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CSCI 338
HW #7
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1. 5.1, p.239

Suppose that EQ_{CFG} is decidable by Turing machine M_1. Construct a decider M_2 for $ALL_{CFG} = \{\langle G\rangle \mid G$ is a CFG and $L(G) = \Sigma^*\}$ as follows:

$M_2 =$ "On input $\langle G\rangle$:

1. Construct a CFG G_1 where $L(G_1) = \Sigma^*$
2. Run M_1 on input $\langle G, G_1\rangle$
3. If M_1 accepts, accept and if M_1 rejects, reject."

Therefore M_2 is a decider for ALL_{CFG} if M_1 exists. But it was proven that ALL_{CFG} is undecidable so M_1 cannot exist and we have a contradiction. Thus, EQ_{CFG} is undecidable.

2. 5.3, p.239

$$\left[\frac{ab}{abab}\right]\left[\frac{ab}{abab}\right]\left[\frac{aba}{b}\right]\left[\frac{b}{a}\right]\left[\frac{b}{a}\right]\left[\frac{aa}{a}\right]\left[\frac{aa}{a}\right]$$

3. 5.4, p.239

No. Define the languages $A = \{a^n b^n \mid n \geq 0\}$ and $B = \{a\}$, both over the alphabet $\Sigma = \{a, b\}$. Define the function $f: \Sigma^* \to \Sigma^*$ as

$$f(w) = \begin{cases} a \; if \; w \in A \\ b \; if \; w \notin A \end{cases}$$

A is a CFL, therefore it is Turing-decidable and *f* is a computable function. So if $w \in A$, the output is $f(w) = a$ and $f(w) \in B$. Thus, *A* is non-regular, *B* is regular, since it is finite, and $A \leq_m B$.

5.

1. False
2. True
3. True

6.

1. True
2. True
3. False