

Queuing Network Model and Visualization for the Patient Flow in the Obstetric Unit of the University of Tsukuba Hospital

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Abstract—A complete data set of every movement of all the inpatients from room to room covering two years was provided us by the Medical Information Department of the University of Tsukuba Hospital in Japan. By focusing on the obstetric patients, who are assumed to be hospitalized rather at random times, we have analyzed the patient flow using our original visualization software. Upon admission, each obstetric patient is assigned to a bed in one of the two wards, one for high-risk delivery and the other for normal delivery, and then she may be transferred between the two wards before discharge. We confirm Little's law of queuing theory for the patient flow in each ward. Then we propose a network model of $M/G/\infty$ and $M/M/m$ queues to represent the flow of these patients, which is used to predict the probability distribution for the number of patients staying in each ward at the nightly census time from the observed data of patient admission rate and the histogram of the length-of-stay (LOS) in that ward. Although our model is a very rough and simplistic approximation of the real patient flow, the predicted probability distribution is shown to be in good agreement with the observed one. Our method can be used for planning the capacity of obstetric units when the patient demand is predicted.

Keywords—OR in healthcare; obstetrics; patient flow; Little's law; queuing network; visualization

I. INTRODUCTION

The University of Tsukuba Hospital (UTH) is affiliated to the University of Tsukuba which is located about 40 miles northeast of Tokyo, Japan. It is esteemed as one of specific functional hospitals in Ibaraki Prefecture. A joint team of hospital staff and OR/MS researchers in the engineering department launched a healthcare service innovation project in April 2011. The goal of our project is to develop a web-based software system for controlling admission and bed allocation of patients using mathematical optimization. In addition, we analyze the inpatient flow from admission to discharge in the hospital by system-scientific techniques [1].

As an enabler of our project, a complete data set of every movement of all the inpatients (with encrypted id's) from room to room, covering a two-year period from April 2010 to March 2012, was provided us by the Medical Information Department of UTH. See Figure 1 at the end of this paper for the sample log data. Orders for patient movement include admission, discharge, ward transfer, room transfer in the same ward, clinical group transfer, overnight stay

outside hospital, and return from it. The statistical treatment of these log data with the resulting publication of research findings in our project has been approved by the Ethics Committee of UTH.

Basic performance measures in hospital management include the bed utilization and the mean length-of-stay (LOS) per patient. The bed utilization is calculated as the ratio of the mean number of patients staying in the entire hospital or in each clinical ward to the total number of beds that the facility has. Also the mean LOS is calculated as the ratio of the mean number of patients present there to the patient admission rate by Little's law from queuing theory. These are simple calculations. We take a step forward.

In this paper, focusing on the obstetric unit of UTH, we propose an approximate queuing network model for the patient flow that makes it possible to predict the probability distribution for the number of patients staying in each ward at the midnight census time from the observed data of patient admission rate and the histogram of the LOS in that ward. Our model is fully validated against the observed data for practical use. We also present visualization techniques for the quantitative display of patient flow in graphics.

There has been extensive work on applying system-scientific approach, in particular queuing theory, to the patient flow in hospitals such as surveys [2–4] and original papers [5–8] just to name a few. Among others, the patients in the obstetric and neonatal units are studied in [9, 10]. Other techniques may be discrete-event simulation and agent-based simulation. Hospital is a typical service system for which a book by Hall [11] introduces general queuing-theoretic methods in detail.

The rest of this paper is organized as follows. In Section II, we mention the statistical data related to the number and LOS of patients in the UTH and its obstetric unit in particular. These data are analyzed by Little's law in Section III. The construction of a queuing network model is started with the extraction and visualization of dominant patient routes in Section IV. Relevant queuing models are discussed in Section V. The queuing network model of the obstetric patient flow is finally presented in Section VI, where comparison is made in detail between theoretical results and observed values. We conclude the paper with a brief remark on the success of modeling and a plan of our future research in Section VII.

II. STATISTICS ABOUT THE UNIVERSITY OF TSUKUBA HOSPITAL AND ITS OBSTETRIC UNIT

As of April 1, 2012, the UTH has a total of 1,569 staff members, including 570 doctors and 704 nurses [12]. It has 23 wards holding 800 beds, out of which 759 beds are in the general wards and 41 beds are in the psychiatric ward as shown in Table I. The critical care units with 36 beds are divided into intensive care units (ICUs) with 10 beds and high care units (HCUs) with 26 beds. A ward is often shared by several clinical departments, while each department may use a few wards. There are twelve operating rooms. In the fiscal year 2011 (from April 1 2011 to March 31 2012), the hospital conducted 14,303 operations. It admitted 13,515 patients who stayed in the hospital amounting to 239,100 patient-days with 81.7% bed utilization. The average length-of-stay (LOS) of a patient was 16.4 days (Note that this is extremely long compared to any other OECD countries). See Table II for the statistics in each year during 2010–2012.

The wards in UTH moved to a new site in December 2012. We note that all the numerical data in this paper refer to the statistics before this recent movement.

TABLE I. NUMBER OF WARDS AND BEDS IN THE UNIVERSITY OF TSUKUBA HOSPITAL.

			Wards	Beds
General wards	Adults	Critical care units	2	36
		Semi-critical care units	13	563
		General units	1	45
	Obstetrics		2	32
	Child health		4	83
Psychiatric ward			1	41
Total			23	800

TABLE II. STATISTICS ON THE NUMBER OF PATIENTS IN THE UNIVERSITY OF TSUKUBA HOSPITAL IN THE RECENT FISCAL YEARS.

Fiscal Year	2010	2011	2012
Total count of patient-days	242,003	239,100	227,718
Mean number of patients staying in bed on a day	663	653	624
Bed utilization	82.9%	81.7%	78.0%
Total number of admitted patients	13,897	13,515	13,725
Patients arrival rate per day	38	37	38
Mean LOS in days	16.3	16.4	15.6

The obstetric unit of UTH is called the Center for Maternal, Fetal and Neonatal Health for treatment of normal as well as high-risk childbirth in the Tsukuba and Southern Ibaraki Prefecture areas. There are two wards, numbered 30M and 300, for the obstetric unit:

- Ward 30M is the maternal and fetal intensive care unit (MFICU), which has 6 beds, for the treatment of high-risk delivery.

- Ward 300 with 26 beds accommodates patients with normal delivery and also plays a role of backup (waiting room) for 30M.

In Table III, we show several statistics on the number of patients in the two wards of the obstetric unit during two fiscal years 2010–2011, more precisely, a period from the night of April 1, 2010, to the night of March 31, 2012, both inclusively. In this table, for a patient who was assigned to Ward 300 and then transferred to Ward 30M, we count one admission to Ward 300 and another admission to Ward 30M. For convenience sake, we assume that all the patients who stayed on April 1, 2010 were admitted on that day, and that all the patients who stayed on March 31, 2012 were discharged on the next day. Also those patients who were admitted and discharged on the same day are counted in the total number of admitted patients but they do not contribute to the count of patient-days. Since the year 2012 was a leap year, there were 731 nights in the period of our study.

Let us make a few comments on the numbers in Table III. For example, take Ward 300. The number of patients in bed is counted at 0:00 a.m. (midnight census) on each night, the sum of which over the two years amounts to 12,630. Therefore, the mean number of patients staying in bed on each night is $12,630/731=17.28$, which leads to the bed utilization $17.28/26=0.665$. On the other hand, since 1,963 patients were admitted during these two years, the arrival rate was $1,963/731=2.685$ patients/day. The mean LOS for each patient was 6.43 days. This is reasonable because, in Japan, a mother with normal delivery usually stays 5 days in hospital after childbirth. Therefore, if a baby is born on the day of admission, the mother stays 5 nights. However, if a baby is born on the day following the admission, she stays 6 days in hospital. Mothers with abnormal delivery may stay a few more days in the hospital.

Similar calculation for Ward 30M gives the numbers on the rightmost column in Table III. The LOS in Ward 30M is longer than that in Ward 300, because mothers with high-risk delivery are accommodated in Ward 30M.

TABLE III. STATISTICS ON THE NUMBER OF OBSTETRIC PATIENTS IN THE UNIVERSITY OF TSUKUBA HOSPITAL IN TWO FISCAL YEARS 2010–2011.

	Ward 300	Ward 30M
Number of beds	26	6
Total number of patients-day	12,630	3,600
Mean number of patients staying in bed of the ward on a day	17.28	4.92
Bed utilization	66.5%	82.1%
Total number of admitted patients	1,963	338
Patients arrival rate per day	2.685	0.462
Mean LOS in days	6.43	10.65

Table III shows only the mean of the number of patients staying in bed of each ward on a day. Figure 2 and Figure 3 show the probability distribution for the number of patients

who stayed in Ward 300 and Ward 30M, respectively, in 2010–2011. These figures are obtained from the histogram for the number of days each patient stayed continuously during the two years. It is interesting to observe that the distribution for Ward 300 in Figure 2 is symmetric about the mean value 17.28 but that the distribution for Ward 30M in Figure 3 is monotonically increasing up to the capacity of 6 patients. The probability that all the beds are occupied on an arbitrary day is only about 1% for Ward 300, while it is nearly equal to 50% for Ward 30M.

One of the objectives in this paper is to predict the distributional forms in Figures 2 and 3 from the measured statistics of (i) the patient arrival process and (ii) the distribution of the LOS of patients by using a queuing network model for the patient flow.

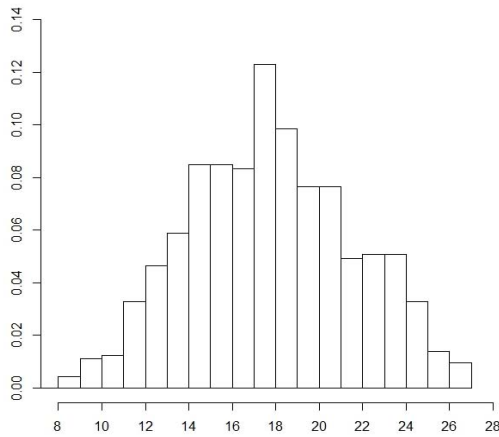


Figure 2. Probability distribution for the number of obstetric patients who stayed in Ward 300 in 2010–2011.

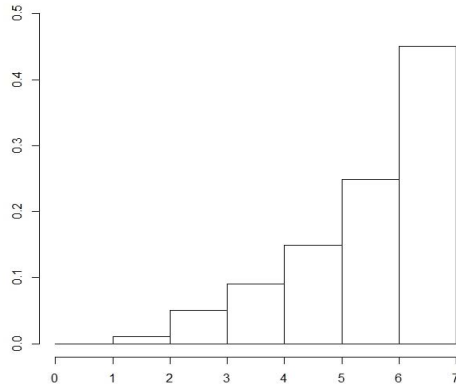


Figure 3. Probability distribution for the number of obstetric patients who stayed in Ward 30M in 2010–2011.

III. MEAN LENGTH-OF-STAY CALCULATED FROM THE NUMBER OF HOSPITALIZED PATIENTS BY LITTLE'S LAW

In this section, we show an application of a simple, but far-reaching, mathematical formula called *Little's law* [13] to calculate the mean length of stay (LOS) of an arbitrary patient from the mean number of patients being hospitalized on each day during a given observation period and the number of new patients admitted per day in the same period.

Little's law is concerned about a generic service system in which customers arrive, get served and depart as shown in Figure 4 [11, p.30]. It is simply written as

$$L = \lambda W,$$

where L denotes the time average of the number of customers present in the system, λ denotes the average number of customers who arrive in the system per unit time (the arrival rate), and W denotes the average time that an arbitrary customer spends in the system. The only condition for this law is that the system is stable in that the number of customers present in the system does not grow indefinitely.

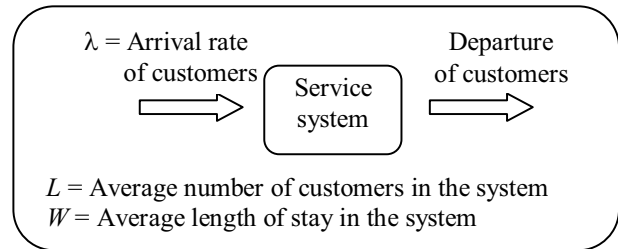


Figure 4. Generic service system in equilibrium.

Several articles have been published on the application of Little's law to the patient flows in hospitals, e.g. [8]. If we apply Little's law to all the patients of UTH in year 2011 in which the arrival rate is $\lambda = 37$ patients/day and the mean number of patients is $L = 653$ (see Table II), we get the mean LOS as

$$W = L / \lambda = 653/37 = 17.6 \text{ days},$$

which overestimates the reported value $W = 16.4$ days. This difference attributes to the following official manipulation rule in the reported values: (i) the LOS longer than 90 days is always reported as 90 days, and (ii) patients in psychiatric units are not counted in the report.

However, if we apply Little's law to the obstetric patients in Ward 300 during a period of 2010–2011 in which the arrival rate is $\lambda = 2.685$ patients/day and the mean number of patients is $L = 17.28$ (see Table III), we get the mean LOS as

$$W = L / \lambda = 17.28/2.685 = 6.436 \text{ days},$$

which virtually agrees with the reported value $W = 6.43$ days as it should be.

IV. EXTRACTION AND VISUALIZATION OF DOMINANT OBSTETRIC PATIENT FLOW

We now proceed to present the method to identify the dominant routes in the patient flow. Our method is supplemented by the visualization techniques which display the volume of patient flow quantitatively in graphics.

A. Visualization of the Obstetric Patient Flow

Our first step is to find a group of major wards in which patients stay during a period of their hospitalization from the available log data. Figure 5 presents a graph showing a bird-eye view for the transitions of obstetric patients between relevant wards, where the nodes represent those wards in the hospital that held at least one patient during the two years (the label “in” means the admission, and the label “out” means the discharge, while the numbers identify the wards). We would like to make the width of each link indicate the frequency of transitions, i.e. the number of patients who moved on that link in each direction. The direction of patient movement is reflected in the *counterclockwise* curvature of the link.

However, a difficulty arises if we try to draw links with the width exactly proportional to the frequency, because there is usually order of magnitude difference in the number of patient transitions between primary wards of the clinical group under consideration and the number of transitions to and from other wards. In the case of obstetric patients, since the frequency of going to and from Ward 300 is by far higher than those on other links, the latter must be drawn with the narrowest width (of a single pixel). While the frequency of going to and from Ward 30M is much less than those on the links going to and from Ward 300, it is yet by far larger than the frequencies on other links. Thus we have decided to draw the links with width proportional to the *logarithm* of the frequency of transitions. Then we can visually identify a group of major nodes (wards) and links for a given set of patient movements.

In Figure 5, we observe that most obstetric patients are admitted either to Ward 300 or to Ward 30M, and then possibly move back and forth between the two wards before the final discharge. All other wards such as numbered 400, 901 and so on may be neglected in our treatment of obstetric patient flow without much influence on the result of analysis. Thus we identify five major routes of obstetric patients as shown in Figures 6 and 7.

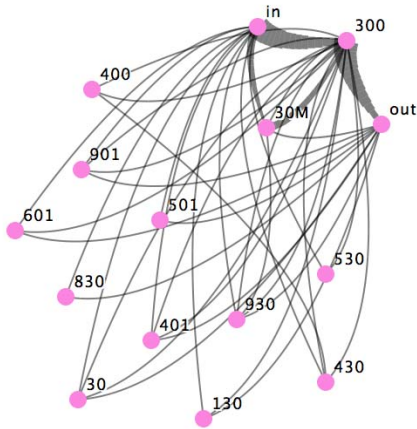


Figure 5. Visual finding of the major transition links of obstetric patients.

B. Selection of Dominating Routes for Obstetric Patients

The visual method of finding the major transition links may bear ambiguity if there are many relevant wards and links. Therefore we confirm the dominating routes in the patient flow by means of numerical evaluation. To do so, we calculate the *load* (patient-days) for each route by summing the *load* of patients over all the wards on that route, while the latter is given by the sum of length-of-stay (LOS) of all the patients who stayed in that ward on the route. Figure 6 shows the load of obstetric patients on routes 1 through 5 and on other routes. Here we exclude those patients who are hospitalized on April 1, 2010 and on March 31, 2012 in order to consider their length of stay exactly. Since the set of patients on routes 1 through 5 accounts for 97.8% of all obstetric patients, we will consider only them in our analysis of obstetric patient flow. We note in passing that the load in ward 300 on all the routes shown in Figure 6 sums up to

$$10,317 + 60 + 1,254 + 303 + 270 = 12,204,$$

which is less than 12,630 shown in Table III. The reason is the exclusion of those patients who were hospitalized on the first and last days of the observation period.

Figure 6. Dominating routes of obstetric patients in terms of patient load (patient-days).

Routes	Admission	Discharge	Load(patient-days)	
			300	30M
Route 1	→ 300	→	10317 (84.5%)	0
Route 2	→ 30M	→	0	621 (18.8%)
Route 3	→ 300 → 30M	→	60 (0.5%)	354 (10.7%)
Route 4	→ 30M → 300	→	1254 (10.3%)	1864 (56.5%)
Route 5	→ 300 → 30M → 300	→	303 (2.5%)	422 (12.8%)
Others			270 (2.2%)	37 (1.1%)

After selecting routes 1 through 5 shown in Figure 6, we show in Figure 7 the five routes and the number of patients who take each route. In total, we consider

$$1,582 + 81 + 25 + 176 + 30 = 1,894 \text{ patients,}$$

which account for 96.8% of a total of the 1,956 obstetric patients.

Routes	Admission	Discharge	Numbers
Route 1	→ 300	→	1582 (83.5%)
Route 2	→ 30M	→	81 (4.3%)
Route 3	→ 300 → 30M	→	25 (1.3%)
Route 4	→ 30M → 300	→	176 (9.3%)
Route 5	→ 300 → 30M → 300	→	30 (1.6%)

Figure 7. Dominating routes of obstetric patients in terms of numbers of patients.

C. Visualization of the Load of Dominant Routes

Let us present some visualization of the load of dominating routes showing the number of patients and LOS in two-dimensional graphics. Such visualization would be

useful for hospital management to get a rough quantitative view of patient flow.

Figure 8 shows the load of dominating routes of obstetric patients in a fashion similar to the *icicle plot* in [14]. Each rectangle represents the patient-days in a ward. A horizontal sequence of rectangles corresponds to the wards on a route of patients. The height of each rectangle expresses the number of patients in the ward, and its width stands for the mean LOS in days.

Figure 9 deals with the same data as in Figure 8 in the *sunburst diagram* [15], which is a radial style of the icicle plot. It is a variation of doughnut chart and is convenient to display the ratios of the number of patients by angles and the mean LOS in days in the wards on different routes by the radial distance.

In Figure 10, instead of the mean LOS in the width of each rectangle in Figure 8, the histogram of LOS is shown in the horizontal direction. In order to show a histogram in a limited rectangular area, we have exploited the idea of *horizon graph* reported in [16].

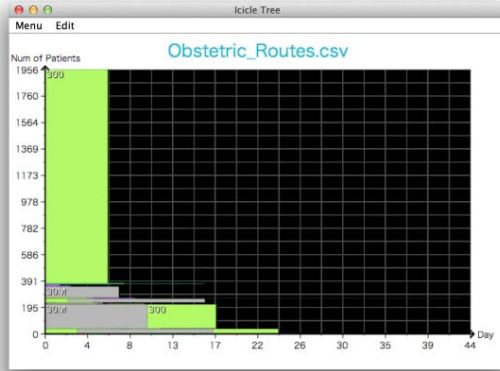


Figure 8. Visualization of the mean number of patients and mean LOS in the wards on each route of obstetric patients.

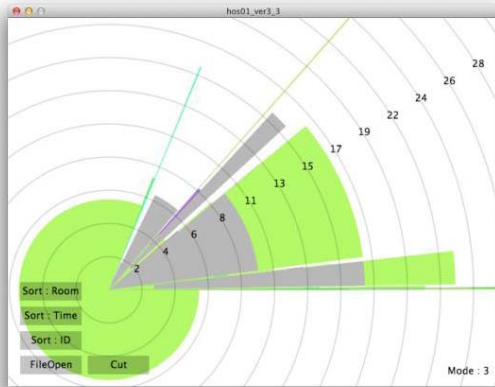


Figure 9. Circular visualization of the mean number of patients and mean LOS in the wards on each route of obstetric patients.

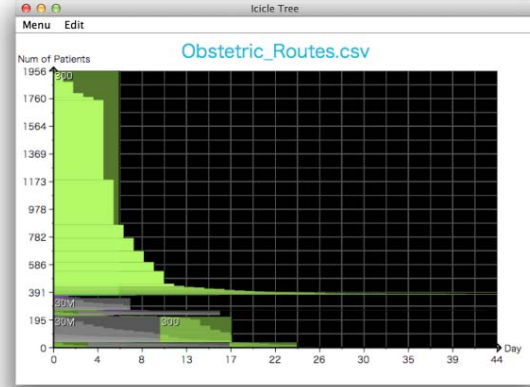


Figure 10. Visualization of the probability distribution for the number of patients and the mean LOS in the wards on each route of obstetric patients.

V. RELEVANT QUEUING MODELS

It is very difficult to construct a mathematically precise queuing network model to represent a patient flow shown in Figure 7. However, a sophisticated model is not needed for our purpose of practical estimation. The following queuing models are relatively simple and robust, and the formulas for the probability distribution of the number of customers present in the system are available explicitly.

A. $M/G/\infty$

A model denoted by $M/G/\infty$ in *Kendall's notation* of queuing theory is simply a system with sufficiently many servers to which customers arrive in a Poisson process and spends there a random amount of service time which is generally distributed probabilistically [11, p.145]. There is no contention for servers among customers. If λ denotes the arrival rate and b denotes the mean service time, the number N of customers present in the system at an arbitrary time has the Poisson distribution with mean $\rho = \lambda b$:

$$P\{N = k\} = \frac{\rho^k}{k!} e^{-\rho} \quad k \geq 0$$

Note that this distribution depends only on the mean of the service time.

A useful property in modeling is that the output of an $M/G/\infty$ system is a Poisson process [17]. Another nice property about the Poisson process is that the superposition of independent Poisson processes forms another Poisson process with added rates.

B. $M/M/m$

A model denoted by $M/M/m$ in Kendall's notation is a queuing system with m servers to which customers arrive in a Poisson process at rate λ with service time exponentially distributed with mean $1/\mu$ [11, p.142]. Then the probability

distribution for the number N of customers present in the system at an arbitrary time is given by

$$P\{N = k\} = \begin{cases} P_0 \frac{\rho^k}{k!} & 0 \leq k \leq m \\ P_0 \frac{\rho^m}{m!} \left(\frac{\rho}{m}\right)^k & k \geq m+1 \end{cases}$$

where $\rho = \lambda/\mu$ and

$$\frac{1}{P_0} = \sum_{i=1}^{m-1} \frac{\rho^i}{i!} + \frac{\rho^m}{(m-1)!(m-\rho)}$$

The output of an M/M/m system is also a Poisson process.

C. $M^X/G/\infty$

A model denoted by $M^X/G/\infty$ in extended Kendall's notation is a system with sufficiently many servers to which customers arrive in a Poisson process possibly in a batch of random size and each customer spends there a random amount of service time which is generally distributed probabilistically. Keilson and Seidmann [18] give the probability distribution for the number N of customers present in the system at an arbitrary time in terms of the generating function:

$$P(z) = \exp \left[-\lambda \int_0^\infty \{1 - G[z + (1-z)B(y)]\} dy \right]$$

where

$$P(z) := \sum_{k=0}^{\infty} P\{N = k\} z^k,$$

$$G(z) := \sum_{j=1}^{\infty} P\{\text{batch size} = j\} z^j$$

and $B(y)$ is the distribution function for the service time. The output of an $M^X/G/\infty$ system is *not* a Poisson process.

A special case of single arrivals $G(z) = z$ yields $P(z) = \exp[(z-1)\rho]$, where $\rho = \lambda b$, and it leads to the result for the M/G/ ∞ system. However, it is not usually easy to obtain the explicit probability distribution from its generating function. Therefore we do not use $M^X/G/\infty$ system in our modeling.

VI. MODELING AND ANALYSIS OF THE OBSTETRIC PATIENT FLOW

We are now in a position to present a queuing network model for the selected dominant routes of obstetric patient flow by using theoretical M/G/ ∞ and M/M/m systems. Our model is approximate from queuing-theoretic viewpoint in the following sense:

- The stochastic process for the patient flow under study is essentially a discrete-time system in which the patient arrivals and the length of stay are counted on daily basis, while the M/G/ ∞ and M/M/m systems work in the continuous-time framework. The fitting of Poisson distribution for the number of admissions per day in Figures 12 and 14 shown below is only a

necessary condition for the Poisson arrival process assumed in the M/G/ ∞ and M/M/m systems.

- The residence of patients in each ward is treated as independent while the admission of route 5 patients twice in Ward 300 clearly violates this assumption.

Nevertheless, we propose a queuing network model of M/G/ ∞ and M/M/m systems because of its simplicity in modeling and computation in addition to our belief that the mathematical rigor should not be overly requested for practical purpose. The justification of our method is partly provided from the good agreement of the calculated results with the observation as demonstrated below.

A. Special Attention to Routes 3 and 5

In preparation for building a queuing network model of the obstetric patient flow, we paid special attention to routes 3 and 5 on which 55 patients were first admitted to Ward 300 and then transferred to Ward 30M. There are 48 days on which admissions occurred to Ward 300, out of which Ward 30M was full on 41 days. Therefore we conjectured that Ward 300 is used by patients with high-risk delivery as the “waiting room” for Ward 30M. This conjecture was later confirmed to be true by doctors in the Obstetric Section.

B. Queuing Network Model of the Obstetric Patient Flow

Figure 11 shows a queuing network model we propose for the obstetric patient flow in the University of Tsukuba Hospital. A major portion of the patient flow consists of five routes: Patients on route 1 are supposed to those who have normal childbirth. They are just admitted in Ward 300 and simply leave the hospital in about 6 days. Patients on routes 2 through 5 are supposed to those who have high-risk delivery. They are to be treated in MFICU, i.e. Ward 30M. Upon arrival they are admitted to Ward 30M if beds are available there. After giving birth, they either leave the hospital (route 2) or move to Ward 300 possibly for after-birth treatment (route 4). If there are no beds available in Ward 30M when they arrive, they are accommodated in Ward 300 until beds become available in Ward 30M. Then they are transferred to Ward 30M. After giving birth, they either leave the hospital (route 3) or move back to Ward 300 possibly for after-birth treatment (route 5).

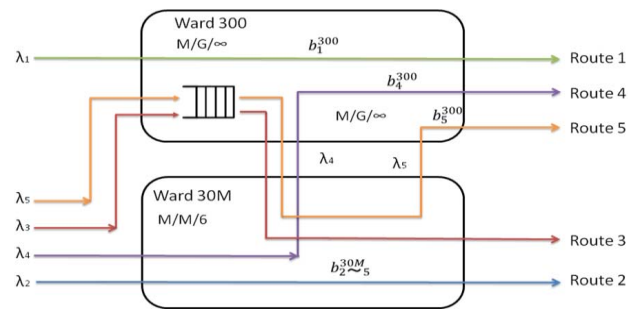


Figure 11. Queuing network model of the obstetric patient flow in the University of Tsukuba Hospital.

C. Arrivals and Length of Stay in the Two Wards

Figure 12 shows the probability distribution for number of route 1 patients newly admitted to Ward 300 per day. The mean is calculated as 2.164 patients per day. The Poisson distribution with mean 2.164 is also plotted which seems to fit with the observed distribution fairly well.

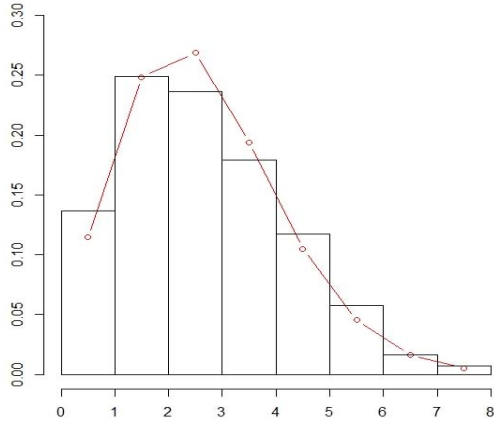


Figure 12. Probability distribution for number of route 1 patients newly admitted to Ward 300 per day.

Figure 13 shows the probability distribution for the LOS (in days) of route 1 patients in Ward 300. The mean is calculated as 6.521 days. We do not try to find the theoretical distribution that may fit to this distribution, because we do not need the distributional form in our modeling of route 1 patients by an M/G/∞ system.

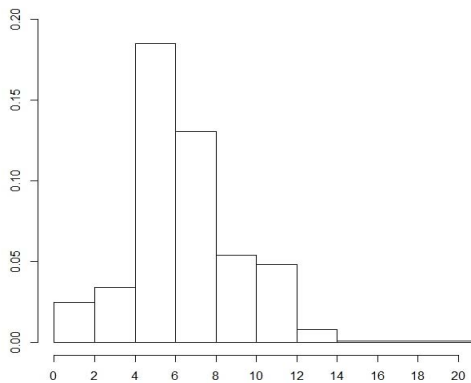


Figure 13. Probability distribution for the LOS (in days) of route 1 patients in Ward 300.

Figure 14 shows the probability distribution for number of route 2–5 patients newly admitted to Wards 300 and 30M per day. The mean is calculated as 0.427 patients per day. The Poisson distribution with mean 0.427 is also plotted which again seems to model the observed distribution fairly well.

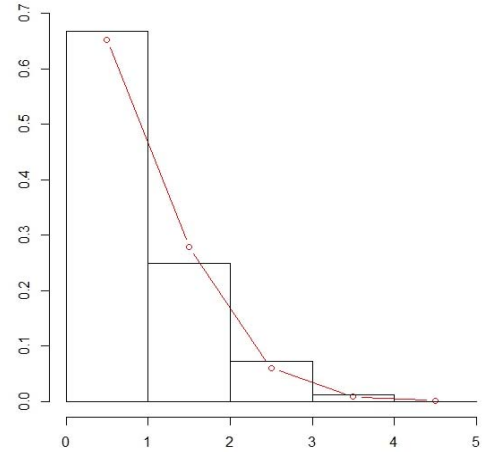


Figure 14. Probability distribution for the number of route 2–5 patients newly admitted to Wards 300 and 30M per day.

Figure 15 shows the probability distribution for the LOS (in days) of route 2–5 patients in Ward 30M. The mean is calculated as 10.452 days. The probability density function for the exponential distribution with mean 10.452 is also plotted in the same coordinates, which looks to agree with the observed distribution very well. This agreement is essential for our usage of an M/M/m queuing model for the patients whose high-risk delivery is treated in Ward 30M.

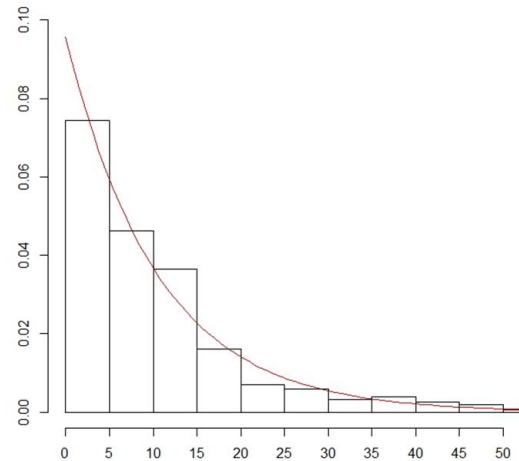


Figure 15. Probability distribution for the LOS (in days) of route 2–5 patients in Ward 30M.

D. Number of Patients in Ward 300

Patients staying in Ward 300 consist of

- Patients going through Ward 300 only (route 1)
- Patients transferred from Ward 30M (route 4)
- Patients transferred from Ward 30M (route 5)

- Patients waiting for Ward 30M (routes 3 and 5)

Therefore, we first obtain the probability distribution for the number of patients in Ward 300 for the four types. We then obtain the probability distribution for the total number of patients in Ward 300 by convoluting the four distributions.

In Figure 2 we observe that the probability that all the beds are occupied is only about 1% for Ward 300. Therefore, we may assume that there are a sufficient number of beds in Ward 300 which can accept all patients at any time. Thus we will use an M/G/∞ system to model the flow of patients on routes 1, 4, and 5 with respectively observed arrival rates and mean LOS. For patients on routes 3 and 5, we assume that the waiting room in an M/M/m queue with $m = 6$ virtually exists in Ward 300, while the service facility of the M/M/m queue exists in Ward 30M.

Now the probability distribution for the number of route 1 patients in Ward 300 is given by

$$P_k^{(1)} = \frac{(\lambda_1 b_1^{300})^k}{k!} e^{-\lambda_1 b_1^{300}} \quad k \geq 0$$

$$\lambda_1 = 2.164 \quad ; \quad b_1^{300} = 6.521$$

This theoretical distribution is plotted in Figure 16 along with the observed distribution, which seems to agree with each other very well.

Similarly, the probability distribution for the number of route 4 patients in Ward 300 is given by

$$P_k^{(4)} = \frac{(\lambda_4 b_4^{300})^k}{k!} e^{-\lambda_4 b_4^{300}} \quad k \geq 0$$

$$\lambda_4 = 0.241 \quad ; \quad b_4^{300} = 7.125$$

This distribution is plotted in Figure 17 along with the observed distribution, which also seems to agree with each other very well.

The probability distribution for the number of route 5 patients in Ward 300 is given by

$$P_k^{(5)} = \frac{(\lambda_5 b_5^{300})^k}{k!} e^{-\lambda_5 b_5^{300}} \quad k \geq 0$$

$$\lambda_5 = 0.041 \quad ; \quad b_5^{300} = 6.700$$

This distribution is plotted in Figure 18 along with the observed distribution, which again seems to agree with each other very well.

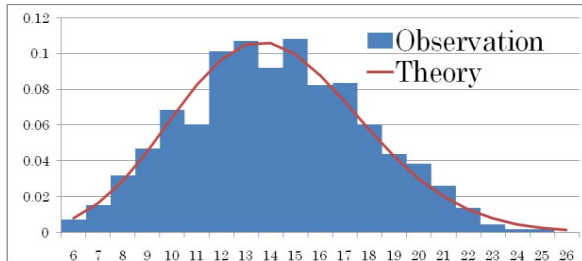


Figure 16. Probability distribution for the number of route 1 patients in Ward 300.

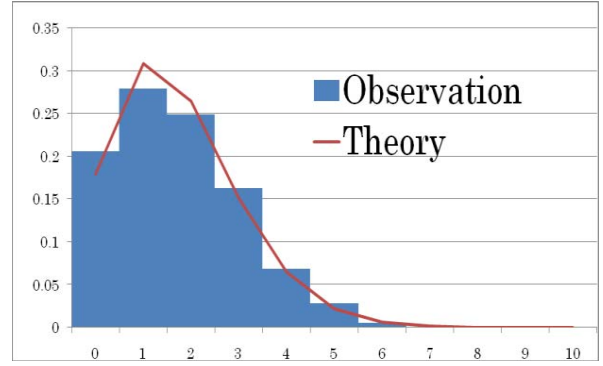


Figure 17. Probability distribution for the number of route 4 patients in Ward 300.

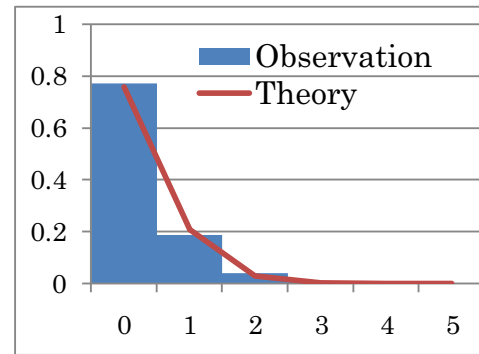


Figure 18. Probability distribution for the number of route 5 patients in Ward 300.

The patients on routes 3 and 5 in Ward 300 has the distribution for the number of customers *in the waiting room* in an M/M/m queuing system with $m = 6$ servers, which is given by

$$P\{L^{(2-5)} = k\} = P\{N = k + m\}$$

$$= \begin{cases} 1 - \frac{\rho}{m} C(m, \rho) & k = 0 \\ C(m, \rho) \left(1 - \frac{\rho}{m}\right) \left(\frac{\rho}{m}\right)^k & k \geq 1 \end{cases}$$

where $\rho = \lambda_{2-5} \times b_{2-5}^{30M}$ and

$$C(m, \rho) := \frac{\frac{\rho^m}{m!}}{\left(1 - \frac{\rho}{m}\right) \sum_{i=1}^{m-1} \frac{\rho^i}{i!} + \frac{\rho^m}{m!}}$$

which is the famous Erlang's C formula for the probability that a customer is forced to wait upon arrival. We use

$$\lambda_{2-5} = 0.427 \quad ; \quad b_{2-5}^{30M} = 10.452$$

so that $\rho = 4.463$. This distribution is plotted in Figure 19 along with the observed distribution, where the agreement is acceptable.

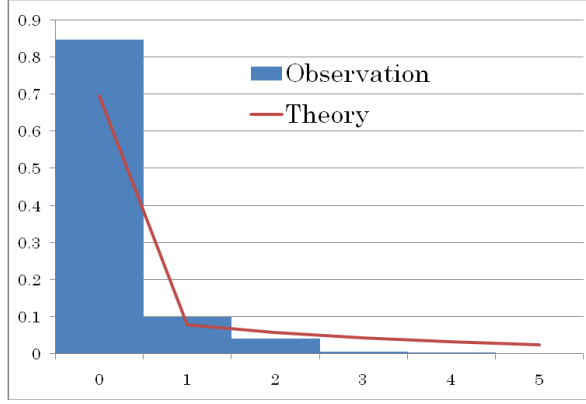


Figure 19. Probability distribution for the number of route 3 and 5 patients in Ward 300.

Finally, the probability distribution for the total number of patients in Ward 300 is obtained by the convolution of the above-mentioned four distributions. Since the first three are independent Poisson distributions, their convolution is given by another Poisson distribution with mean

$$\lambda_1 b_1^{300} + \lambda_4 b_4^{300} + \lambda_5 b_5^{300} = 16.103$$

Therefore, we only have to calculate the convolution of this distribution with the fourth distribution $P\{L^{(2-5)} = k\}$. However, we must note that the admission of route 5 patients twice in Ward 300 violates the independence assumption required for the convolution of the fourth distribution. Nevertheless, we carry out this convolution as if these distributions were independent. The result is shown in Figure 20, where we see fairly good agreement between the theory and observation except near the capacity 26 of Ward 300. The reason for this discrepancy is that the Poisson distribution has a support until ∞ while the observed distribution must be truncated at the capacity of Ward 300.

E. Number of Patients in Ward 30M

The probability distribution for the number of patients in Ward 30M is obtained as that for the number of customers in the service facility in an M/M/m queuing system with $m = 6$. It is given by

$$P\{S^{(2-5)} = k\} = \begin{cases} P\{N = k\} & 0 \leq k \leq m-1 \\ P\{N \geq m\} & k = m \end{cases} = \begin{cases} P_0 \frac{\rho^k}{k!} & 0 \leq k \leq m-1 \\ C(m, \rho) & k = m \end{cases}$$

where P_0 and $C(m, \rho)$ are given above with $\rho = 4.463$. This is shown in Figure 21. While the theoretical result still captures major characteristics of the observed values, there remains a challenge for better agreement.

This completes our modeling and analysis of the obstetric patient flow.

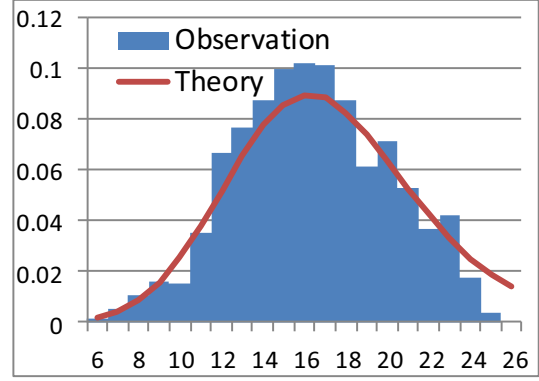


Figure 20. Theoretical versus observed values for the probability distribution of the number of patients in Ward 300.

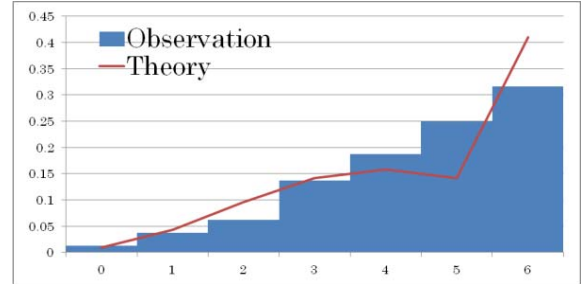


Figure 21. Theoretical versus observed values for the probability distribution of the number of patients in Ward 30M.

VII. CONCLUDING REMARK AND FUTURE RESEARCH

In this paper, we have proposed a network model of M/G/ ∞ and M/M/m queues for the obstetric patient flow in the University of Tsukuba Hospital (UTH). In spite of very rough approximation, we have “explained” the patient flow with acceptable accuracy. The reason for this success may be (i) the flow of obstetric patients is rather isolated from the flow of other patients as two wards are almost dedicated to the obstetric unit, (ii) the process in which obstetric patients tend to arrive at random times may well be modeled as a Poisson process, and (iii) the length of stay in the MFICU (Ward 30M) has an exponential distribution which makes it possible to use an M/M/m queuing model.

We plan to extend our study of patient flows to those in other clinical units of the UTH. It is also interesting to see if the modeling technique proposed in this paper can be applied to the obstetric patient flow in other hospitals.

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Encrypted Patient ID	Gender	Time	Order	Clinical Group	From Ward	From Room	To Ward	To Room
000958b2ca6309*****bf35643d8	Male	2011/5/9 10:30	Admission	Cardiovascular Surgery	Admission		001	B1012
000958b2ca6309*****bf35643d8	Male	2011/5/13 9:40	Transfer of Ward/Room	Cardiovascular Surgery	001	B1012	400	E0407
000958b2ca6309*****bf35643d8	Male	2011/5/13 13:54	Transfer of Room	Cardiovascular Surgery	400	E0407	400	E0403
000958b2ca6309*****bf35643d8	Male	2011/5/14 15:38	Transfer of Ward/Room	Cardiovascular Surgery	400	E0403	001	B1012
000958b2ca6309*****bf35643d8	Male	2011/5/27 11:47	Discharge	Cardiovascular Surgery	001	B1012	Discharge	
000b6ed887155*****2b614deb5	Female	2011/7/1 9:13	Admission	Orthopedics	Admission		501	B0506
000b6ed887155*****2b614deb5	Female	2011/7/28 9:30	Discharge	Orthopedics	501	B0506	Discharge	

Figure 1. Sample log data of patient flow provided by the Medical Information and Medical Records Department of the University of Tsukuba Hospital.