

Day 15.

N1.

Dado:

$x_1 = 2$ - peça e gp - 8 $x^2 - 54x + 104 = 0$

$x_2 = ?$

$$x^2 - 54x + 104 = 0$$

$$x_1, x_2 = 104 \quad | \Rightarrow \quad x_1 = 52$$

$$x_1 + x_2 = 54 \quad | \quad x_2 = 2$$

Omkem: $x_2 = 52$

Day 15

Nd. a u b kopru $x^2 - 10x + 5 = 0$

Найти: $\frac{a^2+b^2}{a^2b+ab^2}$

$$x^2 - 10x + 5 = 0$$

$$a+b = -\frac{b}{a} = -\left(-\frac{10}{1}\right) = 10$$

$$a \cdot b = \frac{c}{a} = \frac{5}{1} = 5$$

$$a^2+b^2 = a^2+2ab+b^2-2ab = (a+b)^2-2ab = 10^2-2 \cdot 5 = 100-10 = 90$$

$$a^2b+ab^2 = ab(a+b) = 5 \cdot 10 = 50$$

$$\frac{a^2+b^2}{a^2b+ab^2} = \frac{90}{50} = \frac{9}{5} = 1 \frac{4}{5} = 1 \frac{8}{10} = 1,8$$

Дано:

N1

$$x, u x_2 \text{ - корни } x^2 + 5x - 3 = 0$$

$$\text{Найти } x_1^2 + x_2^2$$

$$x^2 + 5x - 3 = 0$$

$$x_1 x_2 = -3$$

$$x_1 + x_2 = -5$$

$$x_1^2 + x_2^2 = (x_1^2 + 2x_1x_2 + x_2^2) - 2x_1x_2 = (x_1 + x_2)^2 - 2x_1x_2 = (-5)^2 - 2 \cdot (-3) = 25 + 6 = 31$$

Ответ: 31

Дано:

N2

$x_1, u x_2$ - корни

$$x^2 + 11x + 12 = 0$$

$$x_1^2 + x_2^2 = ?$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = (-11)^2 - 2 \cdot 12 = 121 - 24 = 97$$

$$x^2 + 11x + 12 = 0$$

$$x_1 x_2 = 12 ; x_1 + x_2 = -11$$

Ответ: 97

Дано:

N3

$$x^2 + 9x + 33 = 0$$

$$\frac{1}{x_1} + \frac{1}{x_2} = ?$$

$$x^2 + 9x + 33 = 0$$

$$x_1 x_2 = 33 ; x_1 + x_2 = -9$$

Ответ: $-\frac{3}{11}$

Дано:

N4

$$x^2 - 8x + 11 = 0$$

$$x_1^3 + x_1^2 + x_1 + x_2^3 + x_2^2 + x_2 = ?$$

$$x_1^3 + x_1^2 + x_1 + x_2^3 + x_2^2 + x_2 = (x_1^3 + x_1^2 + x_1) + (x_2^3 + x_2^2 + x_2)$$

$$= (x_1 + x_2)(x_1^2 + x_1x_2 + x_2^2) + (x_1 + x_2) + (x_1 + x_2)^2 - 2x_1x_2 =$$

$$x^2 - 8x + 11 = 0$$

$$x_1 x_2 = 11 ; x_1 + x_2 = 8$$

$$x_1^3 + x_1^2 + x_1 + x_2^3 + x_2^2 + x_2 =$$

$$= (x_1^3 + x_1^2 + x_1) + (x_2^3 + x_2^2 + x_2)$$

$$= (x_1 + x_2)(x_1^2 + x_1x_2 + x_2^2) + (x_1 + x_2) + (x_1 + x_2)^2 - 2x_1x_2 =$$

day 15

$$= (x_1 + x_2) / ((x_1^2 + x_2^2 + 2x_1x_2) - 9x_1x_2) + (x_1 + x_2) / ((x_1 + x_2)^2 - 2x_1x_2)$$

$$= (x_1 + x_2) / ((x_1 + x_2)^2 - 9x_1x_2) + (x_1 + x_2) / ((x_1 + x_2)^2 - 2x_1x_2) =$$

$$= 8(8^2 - 9 \cdot 11) + 8 + 8^2 - 2 \cdot 11 =$$

$$= 8(64 - 99) + 8 + 64 - 22 = 248 + 50 = 298$$

Answer: 298

N5.

Dano:

$$x^2 - 15x + 36 = 0$$

$$|a| = \sqrt{a^2}$$

$$x^2 - 15x + 36 = 0$$

$$|x_1 - x_2|$$

$$x_1x_2 = 36; x_1 + x_2 = 15$$

$$|x_1 - x_2| = \sqrt{(x_1 - x_2)^2} = \sqrt{x_1^2 - 2x_1x_2 + x_2^2} =$$

$$= \sqrt{(x_1^2 + 2x_1x_2 + x_2^2) - 2x_1x_2 - 2x_1x_2} = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} =$$

$$= \sqrt{15^2 - 4 \cdot 36} = \sqrt{225 - 144} = \sqrt{81} = \sqrt{9^2} = 9$$

Answer: 9

N6.

Dano:

$$x^2 - 12x + 19 = 0$$

$$x^2 - 12x + 19 = 0$$

$$x_1x_2 = 19$$

$$x_1 + x_2 = 12$$

$$x_1(1 - x_1) + x_2(1 - x_2)$$

day 15

$$x_1(1 - x_1) + x_2(1 - x_2) = x_1 - x_1^2 + x_2 - x_2^2 =$$

$$= (x_1 + x_2) - (x_1^2 + x_2^2) = (x_1 + x_2) - ((x_1 + x_2)^2 - 2x_1x_2) =$$

$$= 12 - (12^2 - 2 \cdot 19) = 12 - (144 - 38) = 12 - 106 = -94$$

Answer: -94

N7

Dano:

$$x^2 - 4x + 1 = 0$$

$$x^2 - 4x + 1 = 0$$

$$(x_1 - \frac{1}{x_1})^2 + (x_2 - \frac{1}{x_2})^2$$

$$x_1x_2 = 1; x_1 + x_2 = -B = 4 = 4$$

$$(x_1 - \frac{1}{x_1})^2 + (x_2 - \frac{1}{x_2})^2 = x_1^2 - 2x_1 \cdot \frac{1}{x_1} + (\frac{1}{x_1})^2 + x_2^2 - 2x_2 \cdot \frac{1}{x_2} + (\frac{1}{x_2})^2 =$$

$$= (x_1^2 + x_2^2) + \left(\frac{1}{x_1}\right)^2 + \left(\frac{1}{x_2}\right)^2 - 4 =$$

$$= ((x_1 + x_2)^2 - 2x_1x_2) + \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - \frac{2}{x_1x_2} - 4 =$$

$$= ((x_1 + x_2)^2 - 2x_1x_2) + \left(\frac{x_2 + x_1}{x_1x_2}\right)^2 - \frac{2}{x_1x_2} - 4 =$$

$$= (4^2 - 2 \cdot 1) + \left(\frac{4}{1}\right)^2 - \frac{2}{1} - 4 = 14 + 14 - 4 = 24$$

ay 15

18

Dano:

$$x^2 - 5x + a^2 - 2a + 1 = 0$$

$$a \in \mathbb{R}$$

a - min?

 x_1, x_2 - min. yu-e.

Теорема Виета

$$x^2 - 5x + \underbrace{a^2 - 2a + 1}_c = 0$$

$$x_1 x_2 = c = a^2 - 2a + 1 = (a-1)^2$$

$$(a-1)^2 \geq 0, \text{ т.к. квадрат.}$$

$$a-1 \geq 0$$

$$a \geq 1, \text{ т.е. } a \in [1; +\infty)$$

Т.к. квадрат. наимень min. yu-e a, то a = 1.

$$\text{Если min } a = 1 \text{ yu-e } a^2 - 2a + 1 = x_1 x_2 = 0$$

$$\text{Отсюда: } x_1 x_2 = 0 \text{ -- min yu-e}$$