$$= \lim_{n\to\infty} \frac{\sqrt[3]{7}\cdot\underbrace{\frac{1}{\sqrt[3]{n^2}}}_{0} - \sqrt[4]{81 - \underbrace{\frac{1}{n^2}}_{0}}}{\Big(1+\underbrace{\frac{4}{\sqrt{n}}}_{0}\Big)\Big(\sqrt{1-\underbrace{\frac{5}{n^2}}_{0}}\Big)} = \frac{-\sqrt[4]{3^4}}{1} = -3$$

Ответ: -3.

$$\lim_{n\to -2} \frac{\sqrt[3]{x-6}+2}{x+2} = \left[\frac{0}{0}\right] = (*) =$$

ИШ 01.01 №4

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$= \lim_{n\to -2} \frac{(\sqrt[3]{x-6}+2)\left((\sqrt[3]{x-6})^2 - 2\sqrt[3]{x-6}+2^2\right)}{(x+2)\left((\sqrt[3]{x-6})^2 - 2\sqrt[3]{x-6}+2^2\right)} =$$

$$= \lim_{n\to -2} \frac{(\sqrt[3]{x-6})^3 + 2^3}{(x+2)\ \text{и-и}} = \lim_{x\to -2} \frac{x-6+8}{(x+2)\ \text{и-и}} = \cancel{\lim_{x\to -2} \frac{x-6+8}{(x+2)(\ldots)}} =$$

$$= \lim_{x\to -2} \frac{x+2}{(x+2)\left((\sqrt[3]{x-6}) - 2\sqrt[3]{x-6}+4\right)} = \lim_{x\to -2} \frac{1}{(\sqrt[3]{x-6})^2 - 2\sqrt[3]{x-6}+4}$$

$$\frac{1}{(\sqrt[3]{-2-6})^2 - 2\sqrt[3]{-2-6}+4} = \frac{1}{(\sqrt[3]{-8})^2 - 2\sqrt[3]{-8}+4} = \frac{1}{(\sqrt[3]{(-2)^3})^2 - 2\sqrt[3]{(-2)^3}+4}$$

$$= \frac{1}{(-2)^2 - 2\cdot(-2)+4} = \frac{1}{4+4+4} = \frac{1}{12}$$

Демидович

5.231

$$\lim_{n\to\infty}\frac{n-1}{3n}=\left[\frac{\infty}{\infty}\right]=\lim_{n\to\infty}\frac{\frac{n}{n}-\frac{1}{n}\to 0}{\frac{3n}{n}}=\frac{1}{3}$$

5.232

$$\lim_{n\to\infty}\frac{5n+1}{7-9n}=\lim_{n\to\infty}\frac{\frac{5n}{n}}{\frac{-9n}{n}}=-\frac{5}{9}$$

5.233

$$\lim_{n\to\infty}\frac{(n+1)^2}{2n^3}=\left[\frac{\infty}{\infty}\right]=\frac{\frac{n^2}{n^3}\to 0+\frac{2n}{n^3}\to 0+\frac{1}{n^3}\to 0}{\frac{2n^3}{n^3}}=\frac{0}{2}=0$$

5.234

$$\lim_{n\to\infty}\frac{3n^2-7n+1}{2-5n-6n^2}=\frac{\frac{3n^2}{n^2}}{\frac{-6n^2}{n^2}}=\lim_{n\to\infty}-\frac{1}{2}$$

5.235

$$\lim_{n\to\infty}\frac{(n+2)^3-(n-2)^3}{95n^3+39n}=\left[\frac{\infty-\infty}{\infty+\infty}\right]=$$

$$=\lim_{n\to\infty}\frac{n^3+3n^2\cdot 2+3\cdot 2^2n+2^3-(n^3-3\cdot 2n^2+3\cdot 2^2n-2^3)}{\text{″}-\text{″}}=$$

$$=\lim_{n\to\infty}\frac{\cancel{n^3}+6n^2+\cancel{12n}+8-\cancel{n^3}+6n^2-\cancel{12n}+8}{\text{″}-\text{″}}=$$

$$=\lim_{n\to\infty}\frac{12n^2+16}{n(95n^2+39)}=\left|:n^3\right|=\frac{\frac{12n^2}{n^3}\to 0+\frac{16}{n^3}\to 0}{\frac{95n^3}{n^3}+\frac{39n}{n^3}\to 0}=0$$

$$\lim_{n\to\infty} n^{\frac{3}{2}}\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)=$$

$$=\lim_{n\to\infty} n^{\frac{3}{2}}\cdot\lim_{n\to\infty}\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)=$$

$$=\lim_{n\to\infty}\frac{n^{\frac{3}{2}}}{n^{\frac{3}{2}}}\cdot\lim_{n\to\infty}\frac{\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=$$

$$:n^{\frac{3}{2}}$$

$$=1\cdot\lim_{n\to\infty}\frac{n^3+1-n^3+2}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=\lim_{n\to\infty}\frac{3}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=$$

$$=\lim_{n\to\infty}\frac{3}{\left(\sqrt{\frac{n^3}{n^3}+\underbrace{\frac{1}{n^3}}_{0}}+\sqrt{\frac{n^3}{n^3}-\underbrace{\frac{2}{n^3}}_{0}}\right)}=\frac{3}{2}$$

Ответ: $\frac{3}{2}$

№ 5. 238.

$$\lim_{n\to\infty}\left(\sqrt{n+2}-\sqrt{n}\right)=\lim_{n\to\infty}\frac{\left(\sqrt{n+2}-\sqrt{n}\right)\left(\sqrt{n+2}+\sqrt{n}\right)}{\left(\sqrt{n+2}+\sqrt{n}\right)}=$$

$$=\frac{n+2-n}{\left(\sqrt{n+2}+\sqrt{n}\right)}=\cancel{\frac{2}{\left(\frac{\sqrt{n+2}}{\sqrt{n}}+\frac{\sqrt{n}}{\sqrt{n}}\right)}}=\frac{2}{\left(\sqrt{\frac{n}{n}+\underbrace{\frac{2}{n}}_{0}}+1\right)}=$$

$$=\cancel{\frac{2}{2}=1}$$

$$=\frac{2}{\left(\sqrt{\infty}+\infty\right)}=\frac{2}{\infty}=0$$

№5.240

$$\lim_{n\to\infty} \frac{2^n+3^n}{2^n-3^n} = \Big| \; n\in\infty \;\; n\in N \;\; n>0 \;\; \Big[\; \text{д. макс. ст.}$$

Выбираем наиб. основ. с 3^n

$$\lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n+\left(\frac{3}{3}\right)^n}{\left(\frac{2}{3}\right)^n-\left(\frac{3}{3}\right)^n} = \frac{\overset{0}{\left(\frac{2}{3}\right)^n}+1}{\underset{0}{\left(\frac{2}{3}\right)^n}-1} = -1$$

т.к. $\frac{2}{3}$ стрем-ся к 0 (т.к. $2<3$) $\Rightarrow \left(\frac{2}{3}\right)^\infty$ стрем-ся к нулю.

Ответ: неопр-сть вида $\frac{\infty}{\infty}$ -1

№5.241

$$\boxed{S=\frac{a_1+a_n}{2}\cdot n} \qquad \lim_{n\to\infty} \Big(\underset{1}{\overset{\text{пр.}}{\frac{1}{n^2}}} + \frac{2}{n^2} + \dots + \overset{n\text{-ный}}{\frac{n-1}{n^2}} \Big)$$

У всех дробей знам-ль n^2. В числ. Σ арифмет. прогрессии. Ее преобраз-м по формуле:

$$S=\frac{a_1+a_n}{2}\cdot n$$

$$\lim_{n\to\infty} \frac{\frac{(\overset{a_1=n_1}{1}+\overset{a_n=n\text{-ный}}{n-1})}{2}\cdot(n-1)}{n^2} = \lim_{n\to\infty} \frac{n^2-n}{2n^2} = \lim_{n\to\infty} \frac{\frac{n^2}{n^2}-\overset{0}{\frac{n}{n^2}}}{\frac{2n^2}{n^2}} =$$

$$=\frac{1}{2}$$

Предел послед-сти. (Антидемидович)

№ 66 (44)

$$\cancel{\lim_{n\to\infty}\left(\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\ldots+\frac{2n-1}{2^n}\right)}$$

$$\lim_{x\to\infty}\left(\frac{1}{4}+\frac{1}{4^2}+\ldots+\frac{1}{4^n}\right)=(*)$$ — бесконечно убыв-я геометр. прогр-я.

$$x_1=\frac{1}{4};\ x_2=\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{4^2};\ x_3=\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{4^3};\ x_n=\frac{1}{4^n}$$

$S=\frac{b_1}{1-q}$, где $b_1=\frac{1}{4}$; $q=\frac{1}{4}$ (~~знаменатель~~ знаменат-ль прогрессии)

$$(*)=\lim_{x\to\infty}\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{4}:\frac{3}{4}=\frac{1\cdot 4}{4\cdot 3}=\frac{1}{3}$$

Ответ: $\frac{1}{3}$

$$\lim_{x\to\infty}\frac{1+3+\ldots+(2n-1)}{3n^2}=\frac{n^2}{3n^2}==\frac{1}{3}.\ (\text{Ответ})$$

Рассмотрим числ. отд-но:

$1+3+\ldots+(2n-1)$: арифм. прогр.: $a_1=1$, $a_n=2n-1$, $n=n$.

$a_1=2\cdot1-1=1$
$a_2=2\cdot2-1=3$
$a_3=2\cdot3-1=5$
$a_4=2\cdot4-1=7$

↑n-ый член.

$$S=\frac{(a_1+a_n)\cdot n}{2}$$

$$S=\frac{(1+2n-1)n}{2}=n^2$$

Сканировано с CamScanner

5. 241

$$\lim_{x\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\dots+\frac{n-1}{n^2}\right)=(*)$$

$$=\lim_{x\to\infty}\left(\frac{0+1+2+\dots+n-1}{n^2}\right)$$ ар. прогр.

$n_1=1 \quad a_1=1-1=0$

$n_2=2 \quad a_2=2-1=1$

$n_3=3 \quad a_3=3-1=2$

$d=a_3-a_2=2-1=1$

$$S=\left(\frac{a_1+a_n}{2}\right)n=\left(\frac{0+(n-1)}{2}\right)n=\frac{n(n-1)}{2}$$

$$(*)=\frac{n(n-1)}{2}:n^2==\frac{\cancel{n}(n-1)}{2\cdot n\cdot\cancel{n}}=\frac{n-1}{2n}=$$ $|:n$

$$=\frac{\frac{n}{n}-\frac{1}{n}_0}{\frac{2n}{n}}=\frac{1}{2}$$

$n\to\infty$

Ответ: $\frac{1}{2}$.

№ 5.242

$$\lim_{x\to\infty}\left(\frac{1^2+2^2+\dots+\dots n^2}{n^3}\right)=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)}{n^3}=$$

$$S_n=\frac{n(n+1)(n+2)}{6}$$ – Сумма последов-ти квадратов натур. чисел.

$$= \frac{1}{6} \lim_{x\to\infty} \frac{2n^2+n+2n+1}{n^3} \; :n^3 = \left[\frac{\infty}{\infty}\right] = \frac{1}{6} \cdot \frac{\frac{2n^2}{n^3} + \frac{n}{n^3} + \frac{2n}{n^3} + \frac{1}{n^3}}{\frac{n^3}{n^3}} =$$

$$= \frac{1}{6} \cdot \frac{0}{1} = 0$$

Ответ: 0

№ 5.243.

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^2}\, \sin(n^2)}{n-1} = \frac{\infty \cdot \infty}{\infty} = \frac{\infty}{\infty} =$$

$$\lim_{x\to\infty} \frac{\sin x}{x} = 1$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^2}\, \frac{\sin n^2}{n^2} \cdot n^2}{(n-1)} = \frac{n^{\frac{2}{3}} \cdot n^2}{n-1} = \frac{n^{\frac{2+6}{3}}}{n-1} = \frac{n^{\frac{8}{3}}}{n-1} =$$

$$= \lim_{n\to\infty} \frac{\frac{n^{\frac{8}{3}}}{n^{\frac{8}{3}}}}{\frac{n}{n^{\frac{8}{3}}} - \frac{1}{n^{\frac{8}{3}}}} = \frac{1}{0} = 0$$

Ответ: 0

№ 5.244.

$$\lim_{n\to\infty} \underbrace{\left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \dots + \frac{1}{n(n+1)}\right)}_{n \text{ членов}}$$

$$a_1 = \frac{1}{1(1+1)} = \frac{1}{2}$$

$$a_2 = \frac{1}{2(2+1)} = \frac{1}{2\cdot 3} = \frac{1}{6}$$

$$a_3 = \frac{1}{3(3+1)} = \frac{1}{3\cdot 4} = \frac{1}{12}$$

$$a_4 = \frac{1}{4(4+1)} = \frac{1}{4\cdot 5} = \frac{1}{20}$$

$$a_5 = \frac{1}{5(5+1)} = \frac{1}{30}$$

$d_1 = a_2 - a_1 = 6 - 2 = 4$

$d_2 = a_3 - a_2 = 12 - 6 = 6$

$d_3 = a_4 - a_3 = 20 - 12 = 8$

$d_4 = a_5 - a_4 = 30 - 20 = 10$

d_{n+2}

d_1

$d_1 = a_2 - a_1 = \frac{1}{6} - \frac{1}{2} = \frac{1-3}{6} = \frac{-2}{6} = -\frac{1}{3}$

$d_2 = a_3 - a_2 = \frac{1}{12} - \frac{1}{6} = \frac{1-2}{12} = \frac{-1}{12}$

$d_1 \neq d_2$. $\frac{1}{30} - \frac{1}{20} = \frac{20-30}{60} = \frac{10}{60} = \frac{1}{6}$

$q_1 = \frac{a_2}{a_1} = \frac{1}{6} \cdot \frac{2}{1} = \frac{1}{3}$ $q_2 = \frac{a_3}{a_2} = \frac{1}{12} \cdot \frac{6}{1} = \frac{1}{2}$

$\lim\limits_{n \to \infty} \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{n(n+1)}$

$a_n = \frac{1}{n(n+1)}$ – общий член ряда

$b_n = b_1 \cdot q^{n-1}$ – формула прогрессии

b_1 – 1-й член; q – знаменатель прогр. ($q \neq 0$)

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5.236

$$\lim_{n\to\infty}\left(\frac{2n-1}{5n+7} - \frac{1+2n^3}{2+5n^3}\right) = \left[\frac{\infty}{\infty} - \frac{\infty}{\infty}\right] =$$

$$= \left|:n^3\right| = \lim_{n\to\infty} \frac{\frac{2}{n^2}-\frac{1}{n^3}}{\frac{5}{n^2}+\frac{7}{n^3}} - \frac{\frac{1}{n^3}+\frac{2n^3}{n^3}}{\frac{2}{n^3}+\frac{5n^3}{n^3}} = -\frac{2}{5}$$

Решение 2

$$\lim_{n\to\infty}\left(\frac{2n-1}{5n+1} - \frac{1+2n^3}{2+5n^3}\right) =$$

$$= \lim_{n\to\infty}\left(\frac{2n-1}{5n+1}\right)^{:n} - \lim_{n\to\infty}\left(\frac{1+2n^3}{2+5n^3}\right)^{:n^3} =$$

$$= \lim_{n\to\infty}\left(\frac{2-\frac{1}{n}}{5+1}\right) - \lim_{n\to\infty}\left(\frac{\frac{1}{n^3}+2}{\frac{2}{n^3}+5}\right) = \frac{2}{5} - \frac{2}{5} = 0$$

Ответ: 0

5.237

$\sqrt[3]{n^4} = n^{\frac{4}{3}}$

$\frac{1}{3} > n$

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4+3n+1}}{n-1} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4}{4}-3n+1}}{\frac{n}{n^{\frac{4}{3}}}-\frac{1}{n^{\frac{4}{3}}}} = \frac{1}{0} = 1$$

$$n^1 - n^{\frac{4}{3}} = n^{\frac{3}{3}} - n^{\frac{4}{3}} = n^{-\frac{1}{3}} = \frac{1}{n^{\frac{1}{3}}} \to 0$$

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№ 5.239.

$$\lim_{n\to\infty} n^{\frac{3}{2}}\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)=$$

$$=\lim_{n\to\infty} n^{\frac{3}{2}}\cdot\lim_{n\to\infty}\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)=$$

$$=\lim_{n\to\infty}\frac{n^{\frac{3}{2}}}{n^{\frac{3}{2}}}\cdot\lim_{n\to\infty}\frac{\left(\sqrt{n^3+1}-\sqrt{n^3-2}\right)\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=$$

$$: n^{\frac{3}{2}}$$

$$=1\cdot\lim_{n\to\infty}\frac{n^3+1-n^3+2}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=\lim_{n\to\infty}\frac{3}{\left(\sqrt{n^3+1}+\sqrt{n^3-2}\right)}=$$

$$=\lim_{n\to\infty}\frac{3}{\left(\sqrt{\frac{n^3}{n^3}+\underbrace{\frac{1}{n^3}}_{0}}+\sqrt{\frac{n^3}{n^3}-\underbrace{\frac{2}{n^3}}_{0}}\right)}=\frac{3}{2}$$

Ответ: $\frac{3}{2}$

№ 5.238.

$$\lim_{n\to\infty}\left(\sqrt{n+2}-\sqrt{n}\right)=\lim_{n\to\infty}\frac{\left(\sqrt{n+2}-\sqrt{n}\right)\left(\sqrt{n+2}+\sqrt{n}\right)}{\left(\sqrt{n+2}+\sqrt{n}\right)}=$$

$$=\frac{n+2-n}{\left(\sqrt{n+2}+\sqrt{n}\right)}=\cancel{\frac{2}{\left(\frac{\sqrt{n+2}}{\sqrt{n}}+\frac{\sqrt{n}}{\sqrt{n}}\right)}}=\frac{2}{\left(\sqrt{\frac{n}{n}+\underbrace{\frac{2}{n}}_{0}}+1\right)}=$$

$$=\cancel{\frac{2}{2}=1}$$

$$=\frac{2}{\left(\sqrt{\infty}+\sqrt{\infty}\right)}=\frac{2}{\infty}=0$$

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