Exercises Variance & Co

mercoledì 18 ottobre 2023 10:20

Ex2 $\rho(X,Y) = \dfrac{3.148}{0.115 \cdot 29.91} = 0.9157$

X and Y are strictly positively correlated since $\rho(X,Y) \sim 1$ and $\rho(X,Y) > 0$

Ex 3 $Cov(X,Y) = 8750$

$\rho(X,Y) = 0.9899$

X and Y are strictly correlated. Moreover, since $\rho(X,Y) > 0$, we conclude that when X increases also Y is expected to increase

Ex4 $Cov(X,Y) = -2.8776$

$\rho(X,Y) = -0.7983$

X and Y are negatively correlated with a moderate correlation

Ex 5 Let X be the r.v. that represents the outcome of the roll of the die

" Y " " " " the score associated with the outcome of the coin toss

$X \to \{1,2,3,4,5,6\}$

$Y \to \{\underset{\text{head}}{0}, \underset{\text{cross}}{1}\}$

$Z = X + Y$ $\quad$ $\mathbb{E}[Z]$, $Var[Z]$?

$\mathbb{E}[Z] = \mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y] = .. = 4$

$Var(Z) = Var(X) + Var(Y) + 2\underbrace{Cov(X,Y)}_{=0} = .. = 3.1667$

$\mathbb{E}[Z] = \sum_{x=1}^{6} x P(X=x) + \sum_{y=0}^{1} y \underbrace{P(Y=y)}_{=1/2}$

$= \frac{1}{6} \sum_{x=1}^{6} x + \frac{1}{2} = \frac{21}{6} + \frac{1}{2} = 4$

$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = 2.9167$

$Var(Y) = \frac{1}{2} - \frac{1}{4} = 0.25$

Ex6 $Var(Y) = ?$

$$= \sum_{y_i \in T} (y_i - \mu_y)^2 P(y_i)$$

$T = ?$ $\quad$ $Y : \Omega \to \mathbb{R}$

$\Omega = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$Y : (x_1, x_2) \mapsto \frac{x_1 + x_2}{2}$

$T = \{1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$

(1,1)

$P(Y=1) = \frac{1}{9}$ $\quad$ $P(5/2) = 2/9$

$P(Y = 3/2) = 2/9$ $\quad$ $P(3) = 1/9$

$P(Y = 2) = 3/9$

$\mathbb{E}[Y] = \sum_{i=1}^{5} y_i P(Y = y_i) =$

$= 1 \cdot \frac{1}{9} + \frac{3}{2} \cdot \frac{2}{9} + 2 \cdot \frac{3}{9} + \frac{5}{2} \cdot \frac{2}{9} + 3 \cdot \frac{1}{9} = 2 = \mu_y$

$Var(Y) = \sum_{i=1}^{5} (y_i - \mu_y)^2 P(y_i) =$

$= (1-2)^2 \frac{1}{9} + (\frac{3}{2} - 2)^2 \frac{2}{9} + (2-2)^2 \frac{3}{9} + (\frac{5}{2} - 2)^2 \cdot \frac{2}{9}$

$+ (3-2)^2 \frac{1}{9} = \frac{1}{3} =$

$= \underbrace{\mathbb{E}[Y^2]}_{39/9} - (\underbrace{\mathbb{E}[Y]}_{2})^2$

Ex 7 $Cov(X,Y) = \frac{7}{5} = 1.4$

Ex8 The covariance matrix is

$$\begin{pmatrix} 655.6 & 68.62 & -189.6 \\ 68.62 & 13.06 & -25.72 \\ -189.6 & -25.72 & 133.36 \end{pmatrix}$$

The correlation matrix

$$\begin{pmatrix} 1 & 0.7415 & -0.6412 \\ 0.7415 & 1 & -0.6162 \\ -0.6412 & -0.6162 & 1 \end{pmatrix}$$

For X and Z we have $\rho(X,Z) = -0.6412$ so they are negatively correlated, and the correlation is moderate since $|\rho(X,Z)| = 0.6412$

X and Y are positively correlated with a moderate correlation

Y and Z are negatively correlated