Д/з
5.01.

2) $\lim\limits_{x\to 4} \dfrac{2^x - 16}{\sin \pi x} = \dfrac{2^4 - 2^4}{\sin 4\pi} = \dfrac{0}{0} =$

Пусть $x - 4 = t$, т.к. при $x \to 4$, $t \to 0$
$x = t + 4$, то

$\lim\limits_{t\to 0} \dfrac{2^{t+4} - 2^4}{\sin \pi (t+4)} = \lim\limits_{t\to 0} \dfrac{2^t \cdot 2^4 - 2^4}{\sin(\pi t + 4\pi)} =$

$= \lim\limits_{t\to 0} \dfrac{2^4 (2^t - 1)}{\sin(\pi t + 4\pi)} = 16 \lim\limits_{t\to 0} \underbrace{\dfrac{(2^t - 1)}{t}}_{\ln 2} \cdot \dfrac{t}{\sin(\pi t + 4\pi)} =$

$= 16 \ln 2 \lim\limits_{x\to 0} \dfrac{t}{\sin \pi t \cdot \cos 4\pi + \sin 4\pi \cdot \cos \pi t} =$

$= 16 \ln 2 \lim\limits_{x\to 0} \dfrac{t}{\sin \pi t} = 16 \ln 2 \lim\limits_{x\to 0} \dfrac{\pi t}{\pi (\sin \pi t)} =$

$= 16 \ln 2 \lim\limits_{t\to 0} \dfrac{1}{\pi} = \dfrac{16 \ln 2}{\pi}$

$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \sin\beta \cdot \cos\alpha$

$\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \ln a$

1) $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x} - 2}{3 \operatorname{arctg} x} = \dfrac{\sqrt{4+0} - 2}{3 \operatorname{arctg} 0} = \dfrac{0}{0} =$

$= \left| \operatorname{arctg} x = x, \ x \to 0 \right| = \lim\limits_{x\to 0} \dfrac{\sqrt{4+x} - 2}{3 \cdot x} =$

$= \frac{1}{3} \lim_{x \to 0} \frac{\sqrt{4+x^2}-2}{x} = \frac{1}{3} \lim_{x \to 0} \frac{(\sqrt{4+x^2}-2)(\sqrt{4+x^2}+2)}{x(\sqrt{4+x^2}+2)} =$

$= \frac{1}{3} \frac{4+x-4}{x(\sqrt{4+x^2}+2)} = \frac{1}{3} \lim_{x \to 0} \frac{1}{(\sqrt{4+x^2}+2)} = \frac{1}{3} \cdot \frac{1}{\sqrt{4+0}+2} =$

$= \frac{1}{3 \cdot 4} = \frac{1}{12}$

3 $\lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{\sin x} - e^{\sin 4x}} = \frac{\cos\frac{\pi}{2}}{e^{\sin\frac{\pi}{2}} - e^{\sin\frac{4\pi}{2}}} = \frac{0}{0}$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$= \lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{\sin x} - e^{2\sin 2x \underbrace{\cos 2x}_{\cos 2\pi = 1}}} =$

$\sin\frac{\pi}{2} = 1$

$\cos 2\pi = 1$

$\cos \pi = -1$

$\cos\frac{\pi}{2} = 0$

$= \lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{\sin x} - e^{2 \cdot 2\sin x \cdot \cos x}} =$

$= \lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{\sin x} - e^{-4\sin x}} = \lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{2\sin\frac{x}{2}\cos\frac{x}{2}} - e^{-8\sin\frac{x}{2}\cos\frac{x}{2}}}$

$= \lim_{x \to \pi} \frac{\cos\frac{x}{2}}{e^{2\cos\frac{x}{2}} - e^{-8\cos\frac{x}{2}}} = \frac{0}{0} >$ $\left| \begin{array}{l} \cos\frac{x}{2} = t, \ x \to \pi, \ t \to 0 \\ \text{т.к. } \cos\frac{\pi}{2} = 0 \end{array} \right.$

$= \lim_{t \to 0} \frac{t}{e^{2t} - e^{-8t}} = \lim_{t \to 0} \frac{t}{e^{2t}(1 - e^{-10t})} =$

$= -\lim_{t \to 0} \frac{t}{(e^{-10t} - 1)} = \left(-\frac{1}{10}\right) \lim \frac{-10t}{(e^{-10t}-1)} = -\frac{1}{10}$

$\frac{1}{10}$ 3п.

4. $\lim\limits_{x\to 0} \dfrac{e^{2x}-e^{x}}{\sin 3x - \sin 5x} = \dfrac{0}{0} = (*)$

$$\boxed{\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}}$$

$= (*) \lim\limits_{x\to 0} \dfrac{e^{x}(e^{x}-1)}{2\sin\left(\frac{3x-5x}{2}\right)\cdot\cos\left(\frac{3x+5x}{2}\right)} =$

$= \lim\limits_{x\to 0} \dfrac{\overset{1}{e^{x}}(e^{x}-1)}{2\sin(-x)\cdot\underset{1}{\cos 4x}} = \left| \begin{array}{l} \cos 4x = \cos 4\cdot 0 = 1 \\ e^{x} = e^{0} = 1 \end{array} \right| =$

$= -\dfrac{1}{2} \lim\limits_{x\to 0} \cdot \dfrac{e^{x}-1}{\sin x} = -\dfrac{1}{2} \lim \dfrac{(e^{x}-1)}{\underset{1 зп}{x}} \cdot \dfrac{x}{\underset{1 зп}{(\sin x)}} =$

$= -\dfrac{1}{2}$

5. $\lim\limits_{x\to 1} \dfrac{1-x}{\log_2 x} = \dfrac{1-1}{\log_2 1} = \dfrac{0}{0} = \left| \begin{array}{l} 1-x=t, \; x\to 1, \; t\to 0 \\ x = 1-t \end{array} \right.$ $\quad \times(-1)$

$= \lim\limits_{t\to 0} \dfrac{t}{\log_2(1-t)} = \lim\limits_{t\to 0} \dfrac{t}{\log_2(1+(-t))} =$

$$= -1 \cdot \lim_{t \to 0} \frac{-t}{\log_2(1+1-t)} =$$

$$= (-1) \cdot \frac{1}{\ln(2)} \lim_{t \to 0} \frac{1}{\frac{\log_2(1+1-t)}{-t}} = -\frac{1}{\frac{1}{\ln 2}} =$$

$$= -\ln 2$$

$$\lim_{t \to 0} \frac{\log_a(1+t)}{t} = \frac{1}{\ln a}$$

6. $\lim_{x \to 0} \cos x^{\frac{1}{\ln(1+\sin^2 x)}} = \cos 0^{\frac{1}{\ln(1+\sin^2 0)}}$ $\ln(1)=0$

$$= 1^{\frac{1}{0}} = 1^{\infty} =$$

$$= \lim_{x \to 0} \Bigg[\underbrace{\big(1+(\cos x - 1)\big)^{\frac{1}{\cos x - 1}}}_{e \ (\text{II зп})} \Bigg]^{(\cos x - 1) \cdot \frac{1}{\ln(1+\sin^2 x)}} =$$

$$= e^{\lim_{x \to 0} \frac{\cos x - 1}{\ln(1+\sin^2 x)}} = e^{-\lim_{x \to 0} \frac{1-\cos x}{\ln(1+\sin^2 x)}}$$

$$\lim_{x \to 0} \frac{\cos x - 1}{\ln(1+\sin^2 x)} = \frac{\cos 0 - 1}{\ln(1+\sin^2 0)} = \frac{0}{\ln(1)} = \frac{0}{0}$$

$$= -\lim_{x \to 0} \frac{1-\cos x}{\ln(1+\sin^2 x)} = \frac{2\sin^2\frac{x}{2}}{\sin^2 x} \cdot \boxed{\frac{\sin^2 x}{\ln(1+\sin^2 x)}} = \quad 1 \text{ зп} \qquad 1-\cos x = 2\sin^2\frac{x}{2}$$

$$= e^{-2\lim_{x \to 0} \frac{\sin^2\frac{x}{2}}{\sin^2 x}} = \left| \begin{array}{l} \sin\frac{x}{2} = x \\ \sin x = x \end{array} \Rightarrow \right|$$

$$= e^{-2\lim\limits_{x\to 0}\frac{\left(\frac{x}{2}\right)^2}{x^2}} \quad = -2\cdot\frac{x^2}{4}\cdot\frac{1}{x^2} = -\frac{2}{4} = -\frac{1}{2}.$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

7) $$\lim_{x\to 0}\left[\sin(x+2)\right]^{\frac{3}{3+x}} = \sin(0+2)^{\frac{3}{3+0}} =$$

$$= \sin 2^{1} = \sin 2$$

8. $$\lim_{x\to\frac{\pi}{2}} \sin x^{6\,tg x\cdot tg 3x} = \sin\frac{\pi}{2}^{6\,tg\frac{\pi}{2}\cdot tg 3\frac{\pi}{2}} =$$
$$1$$

$$= 1^{\infty} = (*)$$

$$\lim_{x\to a} u(x)^{v(x)} = e^{\lim\limits_{x\to a}\left[(u(x)-1)\cdot v(x)\right]}$$

$$=(*)\ e^{\lim\limits_{x\to\frac{\pi}{2}}\left[(\sin x-1)\cdot(6\,tg x\, tg 3x)\right]} =$$

$$= (*) \; e^{\lim\limits_{x \to \frac{\pi}{2}} (\sin\frac{\pi}{2} - 1)(6 \, tg\frac{\pi}{2} \cdot tg\frac{3\pi}{2})} = 0 \cdot \infty = [*]$$

$$= 6 \lim_{x \to \frac{\pi}{2}} (\sin x - 1)\left(\frac{\sin x}{\cos x}^{-1} \cdot \frac{\sin 3x}{\cos 3x}^{-1}\right) =$$

$$= 6 \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - 1)\cdot(-1)}{\cos x \cdot \cos 3x} = \left| -6 \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - 1)}{\cos x \, (4\cos^3 x - 3\cos x)} \right| =$$

$$\cos 3x = \cos(2x + x) = \cancel{\cos 2x + \cos x} =$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$= \cos 2x \cdot \cos x - \sin 2x \cdot \sin x =$$

$$\cancel{= (2\cos^2 x - 1)\cos x -} \quad \cancel{= (\cos^2 x - \sin^2 x)\cos x -}$$

$$\cancel{- 2\sin x \cos x \cdot \sin x} \quad \cancel{= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x}$$

?

$$= \left| \begin{array}{l} \cos^2 x = \frac{1 + \cos 2x}{2} \\ \cos 2x = 2\cos^2 x - 1 \\ \sin^2 x = \frac{1 - \cos 2x}{2} \\ 2\sin^2 \cancel{x} = 1 - \cos 2x \end{array} \right|$$

$$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x =$$

$$= 2(\cos^2 x - 1)\cos x - \underline{2\sin^2 x} \cos x =$$

$$= 2(\cos^2 x - 1)\cos x - (1 - \underline{\cos 2x})\cos x$$

$$=-6\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\cos^2 x\,(\underbrace{4\cos^2\tfrac{x}{2}-3}_{0})}=\left|\begin{array}{l}4\cos^2\frac{x}{2}=\\ =4\left(\frac{1+\cos 2x}{2}\right)=\\ =4\left(\frac{1+\cos\frac{\pi}{2}}{2}\right)=\\ =4\left(\frac{1+\cos\pi}{2}-1\right)=\frac{4\cdot 0}{2}=0\end{array}\right.$$

$$=-6\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{-3\cos^2 x}=$$

$$=\frac{-6}{-3}\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\cos^2 x}=2\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{1-\sin^2 x}=$$

$$=2\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{(1-\sin x)(1+\sin x)}=-2\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(1-\sin x)(1+\sin x)}=$$

$$=-2\lim_{x\to\frac{\pi}{2}}\frac{1}{1+\sin x}=-2\frac{1}{1+\underbrace{\sin\frac{\pi}{2}}_{1}}=-\frac{2}{2}=-1$$

$$=[*]=e^{-1}$$

9. $$\lim_{x\to 2}\left(\frac{\sqrt{x+2}-2}{x^2-4}\right)^{\frac{1}{x}}=\left(\frac{\sqrt{2+2}-2}{2^2-4}\right)^{\frac{1}{2}}=\frac{0}{0}=$$

$$=\lim_{x\to 2}\left[\frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(x-2)(x+2)(\sqrt{x+2}+2)}\right]^{\frac{1}{2}}=$$

$$=\lim_{x\to 2}\left[\frac{x-4+2}{(x-2)(x+2)(\sqrt{x+2}+2)}\right]^{\frac{1}{2}}=$$

$$=\lim_{x\to 2}\left[\frac{\cancel{x-2}}{(\cancel{x-2})(x+2)(\sqrt{x+2}+2)}\right]^{\frac{1}{2}}=\left(\frac{1}{\underbrace{(2+2)}_{4}\underbrace{(\sqrt{2+2}+2)}_{4}}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

10. $$\lim_{x\to 0} \frac{\sqrt[3]{tg\,x}\ arctg\,\frac{1}{x}+3}{2-tg(1+\sin x)} =$$

$$= \lim_{x\to 0} \frac{\sqrt[3]{tg\,0}^{\to 0}\cdot arctg\,\frac{1}{0}^{\to\infty}+3}{2-tg(1+\sin \underset{\to 0}{0})} = \frac{\overbrace{0\cdot\infty}^{0}+3}{2-tg\,1} = \frac{3}{2-tg\,1}$$