Gravitational Lensing in Modified Einstein-Maxwell Gravity

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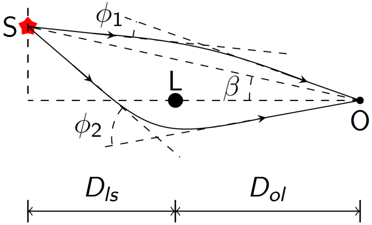
Gravitational Lensing

The idea of gravitational lensing was first proposed by A. Einstein as an important consequence of general relativity.
Photons and other particles bend a significant angle when observed from a distance.



Nowadays, GL is applied to

- probe the mass of celestial bodies and galaxies;
- test dark matter model;
- test various quantum gravity modifications . . .

Deflection Angle in SSS Metrics

The general form of a static spherically symmetric (SSS) metric:

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + C(r)(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$$

Expand the metric functions at weak field limitation ($D_{ol}, D_{ls} \gg r_0 \gg M$) and calculate the change of angular coordinate

$$\begin{aligned}\Delta\phi &= \left[\int_{r_0}^{r_s} + \int_{r_0}^{r_d}\right]\sqrt{\frac{B}{C}}\frac{L}{\sqrt{(E^2/A-\kappa)C-L^2}}\mathrm{d}r \\ &\equiv \left[\int_{\sin\theta_s}^{1} + \int_{\sin\theta_d}^{1}\right] y\left(\frac{u}{b}\right)\frac{\mathrm{d}u}{\sqrt{1-u^2}}\end{aligned}$$

Our final result is expressed in power series form

$$y\left(\frac{u}{b}\right) = \sum_{n=0}^{\infty} y_n \left(\frac{u}{b}\right)^n$$

Modified Einstein-Maxwell Action

Consider a non-minimally coupled Einstein-Maxwell action:

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{|g|} \{R - F_{\mu\nu} F_{\rho\sigma} \chi^{\mu\nu\rho\sigma}\}$$

Instead of taking the usual form $\chi^{\mu\nu}{}_{\rho\sigma} = \delta^{\mu\nu}{}_{\rho\sigma}$, we choose this tensor in the following way

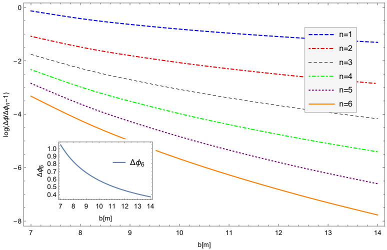
$$\chi^{\mu\nu}{}_{\rho\sigma} = 6\delta^{[\mu\nu}{}_{\rho\sigma} (\mathcal{Q}^{-1})^{\alpha\beta]}{}_{\alpha\beta}$$

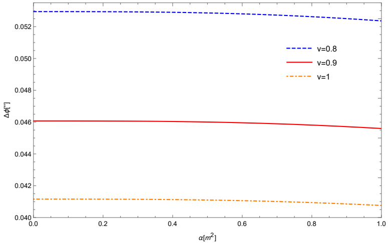
the correction of $\mathcal{Q}^{\mu\nu}{}_{\rho\sigma}$ with respect to $\delta^{\mu\nu}{}_{\rho\sigma}$ is measured by a small constant α with units of length squared.
A general Ansatz for the metric given by this action

$$\mathrm{d}s^2 = -N(r)^2 f(r) \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega^2_{(2)}$$

Deflection Angle for Particles with Different Velocities

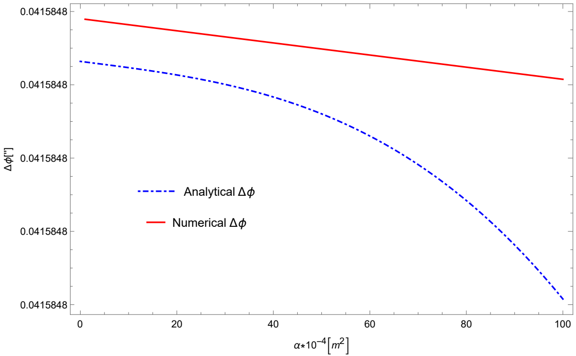
Deflection angle calculated using perturbative and numerical methods (take the analytic results up to the $n-$th order).





- The higher-order result and numerical calculation are really close under a significant precision;
- The deflection angles of particles with different velocities can also be calculated.

Dependence on the Coupling Constant α



The calculation result is valid at a small α. Its contribution reads

$$y_4 \sim -\left(Q^2 + \frac{4Q^2}{v^2}\right)\alpha$$

Prospect

- Extend the theory to more spacetime metrics with or without quantum corrections;
- Calculate the time delay for different particles;
- ...

Reference

1. P. A. Cano, Á. Murcia, arXiv:2006.15149 [hep-th];
2. Junji Jia, arXiv:2001.02038 [gr-qc].

Thank You!