BUL FUNKSIYALARINI O‘ZGARUVCHILAR BO‘YICHA YOYILMASI. MDNSH VA MKNSH

$\{0,1\}$ Bul algebrasidagi xy kon'yunksiya amali oddiy arifmetikadagi 0 va 1 sonlar ustidagi ko'paytma amaliga mos keladi. Ammo 0 va 1 sonlarni qo'shish natijasi $\{0,1\}$ to'plam doirasidan chetga chiqadi. Shuning uchun I.I.Jegalkin (3.8.1869-28.3.1947) 2 moduliga asosan qo'shish amalini kiritadi (I.I.Jegalkin 30-yillarning boshida Moskva davlat universitetida birinchi bo'lib matematik mantiq bo'yicha ilmiy seminar tashkil etgan). x va y mulohazalarning 2 moduli bo'yicha qo'shishni $x+y$ sifatida belgilaymiz va u quyidagi chinlik jadvali bilan beriladi:

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	0

Chinlik jadvalidan ko‘rinib turibdiki, $x+y=\overline{x \leftrightarrow y}$. Mantiq algebrasidagi ko‘paytma va 2 moduli bo‘yicha qo‘shish mantiq amallari uchun kommutativ, assotsiativ va distributiv arifmetik qonunlar o‘z kuchini saqlaydi.

Bul algebrasidagi asosiy mantiqiy amallarni kiritilgan arifmetik amallar orqali quyidagicha ifodalash mumkin:

1. $\bar{x}=x+1$; 2. $x \wedge y = xy$; 3. $x \vee y = xy+x+y$; 4. $x \to y = xy+x+1$;

5. $x \leftrightarrow y = x+y+1$.

2 moduli bo‘yicha qo‘shish amalining ta’rifiga asosan $x+x=0$ va $xx = x$ $(x^n = x)$.

Bul funksiyalarining o‘zgaruvchilar bo‘yicha yoyilmasi

Teorema 1. *E_2 to‘plamdagi har bir $f(x_1,\dots,x_n)$ funksiya quyidagi ko‘rinishda ifodalanishi mumkin:*

1. $f(x_1,\dots,x_n)=x_1\cdot f(1,x_2,\dots,x_n)\vee\bar{x}_1\cdot f(0,x_2,\dots,x_n)$,
2. $f(x_1,\dots,x_n)=x_1\cdot f(1,x_2,\dots,x_n)\oplus\bar{x}_1\cdot f(0,x_2,\dots,x_n)$,
3. $f(x_1,\dots,x_n)=(x_1\vee f(0,x_2,\dots,x_n))\,\&\,(\bar{x}_1\vee f(1,x_2,\dots,x_n))$.

Isbot. Ushbu formulalarni isbotlash uchun

1) Agar $x_1=0$ bo‘lsa, unda

$$f(0,x_2,\dots,x_n)=0\vee f(0,x_2,\dots,x_n)=f(0,x_2,\dots,x_n).$$

2) Agar $x_1=1$ bo‘lsa, unda

$$f(1,x_2,\dots,x_n)=1\,\&\,f(1,x_2,\dots,x_n)\vee 0\,\&\,f(0,x_2,\dots,x_n)=f(1,x_2,\dots,x_n).$$

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

Teorema 2. *Har bir E_2 dan olingan $f(x_1,\dots,x_n)$ funksiyani quyidagicha ifodalash mumkin, bunda $\forall \kappa, 1 \le k \le n$:*

1. $f(x_1,\dots,x_n) = \underset{(\sigma_1,\dots,\sigma_k)}{V} \; x_1^{\sigma_1} \& \dots \& x_k^{\sigma_k} \& f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$,

bunda $x^{\sigma} = \begin{cases} x, \; agar \; \sigma = 1; \\ \bar{x}, \; agar \; \sigma = 0; \end{cases}$ $\quad \overset{n}{\underset{i=1}{V}} x_i = x_1 \vee x_2 \vee \dots \vee x_n$.

2. $f(x_1,\dots,x_n) = \sum\limits_{(\sigma_1,\dots,\sigma_k)} x_1^{\sigma_1} \& \dots \& x_k^{\sigma_k} \& f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$, bunda

$\sum\limits_{i=1}^{n} x_i = x_1 \oplus x_2 \oplus \dots \oplus x_n$.

3. $f(x_1,\dots,x_n) = \underset{(\sigma_1,\dots,\sigma_k)}{\&} \; (x_1^{\bar{\sigma}_1} \vee \dots \vee x_k^{\bar{\sigma}_k} \vee f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n))$;

bunda $\overset{n}{\underset{i=1}{\&}} x_i = x_1 \; \& \; x_2 \; \& \; \dots \; \& \; x_n$, $\quad x^{\bar{\sigma}} = \begin{cases} x, \; agar \; \sigma = 0; \\ \bar{x}, \; agar \; \sigma = 1; \end{cases}$

MDNSH va MKNSH.

Natija 1. *2-teoremaning 1-tasdiqida* $k=n$ *bo'lsa, unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n)=\underset{(\sigma_1,\ldots,\sigma_n)}{V}\quad x_1^{\sigma_1}\ldots x_n^{\sigma_n}\cdot f(\sigma_1,\ldots,\sigma_n)\cdot \tag{1}$$

Agar $f\neq 0$, *unda (1) formuladan*

$$f(x_1,\ldots,x_n)=\underset{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=1}}{V}\quad x_1^{\sigma_1}\,\&\ldots\&\,x_n^{\sigma_n}$$ – *Mukammal diz'yunktiv normal shakl (MDNSh).*

Natija 2. *Agar 2-teoremaning 3-tasdiqida* $k=n$ *bo'lsa, unda unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n)=\underset{(\sigma_1,\ldots,\sigma_n)}{\&}\quad (x_1^{\bar\sigma_1}\vee\ldots\vee x_n^{\bar\sigma_n}\vee f(\sigma_1,\ldots,\sigma_n)). \tag{2}$$

Agar $f\neq 1$, *unda (2) formuladan*

$$f(x_1,\ldots,x_n)=\underset{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=0}}{\&}\quad (x_1^{\bar\sigma_1}\vee\ldots\vee x_n^{\bar\sigma_n})$$ – *Mukammal kon'yunktiv normal shakl (MKNSh).*

Natija 3. *Agar 2-teoremaning 2-tasdiqida* $k = n$ *bo'lsa, unda unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n) = \sum_{(\sigma_1,\ldots,\sigma_n)} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n} \& f(\sigma_1,\ldots,\sigma_n) \qquad (3)$$

Agar $f \neq 0$, *unda (3) formuladan*

$$f(x_1,\ldots,x_n) = \sum_{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n}\,.$$