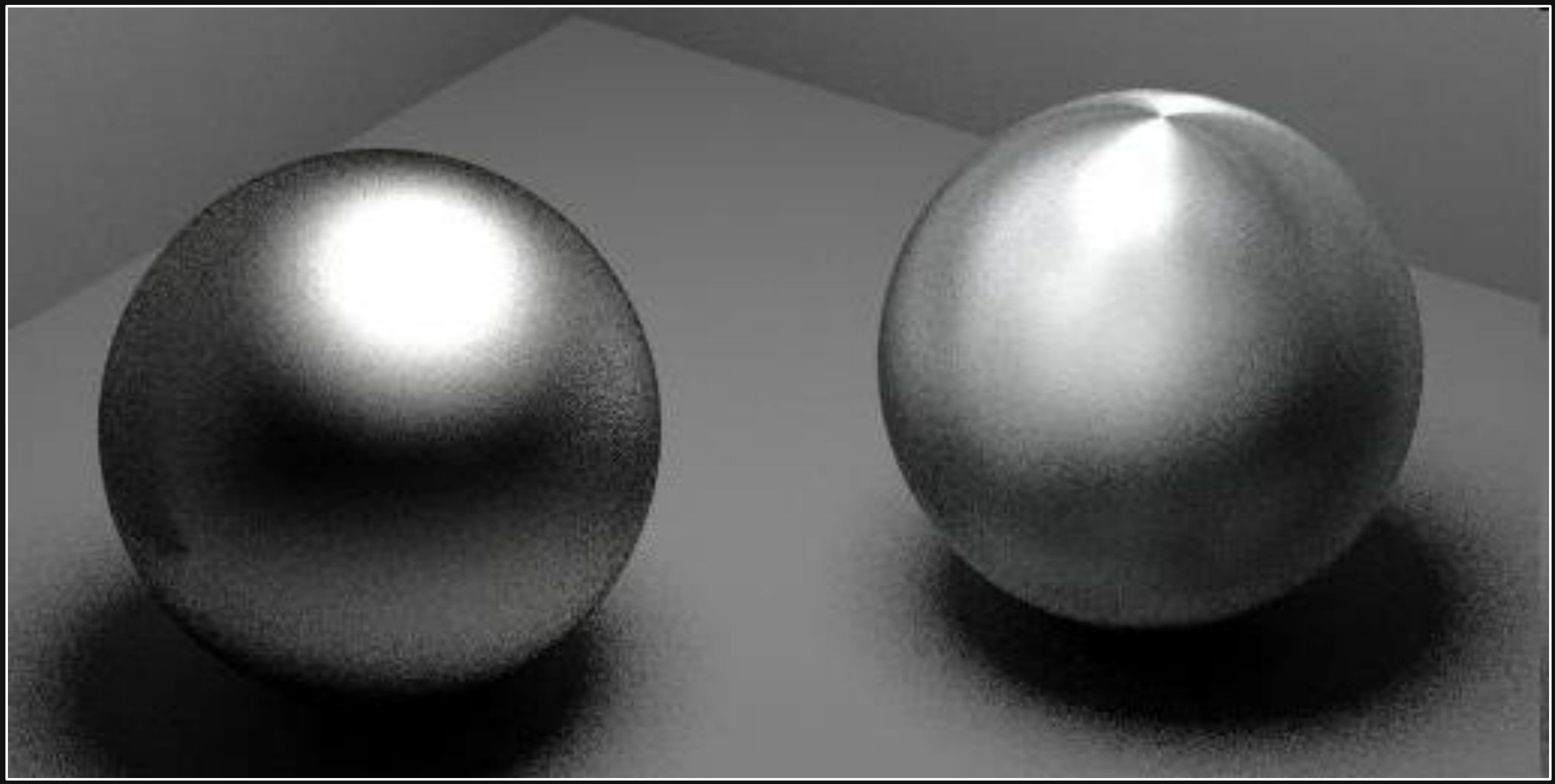


In general, BRDF is a 4-D function: $f(\theta_i, \phi_i, \theta_r, \phi_r)$

For rotationally symmetric reflectance (Isotropic Surfaces), BRDF is simplified as a 3-D function:

$$f(\theta_i, \theta_r, \phi_i - \phi_r)$$

Isotropic BRDF and Anisotropic BRDF



Isotropic BRDF

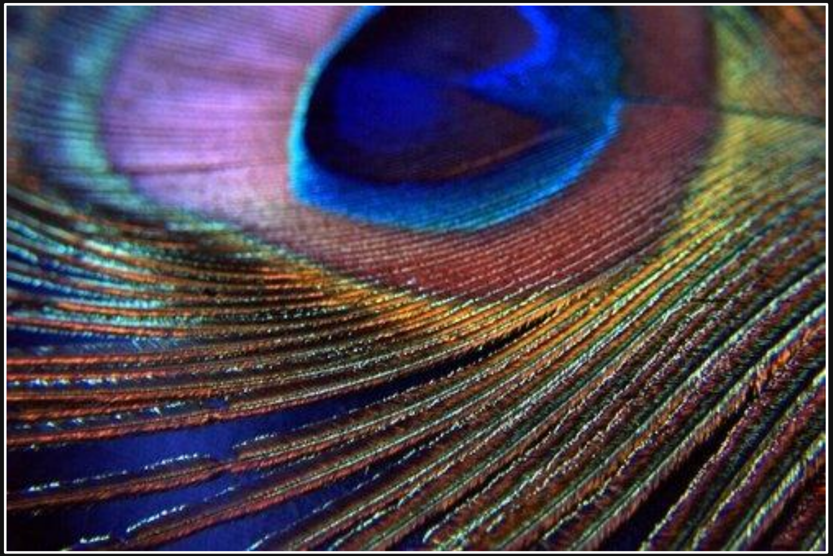
Anisotropic BRDF

[Westin 1992]

Isotropic BRDF and Anisotropic BRDF

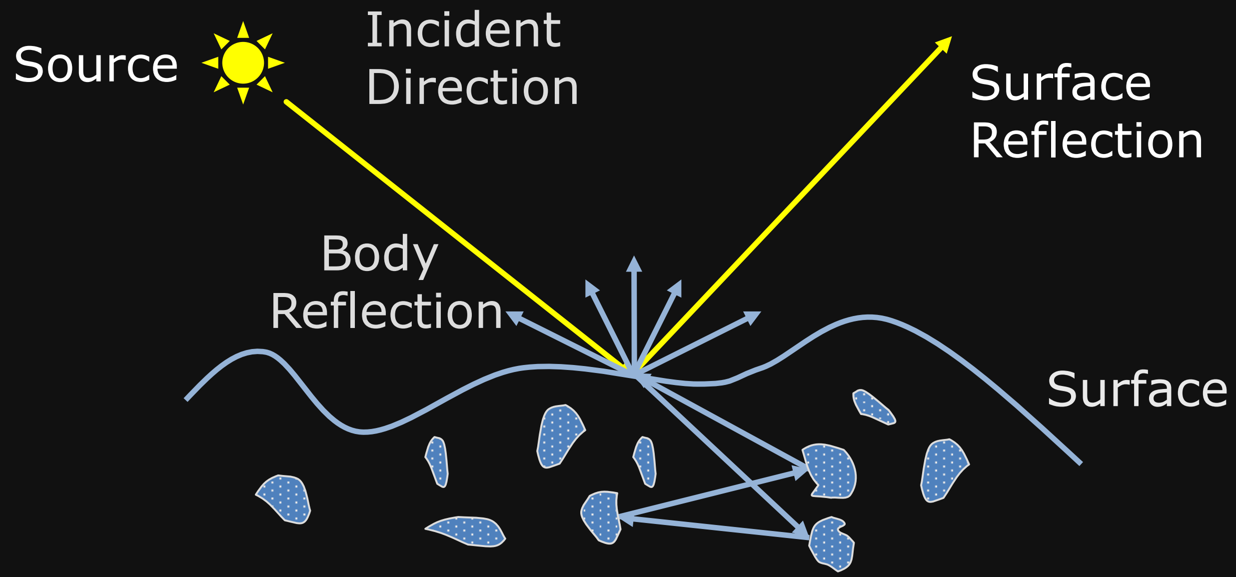
Anisotropic BRDFs in Real World

Butterfly wings



Peacock feathers

Reflection Mechanisms



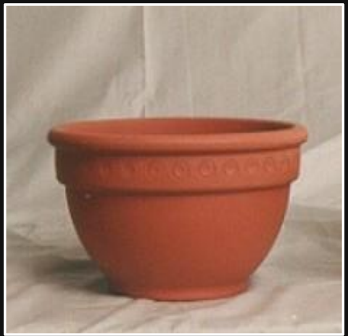
Surface Reflection
- Specular Reflection
- Glossy Appearance
- Smooth Surfaces (e.g., mirror, glass)

Body Reflection
- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium (e.g., clay, paper)

Image Intensity = Diffuse Reflection + Specular Reflection

Examples

Body Reflection:

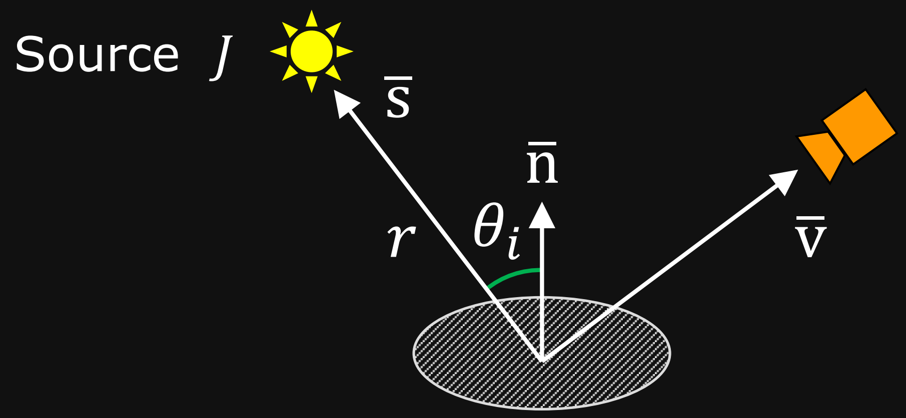


Surface Reflection:

Hybrid (Body + Surface) Reflection:

Diffuse Reflection: Lambertian Model

Surface appears equally bright from ALL directions



Lambertian BRDF:

$$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\rho_d}{\pi}$$

ρ_d : Albedo

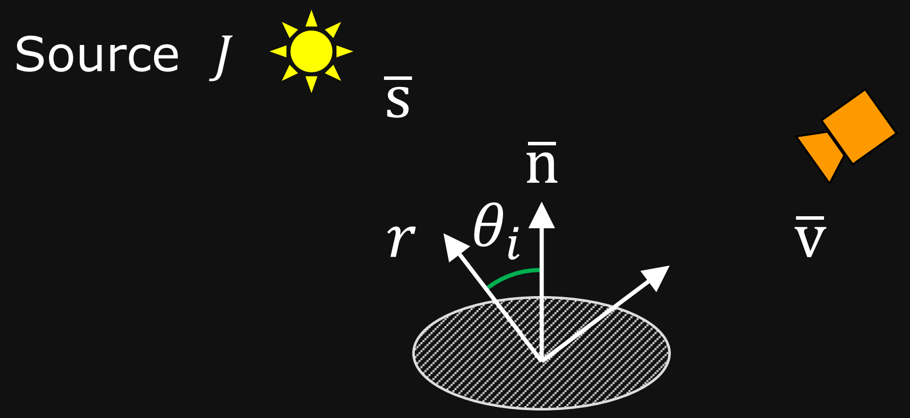
$(0 \leq \rho_d \leq 1)$

Commonly used in Computer Vision and Graphics.

[Lambert 1760]

Diffuse Reflection: Lambertian Model

Surface appears equally bright from ALL directions



Radiance is proportional to Irradiance

$$L = \frac{\rho_d}{\pi} E \qquad E = \frac{J \cos\theta_i}{r^2} = \frac{J}{r^2}(\bar{n} \cdot \bar{s})$$

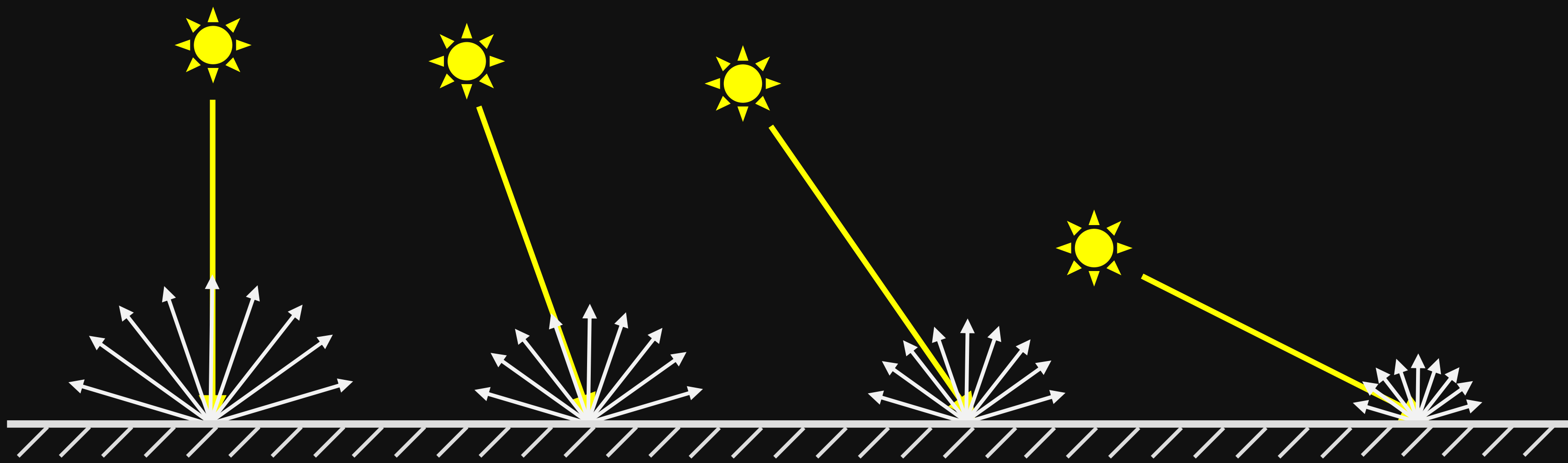
$$\boxed{L = \frac{\rho_d}{\pi} \frac{J}{r^2}(\bar{n} \cdot \bar{s})}$$

[Lambert 1760]

Diffuse Reflection: Lambertian Model

L is independent of viewing direction!

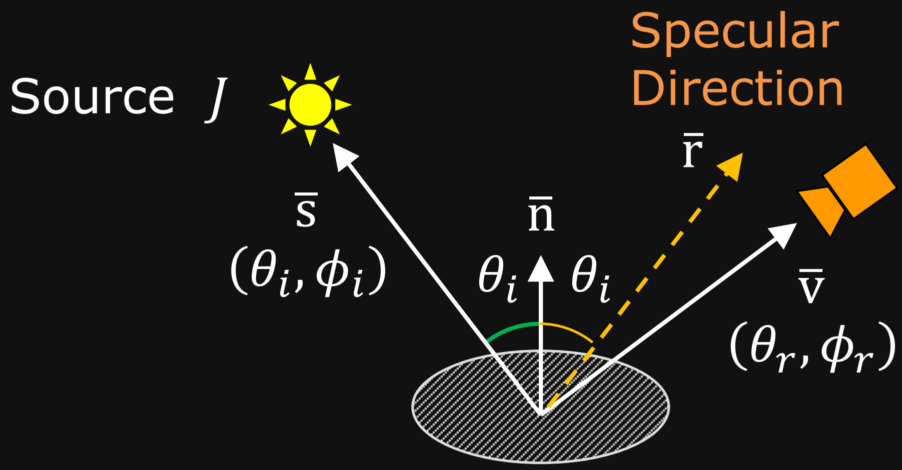
$$L = \frac{\rho_d}{\pi} \frac{J}{r^2} (\bar{\mathrm{n}} \cdot \bar{\mathrm{s}})$$



Lambertian Surface

Ideal Specular Model

Applicable to Only Perfect Mirrors. All incident energy is reflected in a single direction.



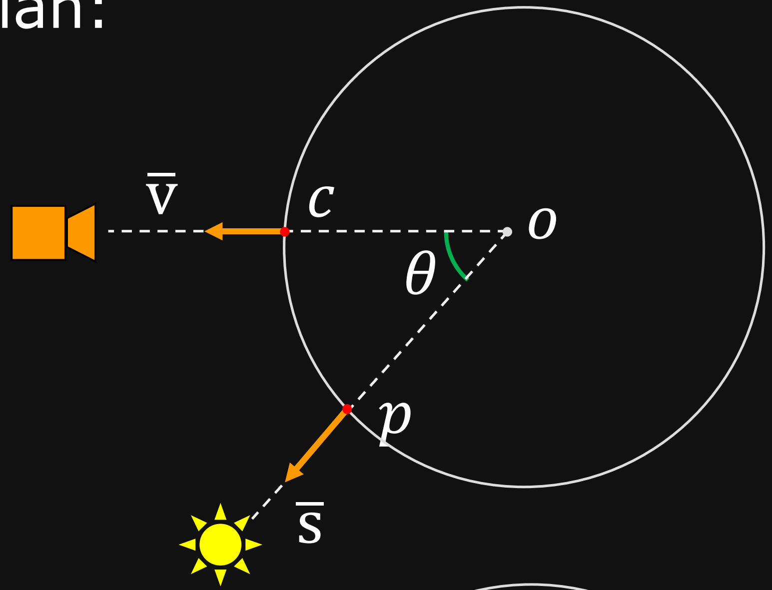
Mirror BRDF:

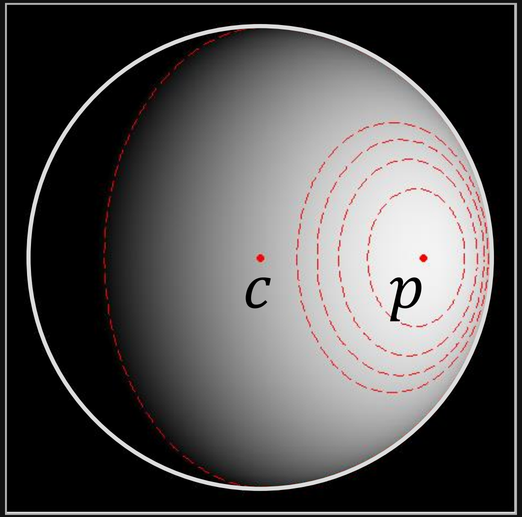
$$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\delta(\theta_i - \theta_r)\,\delta(\phi_i + \pi - \phi_r)}{\cos\theta_i \sin\theta_i}$$

Viewer receives light only when $\bar{\mathrm{v}} = \bar{\mathrm{r}}$

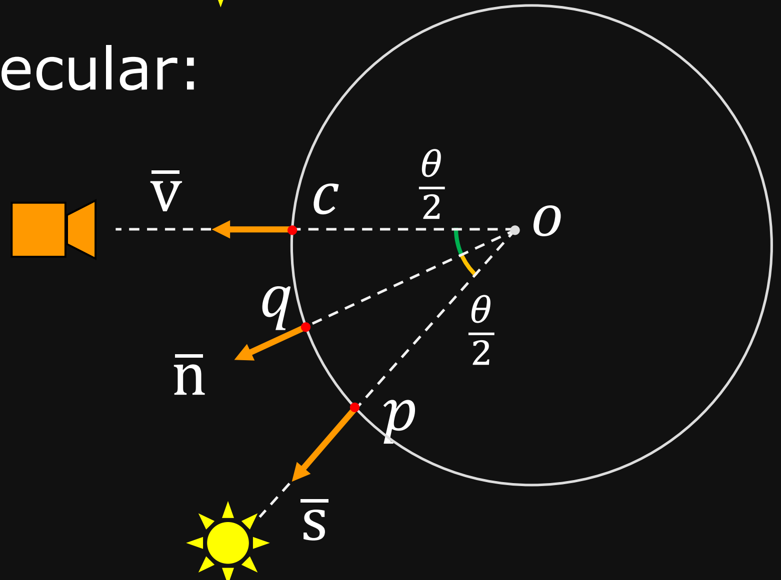
Examples

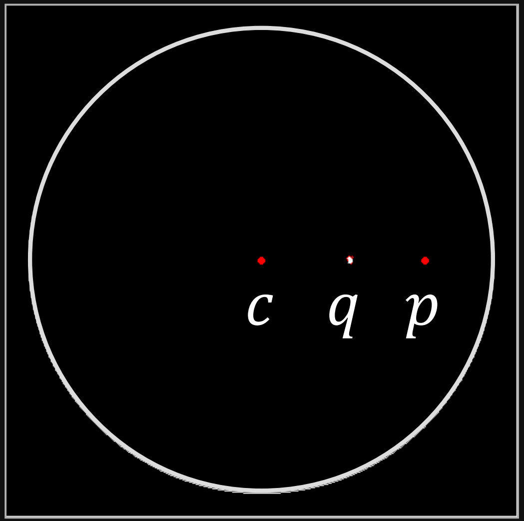
Lambertian:



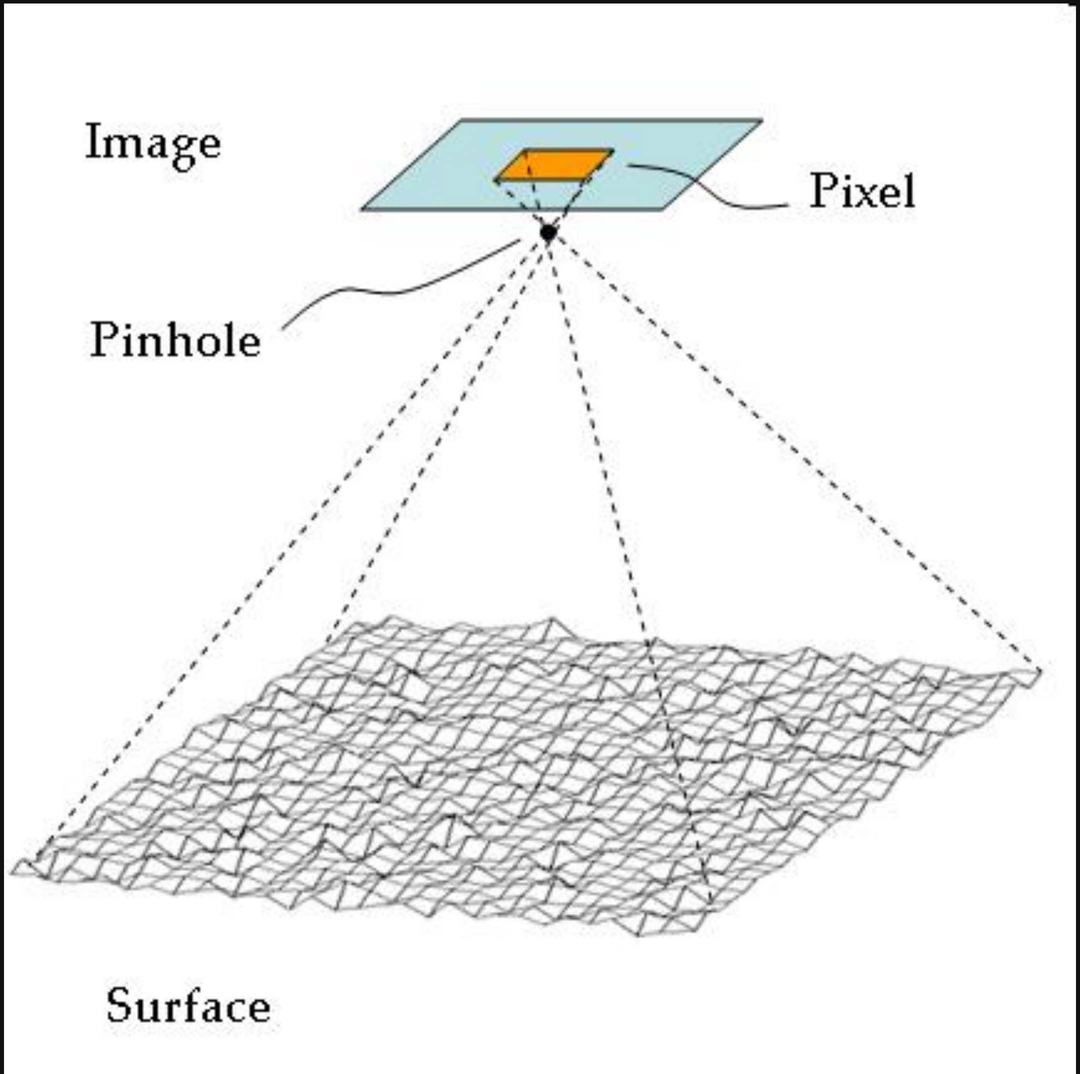


Ideal Specular:



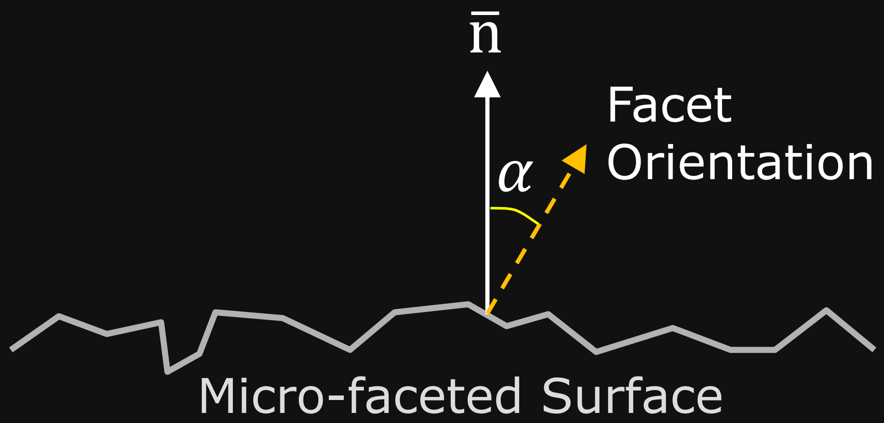


Surface Roughness



Modeling Surface Roughness

Micro-facet Structure for Rough Surfaces



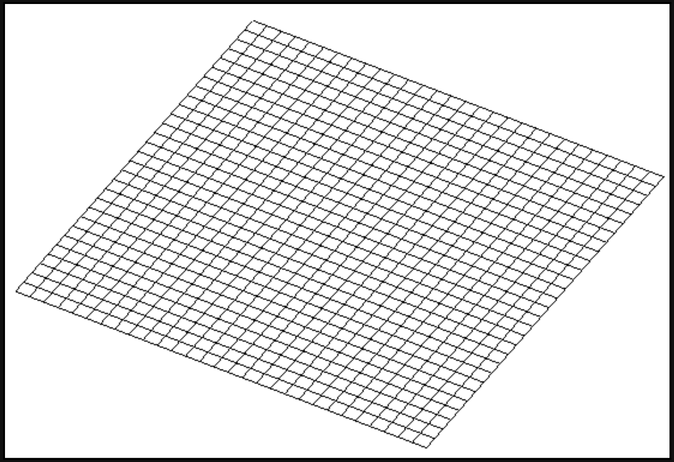
$\bar{n}$: Mean Orientation

Gaussian Micro-facet Model:

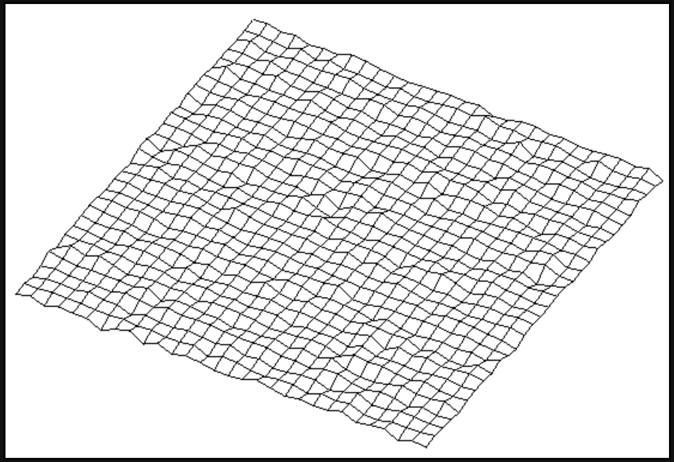
$$p(\alpha, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\alpha^2}{2\sigma^2}}$$

where σ: Roughness Parameter

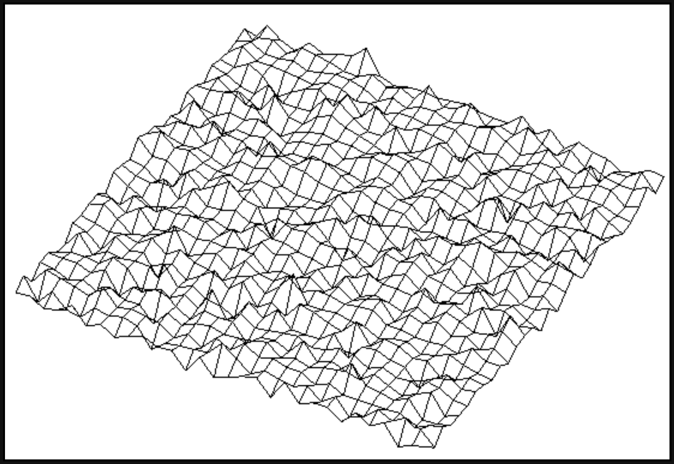
Rough Surfaces



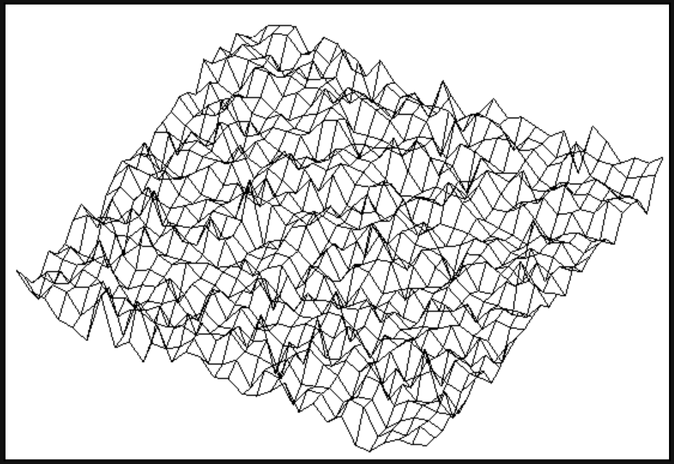
$\sigma = 0$



$\sigma = 0.1$



$\sigma = 0.3$



$\sigma = 0.6$

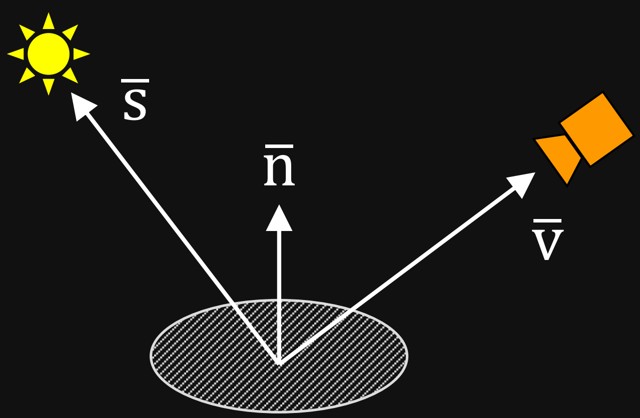
Specular Reflection of Rough Surfaces

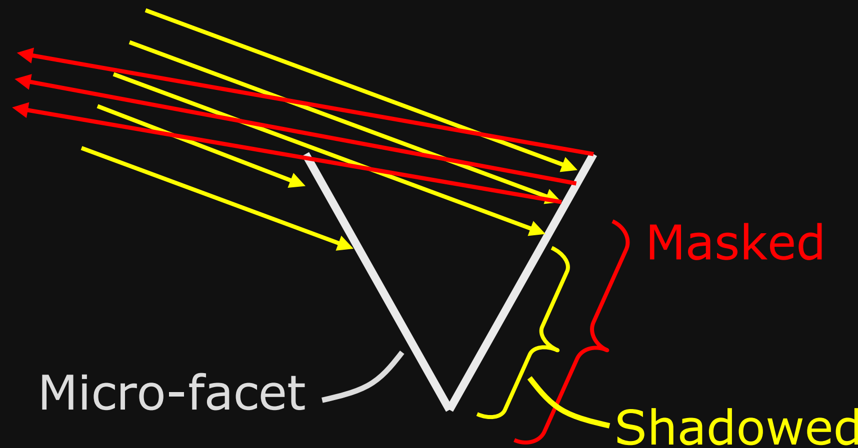
Torrance-Sparrow BRDF Model:

$$f(\bar{\mathrm{s}}, \bar{\mathrm{v}}) = \frac{\rho_s}{(\bar{\mathrm{n}} \cdot \bar{\mathrm{s}})(\bar{\mathrm{n}} \cdot \bar{\mathrm{v}})} \, p(\alpha, \sigma) \, G(\bar{\mathrm{s}}, \bar{\mathrm{n}}, \bar{\mathrm{v}})$$

where $p(\alpha, \sigma)$: surface roughness

$G(\bar{\mathrm{s}}, \bar{\mathrm{n}}, \bar{\mathrm{v}})$: geometric factor (masking, shadowing)

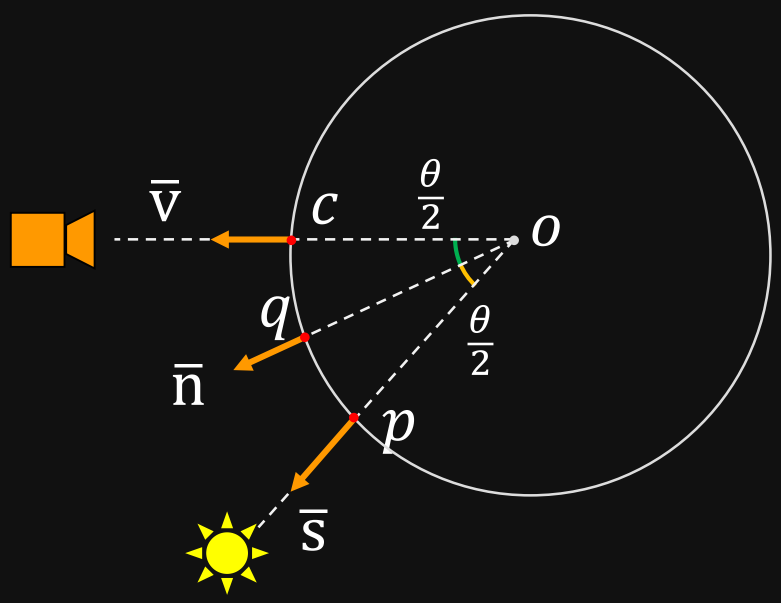


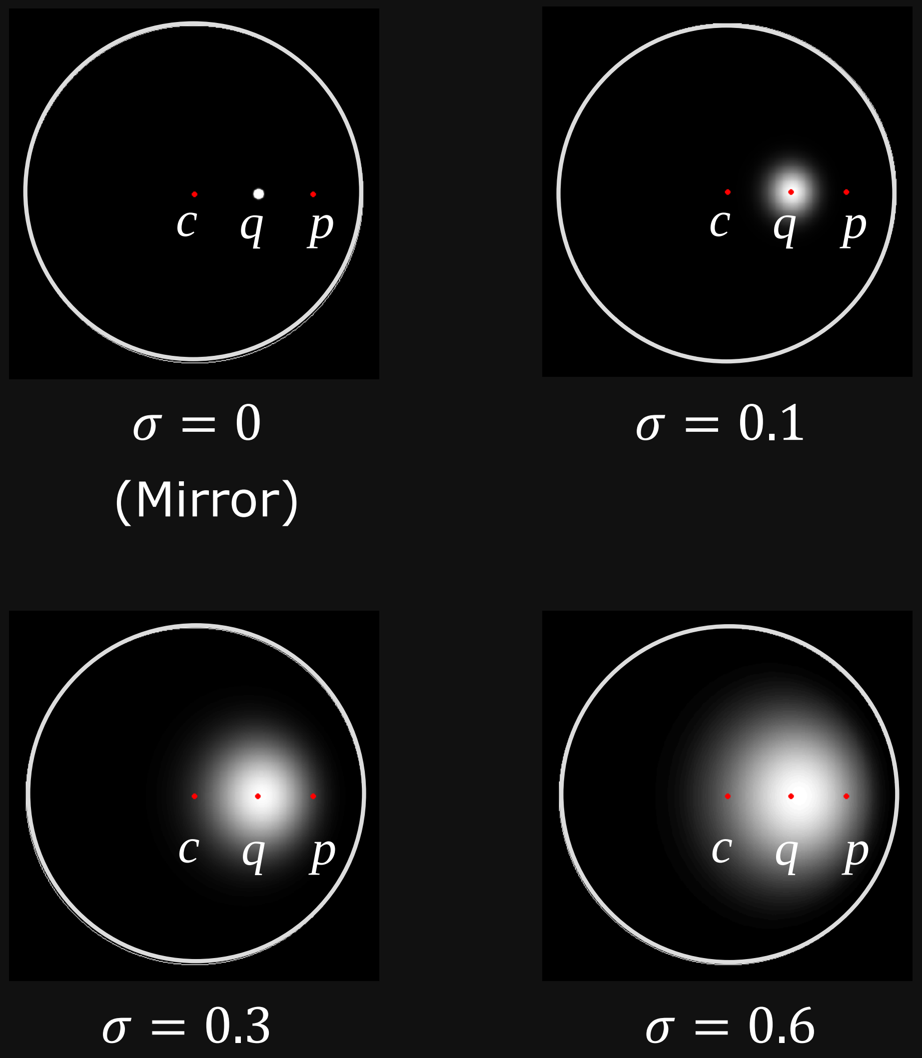


When $\sigma = 0$, it is the Perfect Mirror model.

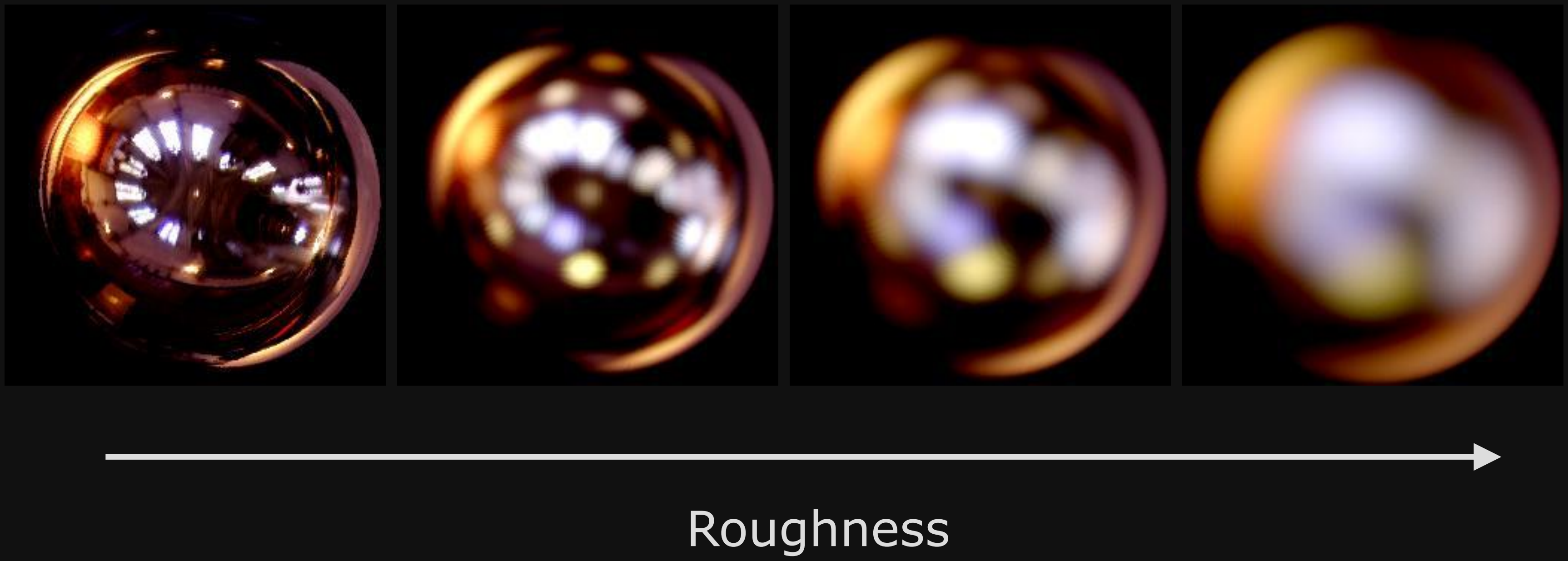
[Torrance 1967]

Specular Reflection of Rough Surfaces





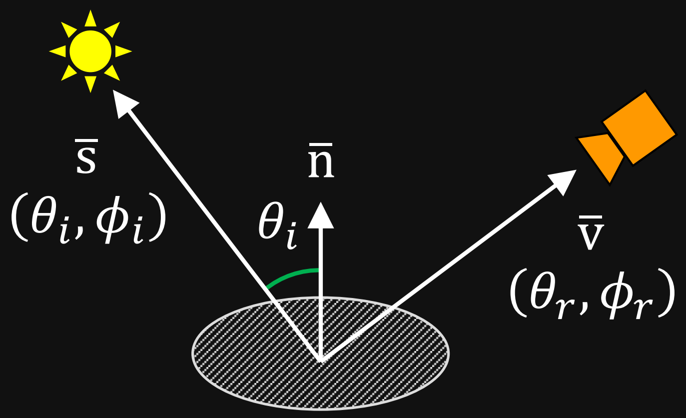
Specular Reflection of Rough Surfaces



Diffuse Reflection of Rough Surfaces

Oren-Nayar BRDF Model:

$$f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{\rho_d}{\pi}\left(A + B \cdot \max(0, \cos(\phi_r - \phi_i)) \cdot \sin\alpha \cdot \tan\beta\right)$$



$$A = 1 - \frac{\sigma^2}{2(\sigma^2 + 0.33)} \qquad \alpha = \max(\theta_i, \theta_r)$$

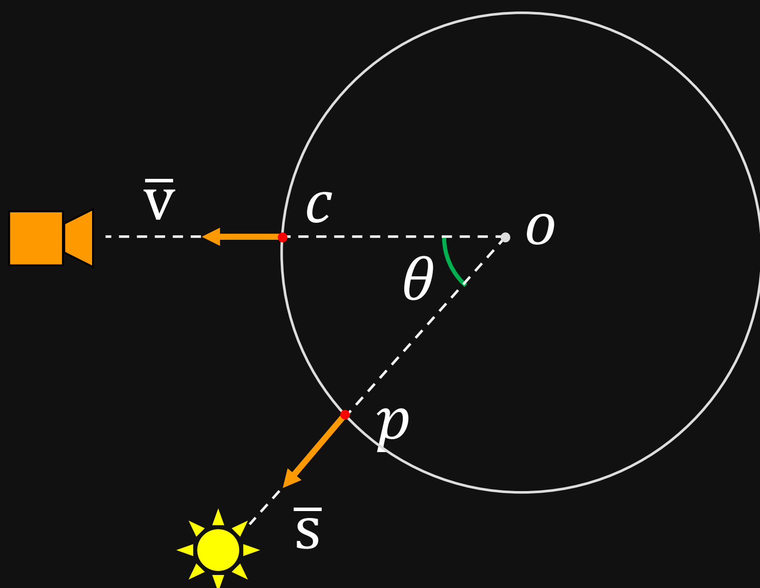
$$B = \frac{0.45\sigma^2}{(\sigma^2 + 0.09)} \qquad \beta = \min(\theta_i, \theta_r)$$

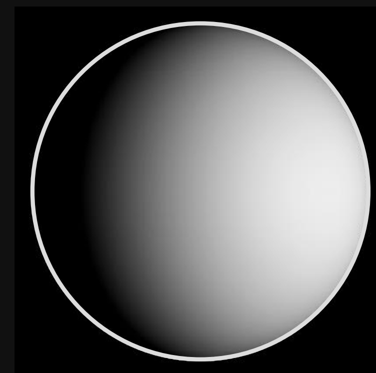
σ - surface roughness

When $\sigma = 0$, it is the Lambertian model.

[Oren 1993]

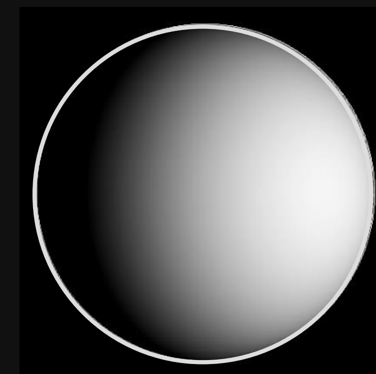
Diffuse Reflection of Rough Surfaces



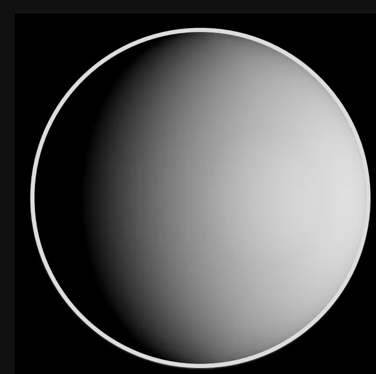


$\sigma = 0$

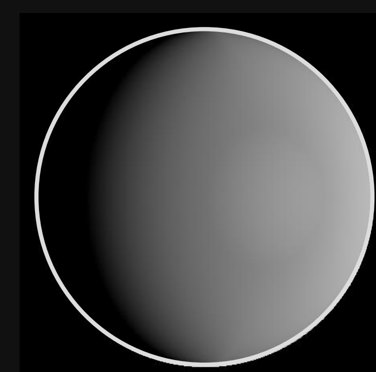
(Lambertian)



$\sigma = 0.1$

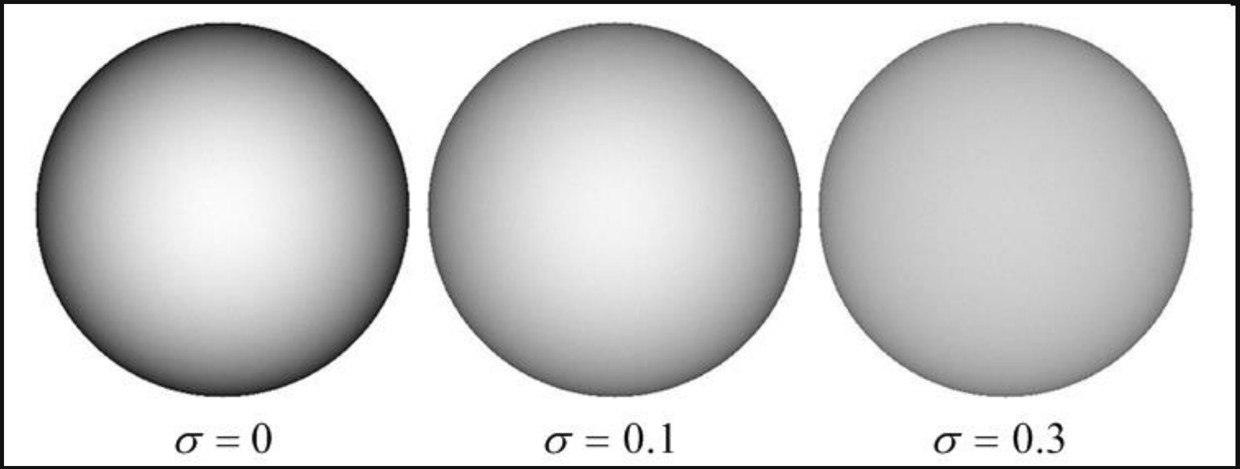


$\sigma = 0.3$



$\sigma = 0.6$

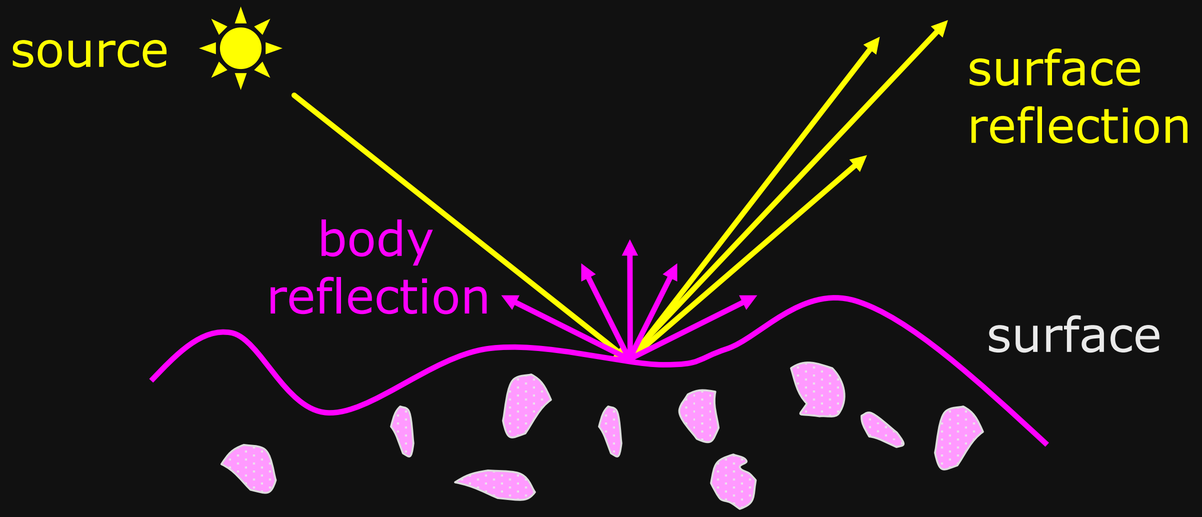
Diffuse Reflection of Rough Surfaces



Full Moon

[Oren 1993]

Color Reflectance Model



- Color of body (diffuse) reflection

  = color of object x color of illumination

- Color of surface (specular) reflection

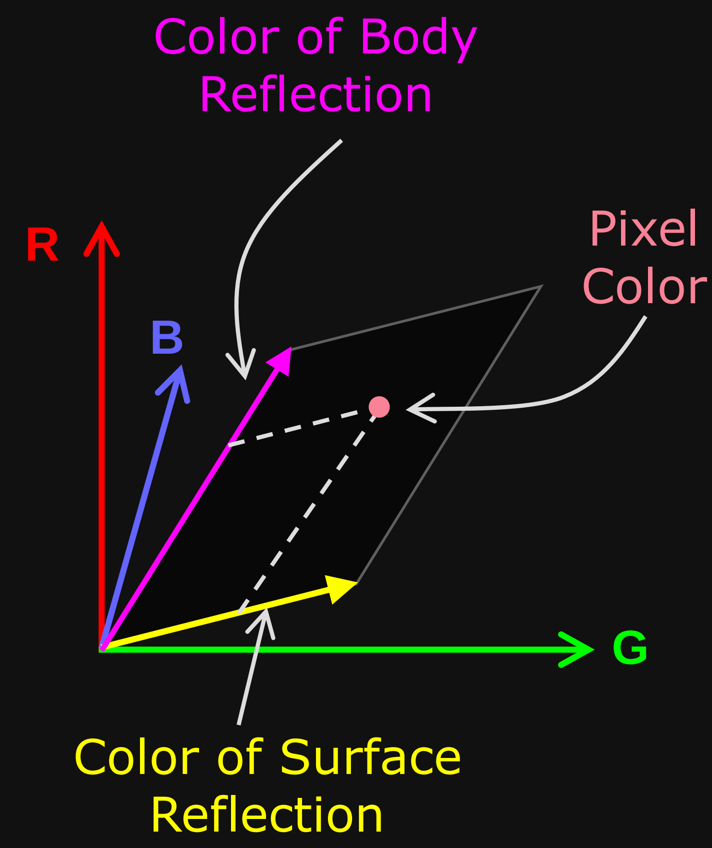
  = color of illumination

Color Reflectance: Dichromatic Model

Pixel color is a linear combination of the color of body reflection and the color of surface reflection.

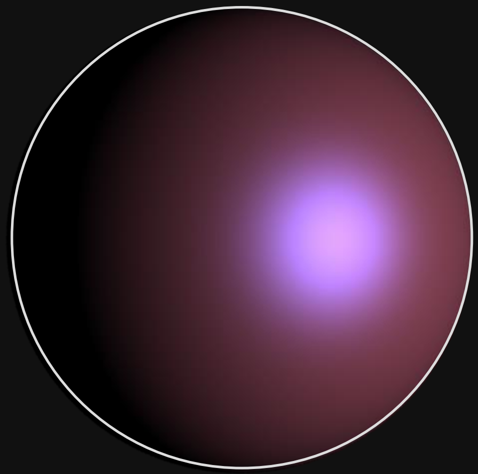
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = m_b \begin{bmatrix} R_b \\ G_b \\ B_b \end{bmatrix} + m_s \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix}$$

Color of Body Reflection   Color of Surface Reflection

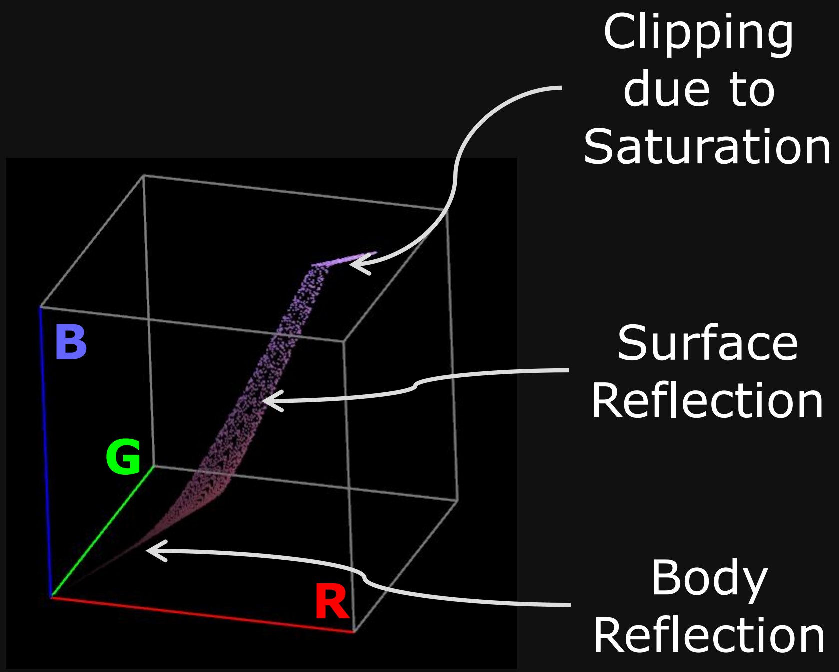


[Shafer 1985]

Color Reflectance: Dichromatic Model

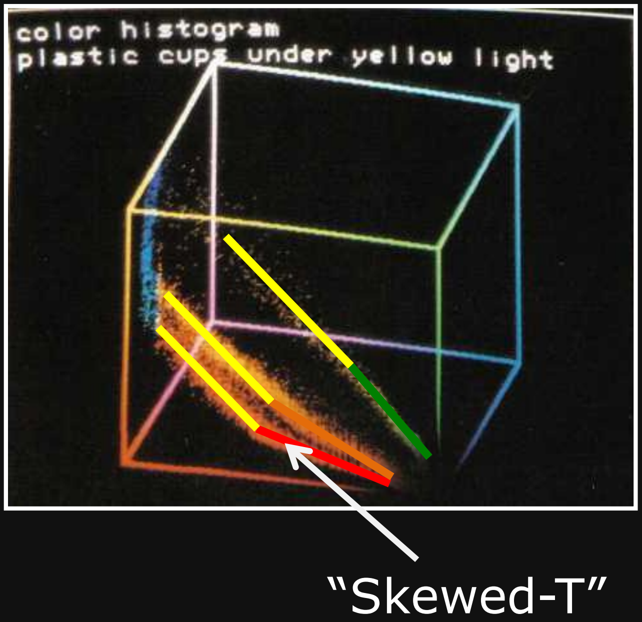


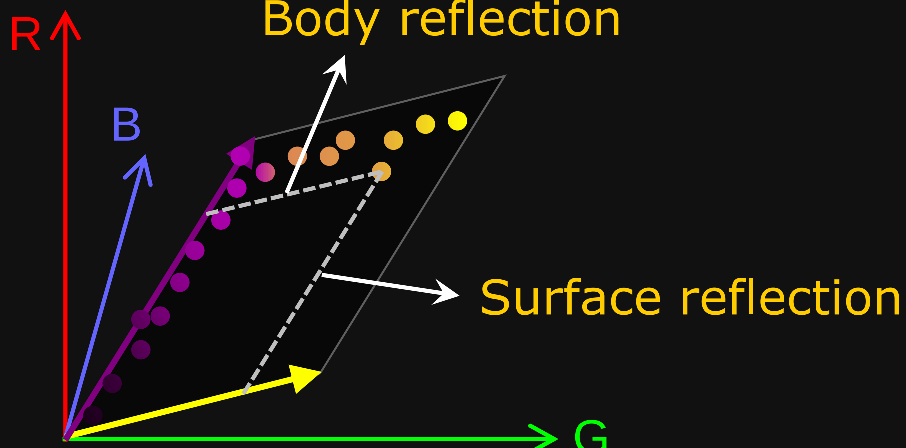
Illumination Color:



Color Histogram

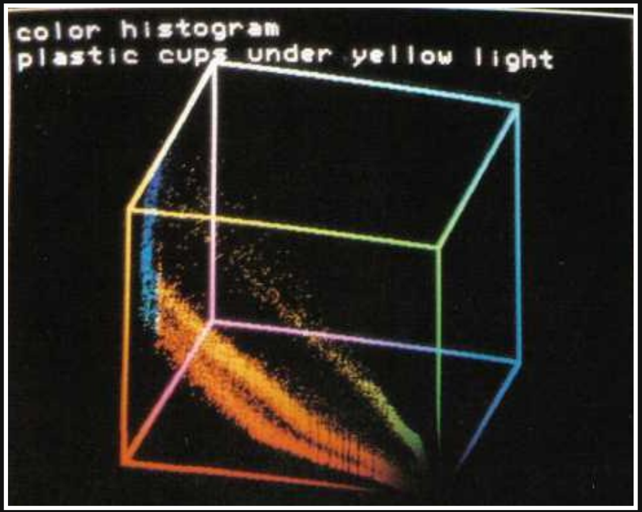
Separating Body and Surface Reflection

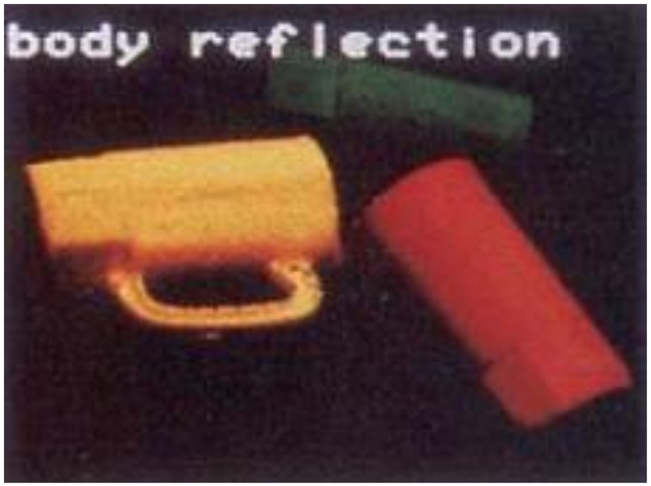




[Klinker 1990]

Separating Body and Surface Reflection





[Klinker 1990]

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