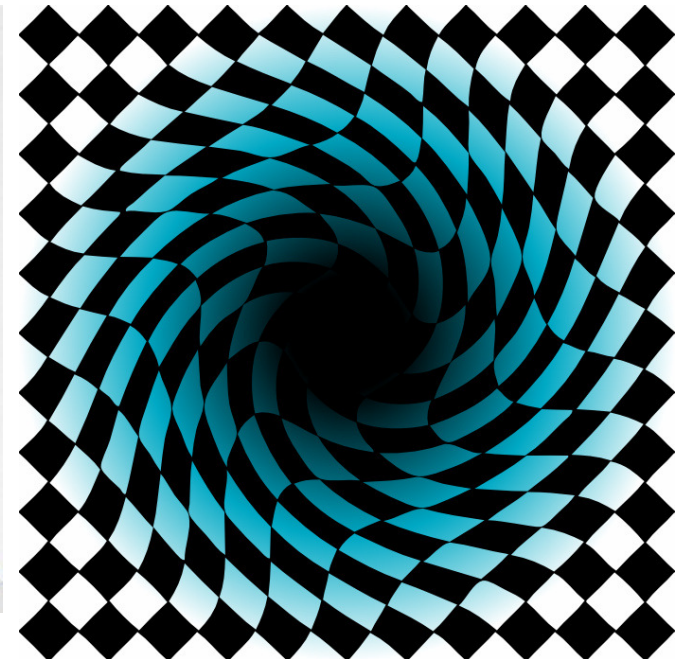


Dimension Reduction

Principal Components Analysis



Mechanics

Input: X_1, X_2, \dots, X_p

Output: PC_1, PC_2, \dots, PC_p (Ultimately we'll use a subset)

The i^{th} principal component is a weighted average:

$$PC_i = a_{i1} X_1 + a_{i2} X_2 + \dots + a_{ip} X_p$$

Weights chosen such that:

1. PCs are ordered by their variance (PC_1 has largest variance)
2. Pairs of PCs have correlation = 0
3. For each PC, sum of squared weights = 1

Example: Business School Programs

Univ	SAT	Top10	Accept	SFRatio	Expenses	GradRate
Brown	1310	89	22	13	22,704	94
CalTech	1415	100	25	6	63,575	81
CMU	1260	62	59	9	25,026	72
Columbia	1310	76	24	12	31,510	88
Cornell	1280	83	33	13	21,864	90
Dartmouth	1340	89	23	10	32,162	95
Duke	1315	90	30	12	31,585	95
Georgetown	1255	74	24	12	20,126	92
Harvard	1400	91	14	11	39,525	97
JohnsHopkins	1305	75	44	7	58,691	87
MIT	1380	94	30	10	34,870	91
Northwestern	1260	85	39	11	28,052	89
NotreDame	1255	81	42	13	15,122	94
PennState	1081	38	54	18	10,185	80
Princeton	1375	91	14	8	30,220	95
Purdue	1005	28	90	19	9,066	69
Stanford	1360	90	20	12	36,450	93
TexasA&M	1075	49	67	25	8,704	67
UCBerkeley	1240	95	40	17	15,140	78
UChicago	1290	75	50	13	38,380	87
UMichigan	1180	65	68	16	15,470	85
UPenn	1285	80	36	11	27,553	90
UVA	1225	77	44	14	13,349	92
UWisconsin	1085	40	69	15	11,857	71
Yale	1375	95	19	11	43,514	96

Use PCA to:

- 1) Reduce # columns
- 2) Identify relations between columns
- 3) Visualize universities in 2D

Source: US News & World Report, Sept 18 1995

PCA in XLMiner

Data Reduction & Exploration

Output specifies whether covariance or correlation matrix used (here – correlation matrix).

Principal Components

Variable	Components					
	1	2	3	4	5	6
SAT	0.45774868	0.03968045	0.18703876	0.13124055	0.02064597	-0.8580547
Top10	0.42714444	-0.19993152	0.49780852	0.37489522	0.48201644	0.39607504
Accept	-0.42430812	0.32089293	-0.15627895	0.06128667	0.80109364	-0.21693356
SFRatio	-0.39064837	-0.43256435	0.60608089	-0.50739086	0.07682328	-0.17204805
Expenses	0.3625232	0.63448638	0.20474122	-0.62340063	0.07254726	0.17376293
GradRate	0.37940401	-0.51555371	-0.53247261	-0.43863374	0.33810937	0.00353743
Variance	4.61208487	0.78681612	0.28656188	0.16378011	0.12430621	0.02645062
Variance%	76.86808014	13.11360168	4.77603149	2.72966838	2.07177019	0.44084364
Cum%	76.86808014	89.98168182	94.75771332	97.48738098	99.5591507	100
P-value	0	0.00000004	0.00073126	0.00263538	0.00140999	1

Reducing data dimension

Principal Components

	Components					
Variable	1	2	3	4	5	6
SAT	0.45774868	0.03968045	0.18703876	0.13124055	0.02064597	-0.8580547
Top10	0.42714444	-0.19993152	0.49780852	0.37489522	0.48201644	0.39607504
Accept	-0.42430812	0.32089293	-0.15627895	0.06128667	0.80109364	-0.21693356
SFRatio	-0.39064837	-0.43256435	0.60608089	-0.50739086	0.07682328	-0.17204805
Expenses	0.3625232	0.63448638	0.20474122	-0.62340063	0.07254726	0.17376293
GradRate	0.37940401	-0.51555371	-0.53247261	-0.43863374	0.33810937	0.00353743
Variance	4.61208487	0.78681612	0.28656188	0.16378011	0.12430621	0.02645062
Variance%	76.86808014	13.11360168	4.77603149	2.72966838	2.07177019	0.44084364
Cum%	76.86808014	89.98168182	94.75771332	97.48738098	99.5591507	100
P-value	0	0.00000004	0.00073126	0.00263538	0.00140999	1

PC_1 captures _____ % of the information

The first two PCs capture _____%

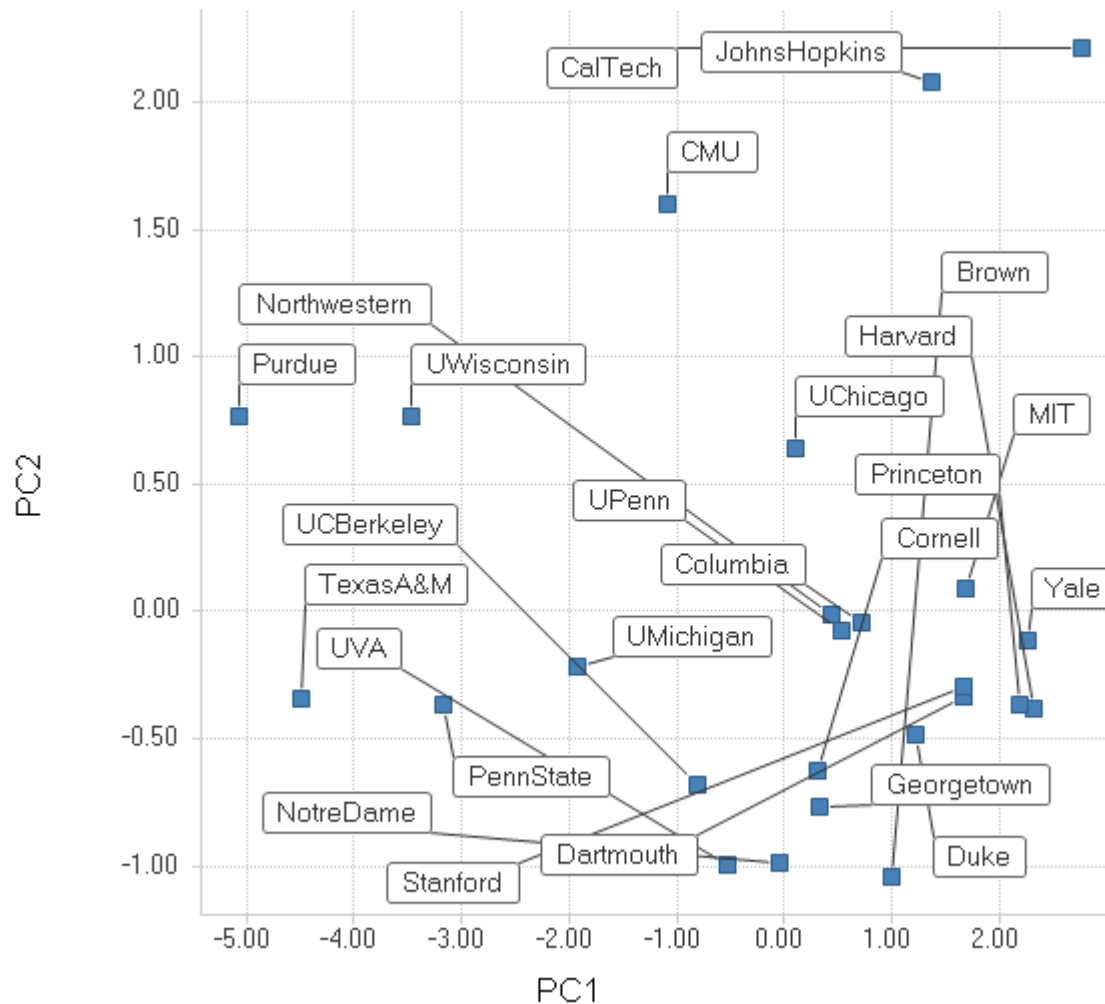
XLMiner: Computing scores

Scores given for each PC

Recall: PC1 & PC2 are uncorrelated ($r = 0$)

Row Id.	1	2
1	0.98947096	-1.04280615
2	2.76521754	2.21340251
3	-1.08998942	1.5982517
4	0.72675508	-0.04133511
5	0.30561018	-0.62240905
6	1.66241097	-0.33740574
7	1.2216301	-0.48106378
8	0.33190566	-0.76930493
9	2.32618284	-0.37872922
10	1.37492549	2.07669187
11	1.69122922	0.08645435
12	0.44174835	-0.01090807
13	-0.03942522	-0.98881435
14	-3.168396	-0.36701241
15	2.19108367	-0.36428159
16	-5.06847715	0.76415795
17	1.66530418	-0.29942313
18	-4.48564911	-0.3405683
19	-0.80598319	-0.68478525
20	0.09578693	0.63730472
21	-1.92351854	-0.22022633
22	0.53133249	-0.07798085
23	-0.52146798	-0.99661624
24	-3.47699881	0.76273364
25	2.25931191	-0.11532623

Score plot (score2 vs score1) using Spotfire



SVD (similar to PCA)

Data Matrix: $M = U \Sigma V'$

Using an example from the Wikipedia page:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \sqrt{0.2} & 0 & 0 & 0 & \sqrt{0.8} \\ 0 & 0 & 0 & 1 & 0 \\ -\sqrt{0.8} & 0 & 0 & 0 & \sqrt{0.2} \end{bmatrix}$$

Books rated by users

	User 1	User 2	User 3	User 4
Book 1	1	4		1
Book 2	2			5
Book 3		4	1	4
Book 4	1			
Book 5	3	5	5	1
Book 6		3	2	

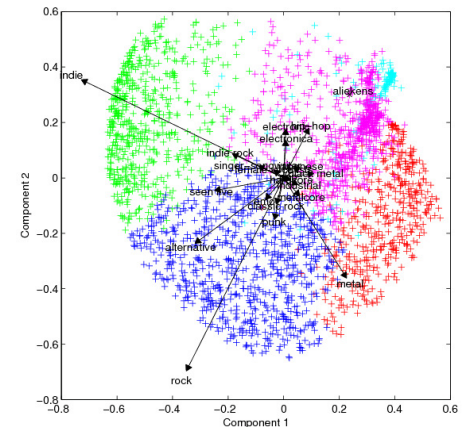
Original matrix:

$$\begin{pmatrix} 1 & 4 & 0 & 1 \\ 2 & 0 & 0 & 5 \\ 0 & 4 & 1 & 4 \\ 1 & 0 & 0 & 0 \\ 3 & 5 & 5 & 1 \\ 0 & 3 & 2 & 0 \end{pmatrix}$$

Rebuilt with $k=4$, the matrix

Approximation in rounded

$$\begin{pmatrix} 0 & 4 & 1 & 1 \\ 2 & 0 & 0 & 5 \\ 0 & 4 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 3 & 5 & 5 & 1 \\ 0 & 3 & 2 & 0 \end{pmatrix}$$



Need only
k columns of U
k values of Σ
k rows of V'

<http://journal.batard.info/post/2009/04/08/svd-fun-profit>

This week's online discussion



Discussion: Reducing Dimension of Mobile Survey Data

Prof. Galit Shmueli

Mar 21 at 3:29pm

16 20

Consider a sample from a dataset on mobile users in India. Take a look at the data set. Choose one of the points below and post a thread with the adequate point.

1. What approaches would you take to reduce the dimensionality of the data? Which method you would apply to which columns?
2. Suppose the goal is *describing* the relationship between the variables. Which potential factors (customer demographics, etc.) would you consider? Would you apply PCA? Which columns would you apply PCA to? Which columns would describe the relationship?
3. For *predicting* service switching (churn), if we have a large number of variables in the space, what information would we need to reduce the dimensionality? Would PCA reduce the number of questions that we need to ask?

Column Name	Description
serialnum	ID of respondent
StartDate	Survey start date/time
EndDate	Survey end date/time
SurveyDuration (Hrs.Min)	Survey Duration
Completed	Whether the survey was completed (1=yes)
Num Mobiles	Q1 Do you currently own one or more Mobiles? <input type="checkbox"/> No. I use landline only. (1) <input type="checkbox"/> One - with single SIM (2) <input type="checkbox"/> One handset with two SIMs (3) <input type="checkbox"/> Two handsets (4) <input type="checkbox"/> More than two handsets or more than two SIMs (5)
Mobile Type (Primary)	What type of mobile phone handset do you own? If you own more than one, please select the type of the primary handset. <input type="checkbox"/> Basic phone without internet capability (1) <input type="checkbox"/> Smartphone (non-touchscreen) (2) <input type="checkbox"/> Smartphone with touch-screen (3) <input type="checkbox"/> Tablet with phone features (4)
Service Provider (Primary)	Who is your current service provider for your primary mobile phone? <input type="checkbox"/> Airtel (1) <input type="checkbox"/> Reliance (2) <input type="checkbox"/> Idea (3) <input type="checkbox"/> Vodafone (4) <input type="checkbox"/> Tata DOCOMO (5) <input type="checkbox"/> Tata Indicom (6) <input type="checkbox"/> Aircel (7) <input type="checkbox"/> BSNL / MTNL (8) <input type="checkbox"/> Uninor (9) <input type="checkbox"/> Virgin Mobile (10) <input type="checkbox"/> Other (11)
Network duration	Q41 How long have you been on this network? <input type="checkbox"/> Less than 6 months (1) <input type="checkbox"/> 6 months to 1 year (2) <input type="checkbox"/> 1 to 2 years (3) <input type="checkbox"/> More than 2 years (4)
Mobile Service	Q42 Which mobile service type do you use for your primary mobile phone? <input type="checkbox"/> GSM (1) <input type="checkbox"/> CDMA (2) <input type="checkbox"/> Not sure/Don't know. (3)
Provider-Network Coverage	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Call quality	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Call charges	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Roaming charges	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Customer support	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Offers and promotions	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)
Provider-Easy bill payment, varied recharge options, etc.	Rate your service provider on this area (1=poor, 2=below average, 3=average, 4=above average, 5=excellent)

Use compressed data in modeling

For predicting?





For explaining?


Data Mining Contest (Crowdanalytics)


Each restaurant, each year


Lots of variables!

MODELING - Olive Garden Restaurant Comparison Analysis

 **15 days left**

 **116 Solvers**

 **USD \$5000**

 **Public**

Perform store comparison analysis to uncover drivers that explain why some Olive Garden ...

Visualization

Statistical Modeling

Consumer Insight

Store-Level

Restaurant

Food and Beverages

Structured

Moderate

Go to Competition

Uncover drivers that impacted performance of Olive Garden restaurants. Drivers identified should explain why some restaurants in the chain perform worse when compared to other restaurants in the chain.

<https://crowdanalytix.com/contests/modeling---olive-garden-restaurant-comparison-analysis>

Uncover drivers that impacted performance of Olive Garden restaurants. Drivers identified should explain why some restaurants in the chain perform worse when compared to other restaurants in the chain.

<https://crowdanalytix.com/contests/modeling---olive-garden-restaurant-comparison-analysis>

Restaurant	City	State_cod	Region	Zip_Code	County	Years	Depender	Depender	Average	Performance	Performance	Performance	Performance	Performance	Performance	Performance	Performance	Performance	Performance	Percentage	Percentage	Percentage	Percentage	Percentage	Percentage	Percentage	
1682	Charlotte	NC	south	28216	Mecklenb	2009	-0.29304	5	5										5	21.87649	25.7739	19.50675	11.23506	16.16007	5.447731	17	
1682	Charlotte	NC	south	28216	Mecklenb	2010	-0.29304	2.333333	3.727273									3.727273	23.04425	24.85875	19.93287	10.58922	16.35827	5.216655	19		
1682	Charlotte	NC	south	28216	Mecklenb	2011	-0.29304	3.25	4.286111									4.066667	22.28177	25.23038	17.95795	11.03463	17.89695	5.598318	19		
1682	Charlotte	NC	south	28216	Mecklenb	2012	-0.29304	3.307692	3.666667									3.666667	21.64029	24.01452	18.01485	11.66289	17.98998	6.677471	20		
1682	Charlotte	NC	south	28216	Mecklenb	2013	-0.29304	3.047619	3.594017									3.166667	21.84293	23.74276	17.24045	11.56268	18.85842	6.752765	2		
1668	Christiana	DE	south	19713	New Castl	2009	0.706364	1	2.979167										19.42077	20.7246	18.57007	14.22739	22.24607	4.811103	27		
1668	Christiana	DE	south	19713	New Castl	2010	0.706364	1	2.979167											19.77732	21.56876	18.34017	13.38821	22.1247	4.800841	28	
1668	Christiana	DE	south	19713	New Castl	2011	0.706364	1	2.979167											17.50994	21.99685	17.94023	13.80547	22.92619	5.821315	27	
1668	Christiana	DE	south	19713	New Castl	2012	0.706364	1	2.979167											17.84492	21.03306	19.83049	13.72193	22.23071	5.338882	2	
1668	Christiana	DE	south	19713	New Castl	2013	0.706364	1	2.979167											16.88951	21.66919	19.51556	13.51097	22.57078	5.843988	27	
1668	Christiana	DE	south	19713	New Castl	2014	0.706364	1	2.979167											23.06847	26.28845	18.46928	12.05746	16.89906	3.217271	28	
1668	Christiana	DE	south	19713	New Castl	2015	0.706364	1	2.979167											25.5064	23.97597	18.25136	12.31818	16.74639	3.201708	28	
1668	Christiana	DE	south	19713	New Castl	2016	0.706364	1	2.979167											25.31734	21.41702	18.60717	12.93667	18.51165	3.210151	29	
1668	Christiana	DE	south	19713	New Castl	2017	0.706364	1	2.979167											25.00739	21.91897	20.37745	11.73751	17.66702	3.291663	30	
1668	Christiana	DE	south	19713	New Castl	2018	0.706364	1	2.979167											26.12985	20.03032	20.02543	12.27324	18.27051	3.270658	30	
1668	Christiana	DE	south	19713	New Castl	2019	0.706364	1	2.979167											12.42528	17.32529	15.45612	13.78432	29.14892	11.86006	27	
1668	Christiana	DE	south	19713	New Castl	2020	0.706364	1	2.979167											15.24754	17.062	15.30982	12.30201	28.31599	11.76265	29	
1668	Christiana	DE	south	19713	New Castl	2021	0.706364	1	2.979167											14.11775	16.65786	15.36285	12.77284	29.41293	11.67577	28	
1668	Christiana	DE	south	19713	New Castl	2022	0.706364	1	2.979167											15.31029	18.40781	17.14873	13.37036	26.19942	9.563396		
1668	Christiana	DE	south	19713	New Castl	2023	0.706364	1	2.979167											4	15.72844	27.29111	16.72754	12.93476	25.02771	2.290437	18
1668	Christiana	DE	south	19713	New Castl	2024	0.706364	1	2.979167											14.75597	24.91524	17.62201	13.85713	23.25948	5.59016	20	
1668	Christiana	DE	south	19713	New Castl	2025	0.706364	1	2.979167											17.61892	14.34694	13.12235	27.82902	5.411197	21		
1119	Irving	TX	south	75062	Dallas	2012	-0.22884	3.166667	3.368421									3.736842	14.74041	19.85927	19.54572	14.4378	23.04944	8.367362	21		
1119	Irving	TX	south	75062	Dallas	2013	-0.22884	2.416667	3.4									3.8	15.68954	14.65118	21.53681	14.38643	24.5026	9.233442	22		
1122	Jensen BEFL	south	34937	Imartin	2013	-0.83	5	3.362934											22.34906	20.23183	18.83002	11.46339	18.96733	7.537776	32		
1091	Merritt Isl FL	south	32952	Brevard	2009	-0.38619	5	3.732292											25.19361	29.38445	19.19288	10.94103	12.75319	2.53484	38		
1091	Merritt Isl FL	south	32952	Brevard	2011	-0.38619	3.166667	2.5											24.47081	28.69044	18.94162	11.04219	14.32232	2.532612	40		
1091	Merritt Isl FL	south	32952	Brevard	2012	-0.38619	2.55	4.071429											25.94752	27.41066	19.59704	10.99494	14.58627	2.565656	40		