Bradley White
HW #4
CSCI 338
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1. 2.30, p.157

a. $L = \{0^n1^n0^n1^n \mid n \geq 0\}$, $s = 0^p1^p0^p1^p \in L$
Since the pumping lemma states $|vy| > 0$ and $|vxy| \leq p$, we can divide s into two cases:

Case 1: Either v or y contains more than one symbol (both 1s and 0s). Consider uv^2xy^2z, the resulting string would consist of extra segments of symbols which is not in L because string need to be of four equal segments.

Case 2: Both v and y contain one symbol. Since $|vxy| \leq p$ pumping s either up or down will result in unequal amounts of either symbol and is not in L.

Therefore, s cannot be pumped and L is not context-free.

2.

$P = \{a^k \mid k \text{ is a prime number}\}$, $s = a^n \in P$ where *n* is a prime number.
Since $|vy| > 0$, $v = a^r$ and $y = a^t$ then it follows that $r + t > 0$. If we pump up once we have $uv^{1+n}xy^{1+n}z$ and therefore $a^{n+rn+tn} = a^{n(1+r+t)}$. However, because $n(1 + r + t)$ is prime it should not factor thus P is not context-free.

3.

The Kleene star can be captured by having the same variable on the left-hand side and right-hand side of a rule within a CFG. Additionally, the rule needs an ε on the RHS to capture 0 repetitions of the CFG.
So, let G_1 be some arbitrary CFG, then to produce $G_2 \Rightarrow G_1^*$:
$G_2 \longrightarrow G_2G_1 \mid \varepsilon$ or $G_2 \longrightarrow G_1G_2 \mid \varepsilon$

4.

DCFLs are of interest to the computer science community because they are more efficient with time and space when compared to non-deterministic CFLs. Non-deterministic CFLs must make copies of the stack every time a nondeterministic step occurs.

5.

$$1aaabbb \mapsto 1aaSbb \mapsto 1aSb \mapsto 1S \mapsto R$$