1) $\max Z = 5x_1 + 6x_2 + 4x_3$

$$\begin{aligned} -x_1 + 3x_2 + 4x_3 &\le 18 \\ 2x_1 + 2x_2 + x_3 &\le 4 \\ 3x_1 + 2x_3 &\le 8 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

slack variables →

$$\begin{aligned} Z - 5x_1 - 6x_2 - 4x_3 &= 0 \quad (0)\\ -x_1 + 3x_2 + 4x_3 + x_4 &= 18 \quad (1)\\ 2x_1 + 2x_2 + x_3 + x_5 &= 4 \quad (2)\\ 3x_1 + 2x_3 + x_6 &= 8 \quad (3)\\ x_1, x_2, x_3, x_4, x_5, x_6 &\ge 0 \end{aligned}$$

Initial tableau

Basic Variable	Eqn	Z	x_1	x_2	x_3	x_4	x_5	x_6	b_i	
Z	(0)	1	−5	−6	−4	0	0	0	0	
x_4	(1)	0	−1	3	4	1	0	0	18	→ 6
x_5	(2)	0	2	2	1	0	1	0	4	→ 2
x_6	(3)	0	3	0	2	0	0	1	8	

The current BFS is $(0,0,0,18,4,8)$ with $Z=0$

Optimality: NO. This BFS is not optimal because in equation (0) there are negative coefficients

x_2 is the entering basic variable. x_5 is the leaving variable

$(2') = \frac{(2)}{2}$:

x_1	x_2	x_3	x_4	x_5	x_6	b_i
1	1	1/2	0	1/2	0	2

$(3') = (3)$

$(0') = (0) + 6\cdot(2')$

	−5	−6	−4	0	0	0	0
+	+6	6	3	0	3	0	12
(0')	1	0	−1	0	3	0	12

$(1') = (1) - 3(2')$

	−1	3	4	1	0	0	18
+	−3	−3	$-\frac{3}{2}$	0	$-\frac{3}{2}$	0	−6
(1')	−4	0	$\frac{5}{2}$	1	$-\frac{3}{2}$	0	12

Iteration 1

Basic V.	Eqn	Z	x_1	x_2	x_3	x_4	x_5	x_6	b_i	
Z	(0)	1	1	0	−1	0	3	0	12	
x_4	(1)	0	−4	0	5/2	1	−3/2	0	12	→ 24/5
x_2	(2)	0	1	1	1/2	0	1/2	0	2	→ 4
x_6	(3)	0	3	0	2	0	0	1	8	→ 4

The current BFS is $(0,2,0,12,0,8)$ with $Z = 12$

I apply the Bland's rule to choose x_2 as leaving variable

$(2') = 2(2)$

2	2	1	0	1	0	4

$(0') = (0) + (2')$

1	0	−1	0	3	0	12	+
2	2	1	0	1	0	4	
3	2	0	0	4	0	16	←

$(1') = (1) - \frac{5}{2}(2')$

−4	0	5/2	1	−3/2	0	12
−5	−5	−5/2	0	−5/2	0	−10
−9	−5	0	1	−4	0	2

$(3') = (3) - 2(2')$

3	0	2	0	0	1	8
−4	−4	−2	0	−2	0	−8
−1	−4	0	0	−2	1	0

Iteration 1

Basic V.	Eqn	Z	x_1	x_2	x_3	x_4	x_5	x_6	b_i
Z	(0)	1	3	2	0	0	4	0	16
x_4	(1)	0	−9	−5	0	1	−4	0	2
x_3	(2)	0	2	2	1	0	1	0	4
x_6	(3)	0	−1	−4	0	0	−2	1	0

The optimal BFS is $(0,0,4,2,0,0)$ in augmented form with $Z^* = 16$

For the original problem the optimal solution is $(0,0,4)$ #

Ex 2 $\max Z = 6x_1 + 4x_2 + x_3 + x_4$

$$\begin{aligned} 2x_1 + x_2 - 3x_3 + 4x_4 &\le 10 \\ 3x_1 + x_2 + 5x_4 &\le 6 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

Initial tableau

Basic Var.	Eqn	Z	x_1	x_2	x_3	x_4	x_5	x_6	b_i	
Z	(0)	1	−6	−4	−1	−1	0	0	0	
x_5	(1)	0	2	1	−3	4	1	0	10	→ 5
x_6	(2)	0	3	1	0	5	0	1	6	→ 2

BFS $(0,0,0,0,10,6)$ $Z=0$ not optimal

2-Phase Simplex Method

$$\begin{aligned} \max Z &= 4x_1 + 2x_2 + x_3 \\ 2x_1 + x_2 &= 4 \\ x_1 + 3x_2 + 3x_3 &= 8 \\ x_1, x_2, x_3 &\ge 0 \end{aligned} \longrightarrow$$

I phase

$$\begin{aligned} \max Z &= -\bar{x}_4 - \bar{x}_5 \\ 2x_1 + x_2 + \bar{x}_4 &= 4 \\ x_1 + 3x_2 + 3x_3 + \bar{x}_5 &= 8 \\ x_1, x_2, x_3, \bar{x}_4, \bar{x}_5 &\ge 0 \end{aligned}$$

II phase $\max Z = 4x_1 + 2x_2 + x_3$

$$\begin{aligned} 2x_1 + x_2 &= 4 \\ x_1 + 3x_2 + 3x_3 &= 8 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

I phase

(0)−(1)−(2)

$$\begin{aligned} (0)\quad & Z + \bar{x}_4 + \bar{x}_5 = 0 \;\rightarrow\; Z - 3x_1 - 4x_2 - 3x_3 = -12 \\ (1)\quad & 2x_1 + x_2 + \bar{x}_4 = 4 \\ (2)\quad & x_1 + 3x_2 + 3x_3 + \bar{x}_5 = 8 \end{aligned}$$

Initial tableau

Basic var	Eqn	Z	x_1	x_2	x_3	$\bar{x}_4$	$\bar{x}_5$	b_i	
Z	(0)	1	−3	−4	−3	0	0	−12	
$\bar{x}_4$	(1)	0	2	1	0	1	0	4	→ 4
$\bar{x}_5$	(2)	0	1	3	3	0	1	8	→ 8/3

Iteration 1

Basic var	Eqn	Z	x_1	x_2	x_3	$\bar{x}_4$	$\bar{x}_5$	b_i
Z	(0)	1	−5/3	0	1	0	4/3	−4/3
$\bar{x}_4$	(1)	0	5/3					
x_2	(2)	0	1/3					

Iteration 2

Z	(0)	1	0	0	0	1	1	0
x_1	(1)	0	1	0	−3/5	3/5	−1/5	4/5
x_2	(2)	0	0	1	6/5	−1/5	2/5	12/5

$\left(\frac{4}{5}, \frac{12}{5}, 0, 0, 0\right)$ $Z = 0$ ✓