MUAMMOLI MASALA VA TOPSHIRIQLAR:

1. Quyidagi bul funksiyalariga teng kuchli bo‘lgan Jegalkin ko‘phadini noma‘lum koeffisientli ko‘phad usuli bilan toping:

1) $x'(yz' \vee y'z)$;

2) $(x \to (y \to z'))(yz' \to x)$;

3) $(x+1)(y+1)z' \vee yz$;

4) $x'z' \vee (x'y \vee xy')z$;

5) $x'z' \vee (x'z \vee xz')y \vee xy'z'$.

Yechim: 5) $F(x,y,z) = x'z' \vee (x'z \vee xz')y \vee xy'z' = a \cdot xyz + b \cdot xy + c \cdot xz +$

$+ d \cdot yz + e \cdot x + f \cdot y + g \cdot z + h$

Funksiyaning qiymatlar jadvalida noma‘lum koeffisientlarni aniqlaymiz:

x	y	z	$x'z' \vee (x'z \vee xz')y \vee$ $\vee xy'z'$	$a \cdot x_1x_2x_3 + b \cdot x_1x_2 + c \cdot x_1x_3 +$ $+ d \cdot x_2x_3 + e \cdot x_1 + f \cdot x_2 + g \cdot x_3 + h$	
0	0	0	1	h	$h=1$
0	0	1	0	$g+h$	$g=1$
0	1	0	0	$f+h$	$f=1$
0	1	1	1	$d+f+g+h$	$d=0$
1	0	0	1	$e+h$	$e=0$
1	0	1	0	$c+e+g+h$	$c=0$
1	1	0	0	$b+e+f+h$	$b=0$
1	1	1	0	$a+b+c+d+e+f+g+h$	$a=1$

Demak, $F(x,y,z) = xyz + y + z + 1$.

2. Tenglikning o‘ng va chap tomoni Jegalkin ko‘phadi ko‘rinishiga keltirib, quyidagilarning to‘g‘rilarini aniqlang:

1) $x\to(y\to z)=(x\to y)\to(x\to z)$;

2) $(xy\to z)\to(x\to z)=x'yz$;

3) $xy\to z=(x\to z)(y\to z)$;

4) $(x\leftrightarrow y)(xy'\vee y)=xy$;

5) $(x\to y)\to z=x\to(y\to z)$.

Yechim: 5) $(x\to y)\to z=(xy+x+1)\to z=$

$=(xy+x+1)z+(xy+x+1)+1=xyz+xz+xy+x+z;$

$x\to(y\to z)=x\to(yz+y+1)=x(yz+y+1)+x+1=xyz+xy+1.$

Bundan, $(x\to y)\to z\neq x\to(y\to z)$.

3. Quyidagi bul funksiyalarini Jegalkin ko‘phadi ko‘rinishini topib, qaysi biri aynan chin -1, yoki aynan yolg‘onligini -0 aniqlang:

1) $(y\to z)\to((x\to y)\to(x\to z))$;

2) $x\to(x'\to y)$;

3) $(x\to y)\to((x\to(y\to z))\to(x\to z)$;

4) $((x\to y)\to x)\to x$;

5) $(x\vee y)'\vee x'y\vee x$.

Yechim: 5) $(x\vee y)'\vee x'y\vee x=(xy+x+y+1)\vee(x+1)y\vee x=(xy+x+y+1)\vee$

$\vee((x+1)xy+(x+1)y+x)=(xy+x+y+1)\vee(xy+x+y)=$

$=(xy+x+y+1)(xy+x+y)+(xy+x+y+1)+(xy+x+y)=xy+x+y+1+xy+$

$+x+y=1$

4. Quyidagi barcha bul funksiyalari chiziqli ekanligini isbotlang:

1) $x'y'z'\vee xy'z'\vee x'yz\vee xyz$;

2) $((x\vee y\vee z)\to xyz')\vee(x+y)z$;

3) $(x'y\vee xy')z'\vee(x+y)z$;

4) $(x+y)z \vee (x'y' \vee xy)z$;

5) $xyz \vee xy'z \vee x'yz' \vee x'y'z'$.

Yechim: 5) $xyz \vee xy'z \vee x'yz' \vee x'y'z' = xz(y \vee y') \vee x'z' \vee (y \vee y') = xz \vee x'z' = xz \vee (x + 1)(z + 1) = xz \vee (xz + x + z + 1) = xz(xz + x + z + 1) + xz + xz + x + z + 1 = xz + xz + xz + xz + x + z + 1 = x + z + 1.$

5. Bul funksiyalarining monotonligini isbotlang.

1) $xyz \vee x'yz \vee xyz'$;

2) $xyz + xy + xz$;

3) $xy \vee xz \vee yz$;

4) $(x \vee y \vee z)(x \vee y' \vee z)(x \vee y \vee z')$;

5) $(x \vee y \vee z)(x' \vee y \vee z)(x \vee y' \vee z)$.

Yechim: 5) avval qiymatlar jadvalini tuzamiz:

x	y	z	$f(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Endi har bir qiymatlar satrini va natijasini taqqoslaymiz:

000 < 001, 000 < 101, 000 < 010, 000 <110, 000 < 011, 000 < 100, 000 < 111,

001 < 011, 001 < 101, 001 < 111, 010 < 110, 010 < 111, 010 < 011, 011 < 111, 100 < 101, 100 < 110, 100< 111, 101<111, 110< 111, 011 < 111.

Demak, berilgan funksiyamiz monoton funksiya.

Berilgan funksiyani taqqoslashning yana bir usuli diagrammada tasvirlash yordamida bajariladi. (chap tomonda o‘zgaruvchilarning qiymatlari diagrammasi tasvirlangan bo‘lsa, o‘ng tomonda funksiyaning qabul qilgan qiymatlari diagrammasi tasvirlangan.) Ko‘rinib turibdiki, o‘zgaruvchilarning qiymatlari pastdan yuqoriga o‘sib borayapti va xuddi shunday funksiya qabul qilgan qiymatlar ham pastdan yuqoriga o‘sib boradi. Demak, berilgan funksiyamiz monoton funksiya.