Imaginary Unit (139)

Мнимая единица.

Complex Numbers

Множество чисел — Real numbers

$x^2 = -1$ — the basis of new number syst.

$$\boxed{i = \sqrt{-1} \text{ where } i^2 = -1}$$

the imaginary unit

$$\sqrt{-25} = \sqrt{-1}\sqrt{25} = i\sqrt{25} = 5i$$

$$(5i)^2 = 5^2 i^2 = 25(-1) = -25$$

Complex numbers system

$a + bi$

b — завис. пер.

a — независ. перем.

Real numbers: $a + bi$ with $b = 0$

Imaginary numbers: $a + bi$ with $b \neq 0$

единиц → the $5i$

pure imaginary number - мнимые ч-ла

$\underset{=}{-4} + \underbrace{6i}$ $\quad 2i = \underbrace{0}_{\text{real part}} + \underbrace{2i}_{\text{imaginary part}}$ $\quad 3 = \underset{=}{3} + \underbrace{0i}$

3 можно представить как $\underbrace{3 + 0 \cdot i}$

Real number $\Rightarrow$ imaginary number

как ребёнок мнимую часть в 5i

Упрощенные комплексные числа (simplified form) $\underline{a + bi}$ standard for.

$\cancel{7 + 3\sqrt{5i}}$ $= 7 + 3i\sqrt{5} \neq 7 + 3\sqrt{5i}$

i - ставим before radical ($\sqrt{\ }$)

Equality of Complex Numbers.

$a + bi = c + di$ if and only if $a = c$ & $b = d$

$a + bi$ is like binominal $a + bx$

By contrast

Треугольник паскаля

N строки = N степени

1)	1 1	$(a+b)^1 = 1a + 1b = a + b$
2)	1 2 1	$(a+b)^2 = \overset{2+0=2}{a^2b^0} + \overset{1+1=2}{2a^1b^1} + \overset{2+0=2}{b^2a^0}$
3)	1 3 3 1	$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + b^3$
4)	1 4 6 4 1	$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
5)	1 5 10 10 5 1	$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Не пиши м

$a^0 = 1$

$b^0 = 1$

Adding & Substracting Complex Numbers

1) $(a+bi)+(c+di)=(a+c)+(b+d)i$

2) $(a+bi)-(c+di)=(a-c)+(b-d)i$

$(5-11i)+(7+4i)=$

$=(5+7)+(-11+4)\cdot i=12-7i$

$(-5+i)-(-11-6i)$

$=(-5-(-11))+(1-(-6))\cdot i=6+7i$

Operations with complex numbers are just like operations with polynomials. We're simply <u>combining like terms.</u>

$(5-11x)+(7+4x)=\underline{5}-\underline{\underline{11x}}+\underline{7}+\underline{\underline{4x}}=12-7x$

$(-5+x)-(-11-6x)=\underline{-5}+\underline{\underline{x}}+\underline{11}+\underline{\underline{6x}}=6+7x$

$(5-2i)+(3+3i) = 5-2i+3+3i = 8+i$

$(2+6i)-(12-i) = 2-12+6i+i = -10+7i$

FOIL method. (to find product of polynominals $ax+b$ & $cx+d$)

(First Outside Inside Last)

first, last

$(ax+b)(cx+d) = ax \cdot cx + ax \cdot d + b \cdot cx + b \cdot d$

inside, outside

Product of First Terms — Outside "–" — Inside "–" — last "–"

Multiplying Complex numbers

$4i(3-5i) = 4i \cdot 3 + 4i \cdot (-5i) = 12i - 20\underbrace{i^2}_{=-1} =$

$= 12i+20(-1) = 12i+20$

$(7-3i)(-2-5i) = -14 - 35i + 6i + 15i^2 = 15(-1)+(-14) \cancel{-6}\, 29i =$

$= -29-29i$

$7i(2-9i) = 14i - 63i^2 = 14i+63$

$(5+4i)(6-7i) = 30-35i+24i-28i^2 = 30-11i-28(-1) =$

$= 58-11i$

Devide complex numbers.

Complex conjugates & division.

$$(a+bi)(a-bi) = a^2 - abi + abi + b^2i^2 =$$
$$= a^2 - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$$

! the product eliminates i.

Thus $a+bi$ (a complex number), where $a-bi$ - a complex conjugate for it & VS.

$$(a+bi)\cdot(a-bi) = a^2+b^2$$
$$(a-bi)\cdot(a+bi) = a^2+b^2$$

complex №s complex conjugates.

$$\frac{3i}{4+i} = \frac{3i}{4+i}\cdot\frac{(4-i)}{(4-i)} = \frac{12i-3i^2}{16+i^2} = \frac{12i-3(-1)}{16+1} = \frac{12i+3}{17} =$$

$$= \frac{12i}{17} + \frac{3}{17} = \frac{3}{17} + \frac{12}{17}i$$

$(a+bi)$ ← standard form of quotation

$$\frac{5i}{7+i} = \frac{5i(7-i)}{7^2+i^2} = \frac{35i-i^2}{49+1} = \frac{35i}{50} + \frac{1}{50} = \frac{1}{50} + \frac{35i}{50}$$

Using Complex Conjugates to Divide Complex Numbers

$$\frac{7+4i}{2-5i} = \frac{(7+4i)(2+5i)}{(2-5i)(2+5i)} = \frac{14+35i+8i+20i^2}{2^2+5^2i^2} =$$

$$= \frac{14+43i+20(-1)}{4+25\cdot 1} = \frac{-6+43i}{29} = -\frac{6}{29} + \frac{43}{29}i$$

$$\frac{5+4i}{4-i} = \frac{(5+4i)(4+i)}{4^2+i^2} = \frac{20+5i+16i+4i^2}{16-(-1)} = \frac{16+21i}{17} = \frac{16}{17}+\frac{21}{17}i$$

Roots of negative numbers:

$$(4i)^2 = (-4i)^2 = -16$$

$$(4i)^2 = 4^2i^2 = 16\cdot(-1) = -16$$

$$(-4i^2) = (-4)^2\cdot i^2 = 16\cdot(-1) = -16$$

$$-16$$

$$(4i)^2 \quad (-4i)^2$$

$4i$ – principal square roote of -16.

$$\boxed{\sqrt{-b} = i\sqrt{b}}$$

– for any positive realnumber b, the principle square root of the negative number $-b$ is defined by

$$\sqrt{-25} = \sqrt{(-1)\cdot 5^2} = \sqrt{-1}\cdot\sqrt{5^2} = 5i$$

$$\sqrt{-b} = \sqrt{b}\,i, \quad \cancel{\sqrt{-b} = \sqrt{bi}}$$

When performing operations with square roots of negative numbers, expres $\sqrt{}$ with i.

$$\sqrt{-25}\cdot\sqrt{-4} = \sqrt{-1}\cdot\sqrt{25}\cdot\sqrt{-1}\cdot\sqrt{4} = i\sqrt{25}\cdot i\sqrt{4} = 5i\cdot 2i = 10i^2 =$$

$$= 10(-1) = -10$$

$$\sqrt{-18} - \sqrt{-8} = i\sqrt{18} - i\sqrt{8} = i\sqrt{9\cdot 2} - i\sqrt{4\cdot 2} =$$

$$= 3i\sqrt{2} - 2i\sqrt{2} = \sqrt{2}(3i - 2i) = i\sqrt{2}$$

$$(-1+\sqrt{-5})^2 = (-1)^2 + 2(-1)\cdot\sqrt{-5} + (\sqrt{-5})^2 =$$

$$= 1 - 2\sqrt{-5} + (i\sqrt{5})^2 = 1 - 2i\sqrt{5} + 5i^2 = 1 - 2i\sqrt{5} - 5 =$$

$$= -4 - 2i\sqrt{5}$$

$$\frac{-25+\sqrt{-50}}{15} = \frac{-25+i\sqrt{50}}{15} = \frac{-25+i\sqrt{5^2\cdot 2}}{15} =$$

$$= -\frac{25}{15} + \frac{5i\sqrt{2}}{15} = -\frac{5}{3} + \frac{\sqrt{2}}{3}i$$

$$\sqrt{-27} + \sqrt{-48} = i\sqrt{9\cdot 3} + i\sqrt{2^4\cdot 3} = 3i\sqrt{3} + 4i\sqrt{3} = 7i\sqrt{3}$$

$$(-2+\sqrt{-3})^2$$

$i^2 = -1$