

Day 14 NY

$$x^2 - 5x + 4 = 0$$

$$x_1 x_2 = 4 \Rightarrow x_1 = 4$$

$$x_1 + x_2 = 5 \Rightarrow x_2 = 1$$

Orbit: 4 NY

$$x^2 - 5x + 2 = 0$$

$$x_1 x_2 = 2 \Rightarrow x_1 = 2$$

$$x_1 + x_2 = 5 \Rightarrow x_2 = 3$$

Orbit: 1 NY

$$x^2 - 13x + 12 = 0$$

$$x_1 x_2 = 12 \Rightarrow x_1 = 12$$

$$x_1 + x_2 = 13 \Rightarrow x_2 = 1$$

Orbit: 1, 12

NY

$$x^2 + 14x + 45 = 0$$

$$x_1 x_2 = 45 \Rightarrow x_1 = -5$$

$$x_1 + x_2 = -14 \Rightarrow x_2 = -9$$

Orbit: -5, -9

NY

$$x^2 + 3x - 40 = 0$$

$$x_1 x_2 = -40 \Rightarrow x_1 = -10$$

$$x_1 + x_2 = -3 \Rightarrow x_2 = 7$$

Orbit: -10, 7

NY

$$x^2 - 12x + 35 = 0$$

$$x_1 x_2 = 35 \Rightarrow x_1 = 5$$

$$x_1 + x_2 = 12 \Rightarrow x_2 = 7$$

Orbit: 5, 7

$$\frac{2x-1}{x-1} = \frac{7x-1}{2x+2}$$

$$(2x-1)(2x+2) = (7x-1)(x-1)$$

$$4x^2 + 4x - 2 = 7x^2 - 7x - x + 1$$

$$-3x^2 + 10x - 3 = 0 \quad | \cdot (-1)$$

$$3x^2 - 10x + 3 = 0$$

$$D = 10^2 - 4 \cdot 3 \cdot 3 = 64 = 8^2$$

$$x_1 = \frac{-6 \pm 8}{2 \cdot 3} = \frac{10 \pm 8}{6} \Rightarrow \begin{cases} x_1 = \frac{18}{6} = 3 \\ x_2 = \frac{2}{6} = \frac{1}{3} \end{cases}$$

Orbit: 3, 1/3

N7

$$x^4 - 3x^2 - 4 = 0$$

$$(-) (x^2)^2 - 2 \cdot x^2 - 2^2 = 0 \quad \text{or}$$

Binomische Formel: $ax^2 + bx + c = 0$ $\frac{dx}{dx} = \frac{ac}{bx}$ $\frac{de}{dx} = \frac{ac}{bx}$

$$de = 1 \cdot (-4) = -4 \quad \text{so } d = -4$$

$$d + e = -3 \quad e = 1$$

$$x^4 - 4x^2 + x^2 - 4 = 0$$

$$x^2(x^2 - 4) + (x^2 - 4) = 0$$

$$(x^2 - 4)(x^2 + 1) = 0$$

$$(x - 2)(x + 2)(x^2 + 1) = 0$$

$$x = 2; x = -2; \quad x^2 + 1 = 0$$

$$x^2 = -1 \Rightarrow \emptyset$$

Orbit: -2; 2

N9

$$(1+x)(14+x) = 304$$

$$11 \cdot 14 + 11x + 14x + x^2 - 304 = 0$$

$$x^2 + 25x - 150 = 0$$

$$x_1, x_2 = -150 \quad x_1 = -30$$

$$x_1 + x_2 = -25 \quad x_2 = 5$$

Orbit: -30; 5

N2

$$x^2 - 2ax + a^2 = 0$$

$$\frac{x}{a} = ?$$

$$I (x-a)(x-a) = 0$$

$$x-a=0$$

$$II -2ax = -x^2 - a^2$$

$$2ax = x^2 + a^2$$

$$ax = \frac{x^2 + a^2}{2}$$

$$III x^2 - 2ax + a^2 = 0$$

$$\frac{x^2}{a^2} - \frac{2ax}{a^2} + \frac{a^2}{a^2} = 0$$

$$\frac{x^2}{a^2} - \frac{2x}{a} + 1 = 0$$

$$\frac{dx}{dx} = \frac{x^2}{a^2} + 1$$

$$\frac{x}{a} + \frac{x}{a} = \left(\frac{x}{a}\right)^2 + 1$$

$$\frac{x}{a} = \left(\frac{x}{a}\right)^2 + 1 - \frac{x}{a}$$

$$x^2 - 2x + a^2 = 0$$

$$\frac{x^2}{a} - \frac{2x}{a} + \frac{a^2}{a} = 0$$

$$\frac{x^2}{a} - \frac{2x}{a} + a = 0$$

$$-\frac{2x}{a} = \frac{x^2}{a} - a$$

$$\frac{x}{a} =$$

$\frac{x}{a} = ?$

$$a \cdot 6 = 1 \cdot 1 \cdot 3 = 4$$

$$6 \cdot 3 = 2 \cdot 2$$

$$\frac{4 \cdot 6}{3} = \frac{6}{3} =$$

$$\frac{4 \cdot 6}{3} = 4 \Rightarrow$$

$$= 7 \frac{6}{3} \cdot 4 = 2$$

day 14

Найдем сумму кубов 5 чисел

$$a_1 = 1, q = 7$$

$$S_5 = a_1 \cdot \frac{q^5 - 1}{q - 1} = 1 \cdot \frac{7^5 - 1}{7 - 1} =$$

$$= 1 \cdot \frac{16807 - 1}{6} = 1 \cdot \frac{16806}{6} = 2801$$

N5(-)

$$a_1 = 5, q = 7$$

$$a_2 = a_1 \cdot q = 5 \cdot 7 = 35$$

$$a_3 = a_2 \cdot q = 35 \cdot 7 = 245$$

$$a_4 = a_3 \cdot q = 245 \cdot 7 = 1715$$

$$a_5 = a_4 \cdot q = 1715 \cdot 7 = 12005$$

$$a_1 = 5, q = 7$$

$$a_2 = 35, q = 7$$

$$a_3 = 245, q = 7$$

$$a_4 = 1715, q = 7$$

$$a_5 = 12005, q = 7$$

$$a_6 = 84035, q = 7$$

$$a_7 = 588245, q = 7$$

$$a_8 = 4117715, q = 7$$

$$a_9 = 28824005, q = 7$$

$$a_{10} = 201768035, q = 7$$

$$a_{11} = 1412376245, q = 7$$

$$a_{12} = 9886633715, q = 7$$

$$a_{13} = 69206436005, q = 7$$

day 14

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$$= 1 \cdot \frac{16807 - 1}{6} = 1 \cdot \frac{16806}{6} = 2801$$

N5(-)

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$$a_3 = a_2 \cdot q = 35 \cdot 7 = 245$$

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$$a_5 = a_4 \cdot q = 1715 \cdot 7 = 12005$$

$$a_1 = 5, q = 7$$

$$a_2 = 35, q = 7$$

$$a_3 = 245, q = 7$$

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$$a_{11} = 1412376245, q = 7$$

$$a_{12} = 9886633715, q = 7$$

$$a_{13} = 69206436005, q = 7$$

N6(-)

day 14

Найдем сумму кубов 5 чисел

$$a_1 = 1, q = 7$$

$$a_2 = a_1 \cdot q = 1 \cdot 7 = 7$$

$$a_3 = a_2 \cdot q = 7 \cdot 7 = 49$$

$$a_4 = a_3 \cdot q = 49 \cdot 7 = 343$$

$$a_5 = a_4 \cdot q = 343 \cdot 7 = 2401$$

$$a_1 = 1, q = 7$$

$$a_2 = 7, q = 7$$

$$a_3 = 49, q = 7$$

$$a_4 = 343, q = 7$$

$$a_5 = 2401, q = 7$$

$$a_6 = 16807, q = 7$$

$$a_7 = 117649, q = 7$$

$$a_8 = 823543, q = 7$$

$$a_9 = 5724601, q = 7$$

$$a_{10} = 39810067, q = 7$$

$$a_{11} = 278670469, q = 7$$

$$a_{12} = 1950693283, q = 7$$

$$a_{13} = 13654852981, q = 7$$

$$a_{14} = 95583970867, q = 7$$

$$a_{15} = 669087806069, q = 7$$

$$a_{16} = 4683614642483, q = 7$$

$$a_{17} = 32785302497381, q = 7$$

$$a_{18} = 229497117481667, q = 7$$

$$a_{19} = 1606480822371669, q = 7$$

$$a_{20} = 11245365756601683, q = 7$$

$$a_{21} = 78717560296211781, q = 7$$

$$a_{22} = 551022922073482467, q = 7$$

$$a_{23} = 3857160454514377269, q = 7$$

$$a_{24} = 26999123181600640883, q = 7$$

Day 14

13.

$$\begin{array}{l|l}
 r \cdot n = 5 & b_n = b_1 \cdot q^{n-1} \\
 a_3 = 20 & a_3 = a_1 \cdot q^{2-1} \\
 q = ? & 20 = 5 \cdot q^2
 \end{array}$$

$$\begin{aligned}
 q^2 &= 20 : 5 \\
 q^2 &= 4 \\
 q &= \pm 2
 \end{aligned}$$

$q = \pm 2$, т.к. прогрессия decreasing
 decreasing, но по условию $q > 1 > 0$
 $q = 2$

$$\begin{array}{l|l}
 \infty \cap n \rightarrow \infty & \\
 a_1 = 1, & \\
 q = \frac{1}{2} & \\
 \Sigma a_n = ? &
 \end{array}$$

н.д.

$$\begin{aligned}
 S_n &= \frac{b_1(1-q^n)}{1-q} \quad \text{но т.к. } r \cdot n \rightarrow \infty, \text{ то} \\
 S_n &= \frac{b_1}{1-q}
 \end{aligned}$$

$$\cancel{S_n = \frac{b_1(1-q^n)}{1-q}}$$

$$S_n = \frac{1}{1-\frac{1}{2}} = 1 : \frac{1}{2} = 2$$

$$2 \cdot 2 + 2 \equiv 0 \pmod{4} \quad (\text{rem. on } 4 = 0)$$

$$2 \cdot 3 + 2 + 2 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 4 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 5 + 2 \equiv 0 \pmod{4}$$

$$\text{Order: } 2, 6, 10, \dots, 28 \quad (2 \cdot 14 = 28)$$

$$2 \cdot 15 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 16 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 17 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 18 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 19 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 20 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 21 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 22 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 23 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 24 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 25 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 26 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 27 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 28 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 29 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 30 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 31 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 32 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 33 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 34 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 35 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 36 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 37 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 38 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 39 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 40 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 41 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 42 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 43 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 44 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 45 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 46 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 47 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 48 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 49 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 50 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 51 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 52 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 53 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 54 + 2 \equiv 0 \pmod{4}$$

$$2 \cdot 55 + 2 \equiv 0 \pmod{4}$$

Арифметика остатков (м-то при умножении)

Остаток - это то что останется.

Если a на b , то $a = k \cdot b + r$ - то остаток

что r по b - $0, \dots, b-1$

всех чисел, которые

остаток $b-1$

Остаток от деления м-то неограничен (все возможные!)

т.е. $-100 = -14 \cdot 7 - 2$ - не верно! и т.д.

не и. д. делится

Какой наименьший делитель

7-го $k = 100$, кем? на 7 по

мо не требуется! $100 - 105 = -5$

$-100 = -15 \cdot 7 + 5$ на $105 - 100$

5-4-3-2-1-0-1-2-3-4-5-6-7-8-9

7-я ком. числом от 2-х чис. ка? 2, 9, 16... (было)

$2 + 17 = -5 + 17 = 12$ и $17 = -19 + 36$

$2 + 17 = -5 + 17 = 12$ и $17 = -19 + 36$

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$2 + 17 = -5 + 17 = 12$ и $17 = -19 + 36$

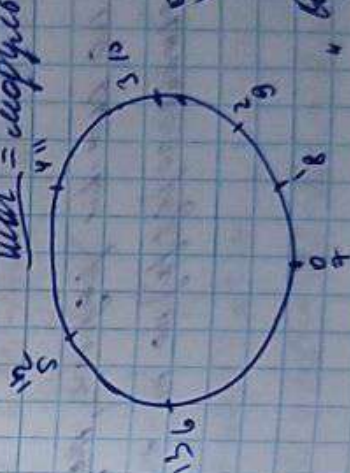
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$2 + 17 = -5 + 17 = 12$ и $17 = -19 + 36$



Сравнение/Равенство $a \equiv b \pmod{m}$
 означает, что a и b дают одинаковый остаток при делении на m .
 т.е. $a = k_1 \cdot m + r$ и $b = k_2 \cdot m + r$, где r - остаток от деления на m .
 А сравнение с 0 по модулю m .

N2.

$$1+4+7+\dots+x=925 \quad \text{— дано: ариф. прогр.}$$

$$\Rightarrow a_1=1; d=7-4=3; a_n=x; a_1+d(n-1)=x$$

$$n = \frac{x+1}{3}$$

$$S_n = \frac{2a_1+d(n-1)}{2} \cdot n =$$

$$= \frac{2 \cdot 1 + 3(n-1)}{2} \cdot n, \text{ м.к. } n = \frac{x+2}{3}, \text{ мо}$$

$$S = \frac{2+3(n-1)}{2} \cdot n = \frac{2+3\left(\frac{x+2}{3}\right)-3}{2} \cdot \frac{x+2}{3} =$$

$$= \frac{2+x+2-3}{2} \cdot \frac{x+2}{3} = \frac{x+1}{2} \cdot \frac{x+2}{3} = \frac{(x+1)(x+2)}{6} = 925$$

$$x^2+2x+x+2=925 \cdot 6$$

$$x^2+3x+2=5550$$

$$x^2+3x-5548=0$$

$$D=3^2-4 \cdot 1 \cdot (-5548)=22201=149^2$$

$$x_1 = \frac{-3 \pm \sqrt{D}}{2a} = \frac{-3+149}{2} = \frac{146}{2} = 73$$

Ответ: $x=73$. Ариф. прогр. не имеет цел. ч. к. с. пр. 20.

N3.

$$\{a_n\}_1^{100} \text{ — ариф. прогр. со 100 ч. и.ч.}$$

$$a_1=5 \Rightarrow d_a = a_2 - a_1 = 8 - 5 = 3$$

$$a_2=8 \quad \{b_n\}_1^{100} \text{ — ариф. прогр. со 100 ч. и.ч.}$$

$$b_1=3 \quad | \Rightarrow d_b = b_2 - b_1 = 7 - 3 = 4$$

$$b_2=7$$

Найдем к-во ч. в.п.х.

$$d_b - d_a = 1, \text{ м.к. } 71 \in a_{ub}, \text{ если}$$

$$8 \quad |k-b|=1$$

$$k \in [0, 99]$$

$$a_k = \frac{a_1+d(k-1)}{1} \Rightarrow a_{k+1} = 5+3k$$

$$b_{L+1} = 3+4L$$

Решим уравнение: $a = b \Rightarrow$

$$3k+5=4L+3$$

$$4L=3k+5-3=3k+2$$

$$L = \frac{3k+2}{4}, \text{ т.к. } L \text{ — целое (но не 0), мо } 3k+2=0 \pmod{4}$$

119

100 ч.

5, 8, 11, 14, 17, 20

100 ч.

3, 7, 11, 15, 19, 23

day 18

Task:

$$a_1 + a_2 + a_3 = 102$$

$$a_1 = 15$$

$$a_{10} = ?$$

$$102 = 15 + a_n \cdot 3$$

$$159 = 3a_n$$

$$a_n = 159 : 3 = 53$$

$$a_3 = 53$$

$$a_3 = a_1 + d(n-1)$$

$$53 = 15 + d(3-1)$$

$$38 = 2d$$

$$d = 19$$

$$a_{10} = a_1 + d(n-1) = 15 + 19(10-1) = 15 + 19 \cdot 9 =$$

$$= 15 + 171 = 186$$

Order 186

Task a.p. n.p.

$$a_{10} = 15$$

$$a_5 = 5$$

$$a_1 = ?$$

$$a_{10} = a_1 + d(n-1); a_5 = a_1 + d(n-1)$$

$$a_1 = a_n - d(n-1) \quad a_1 = 5 - (5-1) = 5-4 = 1$$

$$15 - d(10-1) = 5 - d(5-1)$$

$$15 - 15d + d = 5 - 5d + d$$

$$10 - 10d = -7d = 7$$

$$15 - 10d + d = 5 - 5d + d$$

$$10 = 5d$$

$$d = 2$$

$$a_1 = a_5 - d(n-1) = 5 - 2(5-1) = 5 - 8 = -3$$

$$\text{Order: } a_1 = -3$$

186

Arithmetic progression. Variation.

Long run a.p. n.p., namely so 1-10,

problem given. Arithmetic progression with common difference.

$$\Rightarrow a_n = \frac{a_{n+1} + a_{n-1}}{2}$$

day 17

N4

$$\sum 5 \rightarrow n \text{ rang } n+1 = 100, a_1 = ?$$

$$\sum 5 = 100$$

$$n = 5$$

Princ. 1.

5 rang \rightarrow same after in a.p. $\therefore d = 1 \Rightarrow$

$$S_n = \frac{n}{2} (2a_1 + d(n-1))$$

$$S_n = \frac{n}{2} (2a_1 + d(n-1))$$

$$100 = \frac{5}{2} (2a_1 + 1(5-1))$$

$$100 = \frac{5 \cdot 2a_1 + 5 \cdot 4}{2}$$

$$100 = 5a_1 + 10$$

$$a_1 = \frac{90}{5}$$

$$a_1 = 18$$

Princ. 2.

$$d = 12$$

$$a_3 = 43$$

$$a_1 = ?$$

N6

$$a_n = a_1 + d(n-1)$$

$$a_3 = a_1 + d(3-1)$$

$$43 = a_1 + 12(3-1)$$

$$43 = a_1 + 24$$

$$a_1 = 19$$

day 17

N4

Princ. 1.

$$a_1 = 15$$

$$a_2 = 8$$

$$a_{19} = ?$$

$$d = a_2 - a_1 = 8 - 15 = -7$$

$$a_{19} = a_1 + d(n-1) = 15 + (-7)(19-1) = 15 - 126 = -111$$

N8

Princ. 1.

$$a_1 = 1$$

$$d = 1$$

$$a_{1083} = ?$$

$$a_{1083} = a_1 + d(n-1) = 1 + 1(1083-1) = 1 + 1082 = 1083$$

N9

Princ. 1.

$$a_2 = 5$$

$$a_1 = -11$$

$$d = ?$$

$$d = a_2 - a_1 = 5 - (-11) = 5 + 11 = 16$$

N10

Princ. 1.

$$a_5 = 18$$

$$a_2 = 9$$

$$d = ?$$

$$a_n = a_1 + d(n-1) \Rightarrow d = \frac{a_n - a_1}{n-1}$$

$$d_5 = \frac{18 - 9}{5-2} = \frac{9}{3} = 3$$

$$\frac{18 - a_1}{4} = \frac{9 - a_1}{1}$$

$$18 - a_1 = 36 - 4a_1$$

$$3a_1 = 18 \Rightarrow a_1 = 6$$

$$d = 9 - a_1 = 9 - 6 = 3$$