MUAMMOLI MASALA VA TOPSHIRIQLAR:

1. Quyidagi ta’riflarni predikatlar mantiqi tilida yozing.

a) Chiziqli tartiblangan to‘plam (tartiblangan to‘plam chiziqli deb ataladi, agar shu to‘plamning har qanday x va y elementlari uchun yo $x=y$, yo $x<y$, yoki $x>y$ bo‘lsa).

b) Juft funksiya ($f(x)$ juft funksiya deb ataladi, agar uning aniqlanish sohasi koordinata boshiga nisbatan simmetrik va aniqlanish sohasining har bir x elementi uchun $f(x)=f(-x)$ bo‘lsa).

2. Quyida berilgan jumlalardagi nuqtalar o‘rniga yo «zarur, ammo yetarli emas», yo «yetarli, ammo zarur emas», yo «zarur emas va yetarli emas» yoki, qayerda mumkin bo‘lsa, «zarur va yetarli» so‘zlarini shunday qo‘yingki, hosil bo‘lgan mulohazalar chin bo‘lsin.

a) To‘rtburchak to‘g‘ri burchakli bo‘lishi uchun uning diagonallarining uzunligi teng bo‘lishi ... .

b) $x^2-5x+6=0$ bo‘lishi uchun $x=3$ bo‘lishi ... .

d) $f(x)$ funksiya $[a,b]$ segmentda integrallanuvchi bo‘lishi uchun $f(x)$ chegaralangan bo‘lishi ... .

e) $f(x)$ funksiya $[a,b]$ segmentda integrallanuvchi bo‘lishi uchun $[a,b]$ segmentda $f(x)$ uzluksiz bo‘lishi ... .

f) $\sum_{k=1}^{\infty} a_k$ sonli qator yaqinlashuvchi bo‘lishi uchun $\lim_{n\to\infty} a_n = 0$ bo‘lishi ... .

3. Quyidagi tasdiqlarning (teoremalarning) noto‘g‘riligini isbot qiling.

a) Agar funksiya biror nuqtada uzluksiz bo‘lsa, u holda u shu nuqtada differensiallanuvchi bo‘ladi.

b) Agar sonli qatorning n- hadi nolga teng bo‘lsa, u holda bu qator yaqinlashuvchi bo‘ladi.

d) Agar to‘rtburchakning diagonallari teng bo‘lsa, u holda bu to‘rtburchak to‘g‘ri burchakli bo‘ladi.

e) Agar funksiya $[a,b]$ yopiq intervalda integrallanuvchi bo‘lsa, u holda u shu intervalda uzluksiz bo‘ladi.

4. Ushbu kvantorli mulohazalarning inkorlarini toping:

a) $\forall x\exists y F(x,y)$; b) $\forall x\exists y\forall z A(x,y,z)$;

d) $\forall x[F(x)\vee\overline{\forall y B(x,y)}]$; e) $\exists x\exists y\forall z[\overline{A(x,y)}\wedge B(y,z)]$;

f) $\exists x A(x,z)\wedge\exists x\forall y B(x,y)\to\forall x\forall y\overline{C(x,y,z)}$;

g) $\exists x(A(x)\wedge B(x)\wedge C(x))$; h) $\forall x(A(x)\to\forall y B(y))$;

i) $\forall x(A(x)\to B(x))\wedge\exists x(D(x)\wedge\overline{R(x)})$;

j) $\exists x(R(x)\leftrightarrow P(x))$; k) $\forall x\exists y\forall z(P(x,y,z)\to Q(x,y,z))$.

5. Quyidagi ifodalarning qaysilari predikatlar mantiqining formulasi bo‘lishini aniqlang. Har bir formula uchun erkin va bog‘langan o‘zgaruvchilarni aniqlang.

a) $\exists x\exists y P(x,y)$; b) $\forall x P(x)\vee\forall y Q(x,y)$; d) $\forall x\exists y P(x,y)$;

e) $p\to\forall x P(x,y)$; f) $\exists x P(x,y)\wedge Q(y,z)$.

1. $P(x,y)$: «$x<y$» predikat $M=\boldsymbol{N}\times\boldsymbol{N}$ to‘plamda aniqlangan bo‘lsin. Quyida berilgan predikatlarning qaysilari aynan chin va qaysilari aynan yolg‘onligini aniqlang:

a) $\exists x P(x,y)$; b) $\forall x P(x,y)$; d) $\exists y P(x,y)$;

e) $\forall y P(x,y)$ f) $\exists x\forall y P(x,y)$; g) $\forall x\exists y P(x,y)$;

h) $\forall y\exists x P(x,y)$; i) $\forall x\forall y P(x,y)$; j) $\forall y\forall x P(x,y)$;

k) $\exists y\forall x P(x,y)$; l) $\exists x\exists y P(x,y)$; m) $\exists y\exists x P(x,y)$.

2. Quyidagi teng kuchliliklarning to‘g‘riligini isbot qiling:

a) $\forall x A(x)\equiv\overline{\exists x\overline{A(x)}}$; b) $C\wedge\forall x A(x)\equiv\forall x(C\wedge A(x))$;

d) $\exists x A(x)\equiv\overline{\forall x\overline{A(x)}}$; e) $C\vee\forall x A(x)\equiv\forall x(C\vee A(x))$;

f) $\exists x(A(x)\vee B(x))\equiv\exists x A(x)\vee\exists x B(x)$;

g) $\exists x(C\vee A(x))\equiv C\vee\exists x A(x)$;

h) $\exists x(C\wedge A(x))\equiv C\wedge\exists x A(x)$;

i) $\exists x A(x)\wedge\exists y B(y)\equiv\exists x\exists y(A(x)\wedge B(y))$;

j) $\forall x(A(x)\to C)\equiv\exists x A(x)\to C$;

k) $\exists x(C\to A(x))\equiv C\to\exists x A(x)$;

l) $\exists x(A(x)\to C)\equiv\forall x A(x)\to C$.

3. $A(x)$ va $B(x)$ ixtiyoriy predikatlar bo‘lsin. Quyida berilgan formulalarning qaysilari $A(x)\to\overline{B(x)}$ formulaga teng kuchli bo‘lishini aniqlang.

a) $A(x)\vee B(x)$; b) $\overline{A(x)}\vee\overline{B(x)}$; d) $\overline{A(x)}\to B(x)$;

e) $\overline{B(x)}\to A(x)$; f) $\overline{\overline{A(x)}\wedge B(x)}$; g) $\overline{A(x)\wedge\overline{B(x)}}$;

h) $B(x)\to\overline{A(x)}$.

4. Quyida keltirilgan formulalarning qaysilari umumqiymatli bo‘lishini aniqlang.

a) $\exists x(P_1(x)\wedge P_2(x))\to(\exists xP_1(x)\wedge\exists xP_2(x))$;

b) $\exists x(P_1(x)\wedge P_2(x))\leftrightarrow(\exists xP_1(x)\wedge\exists xP_2(x))$;

d) $(\forall xP_1(x)\vee\forall xP_2(x))\to\forall x(P_1(x)\vee P_2(x))$;

e) $(\forall xP_1(x)\vee\forall xP_2(x))\leftrightarrow\forall x(P_1(x)\vee P_2(x))$;

f) $\forall x(q\to P_1(x))\leftrightarrow(q\to\forall xP_1(x))$;

g) $\forall x(P(x_1)\to P_2(x))\leftrightarrow(\forall xP_1(x)\to\forall xP_2(x))$;

h) $\exists x(P_1(x)\to P_2(x))\to(\exists xP_1(x)\to\exists xP_2(x))$;

i) $\forall x(P_1(x)\to P_2(x))\leftrightarrow(\exists xP(x_1)\to\forall xP_2(x))$;

j) $\forall x(A_1(x)\to A_2(x))\to(\forall xA_1(x)\to\forall xA_2(x))$;

k) $\forall x(A_1(x)\to A_2(x))\to(\exists xA_1(x)\to\exists xA_2(x))$;

l) $\exists x(A_1(x)\to A_2(x))\leftrightarrow(\forall xA_1(x)\to\forall xA_2(x))$;

m) $\exists xQ(x)\to\forall xQ(x)$;

n) $\forall xQ(x)\to\exists xQ(x)$;

o) $\forall xP(x)\wedge\forall xQ(x)\leftrightarrow\forall x(P(x)\wedge Q(x))$;

p) $\forall xP(x)\vee\forall xQ(x)\leftrightarrow\forall x(P(x)\vee Q(x))$;

q) $\exists xP(x)\wedge\exists xQ(x)\leftrightarrow\exists x(P(x)\wedge Q(x))$;

r) $\exists xP(x)\vee\exists xQ(x)\leftrightarrow\exists x(P(x)\vee Q(x))$.

5. Agar M to‘plamda aniqlangan $A(x)$ va $B(x)$ predikatlar chin qiymatli bo‘lsa, u holda quyidagi formulalar uchun ularning chinlik to‘plamlari qanday shartlarni qanoatlantirishi kerakligini aniqlang:

a) $\forall x(A(x)\to B(x))\wedge\exists x(\overline{A(x)}\wedge B(x))$;

b) $\overline{\exists x(A(x)\wedge B(x))}\wedge(\forall(A(x)\to B(x)))$;

d) $\exists x(A(x)\wedge B(x))\to(\forall x(A(x)\to B(x)))$.

6. $M=\{1,2,3,...,20\}$ to‘plamda $A(x)$: «x son 5 ga qoldiqsiz bo‘linmaydi»; $B(x)$: «x – juft son»; $C(x)$: «x – tub son»; $D(x)$: «x 3 ga karrali» predikatlar berilgan. Quyidagi predikatlar uchun chinlik to‘plamlarni toping:

a) $A(x)\wedge B(x)$; b) $C(x)\wedge B(x)$; d) $C(x)\wedge D(x)$;

e) $B(x)\wedge D(x)$; f) $\overline{B}(x)\wedge D(x)$; g) $A(x)\wedge \overline{D}(x)$;

h) $\overline{B}(x)\wedge \overline{D}(x)$; i) $\overline{B}(x)\wedge \overline{D}(x)$; j) $A(x)\vee B(x)$;

k) $B(x)\vee C(x)$; l) $C(x)\vee D(x)$; m) $B(x)\vee D(x)$;

n) $\overline{B}(x)\vee D(x)$; o) $B(x)\vee \overline{D}(x)$; p) $A(x)\vee B(x)\vee D(x)$;

q) $C(x)\to A(x)$; r) $D(x)\to \overline{C}(x)$; s) $A(x)\to B(x)$;

t) $(A(x)\wedge C(x))\to \overline{D}(x)$; u) $(A(x)\wedge D(x))\to \overline{C}(x)$.

12. Ushbu $A\equiv(P(x)\to\overline{Q(x)})\to\overline{\exists xP(x)\wedge\forall xQ(x)}$ formulaning umumqiymatli ekanligini isbotlang.