Assignment 01 - Guidance

Spacecraft Guidance and Navigation

AY 2024-2025

POLITECNICO
MILANO 1863

Exam

Assign. 1

Assign. 2

Oral on course

arithmetic mean

optional

Proposed final mark

- All of you will be asked to come in person during the official exam sessions to check our evaluation and discuss any doubts you/we might have
- In case you are not satisfied with the proposed final mark, you can take an optional oral exam (which will take place during the official exam sessions)

Exam

- Remarks:
  - For those who will not submit the report by the deadlines, you'll be asked to do it before the official exam sessions
  - Once you receive the evaluation of the assignments, please wait for the in-person meeting during the exam sessions to discuss about the evaluation

Laboratory sessions - Assignments

Goals ⇨

- Contribute to the final grade
- Relief the workload of the project by spreading it
- Assets for MSc thesis: Matlab, SPICE, and Latex

Assignment evaluation

Weights	Fail	Poor	Good	Excellent
Report	• Major mismatches w.r.t. the assignment • Report awfully written	• Minor mismatches w.r.t. the assignment • Figures and tables not clear • English is poor	• Answers lengthy, but correct • Figures and tables clear • Good English	• Answers concise and clear • Figures and table clear and meaningful • Good English
Code	• Code does not run • Major algorithmic errors • Code not complete	• Minor algorithmic errors • Code not documented • Code takes unnecessary long to run	• Code runs smoothly • Code is fairly documented • Computational efficiency improvable	• Code runs smoothly • Code is well documented • Care is taken to account computational efficiency

Laboratory sessions

- Laboratory sessions **will not be recorded**
  - we are here to give you answers while you are working

- When writing in the forum:
  - Reply to the proper thread
  - Check if your question was already posted
  - Be patient (do not re-ask a question that still has to receive an answer)
  - Do **not** attach/send code asking for debugging

Timetable		Room
15 Oct	16.15-18.15	LM.1 (+ Morselli virtual room)
18 Oct	11.15-14.15	B8 2.2
22 Oct	16.15-18.15	LM.1
23 Oct	14.15-16.15	BL27 0.3
30 Oct	14.15-16.15	Lecture + Presentation of Ex 3
12 Nov	16.15-18.15	L1.4
13 Nov	14.15-16.15	L1.4
27 Nov	-	Assignment 1 – End of Forum Support
20 Dec	**23.59 CET**	**Assignment 1 deadline**

WeBeep forum

- Organization
    - 1 sub-thread per problem point
    - 1 thread for generic questions (integrators and solvers)

Delivery of assignments

Assigment will be delivered through **WeBeep**:

DEADLINE ⇨ Dec 20, 2024, 23:59:00 CET

1) Click on the link to **load Assignment 1** in your Overleaf

   https://bit.ly/SGN_24_Assignment1

2) **Fill the report** and be sure it is compiled properly
3) **Download the PDF** and merge it in a **zipped file** with MATLAB code. Rename it `lastname123456_Assign1.zip`
4) **Submit the compressed file** by uploading it on Webeep
   - **Max Size**: 10 Mb

Assignment 01 - Topics

3 Exercises

- Periodic Orbits
- Shooting Methods
- Continuous guidance

Suggested MATLAB Version: R2021b (or newer)

Report

1 Impulsive guidance

Exercise 1

Let $\mathbf{x}(t) = \varphi(\mathbf{x}_0, t_0; t)$ be the flow of the geocentric two-body model. 1) Using one of Matlab's built-in integrators, implement and validate[*] a propagator that returns $\mathbf{x}(t)$ for given $\mathbf{x}_0$, t_0, t, and μ. 2) Given the pairs $\{\mathbf{r}_1, \mathbf{r}_2\}$ and $\{t_1, t_2\}$, develop a solver that finds $\mathbf{v}_1$ such that $\mathbf{r}(t_2) = \mathbf{r}_2$, where $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi((\mathbf{r}_1, \mathbf{v}_1)^\top, t_1; t_2)$ (Lambert's problem). To compute the derivatives of the shooting function, use either a) finite differences or b) the state transition matrix $\Phi = \mathrm{d}\varphi/\mathrm{d}\mathbf{x}_0$. Validate the algorithms against the classic Lambert solver. 3) Using the propagator of point 1) in the heliocentric case, and reading the motion of the Earth and Mars from SPICE, solve the shooting problem

$$\min_{\mathbf{x}_1, t_1, t_2} \Delta v \quad \text{s.t.} \begin{cases} \mathbf{r}_1 = \mathbf{r}_E(t_1) \\ \mathbf{r}(t_2) = \mathbf{r}_M(t_2) \\ t_1^L \leq t_1 \leq t_1^U \\ t_2^L \leq t_2 \leq t_2^U \\ t_2 \geq t_1 \end{cases} \tag{1}$$

where $\Delta v = \Delta v_1 + \Delta v_2$, $\boldsymbol{\Delta}\mathbf{v}_1 = \mathbf{v}_1 - \mathbf{v}_E(t_1)$, $\boldsymbol{\Delta}\mathbf{v}_2 = \mathbf{v}(t_2) - \mathbf{v}_M(t_2)$. $\mathbf{x}_1 = (\mathbf{r}_1, \mathbf{v}_1)^\top$, and $(\mathbf{r}(t), \mathbf{v}(t))^\top = \varphi(\mathbf{x}_1, t_1; t_2)$. Define lower and upper bounds, and make sure to solve the problem stated in Eq. (1) for different initial guesses.

Write your answer here

- Develop the exercises in one Matlab script; name the file `lastname123456_Assign1.m`
- Organize the script in sections, one for each exercise; use local functions if needed.
- Download the PDF from the Main menu.
- Create a single .zip file containing both the report in PDF and the MATLAB file. The name shall be `lastname123456_Assign1.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, be concise: to the point.
- **Deadline for the submission: Nov 11 2021, 23:30.**
- **Load the compressed file to the Assignments folder on Webeep.**

Exercise

Answer here

NB: The code is not your report!

Answer the question **in the report** and add any plot you think is relevant for your answers there.

Any description missing in the report but present in the code **will not** contribute to the evaluation!

Script

LastName123456_Assign1.m

```
% Modeling and Simulation of Aerospace Systems (2020/2021)
% Assignment # 1
% Author: XXX YYY

%% Ex 1
clearvars; close all; clc;

% Develop the exercise here

%% Ex 2
clearvars; close all; clc;

% Develop the exercise here

%% Ex N
clearvars; close all; clc;

% Develop the exercise here

%% Functions
% Define your functions at the end of the script

function y = MyFunction(x)

% Content...
y = 2*x;

end
```

Name of
the script
(one per exercise)

One section
per exercise point

Functions at the
end of the script

Exercise 1 – Periodic Orbits

Exercise 1

Consider the 3D Earth–Moon Circular Restricted Three-Body Problem with $\mu = 0.012150$. Note that the CRTBP has an integral of motion, that is, the Jacobi constant

$$J(x, y, z, v_x, x_y, v_z) := 2\Omega(x, y, z) - v^2 = C$$

where $\Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu)$ and $v^2 = v_x^2 + v_y^2 + v_z^2$.

1) Find the coordinates of the five Lagrange points L_i in the rotating, adimensional reference frame with at least 10-digit accuracy and report their Jacobi constant C_i.

Solutions to the 3D CRTBP satisfy the symmetry

$$\mathcal{S} : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \rightarrow (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t).$$

Thus, a trajectory that crosses perpendicularly the $y = 0$ plane twice is a periodic orbit.

2) Given the initial guess $\mathbf{x}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})$, with

$$\begin{aligned} x_0 &= 1.068792441776 \\ y_0 &= 0 \\ z_0 &= 0.071093328515 \\ v_{x0} &= 0 \\ v_{y0} &= 0.319422926485 \\ v_{z0} &= 0 \end{aligned}$$

Find the periodic halo orbit having a Jacobi Constant $C = 3.09$; that is, develop the theoretical framework and implement a differential correction scheme that uses the STM, either approximated through finite differences **or** achieved by integrating the variational equation.
Hint: Consider working on $\varphi(\mathbf{x} + \Delta\mathbf{x}, t + \Delta t)$ and $J(\mathbf{x} + \Delta\mathbf{x})$ and then enforce perpendicular cross of $y = 0$ and Jacobi energy.

The periodic orbits in the CRTBP exist in families. These can be computed by 'continuing' the orbits along one coordinate or one parameter, e.g., the Jacobi energy C. The *numerical continuation* is an iterative process in which the desired variable is *gradually* varied, while the rest of the initial guess is taken from the solution of the previous iteration, thus aiding the convergence process.

3) By gradually decreasing C and using numerical continuation, compute the families of halo orbits until $C = 3.04$.

8 points

Exercise 2 – Impulsive guidance (Shooting methods)

Exercise 2

Consider the two-impulse transfer problem stated in Section 3.1 (Topputo, 2013)*.

1) Using the procedure in Section 3.2, produce a first guess solution using $\alpha = 0.2\pi$, $\beta = 1.41$, $\delta = 4$, and $t_i = 2$. Plot the solution in both the rotating frame and Earth-centered inertial frame (see Appendix 1 in (Topputo, 2013)). Consider the parameters listed in Table 4 and extract the radius and gravitational parameters of the Earth and Moon from the provided kernels and use the latter compute the parameter μ.

Symbol	Value	Units	Meaning
m_s	3.28900541×10^5	-	Scaled mass of the Sun
ρ	3.88811143×10^2	-	Scaled Sun-(Earth+Moon) distance
ω_s	$-9.25195985 \times 10^{-1}$	-	Scaled angular velocity of the Sun
ω_{em}	$2.66186135 \times 10^{-1}$	s^{-1}	Earth-Moon angular velocity
l_{em}	3.84405×10^8	m	Earth-Moon distance
h_i	167	km	Altitude of departure orbit
h_i	100	km	Altitude of arrival orbit
DU	3.84405000×10^5	km	Distance Unit
TU	4.34256461	days	Time Unit
VU	1.02454018	km/s	Velocity Unit

Table 4: Constants to be considered to solve the PCRTBP. The units of distance, time, and velocity are used to map scaled quantities into physical units.

2) Considering the first guess in 1) and using $\{\mathbf{x}_i, t_i, t_f\}$ as variables, solve the problem in Section 3.1 with simple shooting in the following cases

   a) without providing any derivative to the solver, and

   b) by providing the derivatives and by estimating the state transition matrix with variational equations.

4) Perform an n-body propagation using the solution $\{\mathbf{x}_i, t_i, t_f\}$ obtained in point 2), transformed in Earth-centered inertial frame and into physical units. To move from 2-D to 3-D, assume that the position and velocity components in inertial frame are $r_z(t_i) = 0$ and $v_z(t_i) = 0$. To perform the propagation it is necessary to identify the epoch t_i. This can be done by mapping the relative position of the Earth, Moon and Sun in the PCRTBP to a similar condition in the real world:

   a) Consider the definition of $\theta(t)$ provided in Section 2.2 to compute the angle $\theta_i = \theta(t_i)$. Note that this angle corresponds to the angle between the rotating frame x-axis, aligned to the position vector from the Earth-Moon System Barycenter (EMB) to the Moon, and the Sun direction.

   b) The angle θ ranges between $[0, 2\pi]$ and it covers this domain in approximately the revolution period of the Moon around the Earth.

   c) Solve a zero-finding problem to determine the epoch at which the angle Moon-EMB-Sun is equal to θ_i, considering as starting epoch `2024 Sep 28 00:00:00.000 TDB`.
   Hints: Exploit the SPK kernels to define the orientation of the rotating frame axes in the inertial frame for an epoch t. Consider only the projection of the EMB-Sun position vector onto the so-defined x-y plane to compute the angle (planar motion).

Plot the propagated orbit and compare it to the previously found solutions.

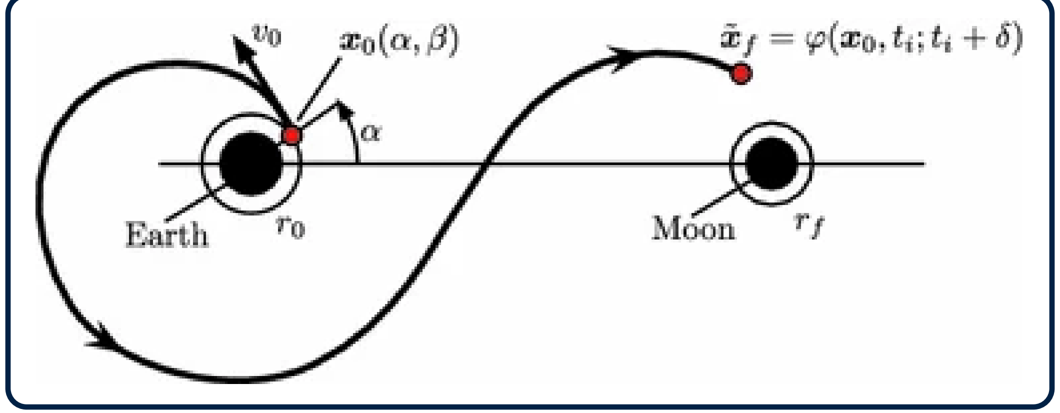
*F. Topputo, "On optimal two-impulse Earth–Moon transfers in a four-body model", Celestial Mechanics and Dynamical Astronomy, Vol. 117, pp. 279–313, 2013, DOI: 10.1007/s10569-013-9513-8.

DOI: 10.1007/s10569-013-9513-8

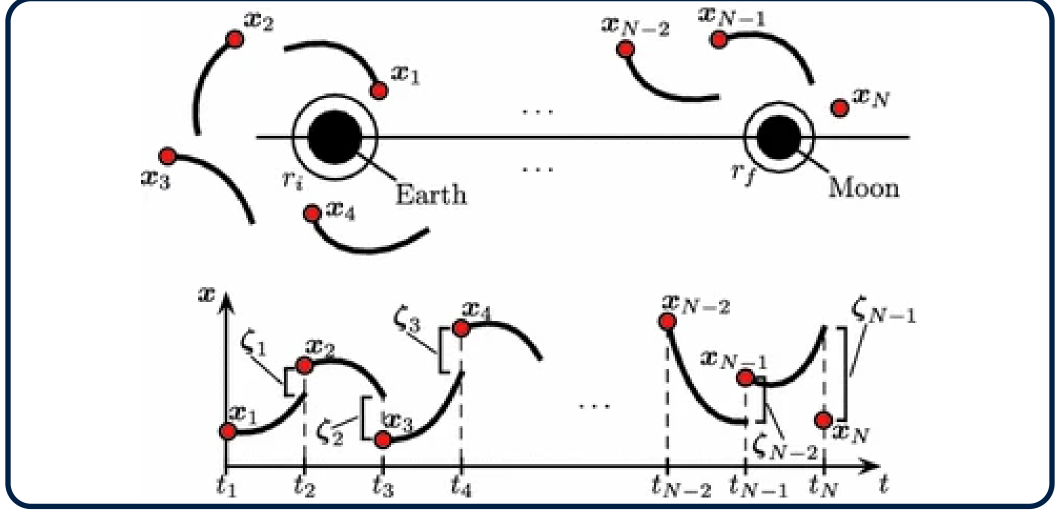
11 points

Exercise 2 – Impulsive guidance (Shooting methods)

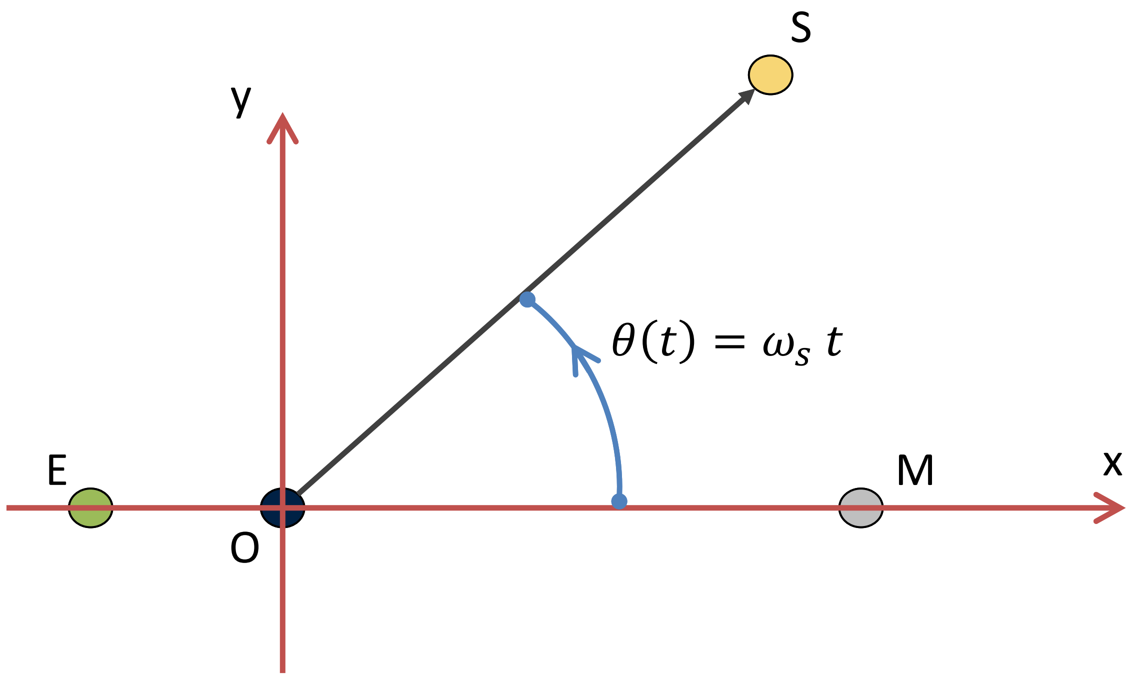
Direct Shooting of Initial Guess



Direct Shooting of Initial Guess



Initial Time – Zero Finding Problem

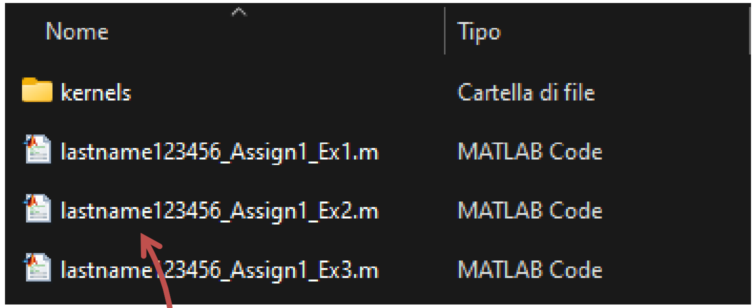



```
[et_i, fval, exitflag] = fzero(@(et)(theta(et, frame)-theta_i),
                                [et_l, et_u]);
```

Refs: MatlabDocs – Fzero, MatlabDocs - Gradients, MatalbDocs - Validation

Supporting material

- Available on **WeBeep**, inside the folder Laboratories
- Compressed folder **Assignment01.zip** contains:
  - Subfolder **`kernels`** with all required SPICE kernels



lastname123456_Assign1_Ex1.m

lastname123456_Assign1_Ex2.m

lastname123456_Assign1_Ex3.m

How to prepare your script

- Unzip the folder and work on your Matlab scripts inside it
- Remember to **load the meta-kernel** (if you created it)
  - Suggestion: use relative paths
- **Obs:** no need to upload the supporting material when preparing your delivery on WeBeep

General hints

Report content

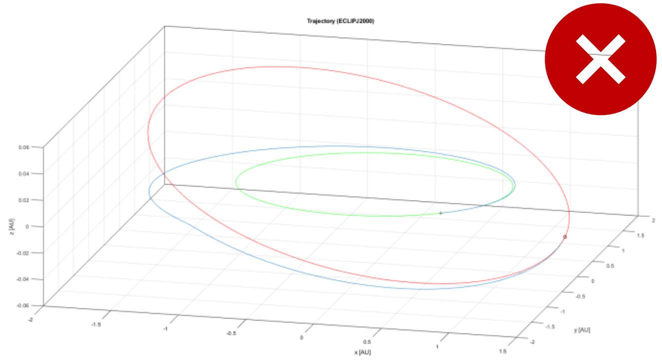
- **State vectors** must be provided with corresponding **reference frame and origin**
- Always put the (correct!) units
- Do not mix position and velocities when reporting errors

$\boldsymbol{r}_x$ [km]	$\boldsymbol{r}_y$ [km]	$\boldsymbol{r}_z$ [km]
1234.567	123.456	234.567

Satellite position (@Sun ECLIPJ2000)

Plots

- Make sure that axis labels and titles are clearly readable with page zoom at 100%
- Remember to put the (correct!) units on each axis
- When plotting trajectories specify the **reference frame and origin** in the plot title or caption



General hints

LaTeX

- Do not put multiplication symbols, especially as asterisks
- Clearly distinguish between vectors and scalar: write vector using underline, arrow or bold
    - Latin alphabet: \mathbf{x} or \mathbf{r}
    - Greek symbols:\boldsymbol{\lambda}
    - **Obs:** No need to use norm with this notation $||\boldsymbol{\lambda}_x|| \leftrightarrow \lambda_x$
- Write scalar product using \cdot
- Prefer the use of \dfrac{}{}over\frac{}{} when writing equations
- When inserting text in an equation remember to use\mathrm{}