Exercises Distributions

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Ex 1
- a) discrete
- b) continuous
- c) discrete
- d) continuous
- e) discrete

Ex 2 This is an example of Binomial distribution

$$P(X=k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$n=6$, $p=0.5$, $q=0.5$

1) $P(X=4) = \frac{6!}{4!\,2!} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$

$= \frac{{}^3 6 \cdot 5 \cdot 4!}{4!\,2} \; \frac{1}{2^6} = 0.2344$

"at most"

2) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{6!}{0!\,6!}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + \frac{6!}{1!\,5!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 + \frac{{}^3 6 \cdot 5 \cdot 4!}{2!\,4!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$

$= \frac{1}{2^6} + 6\,\frac{1}{2^6} + 3\,\frac{1}{2^6} = 0.34375$

3) $P(X \ge 5) = P(X=5) + P(X=6) = 0.1094$

Ex 3 Binomial

$$P(X=k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$n=10$ $\quad p=0.25$ $\quad k=7$

$P(X \ge 7) = P(X=7) + P(X=8)$

$+ P(X=9) + P(10) = 0.0035$

Ex 4 Poisson $\quad p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\lambda = \frac{12}{24} = 0.5$

$P(3;\tfrac{1}{2}) = \frac{e^{-1/2}\left(\frac{1}{2}\right)^3}{6} = 0.0126$

Ex 5 Poisson $\quad \lambda = 1$

a) $P(2;1) = \frac{e^{-1}\,1^2}{2} = 0.1839$

b) $\lambda = 2$

$P(2;2) = \frac{e^{-2}\,2^2}{2} = 0.2707$

c) $\lambda = 2$

$P(X>2) = 1 - (P(0) + P(1) + P(2))$

$= 0.3233$

d) $\lambda = 1$

$P(X \le 2) = P(0) + P(1) + P(2)$

$= e^{-1} + e^{-1} + \frac{e^{-1}}{2} = 0.9197$

Ex 6 Binomial $\quad n=3 \quad p = \frac{1}{4} = 0.25$

$P(X=0) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$

$= \frac{3!}{3!}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^3 = \frac{27}{64} = 0.4219$

$P(1) = 27/64$

$P(2) = 9/64$

$P(3) = 1/64$

Ex 7 Uniform $\quad p = \frac{1}{10}$

$P(7 < X < 10) = P(X=8) + P(9) = \frac{2}{10}$

$P(7 \le X \le 10) = P(7) + P(8) + P(9) + P(10)$

$= \frac{4}{10}$

Ex 8 Geometric $\quad P(X=k) = (1-p)^{k-1} p$

$p = \frac{3}{75}$

$P(X=6) = 0.04 \cdot (0.96)^5 = 0.0326$

Es 9 Binomial $\quad n=4 \quad p=0.7$

$P(X=0) = 0.0081$

$P(1) = 0.0756$

$P(2) = 0.2646$

$P(3) = 0.4116$

$P(4) = 0.2401$