Logic for Knowledge Representation

willie

Announcements

Assignment 0

- outputlog.txt

Get started on Assignment 1

Assignment 2

- Extends Assignment 1
- Some code will not be released until after the late period of Assignment 1
- Instructions will be released on Monday, as scheduled

Wumpus World

Performance measure

- Gold +100, Death -100

Environment

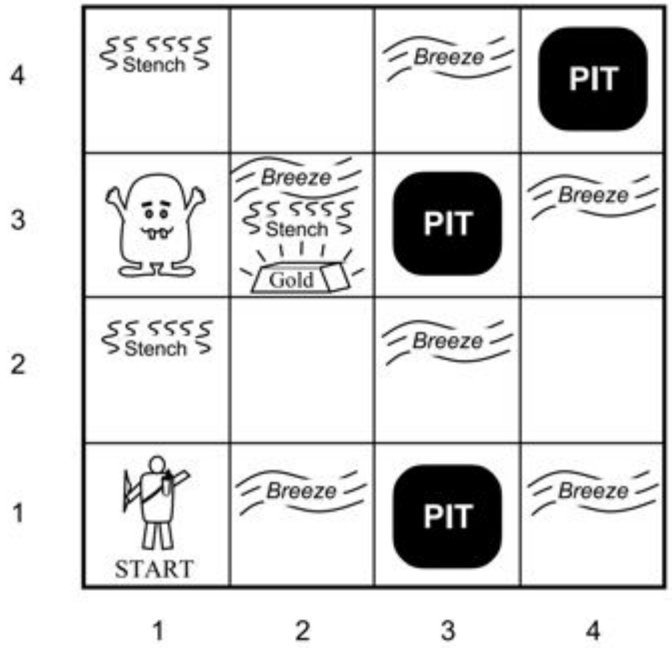
- Squares next to Wumpus are smelly
- Squares next to Pit are breezy

Actuators

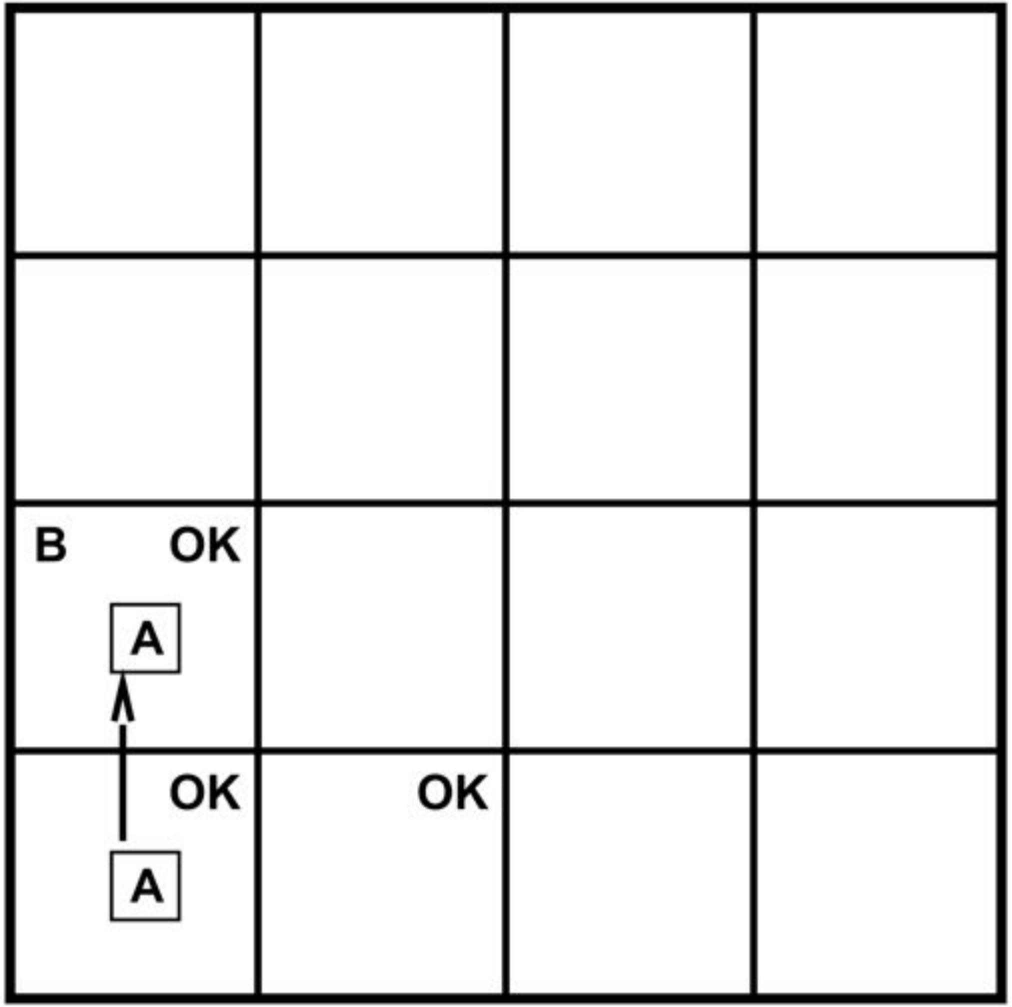
- Move up, down, left, or right
- Grab

Sensors

- Breeze, Glitter, Smell



OK			
OK $\boxed{A}$	OK		


B
OK
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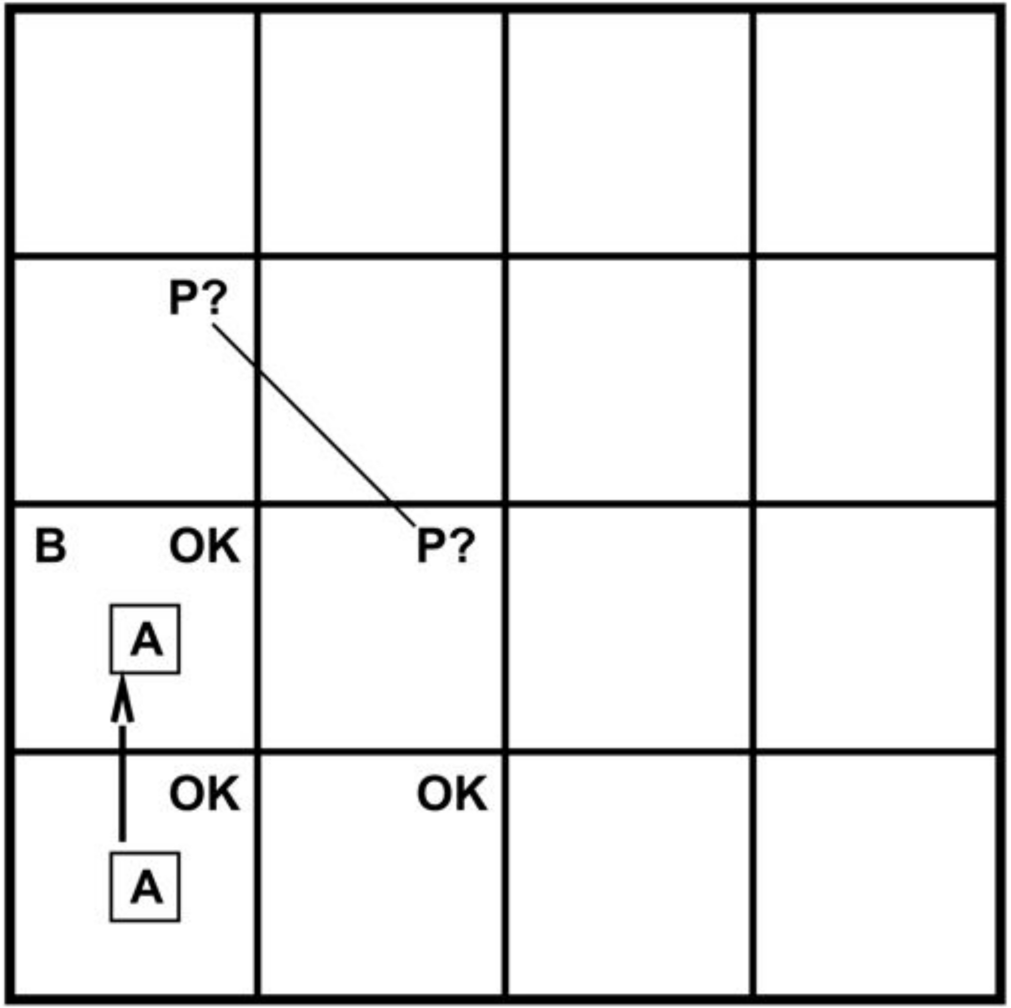
P?
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OK
P?
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OK
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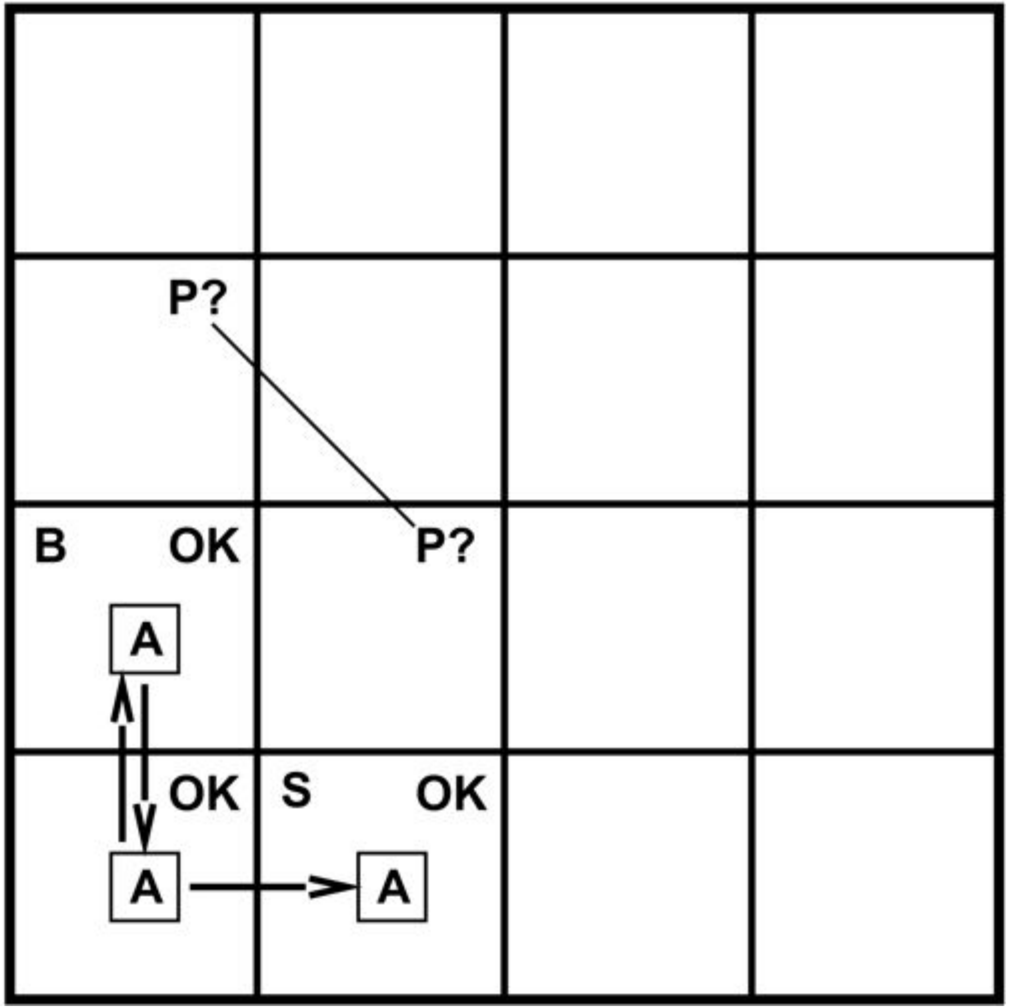
P?
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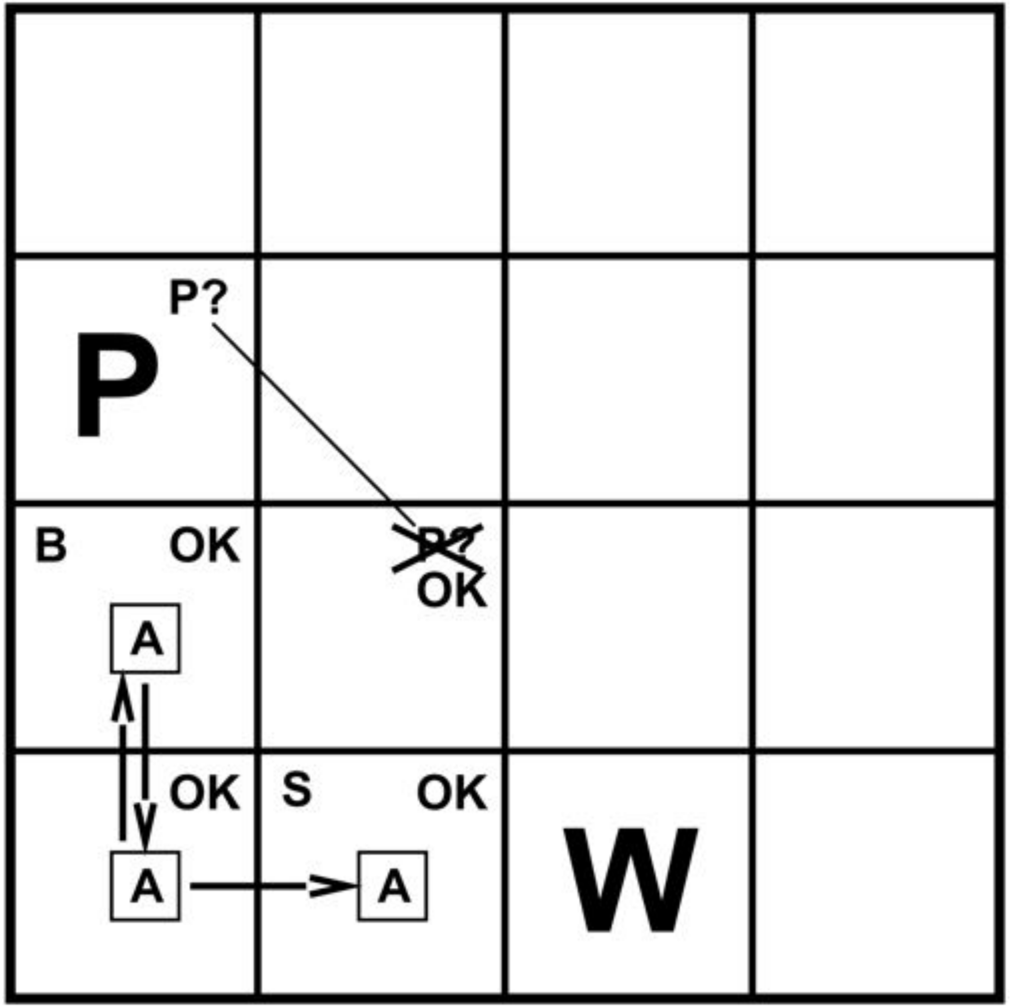
P?
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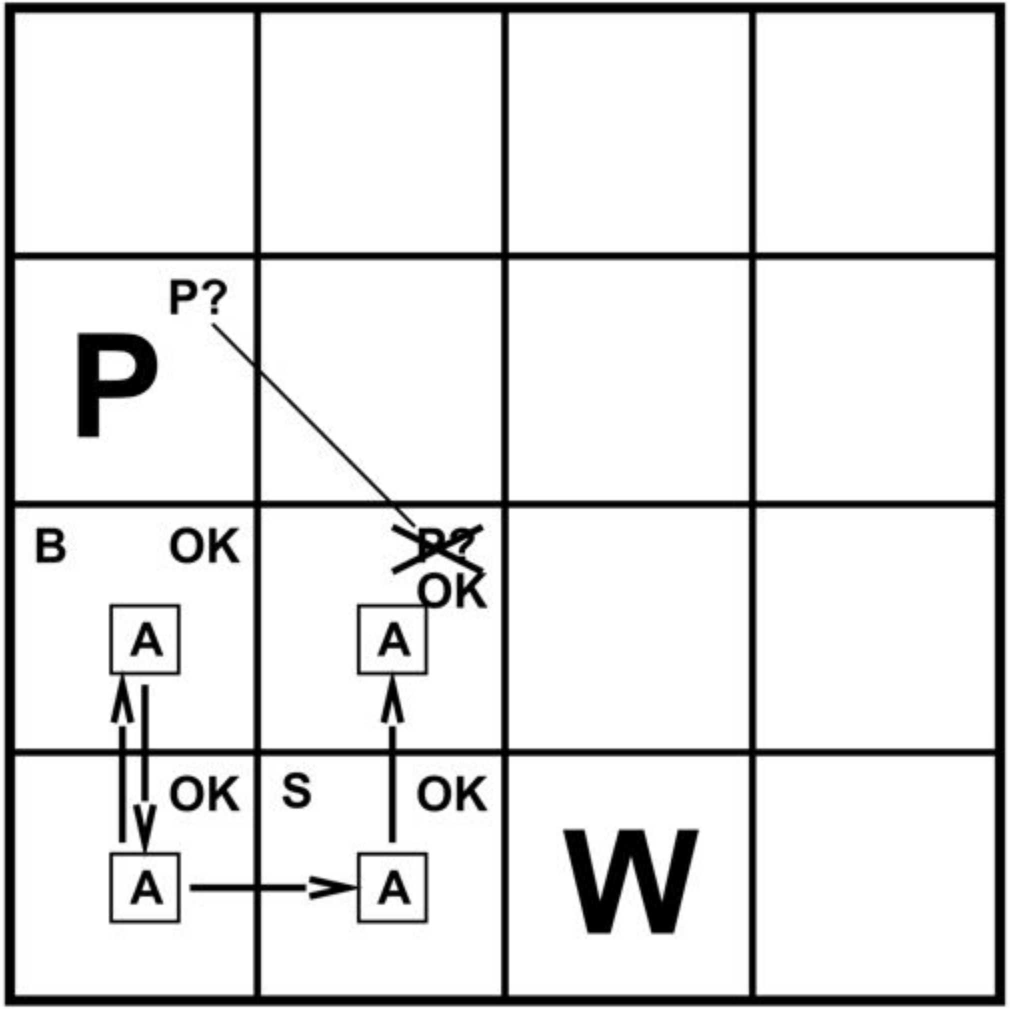
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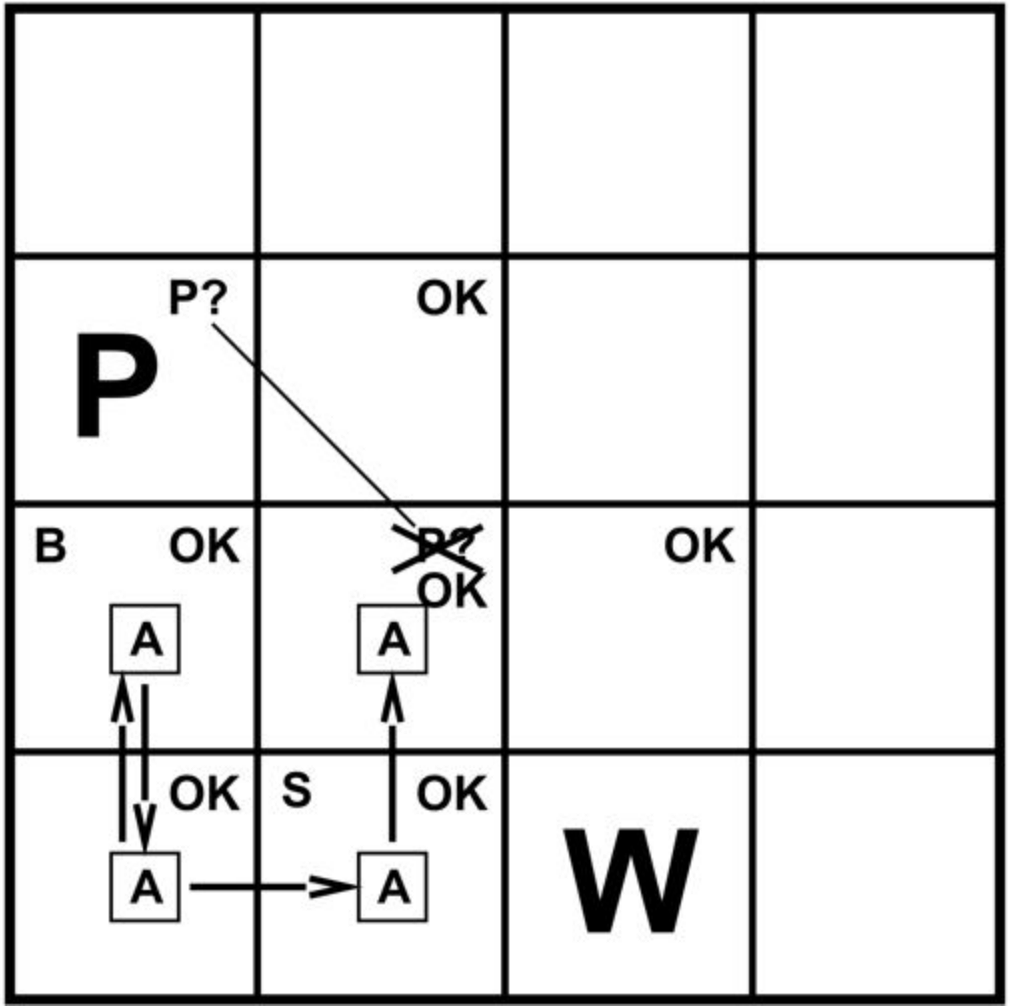
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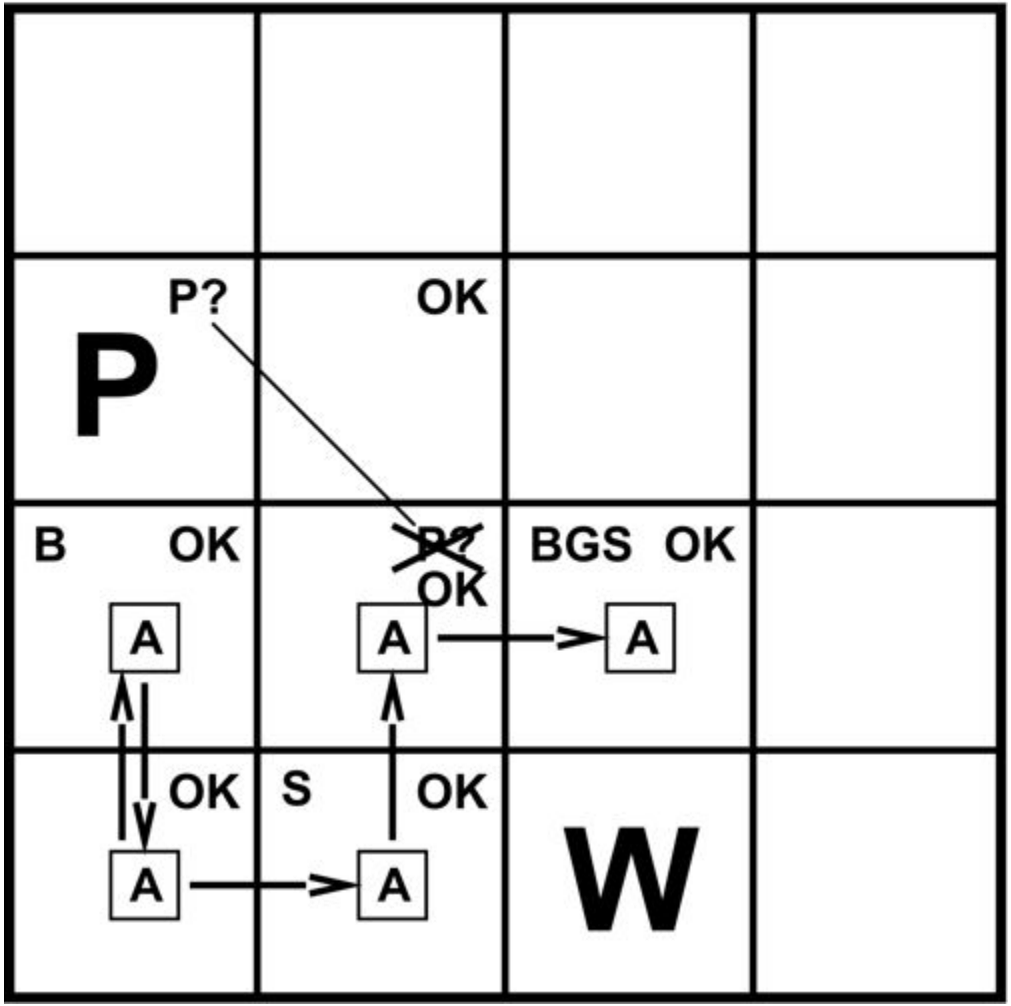
W



P?
OK
P
B
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A
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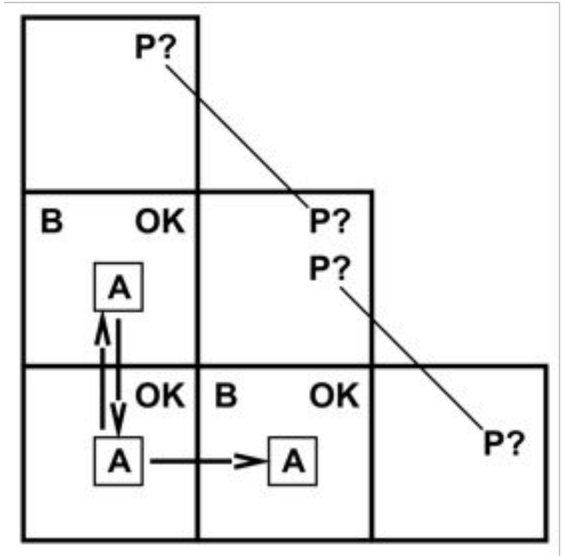
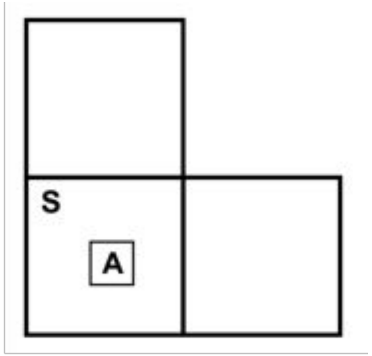
OK
S
OK
A
A
W

Tight Spots





Entailment

Entailment: One thing follows from another

$a \vDash b$

a entails b : in every *model* where a is true, b is also true

For example

If a is $x>4$ and b is $x>3$, then $a \vDash b$

If a is “when it rains it is cloudy” and “it is raining” and b is “it is cloudy” then $a \vDash b$

Propositional Logic

Let’s start to formalize this

Symbols to represent elements of the world

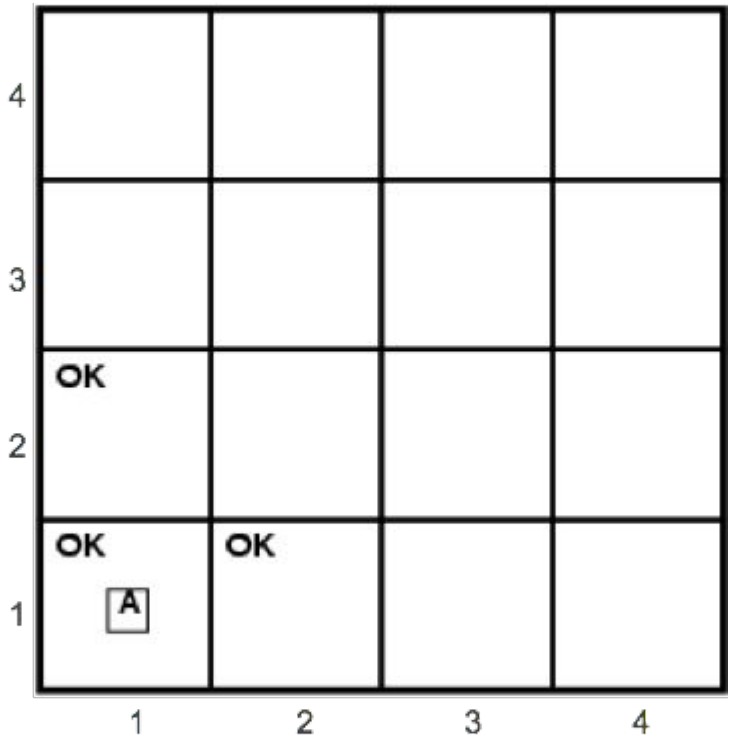
Each proposition

A possible condition of the world that may be true of false

Atomic sentences, single proposition:

```
is_raining
have_umbrella
feel_breeze
```

Representing Wumpus World



128 in total,
8 optional symbols per square (16 squares)

Symbol	Meaning
$a_{1,1}$	Agent is at (1,1)
$k_{1,1}$	(1,1) is OK
$e_{1,1}$	Stench at (1,1)
$z_{1,1}$	Breeze at (1,1)
$g_{1,1}$	Glitter at (1,1)
$w_{1,1}$	Wumpus at (1,1)
$p_{1,1}$	Pit at (1,1)
$d_{1,1}$	Gold at (1,1)

Repeat for each position in Wumpus World

Complex sentences

Use operators to turn simple sentences into complex sentences

Operators:

Not	$\neg$	$\sim$
And	$\wedge$	
Or	$\vee$	
Implies	$\Rightarrow$	$\rightarrow$
Biconditionals	$\Leftrightarrow$	$\leftrightarrow$

<u>Examples</u>

It is not raining

    ¬raining

It is raining and I have an umbrella

    raining ∧ umbrella

It is raining or it is sunny

    raining ∨ sunny

If it is raining, then I am wet

    raining ⇒ wet

It is sunny if and only if it is not cloudy

    sunny ⇔ ¬cloudy

Syntax

```
S := <Sentence> ;

<Sentence> := <AtomicSentence> | <ComplexSentence> ;

<AtomicSentence> := "TRUE" | "FALSE" | <Symbol>;

<Symbol> := “P” | “Q” | “R” | … ;

<ComplexSentence> := "(" <Sentence> ")" |
       <Sentence> <Connective> <Sentence> |
       "¬" <Sentence> ;

<Connective> := "∧" | "∨" | "⇒" | "⇔" ;
```

Complex sentences for Wumpus World

If no stench at (1,1) then adjacent areas do not have the Wumpus

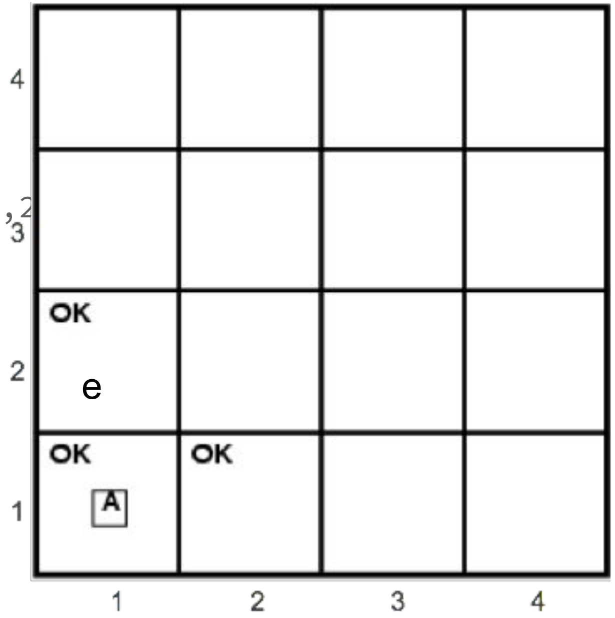
$\neg e_{1,1} \Rightarrow \neg w_{1,1} \land \neg w_{2,1} \land \neg w_{1,2}$

Similarly, if no stench at (2,2)...

$\neg e_{2,2} \Rightarrow \neg w_{2,2} \land \neg w_{2,3} \land \neg w_{3,2} \land \neg w_{2,1} \land \neg w_{1,2}$

Also

$e_{1,2} \Rightarrow w_{1,3} \lor w_{1,2} \lor w_{1,1} \lor w_{2,2}$



Logical Entailment

Goal: Deduce new facts using:

$\left.\begin{array}{l}\text{Rules about the world (background knowledge)}\\ \text{Information we gather through perception}\end{array}\right\}$ KB

$$KB \vDash a$$

KB

Last night I ate either pizza or spam

I did not eat pizza last night

$\vDash$

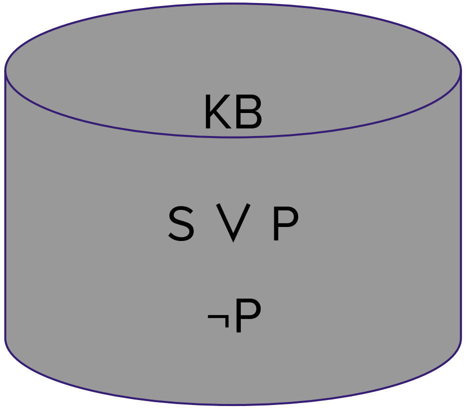
Last night I ate spam

KB entails a : in every *model* where all sentences in KB are true, a is also true

Model

4 possible models b/c
S V P uses both s & p, while s can be T/F and p can be T/F

Assignment of a truth value (true/false) to every atomic sentence



M_1	S = False P = False	M_3	S = True P = False
M_2	S = False P = True	M_4	S = True P = True

A Model *m* is a model of **KB** iff it is a model of all sentences in the **KB**

All sentences in KB are true in *m*

Satisfiability

A KB is **satisfiable** iff it admits at least one model

Otherwise it is **unsatisfiable**

KB1 is {P, ¬Q ∧ R}. Satisfiable? Unsatisfiable? yes

KB2 is {¬P ∨ P}. Satisfiable? Unsatisfiable? yes

KB3 is {P, ¬P}. Satisfiable? Unsatisfiable? no

KB ⊨ a iff every model of KB is also a model of a

KB entails a iff {KB, ¬a} is unsatisfiable

Sound and Complete

Sentence *a* is derived from KB by algorithm *i*.

An algorithm *i* is **sound** (truth preserving) if it derives only entailed sentences

Highly desirable property

Doesn't make up facts

An algorithm *i* is **complete** if it derives all entailed sentences

Also desirable

Entailment in Wumpus World

Nothing detected in [1,1],

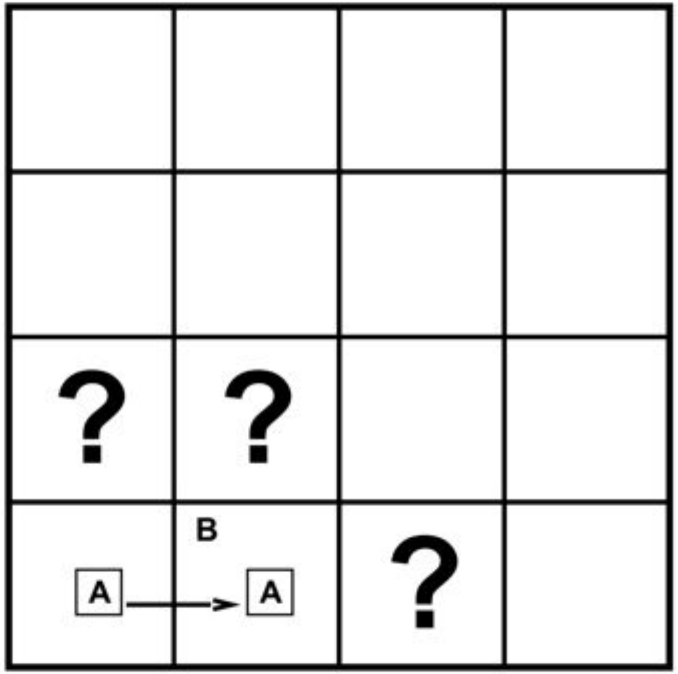
Move right, breeze in [2,1]

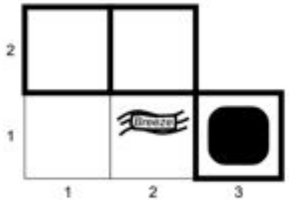
What are the possible worlds for the ?'s

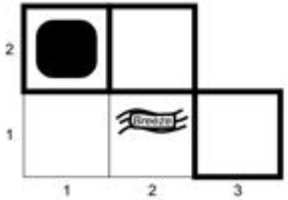
(assuming only pits)

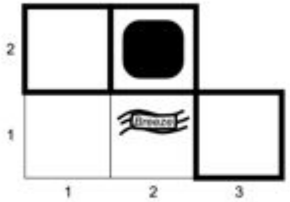
3 Boolean choices

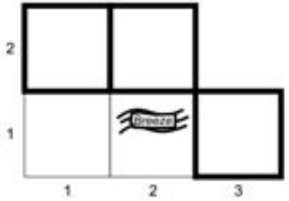
8 models (possible worlds)

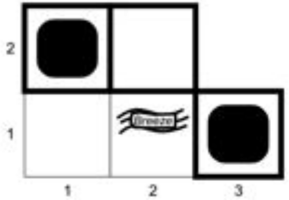


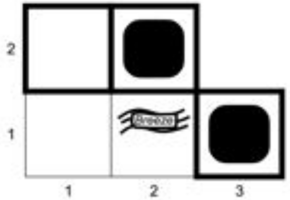

2
1
Breeze
1 2 3

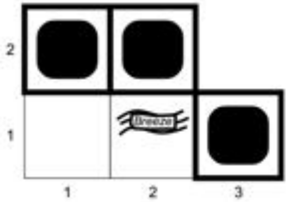

2
1
Breeze
1 2 3

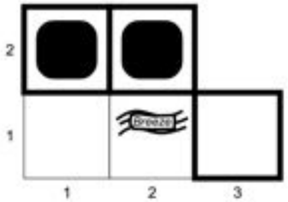

2
1
Breeze
1 2 3


2
1
Breeze
1 2 3


2
1
Breeze
1 2 3

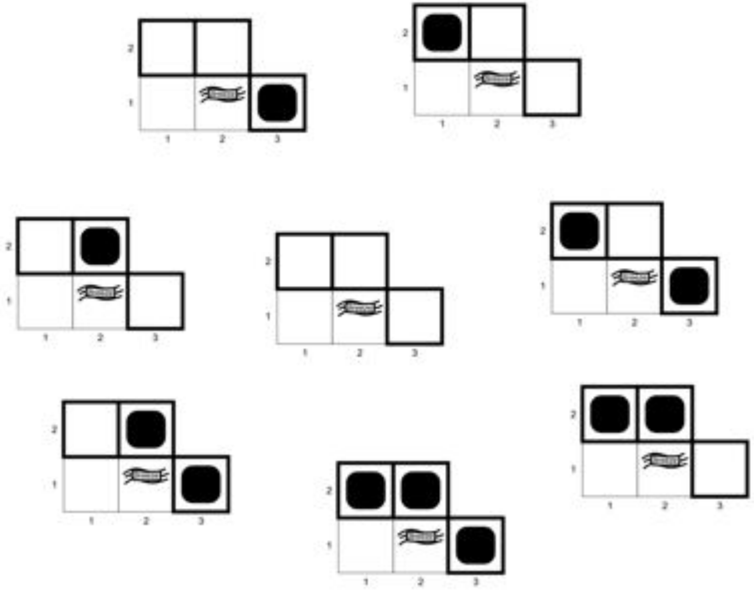

2
1
Breeze
1 2 3


2
1
Breeze
1 2 3


2
1
Breeze
1 2 3

Possible worlds

[1,2]	[2,2]	[3,1]
		pit
	pit	
	pit	pit
pit		
pit		pit
pit	pit	
pit	pit	pit

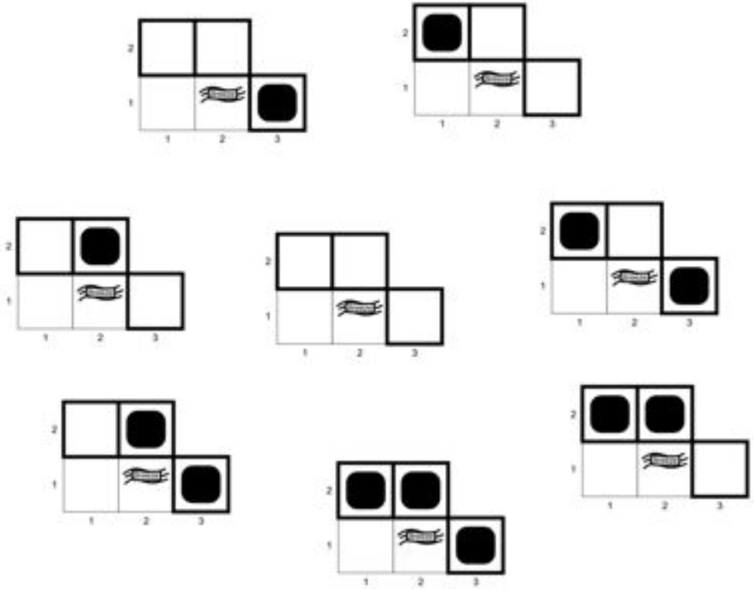


Possible worlds

Nothing detected in [1,1],

Move right, breeze in [2,1]

Which of these is a model of the KB?



Possible worlds

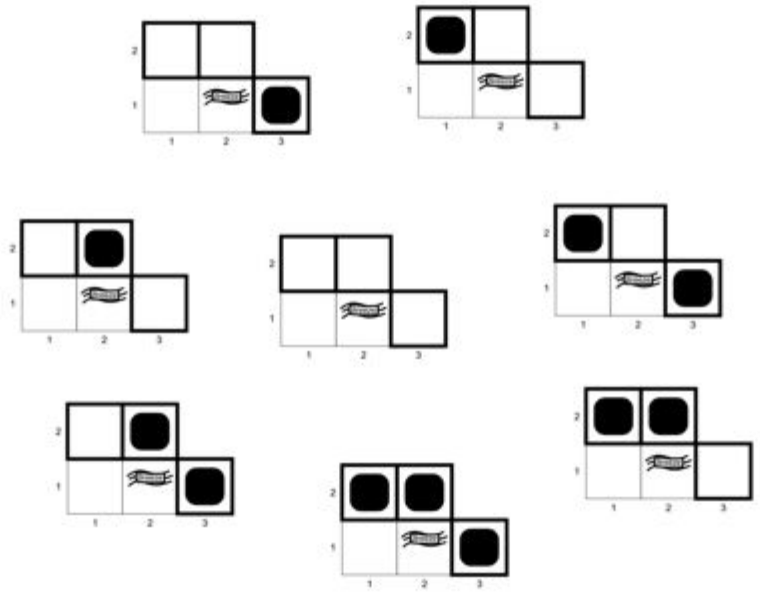
Is it safe to move up to [1 , 2] ?

Let *a* represent “[1 , 2] is safe”

Does KB ⊨ *a* ?

> *a* entails *b*: in every model where *a* is true, *b* is also true

Prove by model checking



Possible worlds

Is it safe to move up to [1 , 2] ?

Let *a* represent “[1 , 2] is safe”

Does KB ⊨ *a* ?

> a entails b: in every model where a is true, b is also true

Prove by model checking

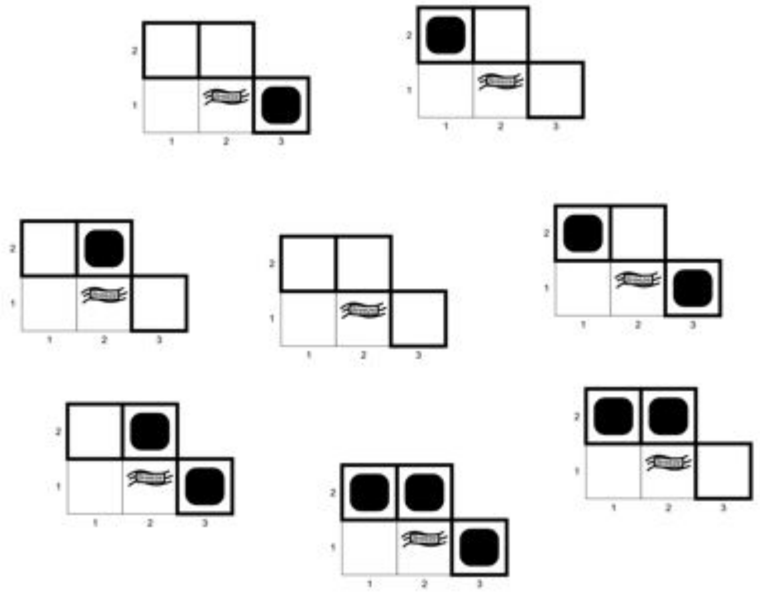
[1,2]	[2,2]	[3,1]	KB	a
			F	T
		pit	T	T
	pit		T	T
	pit	pit	T	T
pit			F	F
pit		pit	F	F
pit	pit		F	F
pit	pit	pit	F	F

Possible worlds

Is [2, 2] safe?

Let *α* represent “[2 , 2] is safe”

Does KB ⊨ *α* ?



Possible worlds

Is [2, 2] safe?

Let *a* represent "[2 , 2] is safe"

Does KB ⊨ *a* ?   No

[1,2]	[2,2]	[3,1]	KB	a
			F	T
		pit	T	T
	pit		T	F
	pit	pit	T	F
pit			F	F
pit		pit	F	F
pit	pit		F	F
pit	pit	pit	F	F

Inference in Wumpus World

Enumerate all combinations of seven symbols (128 possibilities)

To see if KB ⊨ *α* , for all cases where KB is true, a should be true

Does KB ⊨ $P_{1,1}$?

Model Checking

- Sound
- Complete
- Complexity $O(2^n)$

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Problems with Propositional Logic

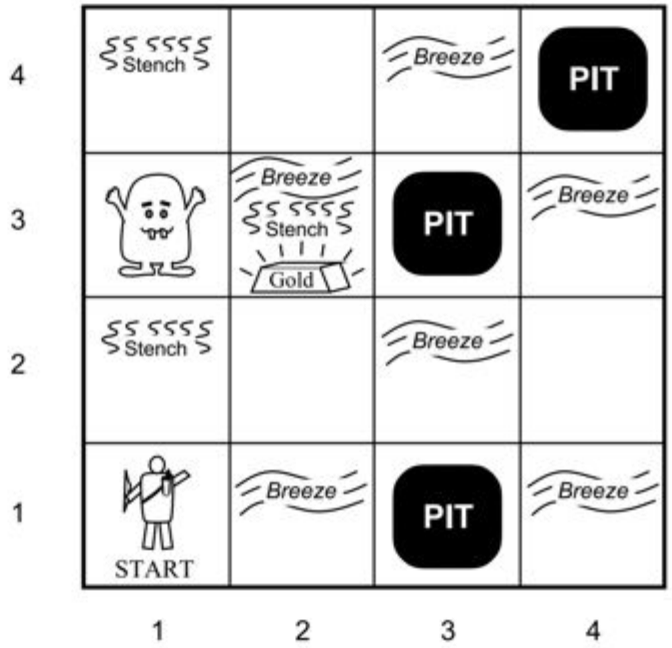
Impossible to make general assertions

"Pits cause breezes in adjacent squares"

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$
$$P_{3,1} \Leftrightarrow (B_{2,1} \vee B_{3,2} \vee B_{4,1})$$

Propositional logic has very limited expressive power (unlike natural language)

E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square



Logics

Propositional logic

- Is simple
- Illustrates important points:
    - Model, soundness, completeness, satisfiability
- Is restrictive: world is a set of facts
- Lacks expressiveness (world contains FACTS)

First-Order Logic

- More symbols (objects, properties, relations)
- More connectives (quantifiers)

First-order Logic

Whereas propositional logic assumes the world contains **facts**,

First-order logic (like natural language) assumes the world contains

- **Objects**: people, houses, numbers, colors, baseball games, wars, ...
- **Properties**: red, round, prime, ...
- **Relations**: brother of, bigger than, part of, comes between, ...
- **Functions**: width, best friend, one more than, plus, ...

Objects

Objects in the world: people, places

Not just physical things: number, events, time

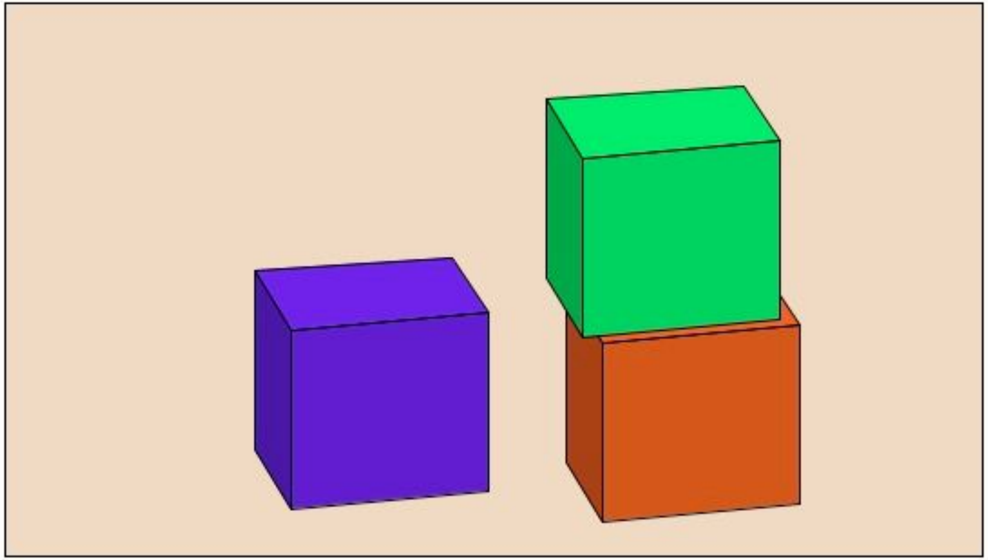
Constants

Table, BlockR, BlockG, BlockB

Variables

x, y, z, a, b, c, etc.

?x, ?y



Properties

BlockB, BlockG, BlockR

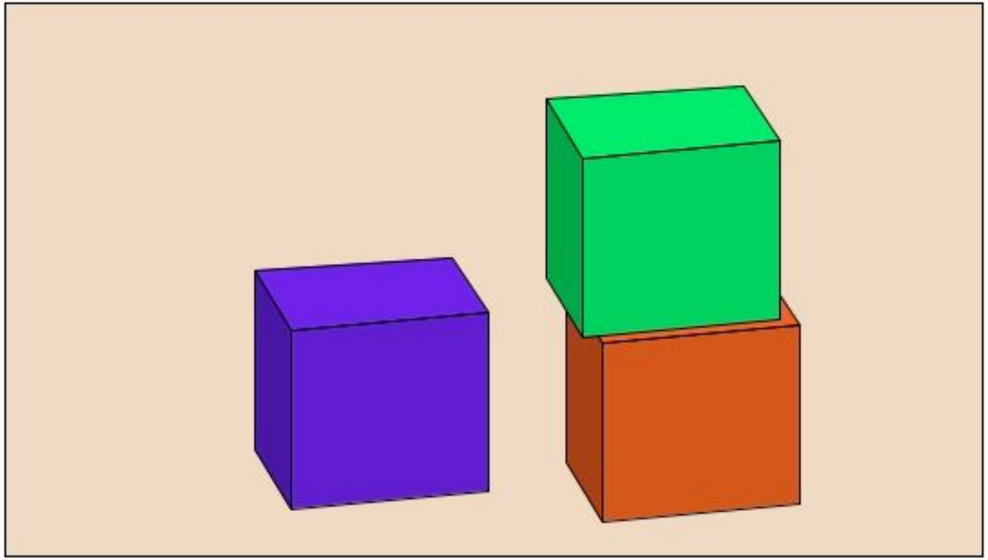
(blue BlockB)   blue(BlockB)

(green BlockG)

(red BlockR)

(six_sided Cube)

(clear BlockG)



Relations

(inst BlockB Block)

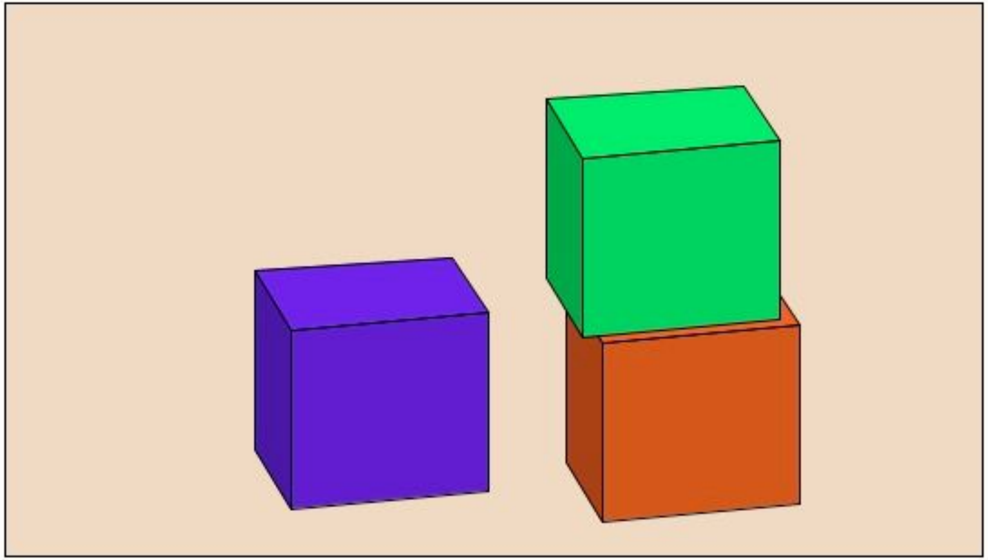
(isa Block PhysicalObject)

(isa PhysicalObject Thing)

(on BlockG BlockR)

(on BlockR Table)

(above BlockG Table)



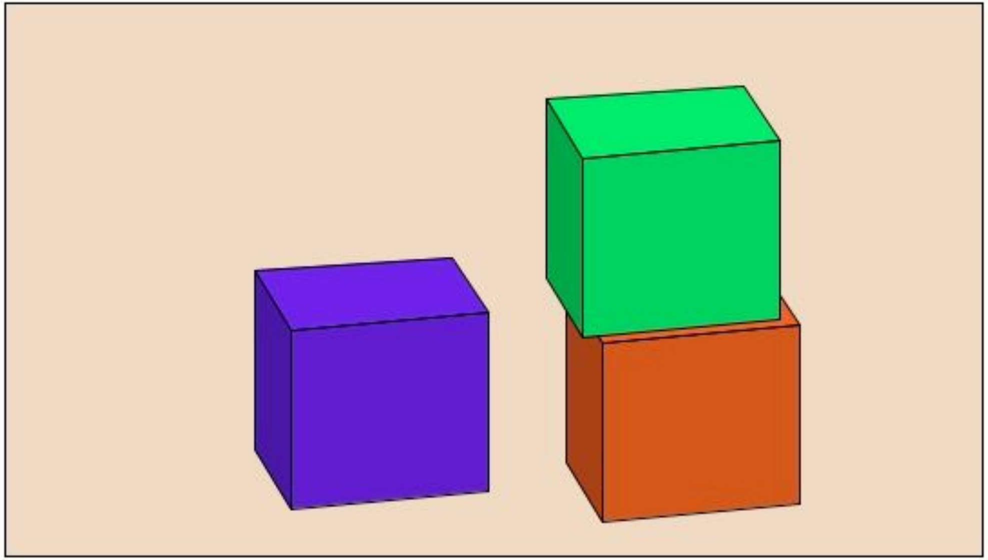
Functions

(mass BlockB) = 200g

(width BlockB) = 40mm

(width BlockB) = (width BlockG)

(price BlockB) = 1_million



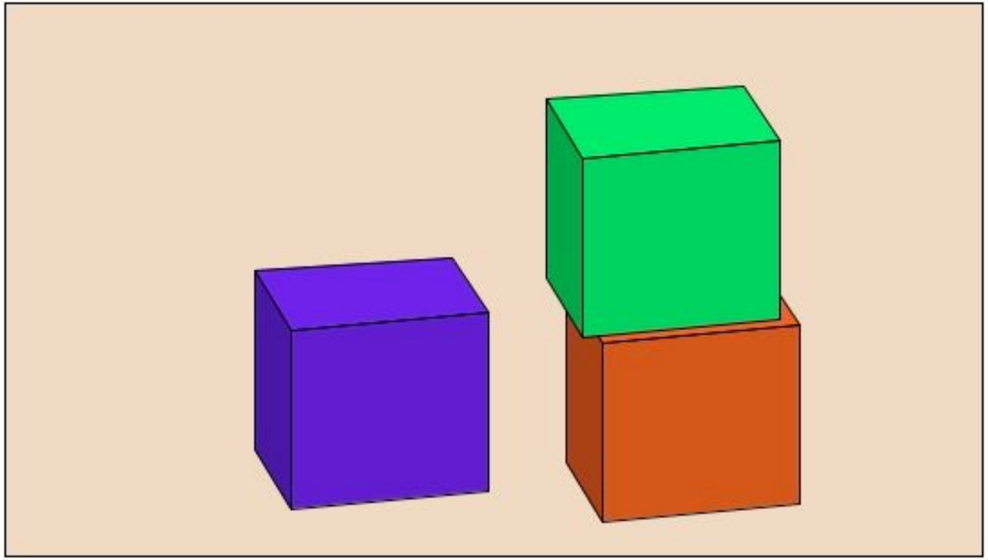
Quantifiers

$\forall x,y$ (on x y) $\Rightarrow$ (above x y)

$\forall x,y,z$ (on x y) $\wedge$ (on y z) $\Rightarrow$ (above x z)

$\forall x\ \exists y$ (isa PhysicalObject x) $\Rightarrow$ (color x y)

$\forall x\ \exists y$ (on x y) is not the same as $\exists y\ \forall x$ (on x y)



Assertions and Queries

ASSERT(KB, (inst BlockB Block))

ASSERT(KB, (inst BlockG Block))

ASSERT(KB, (inst BlockR Block))

ASSERT(KB, $\forall$ x,y (on x y) $\Rightarrow$ (under y x))

ASK(KB, (inst BlockB Block)) : returns True

ASK(KB, $\exists$ x (inst x Block)) : returns {x/BlockB}, {x/BlockG}, {x/BlockR}

FOL for Wumpus World

Objects

Wumpus, Gold, Glitter, Breeze, Stench, 1, 2, 3, 4

Properties

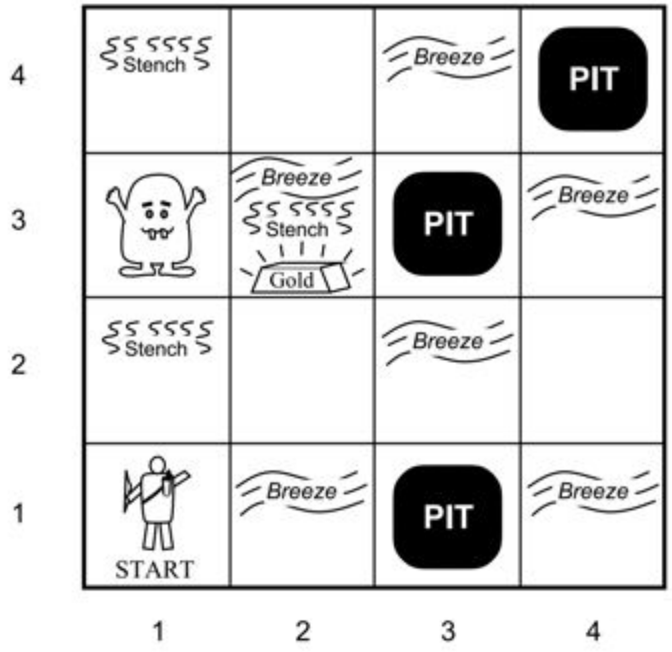
(glitters Gold) (smells Wumpus) (hasPit x)

Relations

(cell 1 1) (adjacent (cell 1 1) (cell 1 2))

Functions

$\forall$ s (breezy s) $\Rightarrow$ $\exists$ r (adjacent r s) $\wedge$ (hasPit r)



Next time

Friday

- More work in logic
    - Truth tables
    - Resolution
- Knowledge representation with FOL

Monday

- Inference