9-MAVZU	BUL FUNKSIYALARINI O‘ZGARUVCHILAR BO‘YICHA YOYILMASI. MDNSH VA MKNSH

$\{0,1\}$ Bul algebrasidagi xy kon’yunksiya amali oddiy arifmetikadagi 0 va 1 sonlar ustidagi ko‘paytma amaliga mos keladi. Ammo 0 va 1 sonlarni qo‘shish natijasi {0,1} to‘plam doirasidan chetga chiqadi. Shuning uchun I.I.Jegalkin (3.8.1869-28.3.1947) 2 moduliga asosan qo‘shish amalini kiritadi (I.I.Jegalkin 30-yillarning boshida Moskva davlat universitetida birinchi bo‘lib matematik mantiq bo‘yicha ilmiy seminar tashkil etgan). x va y mulohazalarning 2 moduli bo‘yicha qo‘shishni $x+y$ sifatida belgilaymiz va u quyidagi chinlik jadvali bilan beriladi:

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	0

Chinlik jadvalidan ko‘rinib turibdiki, $x+y=\overline{x \leftrightarrow y}$. Mantiq algebrasidagi ko‘paytma va 2 moduli bo‘yicha qo‘shish mantiq amallari uchun kommutativ, assotsiativ va distributiv arifmetik qonunlar o‘z kuchini saqlaydi.

Bul algebrasidagi asosiy mantiqiy amallarni kiritilgan arifmetik amallar orqali quyidagicha ifodalash mumkin:

1. $\bar{x}=x+1$; 2. $x \wedge y = xy$; 3. $x \vee y = xy + x + y$; 4. $x \to y = xy + x + 1$;
5. $x \leftrightarrow y = x + y + 1$.

2 moduli bo‘yicha qo‘shish amalining ta’rifiga asosan $x+x=0$ va $xx=x \ (x^n = x)$.

Bul funksiyalarining o‘zgaruvchilar bo‘yicha yoyilmasi

Teorema 1. E_2 *to'plamdagi har bir* $f(x_1,\dots,x_n)$ *funksiya quyidagi ko'rinishda ifodalanishi mumkin:*

1. $f(x_1,\dots,x_n) = x_1 \cdot f(1,x_2,\dots,x_n) \vee \bar{x}_1 \cdot f(0,x_2,\dots,x_n)$,
2. $f(x_1,\dots,x_n) = x_1 \cdot f(1,x_2,\dots,x_n) \oplus \bar{x}_1 \cdot f(0,x_2,\dots,x_n)$,
3. $f(x_1,\dots,x_n) = (x_1 \vee f(0,x_2,\dots,x_n))\,\&\,(\bar{x}_1 \vee f(1,x_2,\dots,x_n))$.

Isbot. Ushbu formulalarni isbotlash uchun

1) Agar $x_1=0$ bo'lsa, unda

$$f(0,x_2,\dots,x_n) = 0 \vee f(0,x_2,\dots,x_n) = f(0,x_2,\dots,x_n).$$

2) Agar $x_1=1$ bo'lsa, unda

$$f(1,x_2,\dots,x_n) = 1\,\&\,f(1,x_2,\dots,x_n) \vee 0\,\&\,f(0,x_2,\dots,x_n) = f(1,x_2,\dots,x_n).$$

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

Teorema 2. *Har bir* E_2 *dan olingan* $f(x_1,\dots,x_n)$ *funksiyani quyidagicha ifodalash mumkin, bunda* $\forall\kappa, 1 \le k \le n$ *:*

1. $f(x_1,\dots,x_n) = \underset{(\sigma_1,\dots,\sigma_k)}{V} x_1^{\sigma_1}\,\&\dots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$,

bunda $x^{\sigma} = \begin{cases} x, \; agar\ \sigma = 1; \\ \bar{x}, \; agar\ \sigma = 0; \end{cases}$ $\quad \overset{n}{\underset{i=1}{V}} x_i = x_1 \vee x_2 \vee \dots \vee x_n$.

2. $f(x_1,\dots,x_n) = \sum_{(\sigma_1,\dots,\sigma_k)} x_1^{\sigma_1}\,\&\dots\&\,x_k^{\sigma_k}\,\&\,f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n)$, bunda

$\sum_{i=1}^{n} x_i = x_1 \oplus x_2 \oplus \dots \oplus x_n$.

3. $f(x_1,\dots,x_n) = \underset{(\sigma_1,\dots,\sigma_k)}{\&} (x_1^{\bar{\sigma}_1} \vee \dots \vee x_k^{\bar{\sigma}_k} \vee f(\sigma_1,\dots,\sigma_k,x_{k+1},\dots x_n))$;

bunda $\overset{n}{\underset{i=1}{\&}} x_i = x_1 \,\&\, x_2 \,\&\dots\&\, x_n$, $\quad x^{\bar{\sigma}} = \begin{cases} x, \; agar\ \sigma = 0; \\ \bar{x}, \; agar\ \sigma = 1; \end{cases}$

1-tasdiqning isboti: $k-(\alpha_1,\dots,\alpha_k)$ bo'lsa, $f(x_1,\dots,x_n)$ funksiya quyidagi ko'rinishga keladi - $f(\alpha_1,\dots,\alpha_k,x_{k+1},\dots x_n)$.

Ta'rifga asosan,

$$\left.\begin{array}{l} \alpha_1^{\sigma_1} = 1 \;\Leftrightarrow\; \alpha_1 = \sigma_1 \\ \alpha_2^{\sigma_2} = 1 \;\Leftrightarrow\; \alpha_2 = \sigma_2 \\ \dots\dots\dots\dots\dots\dots.. \\ \alpha_k^{\sigma_k} = 1 \;\Leftrightarrow\; \alpha_k = \sigma_k \end{array}\right\}$$

Bundan esa, $\alpha_1^{\sigma_1} \& \alpha_2^{\sigma_2} \& \ldots \& \alpha_k^{\sigma_k} = 1 \quad \Leftrightarrow \quad \alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$.

$1 \& x = x$ dan f funksiya berilgan formulaga faqat va faqat $\alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$ bo'lganda o'rinli.

Bundan, formulaning o'ng tomoni

$$\underset{(\sigma_1,\ldots,\sigma_k)}{V} x_1^{\sigma_1} \ldots x_k^{\sigma_k} \ \& \ f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n) = f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n) \text{ ga teng}$$

chunki qolgan barcha kon'yunksiyalar = 0. Formulaning chap tomonining ham $f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n)$ ko'rinishga ega chunki, $\alpha_1 = \sigma_1, \alpha_2 = \sigma_2, \ldots, \alpha_k = \sigma_k$.

Demak, $f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n) = f(\sigma_1,\ldots,\sigma_k, x_{k+1},\ldots x_n)$.

Xuddi shunday 2-3-tasdiqlarni ham isbotlashimiz mumkin.

MDNSH va MKNSH.

Natija 1. *2-teoremaning 1-tasdiqida $k = n$ bo'lsa, unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n) = \underset{(\sigma_1,\ldots,\sigma_n)}{V} x_1^{\sigma_1} \ldots x_n^{\sigma_n} \cdot f(\sigma_1,\ldots,\sigma_n). \tag{1}$$

Agar $f \neq 0$, unda (1) formuladan

$$f(x_1,\ldots,x_n) = \underset{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=1}}{V} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n}$$ *– Mukammal diz'yunktiv normal shakl (MDNSh).*

Natija 2. *Agar 2-teoremaning 3-tasdiqida $k = n$ bo'lsa, unda unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n) = \underset{(\sigma_1,\ldots,\sigma_n)}{\&} (x_1^{\bar{\sigma}_1} \vee \ldots \vee x_n^{\bar{\sigma}_n} \vee f(\sigma_1,\ldots,\sigma_n)). \tag{2}$$

Agar $f \neq 1$, unda (2) formuladan

$$f(x_1,\ldots,x_n) = \underset{\substack{(\sigma_1,\ldots,\sigma_n) \\ f(\sigma_1,\ldots,\sigma_n)=0}}{\&} (x_1^{\bar{\sigma}_1} \vee \ldots \vee x_n^{\bar{\sigma}_n})$$ *– Mukammal kon'yunktiv normal shakl (MKNSh).*

Natija 3. *Agar 2-teoremaning 2-tasdiqida $k = n$ bo'lsa, unda unda yoyilma quyidagi ko'rinishga ega bo'ladi:*

$$f(x_1,\ldots,x_n) = \sum_{(\sigma_1,\ldots,\sigma_n)} x_1^{\sigma_1} \& \ldots \& x_n^{\sigma_n} \ \& \ f(\sigma_1,\ldots,\sigma_n) \tag{3}$$

Agar $f \neq 0$, unda (3) formuladan

$$f(x_1,\ldots,x_n)=\sum_{\substack{(\sigma_1,\ldots,\sigma_n)\\ f(\sigma_1,\ldots,\sigma_n)=1}} x_1^{\sigma_1}\,\&\ldots\&\,x_n^{\sigma_n}\;.$$