Edge And Corner Detection

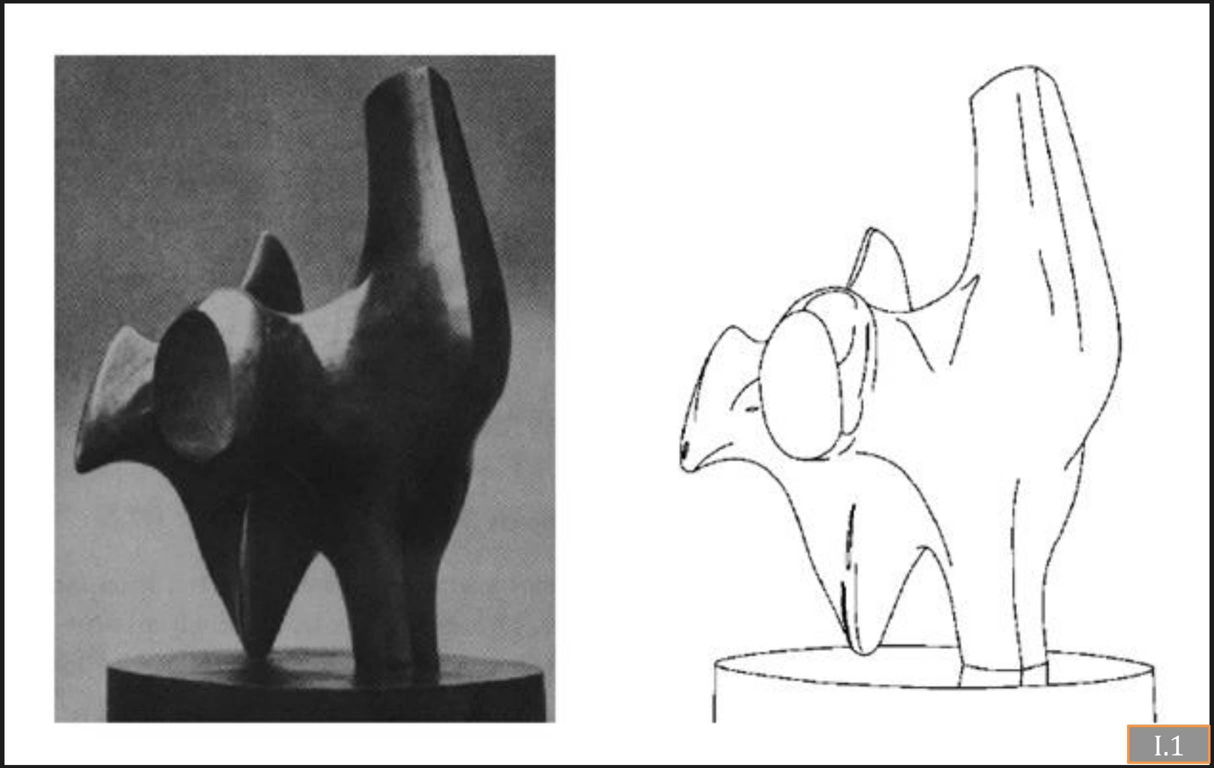
Introduction to Computational Photography:

EECS 395/495

Northwestern University

What Are Edges?

Rapid changes in image intensity within small region



Edge Detection

Convert a 2D Image into a Set of Curves

Topics:

(1) Theory of Edge Detection

(2) Edge Detection Using Gradients

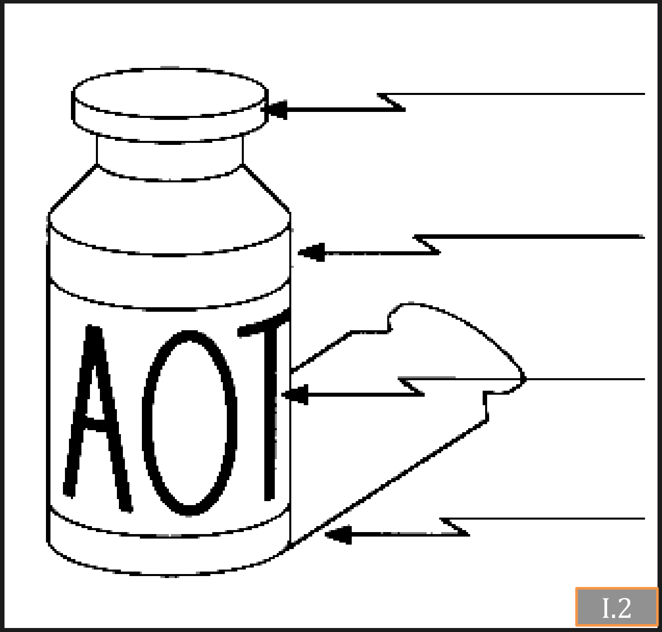
(3) Edge Detection Using Laplacian

(4) Canny Edge Detector

(5) Harris Corner Detector

Causes of Edges

Edges are caused by a variety of factors



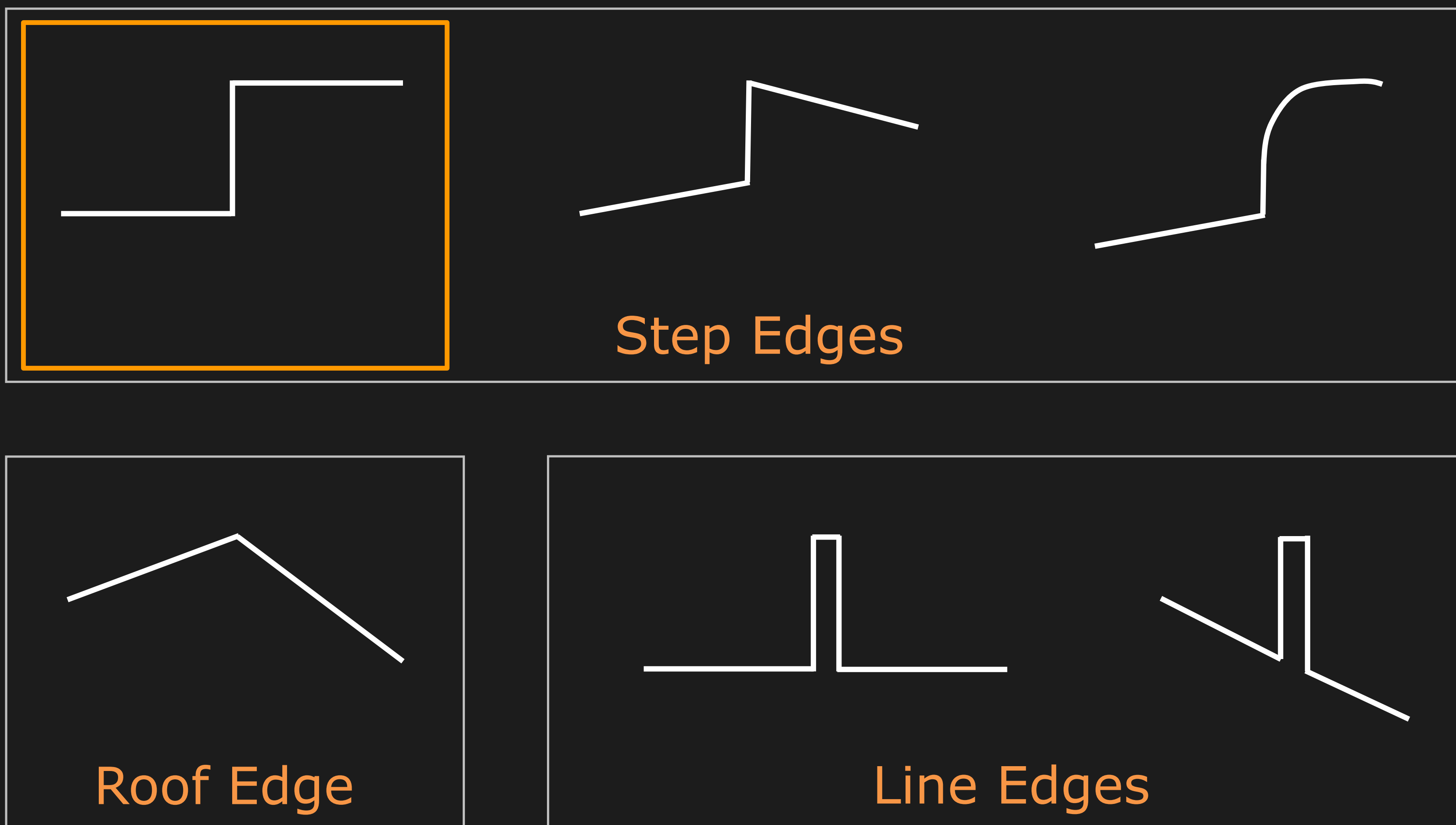
Surface Normal Discontinuity

Depth Discontinuity

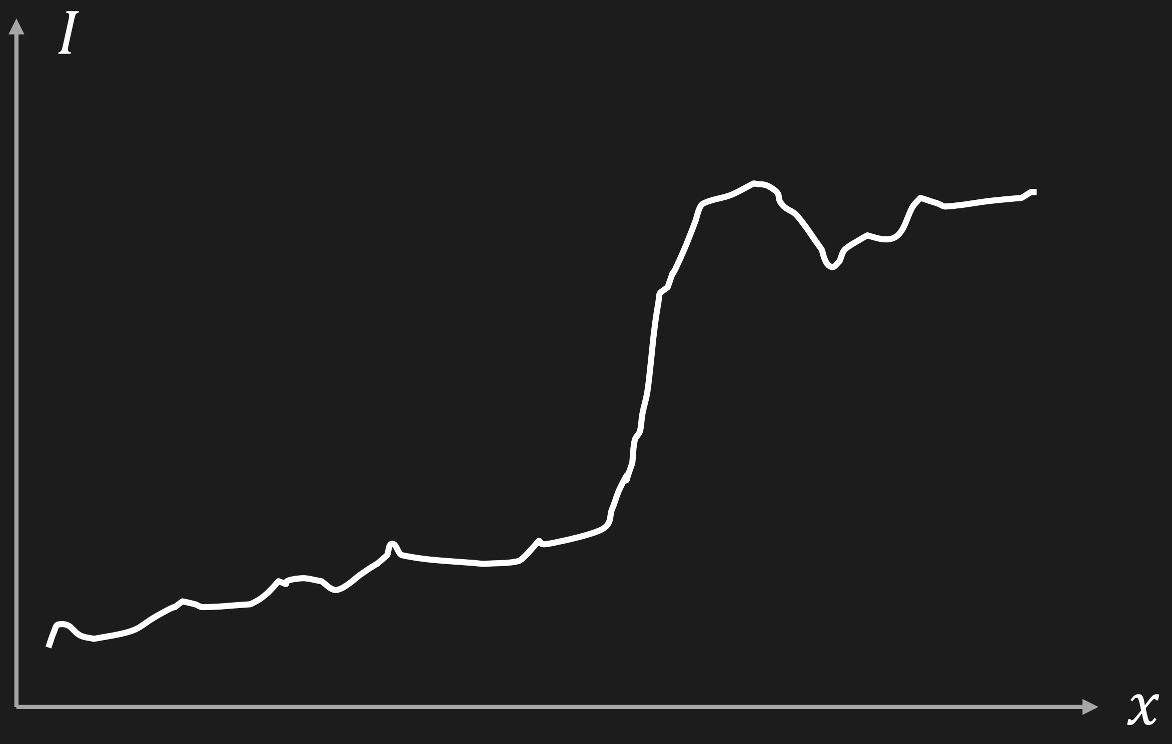
Surface Color Discontinuity

Illumination Discontinuity

Types of Edges



Real Edges



Problems: Noisy Images and Discrete Images

Edge Detector

We want an Edge Operator that produces:

- Edge Position
- Edge Magnitude (Strength)
- Edge Orientation (Direction)

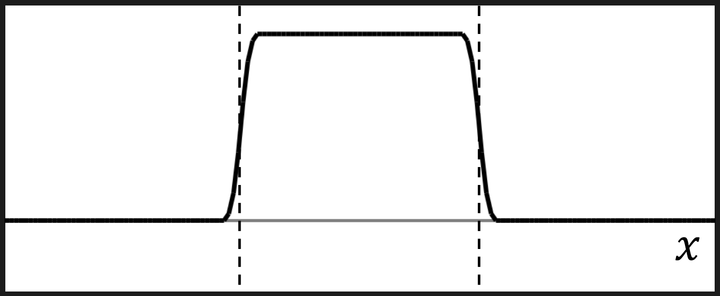
Crucial Requirements:

- High Detection Rate
- Good Localization
- Robust to Noise

1D Edge Detection

Edges are rapid changes in image brightness in a small region.

$f(x)$:



Edge Edge

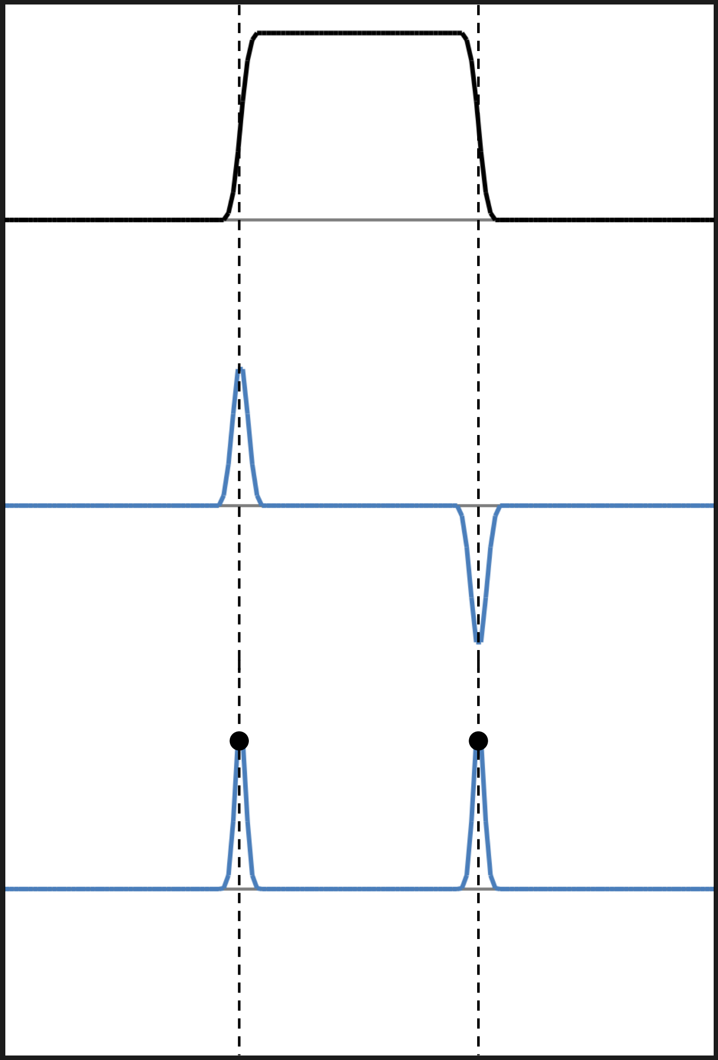
Calculus 101: Derivative of a continuous function represents the amount of change in the function.

Edge Detection Using 1st Derivative

$f(x)$

First Derivative: $\dfrac{\partial f}{\partial x}$

First Derivative Absolute Value: $\left|\dfrac{\partial f}{\partial x}\right|$



Local Extrema Indicate Edges

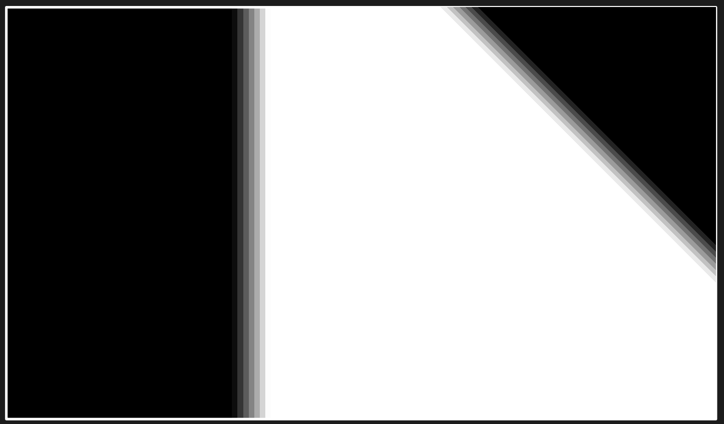
Local Maxima Indicate Edges

Provides Both Location and Strength of an Edge

2D Edge Detection

Edges are rapid changes in image brightness in a small region.

$I(x, y)$:



Edge

Edge

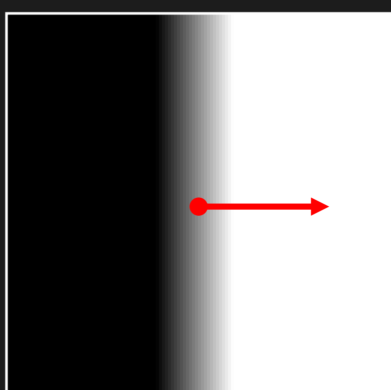
Calculus 101: Partial Derivatives of a 2D continuous function represents the amount of change along each dimension.

Gradient (∇)

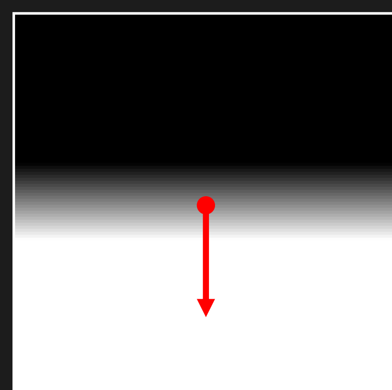
Gradient (Partial Derivative) Represents the Direction of Most Rapid Change in Intensity

$$\boxed{\nabla I = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]}$$

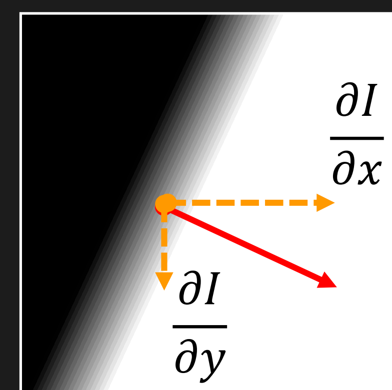
Pronounced as "Del I"



$$\nabla I = \left[\frac{\partial I}{\partial x}, 0\right]$$



$$\nabla I = \left[0, \frac{\partial I}{\partial y}\right]$$

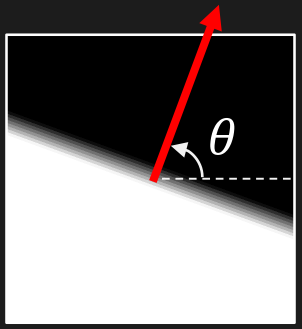


$$\nabla I = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]$$

Gradient (∇) as Edge Detector

Gradient Magnitude $S = \|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$

Gradient Orientation $\theta = \tan^{-1}\left(\frac{\partial I}{\partial y} \Big/ \frac{\partial I}{\partial x}\right)$



Discrete Gradient (∇) Operator

Finite difference approximations:

$$\frac{\partial I}{\partial x} \approx \frac{1}{2\varepsilon}\left(\left(I_{i+1,j+1} - I_{i,j+1}\right) + \left(I_{i+1,j} - I_{i,j}\right)\right)$$

$$\frac{\partial I}{\partial y} \approx \frac{1}{2\varepsilon}\left(\left(I_{i+1,j+1} - I_{i+1,j}\right) + \left(I_{i,j+1} - I_{i,j}\right)\right)$$

$I_{i,j+1}$	$I_{i+1,j+1}$
$I_{i,j}$	$I_{i+1,j}$

ε

Can be implemented as Convolution!

$$\frac{\partial}{\partial x} \approx \frac{1}{2\varepsilon}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \frac{\partial}{\partial y} \approx \frac{1}{2\varepsilon}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

Note: Convolution flips have been applied

Comparing Gradient (∇) Operators

Gradient	Roberts	Prewitt	Sobel (3x3)	Sobel (5x5)
$\frac{\partial I}{\partial x}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -2 & 0 & 2 & 1 \\ -2 & -3 & 0 & 3 & 2 \\ -3 & -5 & 0 & 5 & 3 \\ -2 & -3 & 0 & 3 & 2 \\ -1 & -2 & 0 & 2 & 1 \end{bmatrix}$
$\frac{\partial I}{\partial y}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 3 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & -3 & -5 & -3 & -2 \\ -1 & -2 & -3 & -2 & -1 \end{bmatrix}$

Good Localization
Noise Sensitive
Poor Detection

Poor Localization
Less Noise Sensitive
Good Detection

Gradient (∇) Using Sobel Filter

Image (I)

$\partial I / \partial x$

$\partial I / \partial y$

Gradient Magnitude

Edge Thresholding

Standard: (Single Threshold T)

$\|\nabla I(x,y)\| < T$	Definitely Not an Edge
$\|\nabla I(x,y)\| \geq T$	Definitely an Edge

Hysteresis Based: (Two Thresholds $T_0 < T_1$)

$\|\nabla I(x,y)\| < T_0$	Definitely Not an Edge
$\|\nabla I(x,y)\| \geq T_1$	Definitely an Edge
$T_0 \leq \|\nabla I(x,y)\| < T_1$	Is an Edge if a Neighboring Pixel if Definitely an Edge

Sobel Edge Detector

Image (I)

$\partial I / \partial x$

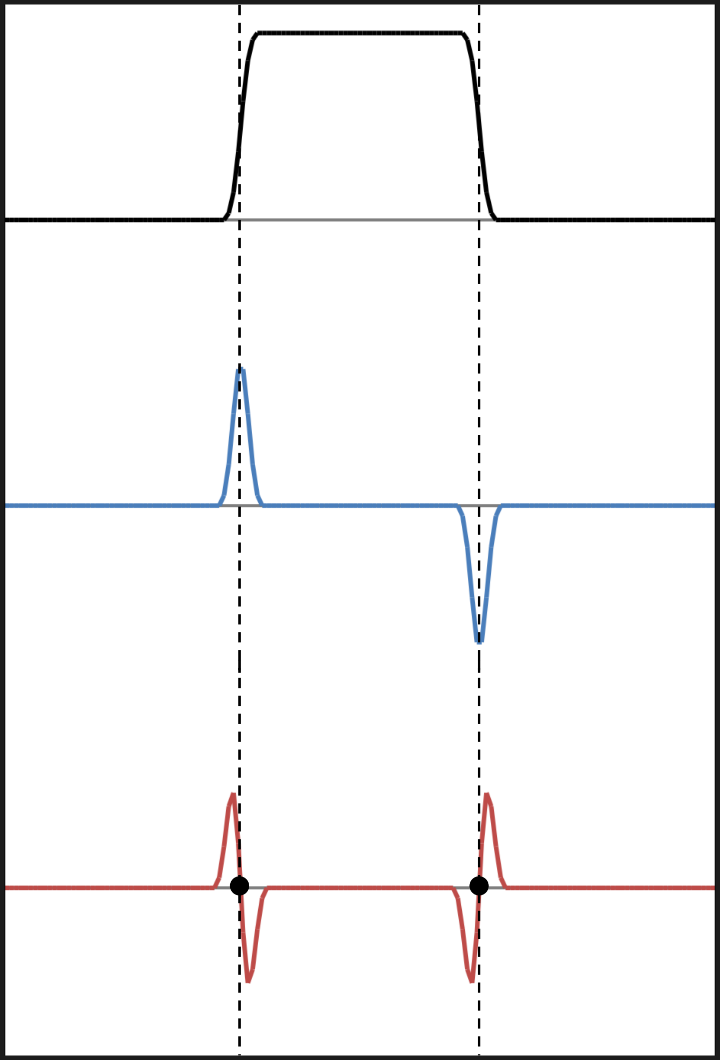
$\partial I / \partial y$

Gradient Magnitude

Thresholded Edge

Edge Detection Using 2^{nd} Derivative

$f(x)$



First Derivative: $\frac{\partial f}{\partial x}$

Local Extrema
Indicate Edges

Second Derivative: $\frac{\partial^2 f}{\partial x^2}$

Zero-Crossings
Indicate Edges

Provides Only the Location of an Edge

Laplacian (∇^2) as Edge Detector

Laplacian: Sum of Pure Second Derivatives

$$\boxed{\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}}$$

Pronounced as "Del Square I"

"Zero-Crossings" in Laplacian of an image represent edges

[Marr1980]

Discrete Laplacian(∇^2) Operator

Finite difference approximations:

$$\frac{\partial^2 I}{\partial x^2} \approx \frac{1}{\varepsilon^2}\left(I_{i-1,j} - 2I_{i,j} + I_{i+1,j}\right)$$

$$\frac{\partial^2 I}{\partial y^2} \approx \frac{1}{\varepsilon^2}\left(I_{i,j-1} - 2I_{i,j} + I_{i,j+1}\right)$$

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

$I_{i-1,j+1}$	$I_{i,j+1}$	$I_{i+1,j+1}$
$I_{i-1,j}$	$I_{i,j}$	$I_{i+1,j}$
$I_{i-1,j-1}$	$I_{i,j-1}$	$I_{i+1,j-1}$

ε

Convolution Mask :

$$\nabla^2 \approx \frac{1}{\varepsilon^2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

OR

$$\nabla^2 \approx \frac{1}{6\varepsilon^2}\begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

(More Accurate)

Laplacian Edge Detector

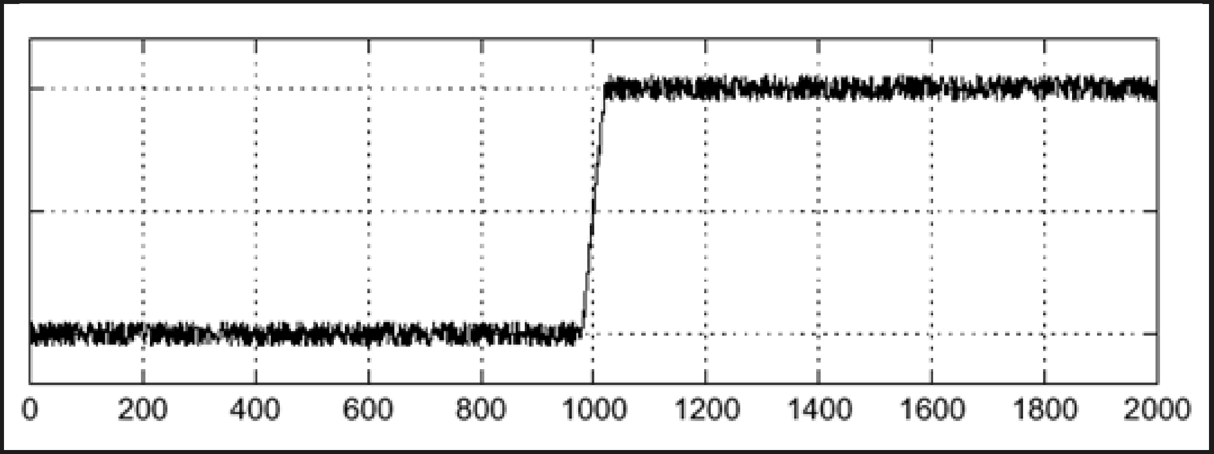
Image (I)

Laplacian
(0 maps to 128)

Laplacian
“Zero Crossings”

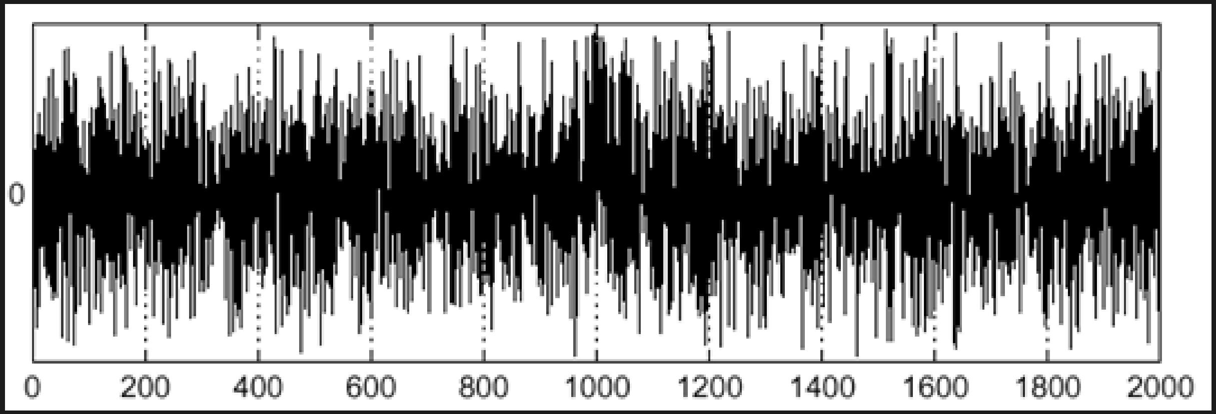
Effects of Noise

$f(x)$



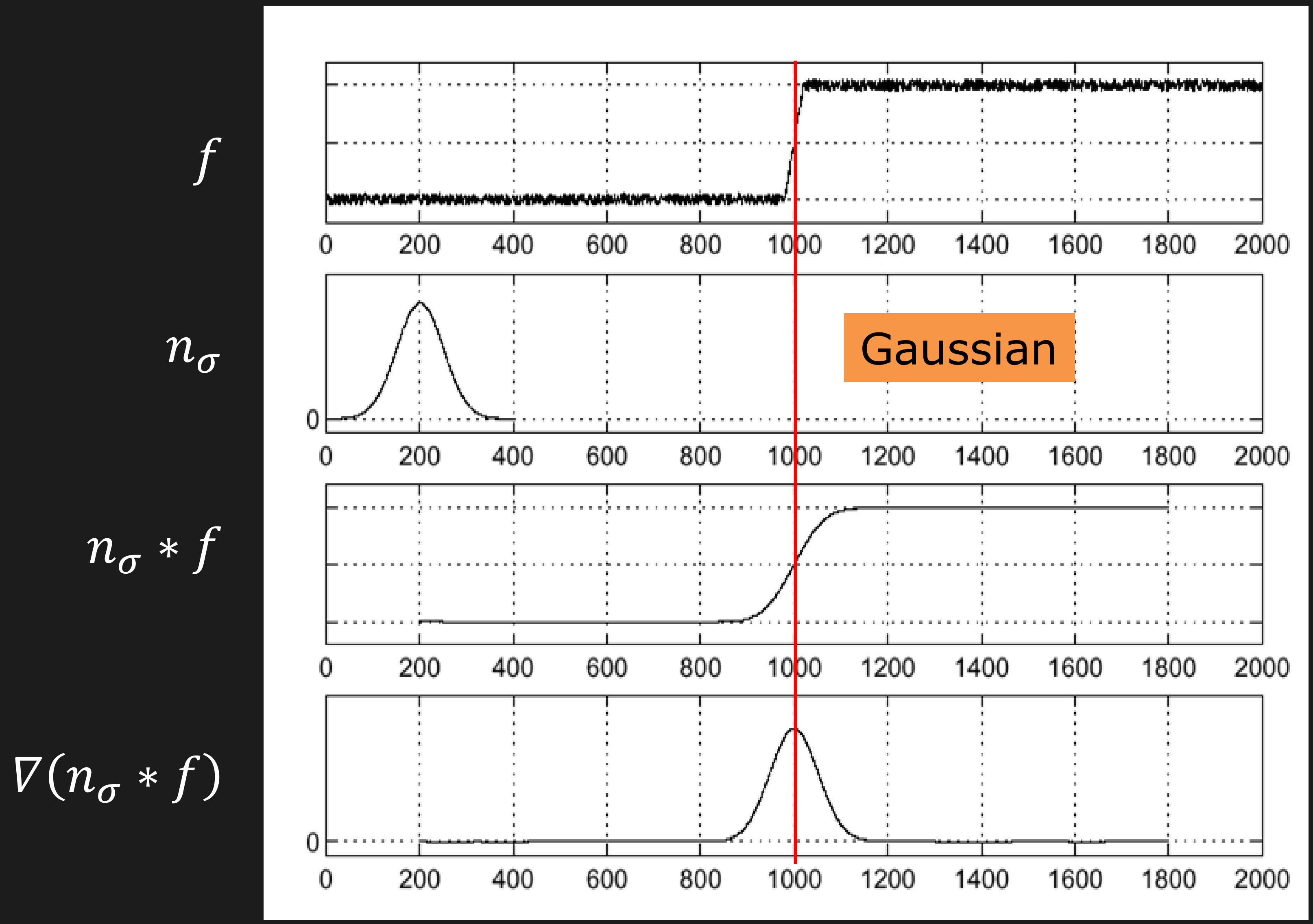
$\nabla f(x)$

(Gradient)



Where is the edge??

Solution: Gaussian Smooth First



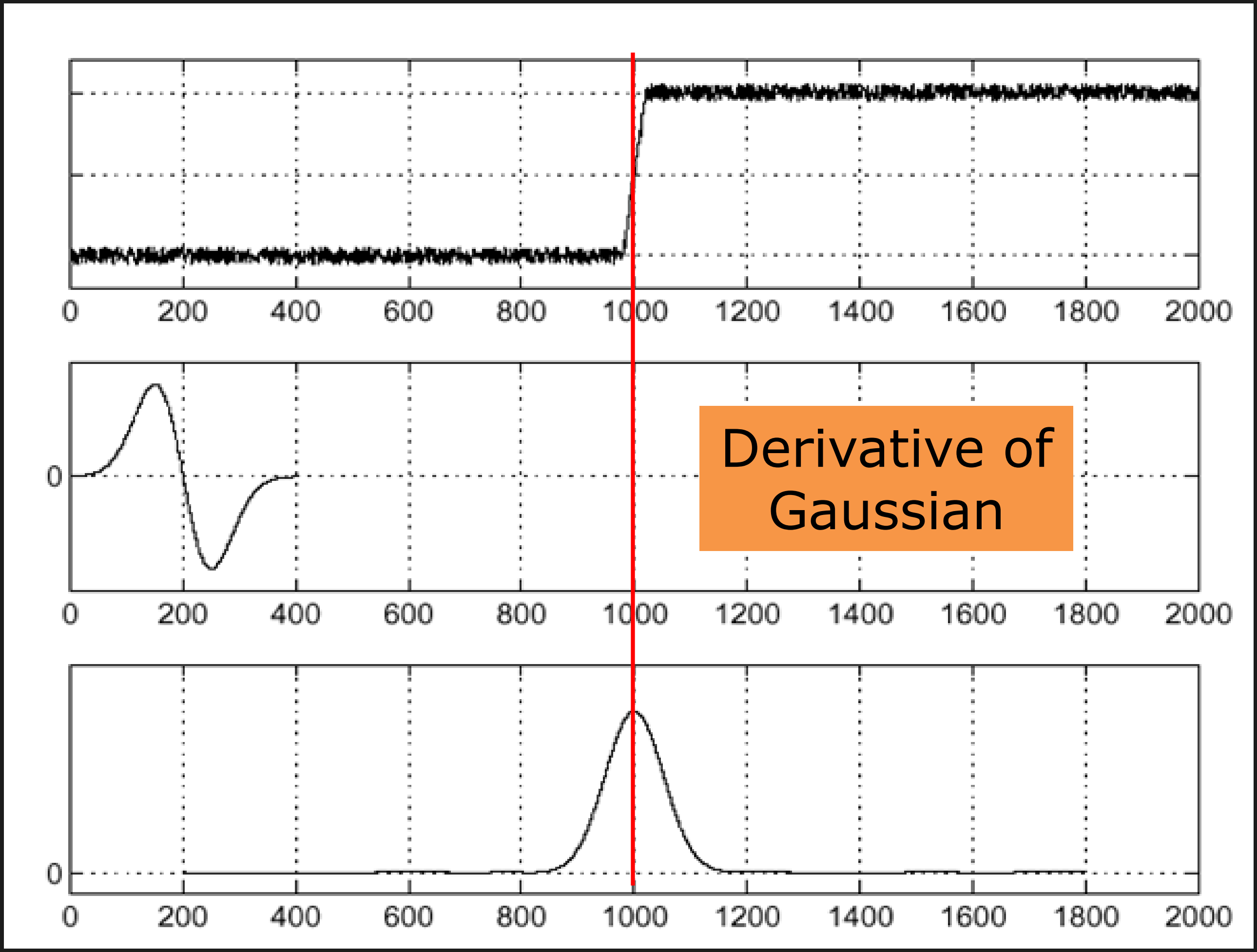
Derivative of Gaussian ($\nabla(n_\sigma)$)

$$\nabla(n_\sigma * f) = \nabla(n_\sigma) * f$$ …saves us one operation.

f

$\nabla(n_\sigma)$

$\nabla(n_\sigma) * f$



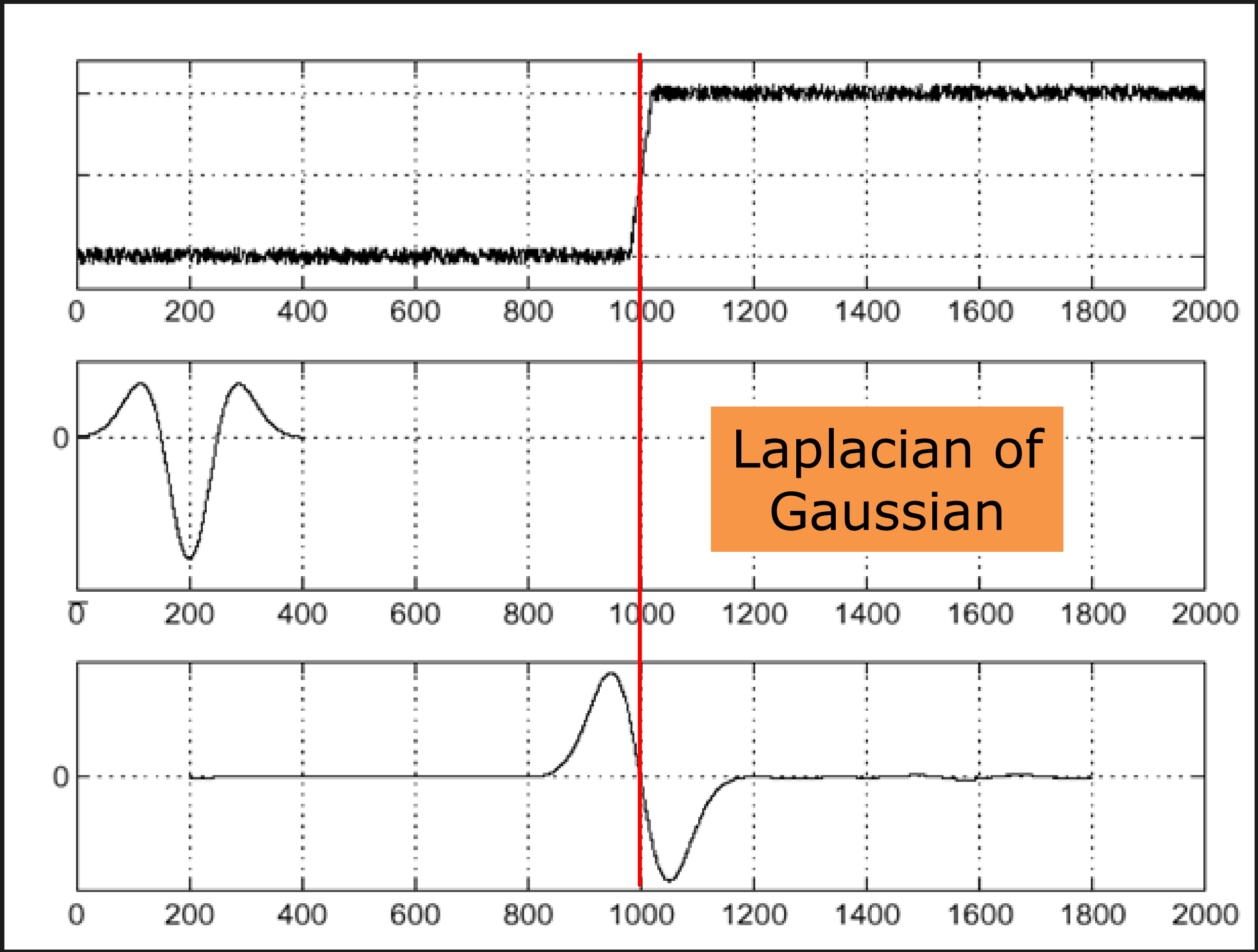
Laplacian of Gaussian ($\nabla^2 n_\sigma$ or $\nabla^2 G$)

$\nabla^2(n_\sigma * f) = \nabla^2(n_\sigma) * f$ …saves us one operation.

f

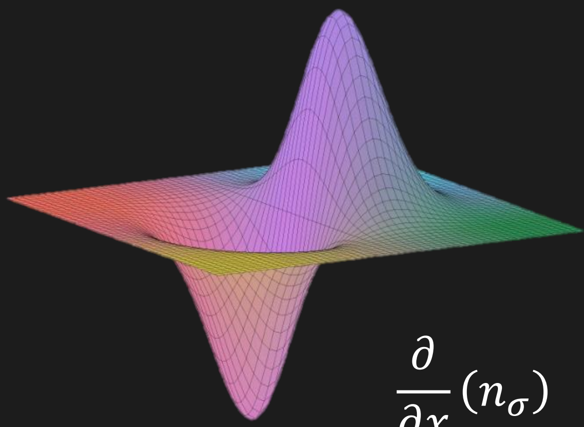
$\nabla^2(n_\sigma)$

$\nabla^2(n_\sigma) * f$

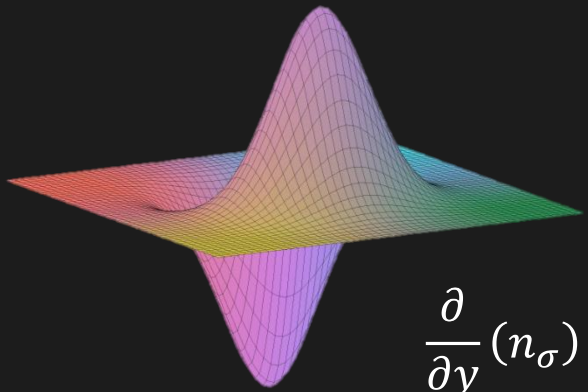


Gradient vs. Laplacian

Derivative of Gaussian (∇G)

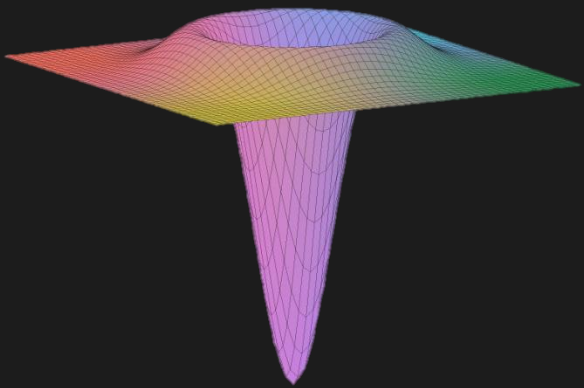


$$\frac{\partial}{\partial x}(n_\sigma)$$



$$\frac{\partial}{\partial y}(n_\sigma)$$

Laplacian of Gaussian ($\nabla^2 G$)



Inverted "Sombrero"
(Mexican Hat)

$$\frac{\partial^2}{\partial x^2}(n_\sigma) + \frac{\partial^2}{\partial y^2}(n_\sigma)$$

Gradient vs. Laplacian

Gradient	Laplacian
Provides location, magnitude and direction of the edge	Provides only location of the edge
Detection using Maxima Thresholding	Detection based on Zero-Crossing
Non-linear operation. Requires two convolutions.	Linear Operation. Requires only one convolution.

An operator that has the best of both?

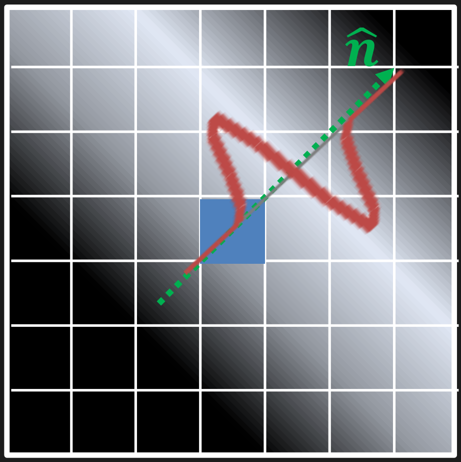
Canny Edge Detector

- Smooth Image with 2D Gaussian: $n_\sigma * I$
- Compute Image Gradient using Sobel Operator: $\nabla n_\sigma * I$
- Find Gradient Magnitude at each pixel: $|\nabla n_\sigma * I|$
- Find Gradient Orientation at each Pixel:

$$\hat{\boldsymbol{n}} = \frac{\nabla n_\sigma * I}{|\nabla n_\sigma * I|}$$

- Compute Laplacian along the Gradient Direction $\hat{\boldsymbol{n}}$ at each pixel

$$\frac{\partial^2 (n_\sigma * I)}{\partial \hat{\boldsymbol{n}}^2}$$



$|\nabla n_\sigma * I|$

[Canny1986]

Canny Edge Detector

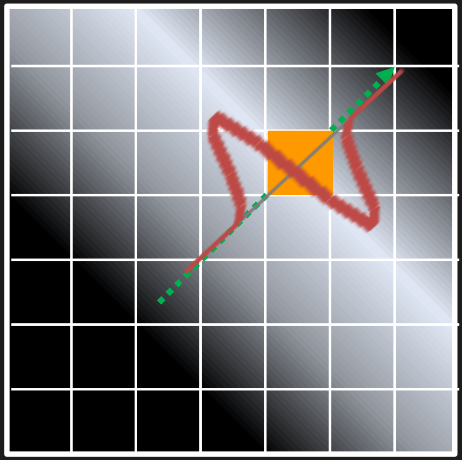
- Smooth Image with 2D Gaussian: $n_\sigma * I$
- Compute Image Gradient using Sobel Operator: $\nabla n_\sigma * I$
- Find Gradient Magnitude at each pixel: $|\nabla n_\sigma * I|$
- Find Gradient Orientation at each Pixel:

$$\hat{\boldsymbol{n}} = \frac{\nabla n_\sigma * I}{|\nabla n_\sigma * I|}$$

- Compute Laplacian along the Gradient Direction $\hat{\boldsymbol{n}}$ at each pixel

$$\frac{\partial^2 (n_\sigma * I)}{\partial \hat{\boldsymbol{n}}^2}$$

- Find Zero Crossings in Laplacian to find the edge location



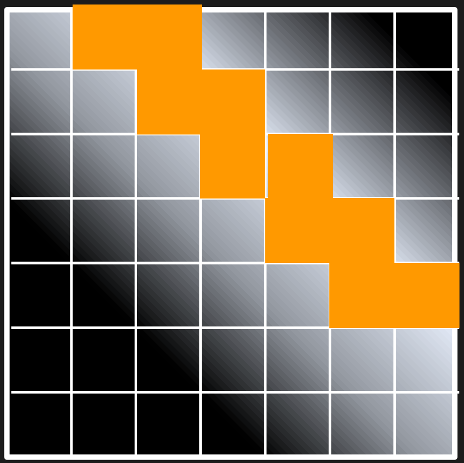
$|\nabla n_\sigma * I|$

[Canny1986]

Canny Edge Detector

- Smooth Image with 2D Gaussian: $n_\sigma * I$
- Compute Image Gradient using Sobel Operator: $\nabla n_\sigma * I$
- Find Gradient Magnitude at each pixel: $|\nabla n_\sigma * I|$
- Find Gradient Orientation at each Pixel:

$$\hat{\boldsymbol{n}} = \frac{\nabla n_\sigma * I}{|\nabla n_\sigma * I|}$$



$|\nabla n_\sigma * I|$

- Compute Laplacian along the Gradient Direction $\hat{\boldsymbol{n}}$ at each pixel

$$\frac{\partial^2 (n_\sigma * I)}{\partial \hat{\boldsymbol{n}}^2}$$

- Find Zero Crossings in Laplacian to find the edge location

[Canny1986]

Canny Edge Detector Results

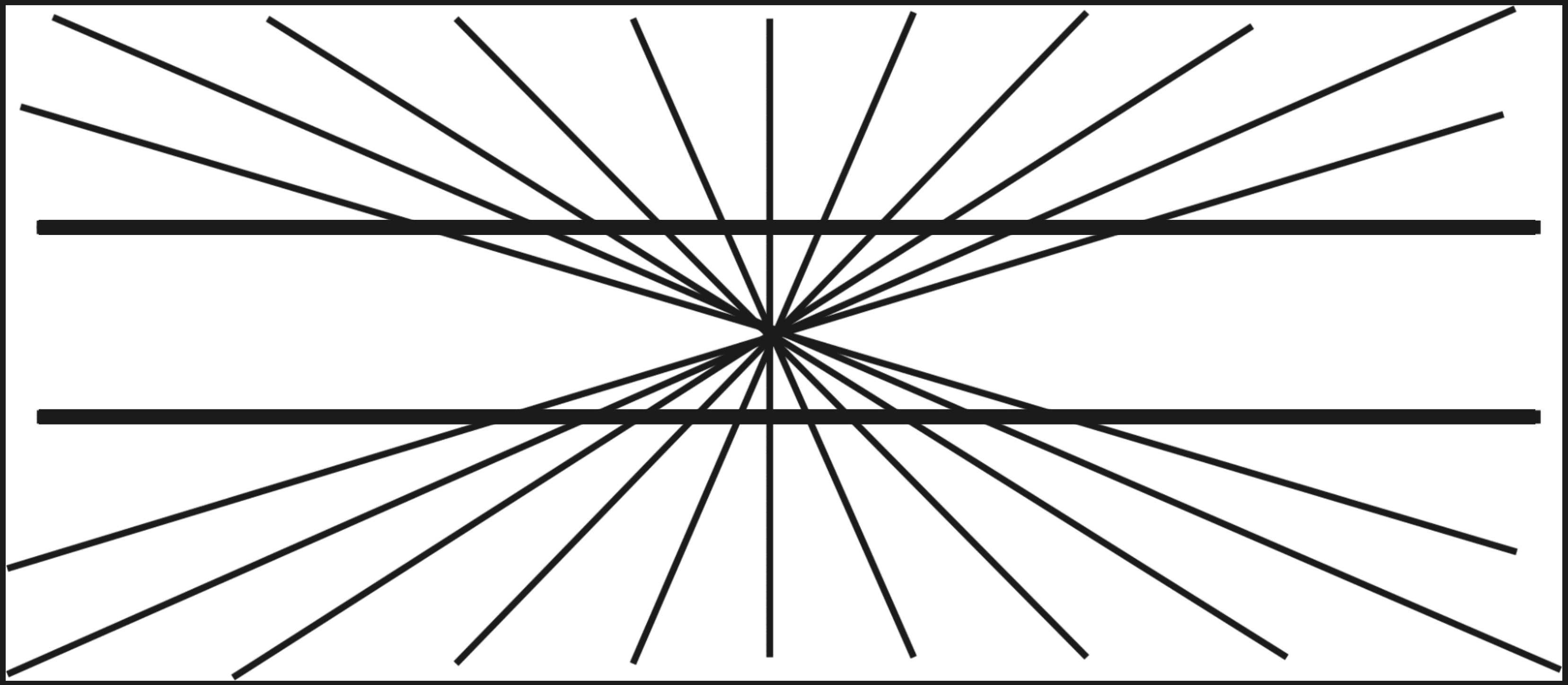
Image

$\sigma = 1$

$\sigma = 2$

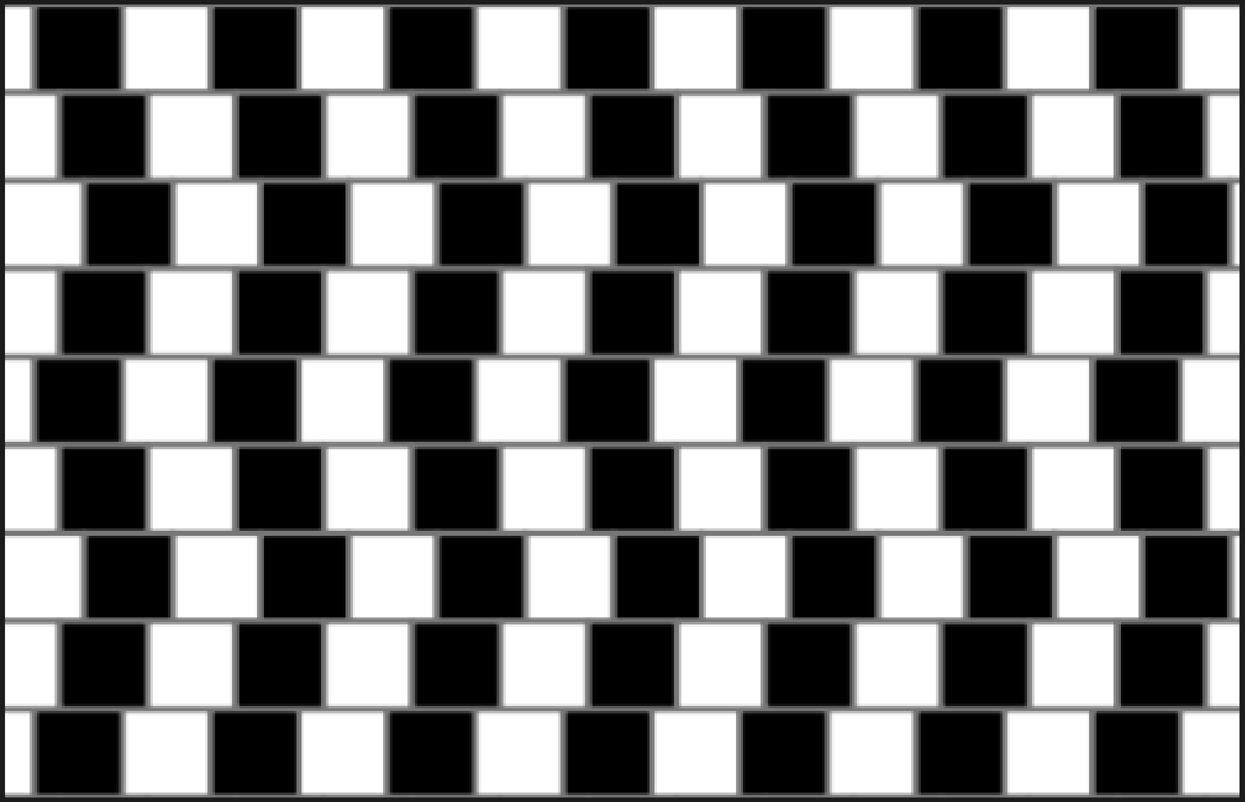
$\sigma = 4$

Edge Illusions: Hering Illusion



Ewald Hering, 1861

Edge Illusions: Café Wall Illusion

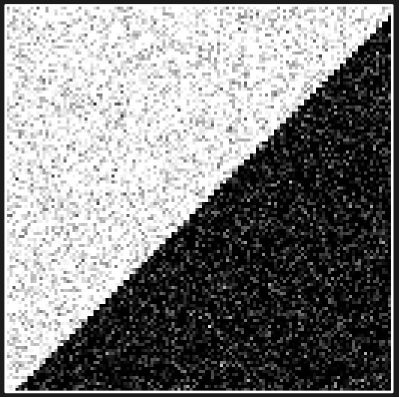


Richard Gregory, 1979

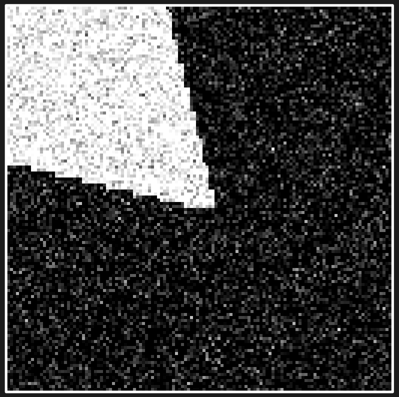
Corners

Corner: Point where Two Edges Meet. i.e., Rapid Changes of Image Brightness in Two Directions within a Small Region

"Flat" Region

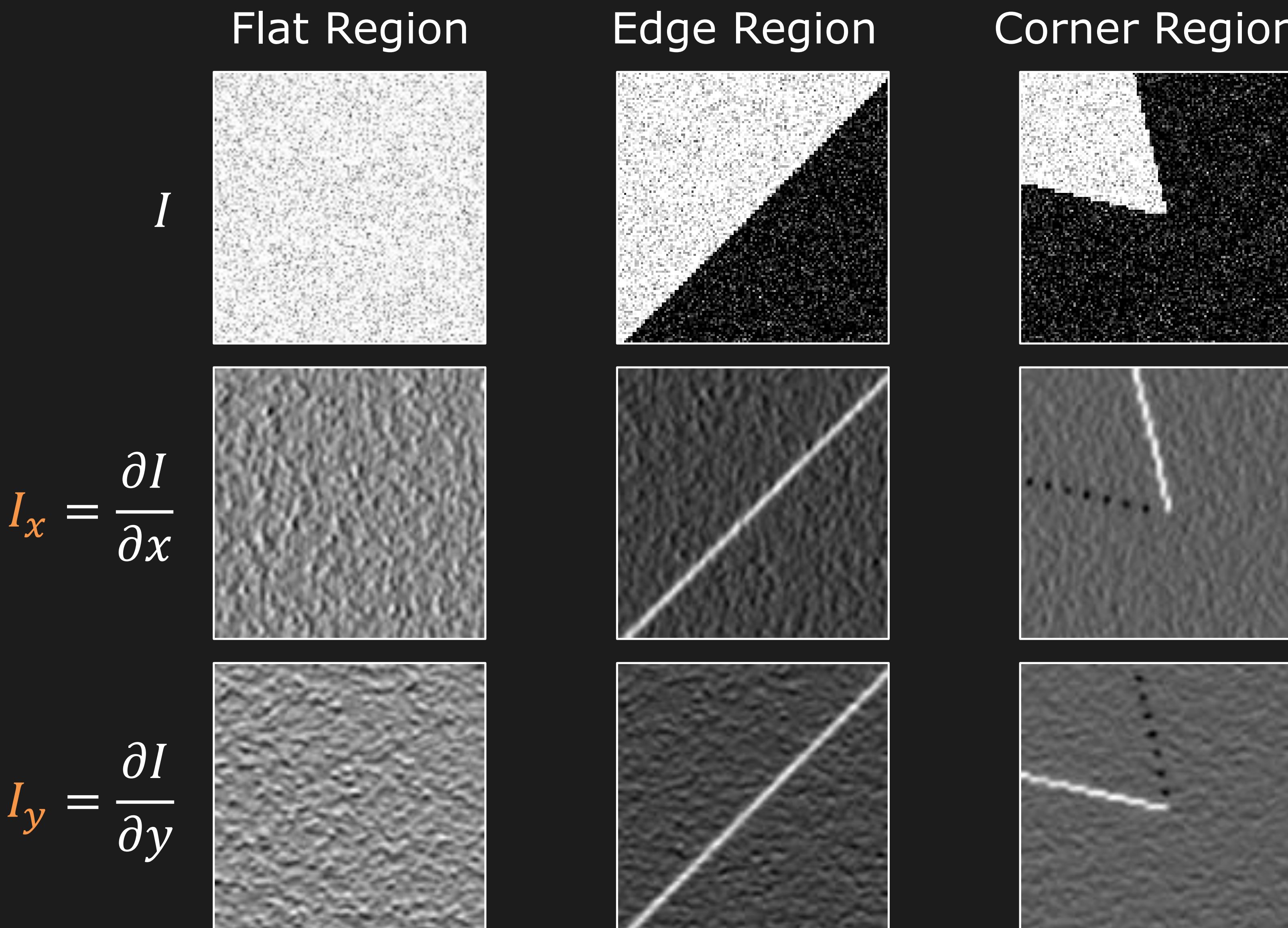


"Edge" Region



"Corner" Region

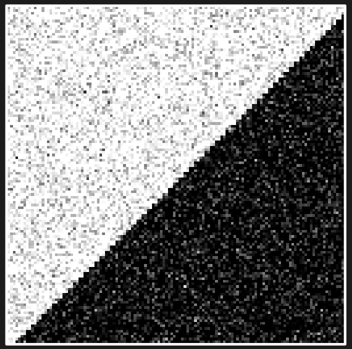
Image Gradients



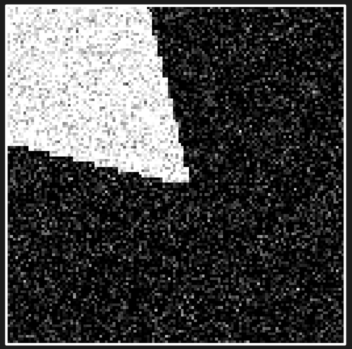
Distribution of Image Gradients

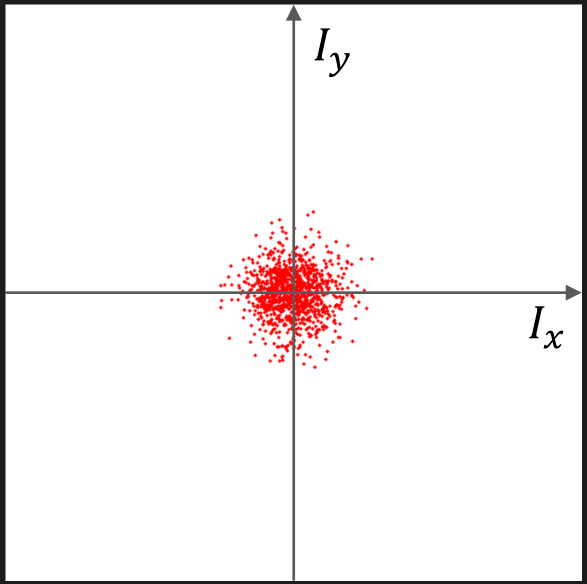
Flat Region

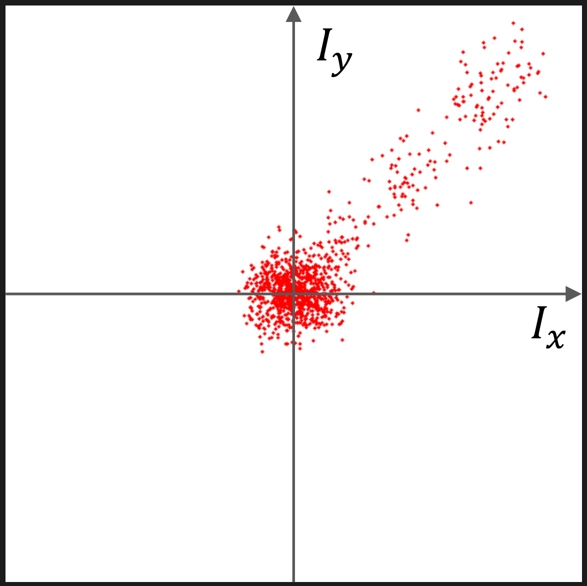
Edge Region

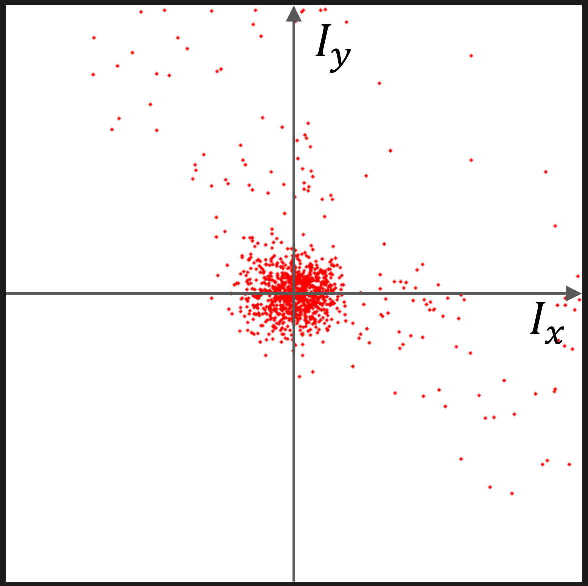


Corner Region



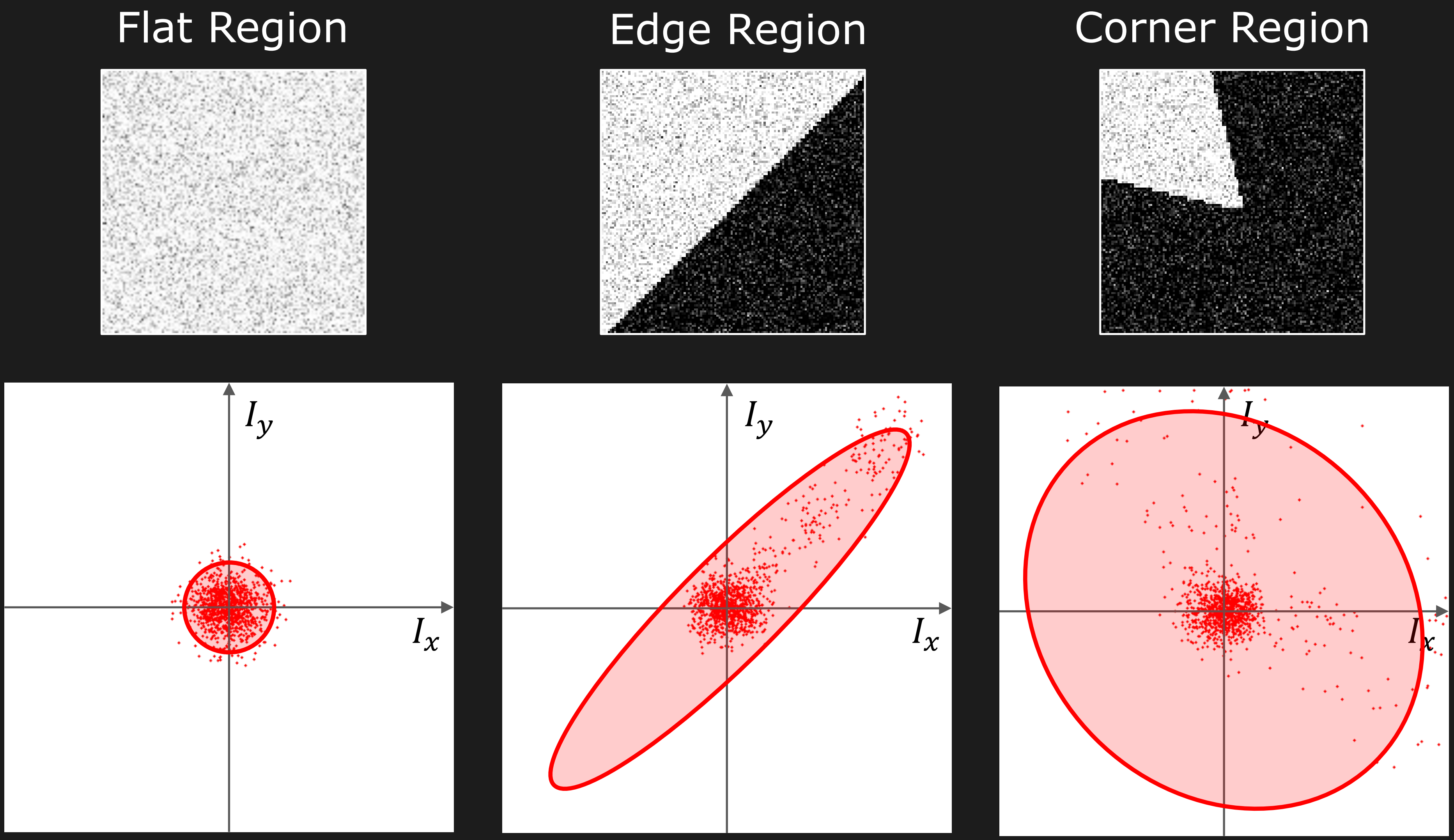






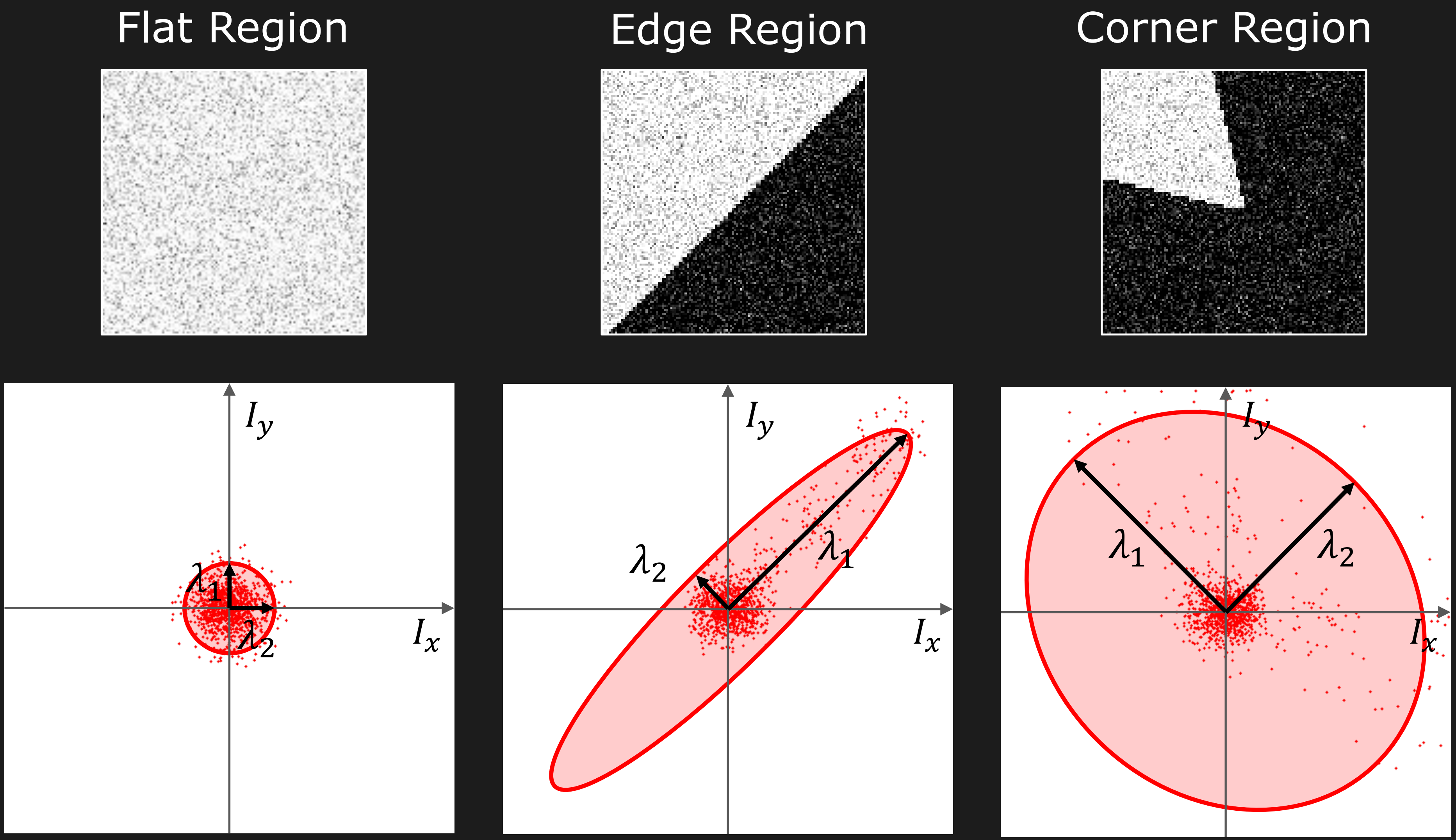
Distribution of I_x and I_y is different for all three regions.

Fitting Elliptical Disk to Distribution



Distribution of I_x and I_y is different for all three regions.

Fitting Elliptical Disk to Distribution

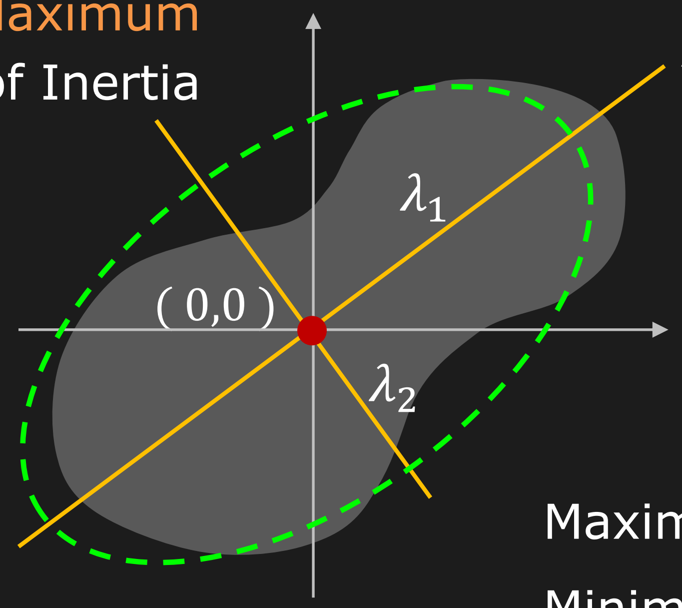


λ_1: Length of Semi-Major Axis λ_2: Length of Semi-Minor Axis

Fitting an Elliptical Disk

Axis of Maximum Moment of Inertia

Axis of Minimum Moment of Inertia



Maximum Moment of Inertia = E_{max}

Minimum Moment of Inertia = E_{min}

Length of Semi-Major Axis = $\lambda_1 = E_{max}$

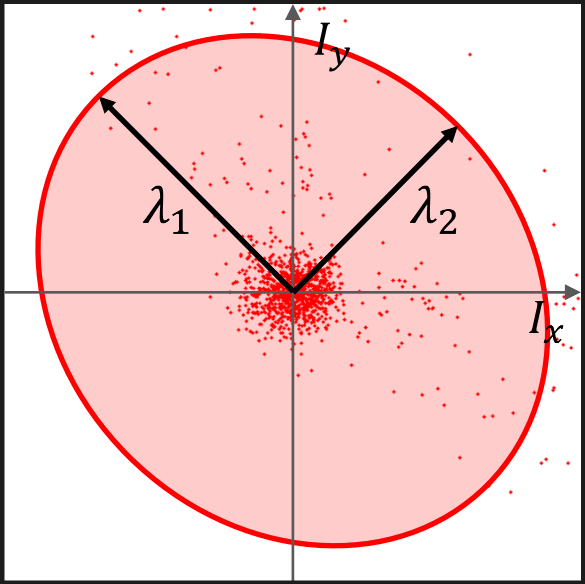
Length of Semi-Minor Axis = $\lambda_2 = E_{min}$

Fitting an Elliptical Disk

Second Moments at each pixel:

$$a = \sum_{i \in W} \left({I_{x_i}}^2 \right) \qquad b = 2 \sum_{i \in W} \left(I_{x_i} I_{y_i} \right)$$

$$c = \sum_{i \in W} \left({I_{y_i}}^2 \right)$$



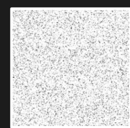
Ellipse Axes Lengths:

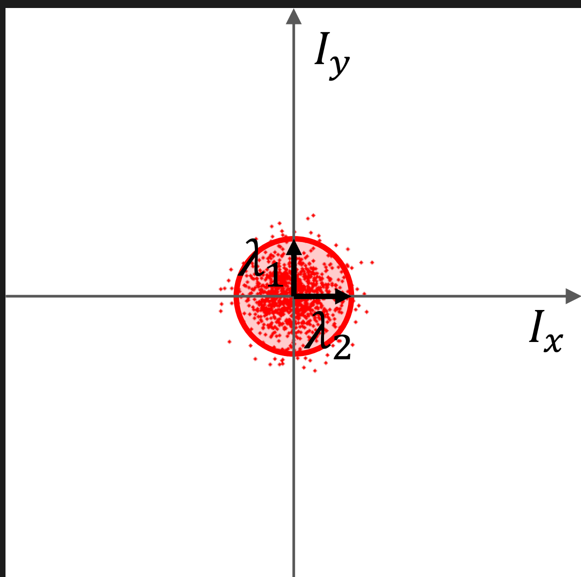
$$\lambda_1 = E_{max} = \frac{1}{2}\left[a + c + \sqrt{b^2 + (a-c)^2}\right]$$

$$\lambda_2 = E_{min} = \frac{1}{2}\left[a + c - \sqrt{b^2 + (a-c)^2}\right]$$

Interpretation of λ_1 and λ_2

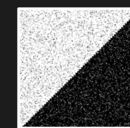
Flat Region

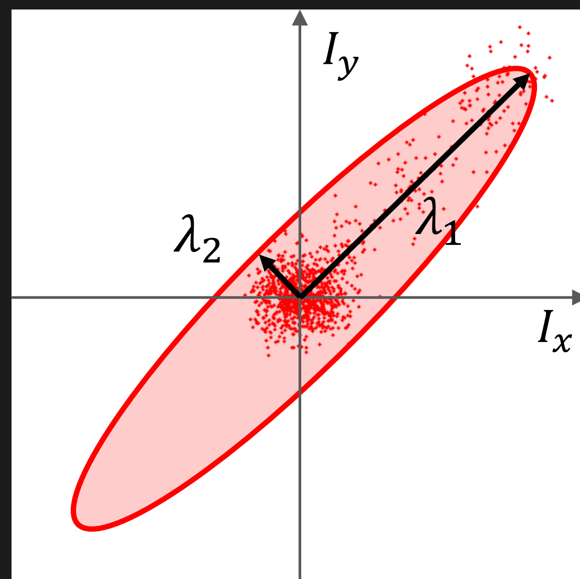




$\lambda_1 \sim \lambda_2$
Both are Small

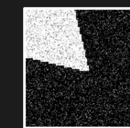
Edge Region

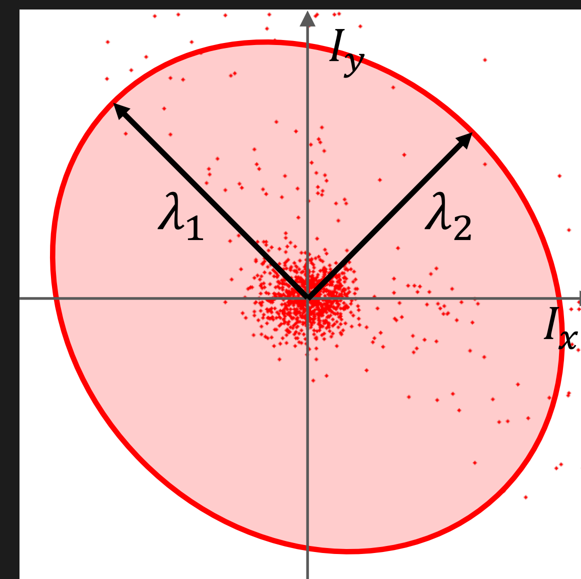




$\lambda_1 \gg \lambda_2$
λ_1 is Large
λ_2 is Small

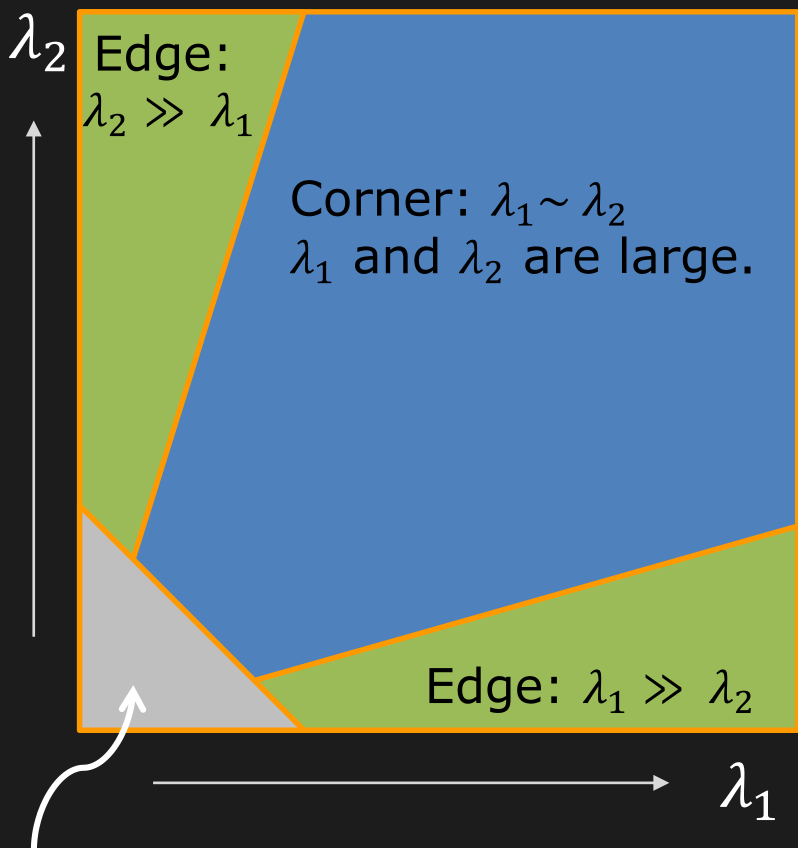
Corner Region



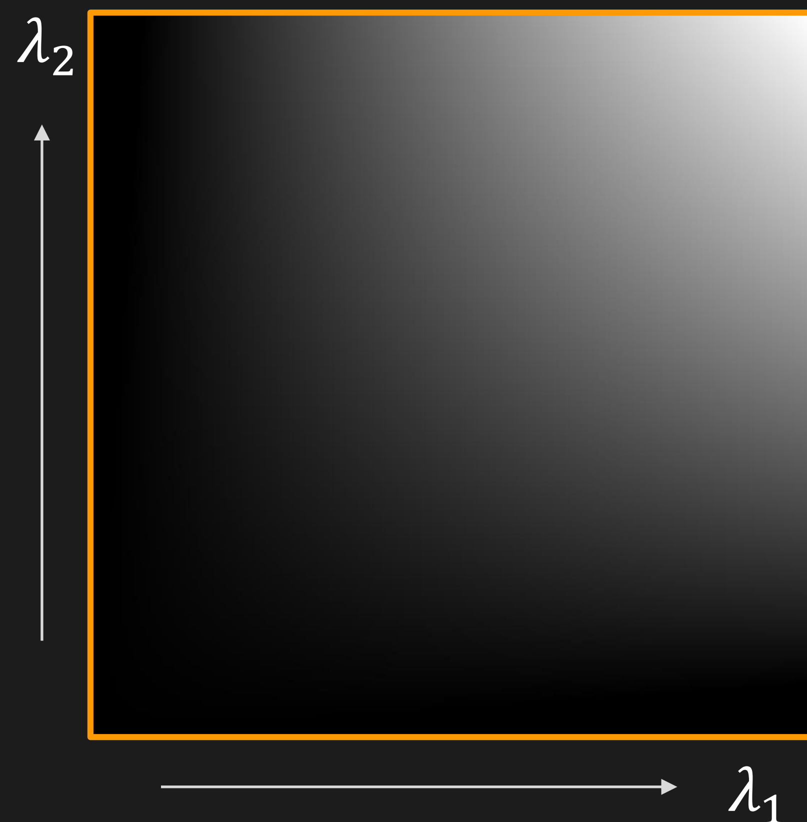


$\lambda_1 \sim \lambda_2$
Both are Large

Harris Corner Response Function



Flat: $\lambda_1 \sim \lambda_2$
λ_1 and λ_2 are small.

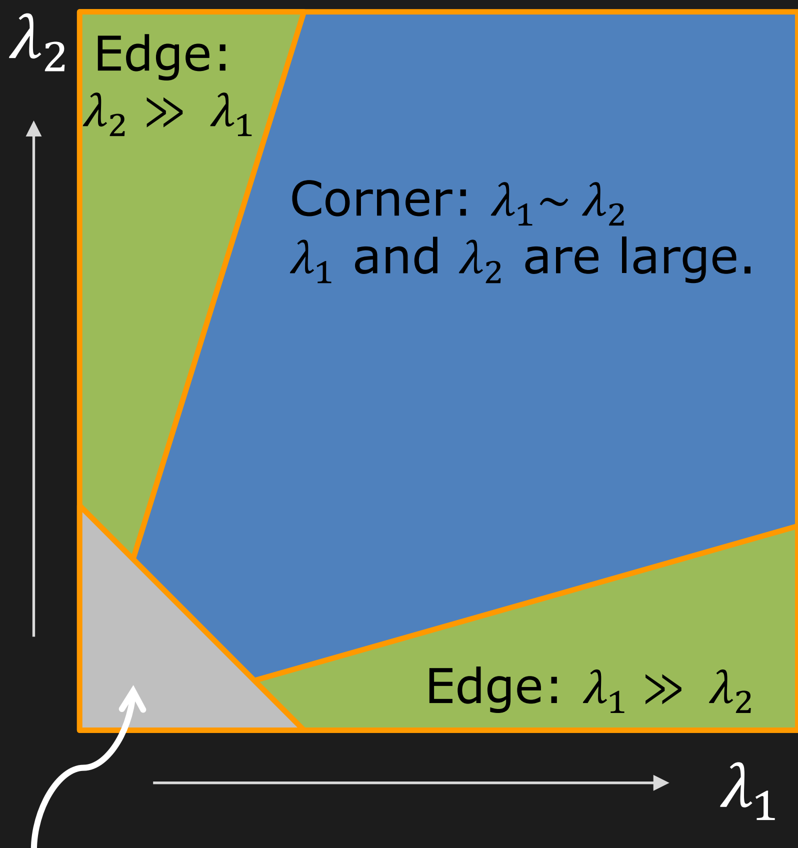


$$\boxed{R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2}$$

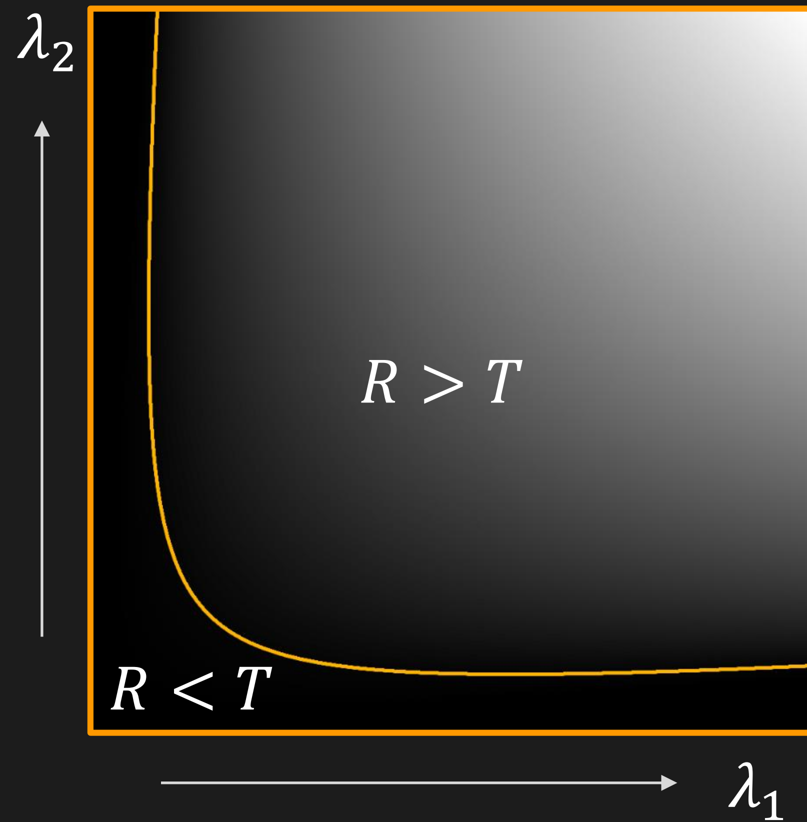
where: $0.04 \leq k \leq 0.06$
(Designed Empirically)

[Harris1988]

Harris Corner Response Function



Flat: $\lambda_1 \sim \lambda_2$
λ_1 and λ_2 are small.



$$\boxed{R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2}$$

where: $0.04 \leq k \leq 0.06$

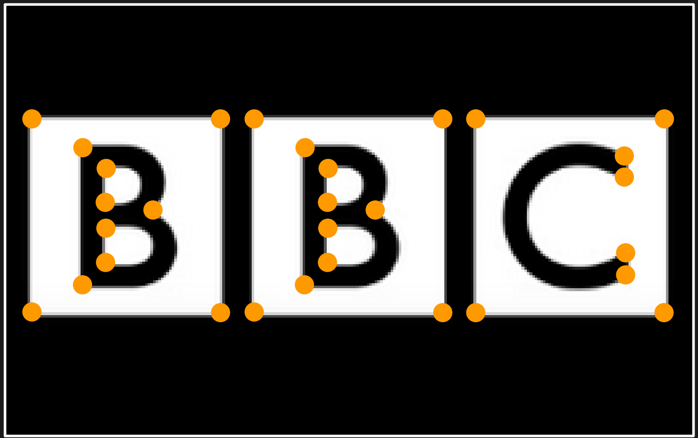
(Designed Empirically)

[Harris1988]

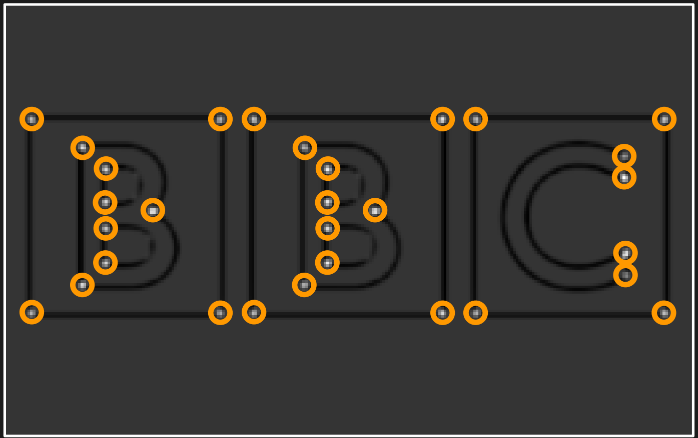
Harris Corner Detection Example

Image

Corner Response R

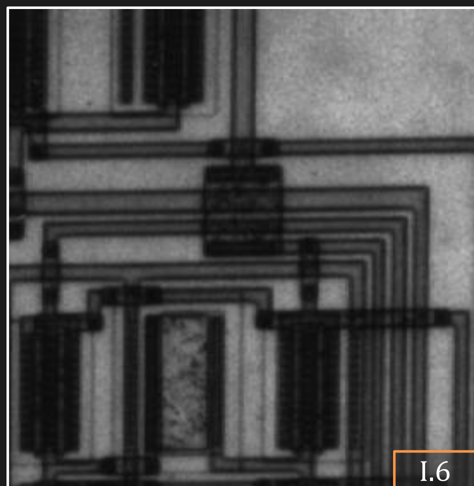


Detected Corners

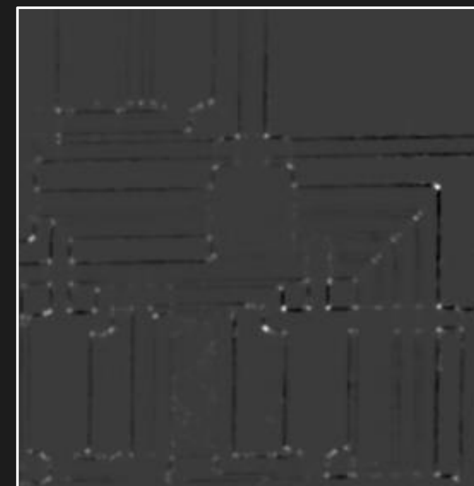


Corner Response $R > T$

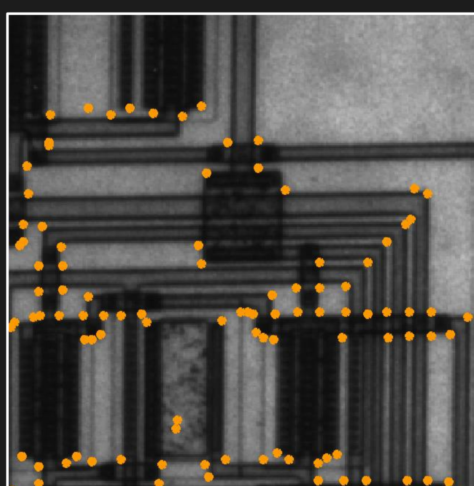
Harris Corner Detection Example



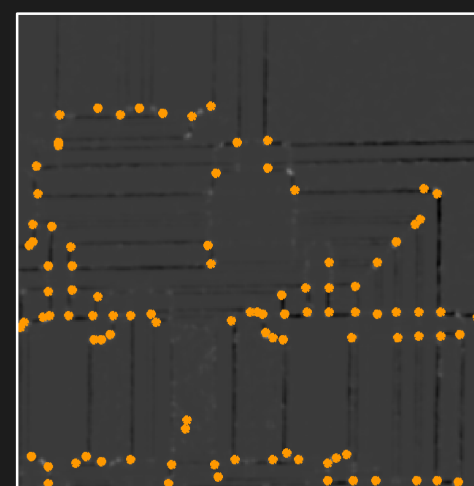
Image



Corner Response R

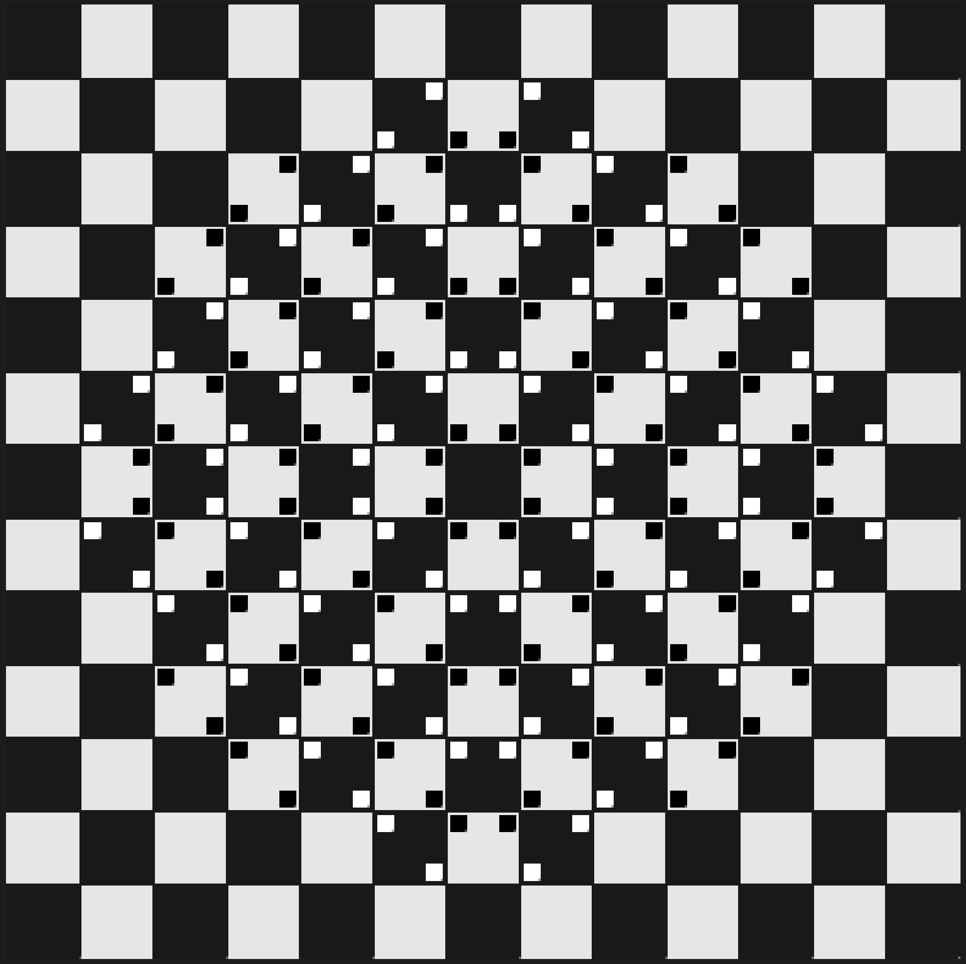


Detected Corners



Corner Response $R > T$

Corner Illusions: The Bulge



Kiyoshi Kitaoka, 1998

References

Textbooks:

Robot Vision (Chapter 8)
Horn, B. K. P., MIT Press

Computer Vision: A Modern Approach (Chapter 8)
Forsyth, D and Ponce, J., Prentice Hall

Digital Image Processing (Chapter 3)
González, R and Woods, R., Prentice Hall

Papers:

[Canny1986] Canny, J., A Computational Approach To Edge Detection, IEEE Trans. Pattern Analysis and Machine Intelligence, 8(6):679–698, 1986.

[Harris1988] Harris, C. and Stephens, M., A combined corner and edge detector. Proceedings of the 4th Alvey Vision Conference. pp. 147–151.

[Marr1980] Marr, D. and Hildreth, E., Theory of Edge Detection," Proc. R. Soc. London,B 207, 187-217, 1980.

Image Credits

I.1	Adapted from Fig 3.1, Nalwa, V., A Guided Tour of Computer Vision.
I.2	Adapted from Fig 3.3, Nalwa, V., A Guided Tour of Computer Vision.
I.3	Matlab Demo Image
I.4	http://en.wikipedia.org/wiki/File:Caf%C3%A9_wall.svg
I.5	http://www.michaelbach.de/ot/geom_KitaokaBulge/index.html
I.6	Matlab Demo Image