9. Negative feedback amplifiers

If you are enthusiastic about semiconductor physics, it is perfectly feasible to understand the operation of basic semiconductor devices such as diodes and transistors from fundamental physical principles. Then, using mainly transistors, it is fully possible to build all of the electronic circuits you are ever likely to need in a lab environment. The problem is that many transistor circuits, such as a simple amplifier, look easy to build (you can see many transistor amplifier circuits in Horowitz & Hill; they are not particularly complicated), but they tend to be quite tricky to get right in practice (hence the title of that book: "The Art of Electronics"...). It is for this reason that the *operational amplifier* was developed, to take the pain and inconsistency out of discrete amplifier circuits.

> **Learn it in lab**
> You will build a simple inverting amplifier in the second lab session.

9.1. The operational amplifier

The op-amp actually predates the transistor: the first op-amps used vacuum valves instead. These were used in early analogue computers where the designers needed simple circuits to perform mathematical *operations* (hence the name) such as addition, subtraction, integration, differentiation, as well as simple amplification. The op-amp provides repeatability and consistency, that is to say, we can take one op-amp out of a circuit (maybe because we have broken it) and replace it with another and the circuit will behave exactly as before. This is often not the case with transistors. The key point of the op-amp is that we need not know what is going on inside the device as long as we have fully characterised its behaviour. We can then use the op-amp to build really quite complex circuits which have a good chance of working first time. There are also a huge number of different op-amps available on the market with many different kinds of specialist properties. It is for these reasons that physicists tend to prefer the op-amp when constructing circuits. As Figure 9.1 shows, the op-amp package integrates quite a lot of complexity into a tiny physical circuit – the details of which the end-user (or the student!) – need not be concerned with.

Active versus passive. All the electronics we have looked at so far has been *passive circuits* composed of resistors, capacitors, etc, which are not able to boost a signal amplitude or power. By contrast, op-amps are *active devices* which are themselves connected to a power supply, which enables them to actively boost the signal power – either by increasing the voltage, the current, or sometimes both.

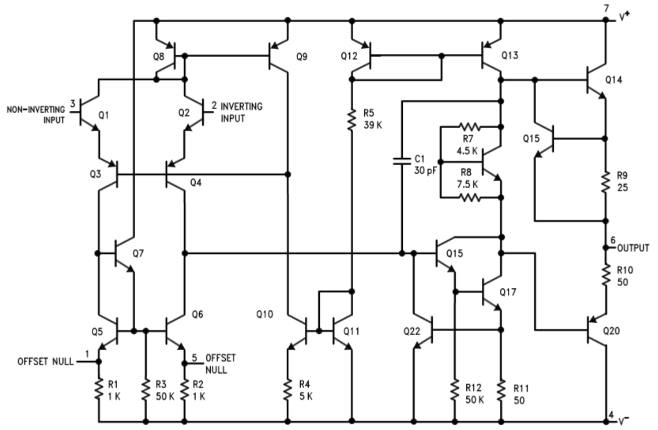

Figure 9.1: Functional circuit of the LM741 op-amp from Texas Instruments (from datasheet: https://www.ti.com/product/LM741).

9.2. Characteristics

The basic op-amp is a *differential amplifier* with a gain A known as the *open-loop gain.* It requires a minimum of 5 connections:

- *Non-inverting input*, often called V_{in+}, and usually labeled simply '+';
- *Inverting input*, often called V_{in-}, labelled '−';
- An output V_{out};
- Two power-supply connections $\pm V_{ss}$, often ±15 V.

Figure 9.2 shows the simplified circuit (with somewhat different labels, in particular the open-loop gain which is more commonly denoted by A).

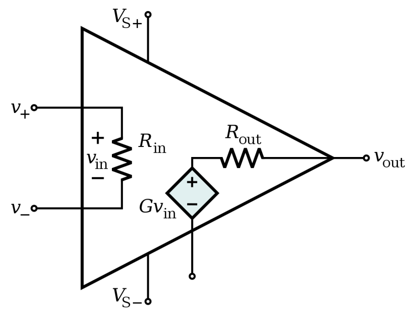

Figure 9.2: The simplified op-amp internal circuit and connections.

The device is governed by three key relations. Firstly,

$$V_{out} = A\left(V_{in+} - V_{in-}\right); \tag{9.1}$$

hence, the idea that this is a differential amplifier. Because it is an active device, the output cannot exceed the power supply "rails" so,

$$-V_{ss} \leq V_{out} \leq +V_{ss}\,. \tag{9.2}$$

If we do try to drive the op-amp above the power supply rails then we will see *clipping*, where the signal is sharply trimmed back to the V_{ss} levels. Finally, while the op-amp is a fast device, the output cannot change instantaneously, and we find there is a maximum *slew rate*

$$S = \left[\frac{\mathrm{d}}{\mathrm{d}t} V_{out}\right]_{\max} , \tag{9.3}$$

typically of the order 1 volt/μs.

The gain A is typically high (at least 100,000), the input impedance is real and high ($> 1\,\mathrm{M}\Omega$) and the output impedance is low (tens of Ω). These facts, and the above relations, are all that is needed to understand the behaviour of any op-amp circuit.

9.3. The comparator

The simplest op-amp circuit is the comparator. This is very effective for comparing two voltage levels and deciding which is greater. Due to the very high gain A and slew-rate S, the op-amp will respond almost instantaneously to any potential difference greater than a microvolt between the two inputs by going straight to one of the two power supply voltages. This is fully consistent with equations (9.1), (9.2) and (9.3). Since the output of the op-amp has the capability to drive some power, we can do something useful with this such as controlling a switch or a relay. For example, if the two voltage inputs represent temperature in an experiment we can do:

```
IF
    temp_A > temp_B
THEN
    switch_off_heater
```

9.4. The voltage-follower or buffer

Simply by looping the output voltage V_{out} back to the inverting input we make a circuit where, to a very good approximation,

$$V_{out} = V_{in} . \tag{9.4}$$

This is illustrated in Figure 9.3. Note that again this is fully consistent with equations (9.1), (9.2) and (9.3). Of course, when we look in detail we see that the output takes some time to track the input due to the finite slew-rate, and also there is some small (μV) difference between V_{out} and V_{in} due to equation (9.1); however, to a very good approximation, we will find that equation (9.4) is valid for most practical purposes.

The practical application of a circuit where $V_{out} = V_{in}$ comes when we consider that the input resistance of the circuit is very high, so it draws negligible current, while its output resistance is very low. The buffer is used as a kind of impedance transformer. This is useful where we have a sensor source which is 'weak' and easily loaded (typically high source resistance). Typically, we will *buffer* or *stiffen* the source using a voltage-follower. The op-amp is then able to drive any current needed for a low-impedance load or a transmission line without pulling any current out of the source. This avoids the problem of loading the sensor.

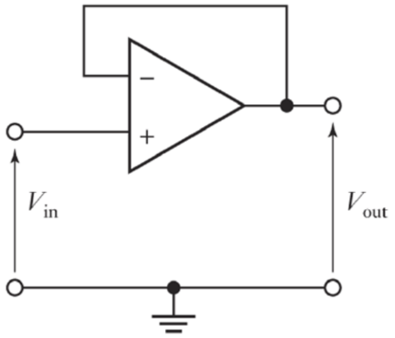

Figure 9.3: The voltage follower, or "buffer".

Feedback. When we connect the output (or some fraction of it) back to the input of a device then we have created a *feedback loop* – see Figure 9.4. In the case of the voltage-follower we have returned all of the output back to the input which is the same as creating a feedback amplifier with a gain of 1. In practice, all real-world op-amp circuits (except the comparator) use this *negative feedback*. So, we can see how the voltage-follower nicely illustrates the concept of the *ideal amplifier*.

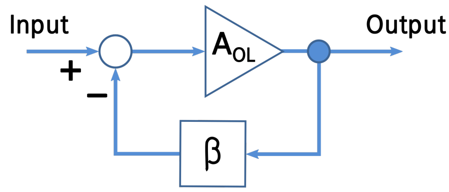

Figure 9.4: Negative feedback.

9.5. The ideal amplifier

Table 9.1 summarises the ideal properties of a perfect amplifier, compared to a readily available device such as the LM741, which will be used during labs. Most of the parameters for the ideal amp are self-explanatory; however, to understand why we want $A \to \infty$ we need to study feedback in more detail. Nevertheless, even in the simple case of the voltage follower, we can see that for $A \to \infty$ we will find $V_{out} \to V_{in}$.

Table 9.1: Ideal amplifier.

Behaviour	Ideal amplifier	Real amplifier
A	$\to \infty$	2×10^5
R_{in}	$\to \infty$	$> 10^6\,\Omega$
R_{out}	0	$\sim 50\,\Omega$
S	$\to \infty$	0.5 V/μs

9.6. Negative feedback

Feedback as applied to operational amplifiers is covered very well in both Horowitz & Hill (Chapter 3) and Diefenderfer & Holton. A illustrated in Figure 9.4 above, the general approach is this: taking an amplifier with very high open-loop gain A and applying negative feedback of *feedback fraction* β will result in an amplifier whose overall *closed-loop gain* is purely a function of the *feedback network*. Given that A is very large, the closed-loop gain will be:

$$G = \frac{A}{1+\beta A} \approx \frac{1}{\beta}\,.$$

Since the feedback network is made from simple passives such as resistors, the closed-loop gain ($G = 1/\beta$) is going to be well determined, stable, and further the circuit will behave the same even if we blow our op-amp and have to put a different one in. You may wonder why we cannot just build transistor amplifiers with sensible values of gain in the first place. There are several answers to this, but the most convincing one is that transistors are tricky things which work best in a very high-gain setup, so it is just easiest to control the gain using feedback. The net result is that the high-gain feedback configuration has the following advantages compared to open-loop amplification:

- Increased linearity and wider bandwidth;
- Increased input impedance and reduced output impedance;
- Reduced gain (or gain brought under 'control');
- Reduced distortion.

Furthermore, since the op-amp is quite close to the 'ideal amplifier' *when connected in negative feedback*, two important *Golden Rules* can be applied:

#1: The output will do whatever is necessary to drive the potential difference between the two inputs to zero;

#2: The inputs draw no current.

These two rules make the analysis of op-amp circuits relatively straightforward, and circuits built using these assumptions and rules will conform to expectations most of the time.

9.7. Control theory

One aspect of feedback not covered in the course texts is *control theory*. A discussion of this may help to illuminate why feedback is such a clever and ubiquitous concept. Indeed, much of control theory is essentially dedicated to the study of feedback. A simple example also shows that feedback is not such a new concept. Consider again the steam engine. The speed of the engine depends on the amount of steam fed in from the boiler, and this is controlled by a valve called the throttle. If the engine is running too slowly we can increase the speed by opening-up the throttle. This is *open-loop control*. Now imagine we ask our engine to do some work such as threshing barley. As the farm-hand puts a pitch-fork of barley into the threshing machine, the engine is *loaded* and will slow-down. The engine operator can compensate for this by opening up the throttle to increase the power; however, once the barley is done then the load will diminish and now the engine will run fast – the operator sees this and closes down the throttle a little. Here we have applied feedback, but the feedback is done by having a human 'in the loop'. Much better to have an automatic system such as a *governor* which is a mechanical system which controls the speed of the engine – see Figure 9.5. If the engine slows down (load applied) then the governor opens-up the throttle, and vice-versa. These usually consist of a pair of rotating weights, which use the centripetal force to regulate the throttle. This is closed-loop control. The key point is that the governor allows the operator to set the desired outcome (i.e. speed) and let the system regulate itself. What is clear is that we can build a really powerful engine and then control this power using the feedback regulator. Under no load the engine will happily idle, and when a big load is requested the engine can respond.

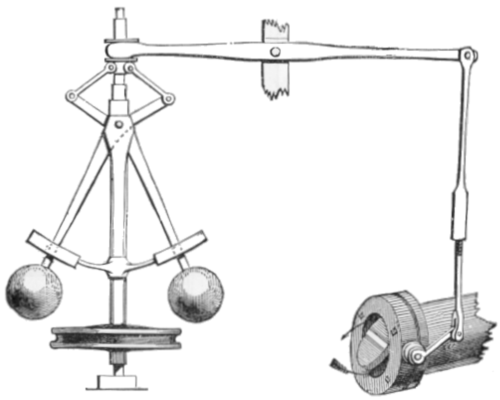

Figure 9.5: The centrifugal "fly-ball" governor: the balls swing out as speed increases, which closes the throttle, until a stable balance is achieved.

We can draw an analogy with the op-amp. Speed becomes output voltage. The governor becomes the feedback loop, whereby we control the net gain of the system. If the amplifier is connected to a low-impedance load then more output current is required and the feedback loop will respond accordingly in order to maintain the desired output voltage. This is another advantage of feedback: the feedback amplifier looks like a very nice voltage source since its output voltage is quite independent of load.

Meaning of negative feedback. Feedback is almost always negative in control theory. This means that the output acts so as to *reduce* the value seen at the input. If the steam engine runs too fast then the governor reduces the amount of steam. For op-amps we find that applying feedback to the negative or inverting input of the amp results in negative feedback according to equation (9.1). This also leads directly to an intuitive understanding that Golden Rule #1 is true: with negative feedback the output will always move to a level which makes the potential difference at the inputs very close to zero. You should ensure that you understand all the circuits in this section with this in mind.

9.8. Common op-amp circuits

The most common op-amp circuits on the syllabus for this course are presented below. You are expected to understand and be able to analyse these (e.g. derive their gains). They are covered also in the usual textbooks.[17]

Non-inverting amplifier. The design in Figure 9.6 illustrates the gain $G = 1/\beta$ relationship, since the feedback fraction is due to the voltage divider formed from two resistors. Note that the voltage-follower or buffer is a non-inverting amplifier with $\beta = 1$. You should try to derive the gain of the non-inverting amplifier as a function of the input and feedback resistors R_1 and R_2.

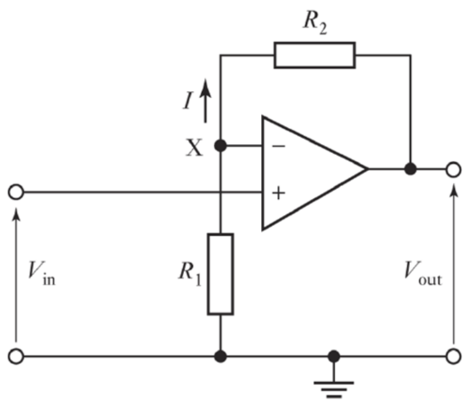

Figure 9.6: The non-inverting amplifier. Note that in this circuit we would avoid the crossing of lines by flipping the amplifier (so the inverting input is at the bottom) – a more common representation.

Inverting amplifier. A basic op-amp configuration is that shown in Figure 9.7, which is expanded on for many of the following designs. This also illustrates well the concept of the *virtual ground*. The virtual ground is not connected to ground (indeed it is isolated from ground by ∼MΩ); however, it is always apparently zero volts, due to Rule #1, which makes circuit analysis easier. The output is inverted (phase shift 180 degrees with respect to input); however, for many AC signals (e.g. audio) this is not important.

[17]Many op-amp datasheets have example circuits such as these, e.g. https://www.ti.com/product/LM741. Although you are not expected to memorise any such datasheet, it is useful to look at some examples to see how parameters of real op-amps are specified.

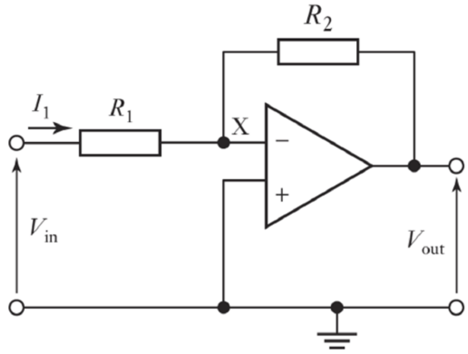

Figure 9.7: The inverting amplifier. Here, 'X' is a virtual ground.

Summing circuit. Kirchhoff's Current Law needs to be invoked here together with the virtual ground, to show that the input current is a function of both input voltages – see Figure 9.8. Whilst the output is inverted this can be fixed by a following inverting amplifier with gain $G = -1$. This circuit could be used, for example, to remove a zero-offset from a sensor's output.

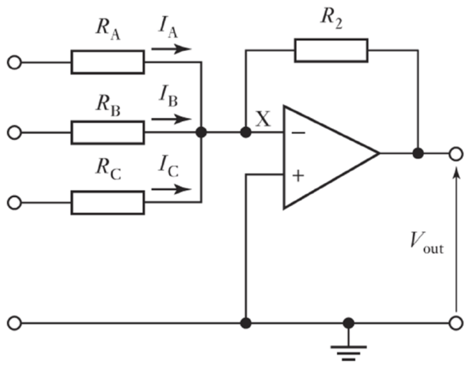

Figure 9.8: The summing amplifier.

Integrator and low-pass filter. Replacing the feedback resistance of the inverting amplifier with a capacitor – see Figure 9.9 (a) – makes the circuit reactive (frequency-dependent gain); in this case we integrate the signal. Compared with the passive RC integrator (low-pass filter) we discussed in Section 7, we have the advantage that here we can choose R and C to give us amplification too. Note, however, that the output cannot exceed the supply limits ($|V_{out}| < V_{ss}$). There are other applications for this circuit beyond filters; for example, you might integrate the output from an accelerometer to obtain the speed of an object and again to measure its displacement.

Differentiator and high-pass filter. With R and C swapped – Figure 9.9 (b) – we have the opposite behaviour: a differentiator and high-pass filter. The interpretation as a filter is intuitive: a low-frequency signal will be 'blocked' by the capacitor, and will not reach the amplifier. Once again the golden rules make it easy to derive the behaviour of this circuit: since 'X' is a virtual ground, the feedback current is $i_f = V_{out}/R$, and the input current is

$$i_i = C\frac{\mathrm{d}}{\mathrm{d}t}V_{in}\,.$$

Since no current enters the op-amp ($i_f = -i_i$) we have

$$V_{out} = -RC\frac{\mathrm{d}}{\mathrm{d}t}V_{in}\,.$$

We can instead use the capacitor impedance explicitly:

$$i_i = \frac{v_{in}}{|Z_C|} = v_{in}\,|j\omega C|$$

$$G = \frac{V_{out}}{V_{in}} = -\frac{R}{|Z_C|} = -\omega RC\,.$$

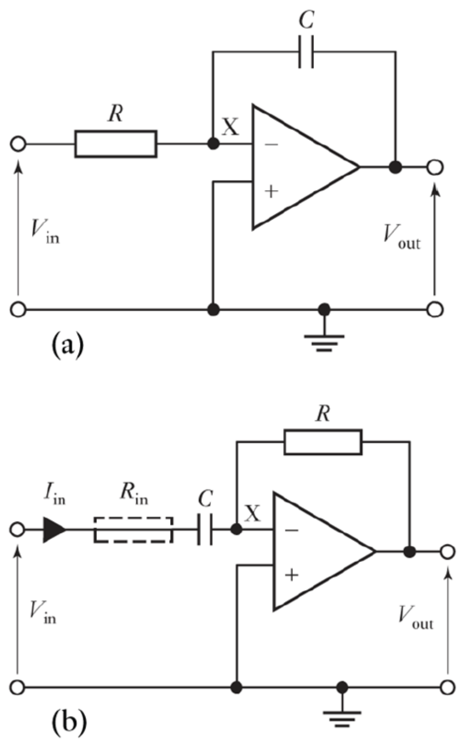

Figure 9.9: The integrating (a) and differentiating (b) amplifiers.

Active filters. Since op-amp filters present a relatively high impedance to the source and can give us amplification, we can chain together lots of such filters without loss of signal amplitude or loading the source. This is a complicated topic (see Horowitz & Hill Chapter 4) and for the purposes of this course it is sufficient to know about the three filter 'figures of merit': (1) *flatness* of the pass-band, (2) *sharpness* of the knee and (3) *steepness* of the roll-off. Even a simple op-amp filter is better than an RC filter, but to design these takes care. Typically, it is easiest to take one of the textbook designs (e.g. those in Figure 8.2) and set the parameters according to the required filter characteristics.

10. Differential signals

In all of the discussions so far to do with electrical signals we have considered our signal to be a voltage relative to ground potential (0 V). This means that we would measure the signals – e.g. with a digital multimeter (DMM) or oscilloscope – by connecting the black lead of the meter to ground and the red lead to the measurement point. These signals are known as *single-ended signals*, and can be carried from A to B on a single piece of wire, that is to say, if A and B are two different units (such as experiment and oscilloscope) then we only need one wire to transmit the signal. This assumes that the zero-volts ground at A and B is the same; this is not always the case, which is why it is good practice to connect the black lead of the scope probe to some ground reference near the point of measurement.

While the single-ended signal is of course always specified as being relative to ground potential, the *differential signal* has the subtle difference that it is relative to *some other potential* (which is itself usually not ground). Consider a sensor which has two terminals and produces an output voltage across these terminals, such as an accelerometer or a strain gauge. We are only interested in the difference between the two terminals, i.e. we do not care what the actual potential of either terminal is relative to ground. This is illustrated in Figure 10.1. We can define the differential signal as

$$V_D = V_+ - V_- \,.$$

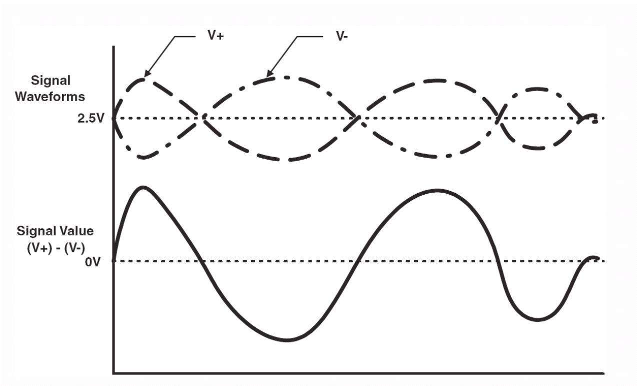

Figure 10.1: Differential signal.

Common-mode signal. The common-mode signal is defined as follows:

$$V_{CM} = \frac{V_+ + V_-}{2} \,.$$

This is the common level about which both terminals vary. In practice, V_{CM} is the average value of the potentials at the two terminals. The main point to consider here is that V_D represents the signal that we want to measure, while V_{CM} represents some kind of background which we are not interested in.

Let us consider the readout of a set of strain gauges using the *Wheatstone bridge* shown in Figure 10.2 – a famous circuit used to measure very small resistance changes by balancing two legs of a bridge circuit. Strain gauges are resistive devices, typically a thin resistive pattern deposited on a malleable substrate. There are many strain-gauge readout configurations with anything between one and four of the resistors in this bridge replaced by such devices in order to measure compressive and tensile stresses in various directions. The point here is to make a very sensitive measurement across terminals A and B which reflects very small changes in resistance which will unbalance the bridge. If all resistors are equal then, under no strain the bridge is balanced and, if the bias voltage is, say, 10 V, then $V_A = V_B = 5$ V. Under some small strain the equivalent resistances of the two arms will vary, and V_A will depart from V_B by some small voltage (typically <1 mV) which we want to measure. So, the signal is differential ($V_D = V_A - V_B = 1$ mV) and must be measured over a large common-mode voltage ($V_{CM} = (V_A + V_B)/2 = 5$ V).

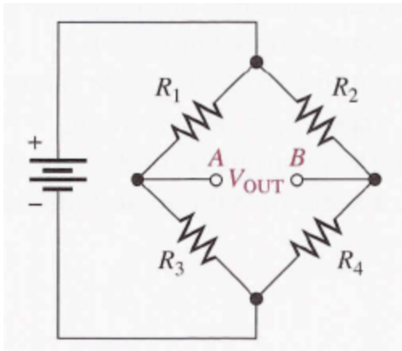

Figure 10.2: Reading a set of strain gauges ($R_{1..4}$ in this example) with a Wheatstone bridge.

Noise rejection. Another advantage of working with differential signals is illustrated in Figure 10.3. Here, the signal is represented by a differential pulse. Wires can act as aerials to pick up noise from radio waves or other signals elsewhere in the circuit – usually called *electromagnetic interference (EMI)*. However, if the two wires which carry the differential signal are reasonably close to each other, then it is reasonable to assume that the amount of noise picked up will be the same on both. That is to say, the noise is a *common-mode signal*. We would like the amplifier to remove this noise. To measure a differential signal, we need a *difference amplifier*.

Common-mode rejection. Some main points to consider when choosing a difference amplifier are:

- Input isolation from ground (high);
- Differential voltage amplification (high);
- Common-mode voltage amplification (low).

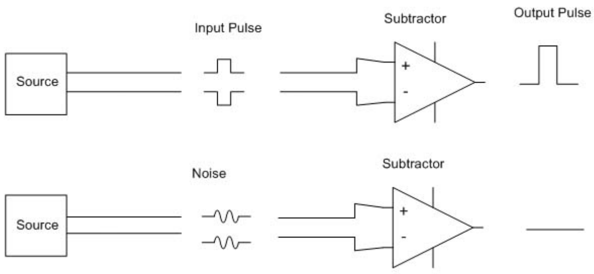

Figure 10.3: Noise rejection.

Since no amplifier is perfect, there will always be some common-mode voltage which gets through the amplifier and appears on the output. In practice, the output voltage is given by

$$V_{out} = A_D(V_+ - V_-) + \frac{1}{2}A_{CM}(V_+ + V_-)\,,$$

where A_D is the *differential gain* and A_{CM} is the *common-mode gain*. For any particular amplifier we can measure the *Common-Mode Rejection Ratio* (CMRR), which is usually given in dB and defined as:

$$\mathrm{CMRR} = 20\log_{10}\frac{A_D}{A_{CM}}\,.$$

The CMRR is a key figure-of-merit for a difference amplifier. The AD620 instrumentation amplifier from Analog Devices has a CMRR of 100 dB, which is very high performance. More specialist devices can achieve 120 dB or more.

10.1. Difference amplifier

The amplifier circuits we have looked at so far are suitable for single-ended signals. Typically, one of the inputs is connected to ground (the inverting amplifier, for example). It is not possible to use these to amplify differential signals since one side of the signal will be shorted to ground. Consider for example a complex experimental setup within which we want to measure the voltage across a single resistor (maybe because we want to be able to calculate the current through it). Connecting the inverting op-amp circuit across the resistor would connect one side to ground, probably stopping our circuit from working properly. The solution to this may be to attach some more resistors to the non-inverting input. The result is the *difference amplifier*, as shown in Figure 10.4.

Circuit analysis. Separating the top and bottom branches from the op-amp (Figure 10.5) and using Kirchhoff's Current Law we get

$$I_1 = \frac{V_1 - V_A}{R_1} = \frac{V_A - V_{out}}{R_f}\,,$$

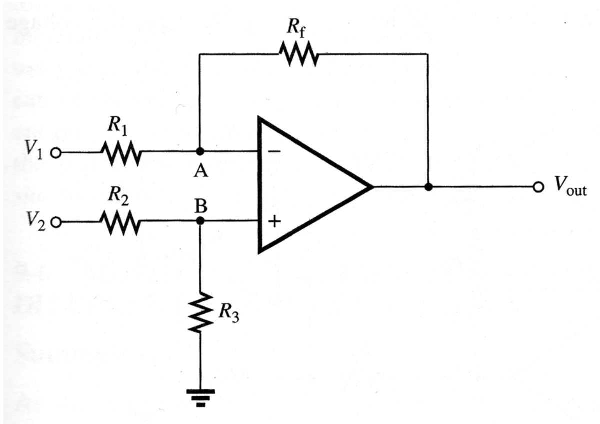

Figure 10.4: The difference amplifier (Diefenderfer Chapter 9); note the non-inverting input is not grounded.

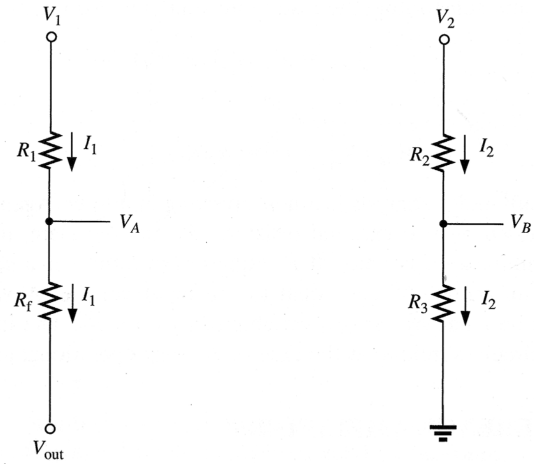

Figure 10.5: Circuit analysis for the difference amplifier (Diefenderfer Chapter 9).

$$V_A = \frac{V_1R_f + V_{out}R_1}{R_1 + R_f}\,.$$

For the other branch we can do the same or, since the bottom of the circuit is connected to ground, we can just treat it as a voltage divider and write

$$V_B = \frac{V_2R_3}{R_2 + R_3}\,.$$

Since Golden Rule #1 requires that $V_A = V_B$, we can solve for V_{out} to yield

$$V_{out} = V_2\frac{R_3}{R_1}\frac{R_1 + R_f}{R_2 + R_3} - V_1\frac{R_f}{R_1}\,.$$

Now, if we choose to set $R_2 = R_1$ and $R_3 = R_f$ (i.e. both signals see the same input impedance) then we get:

$$V_{out} = (V_2 - V_1)\frac{R_f}{R_1}\,. \tag{10.1}$$

The difference amp is commonly used as described, but for high-precision applications there are two problems:

1. If we fail to match $R_1 = R_2$ and $R_3 = R_f$ exactly, then we get a common-mode signal at V_{out} as well as the differential signal that we want. Any common mode signal is an error signal, since it depends on how carefully you build the amplifier circuit;
2. It has a relatively low input impedance (at V_2 this is equal to $R_2 + R_3$). This can cause us to 'load' the signal we are trying to measure by drawing current from it. It is tempting to make the values of the resistors very large, say some MΩ, but the inputs of the op-amp inevitably have some stray-capacitance, so we would have a time-constant effect which would slowdown voltage swings at the input. We need to keep $R_2 + R_3$ to be some kΩ.

Recalling that op-amps are small and cheap, we can make use of the high-input impedance buffer or voltage follower circuit. The simplest thing to do would be to just attach a buffer to each of the inputs at V_1 and V_2. This completely solves the input impedance problem – the points in the circuit we are trying to measure are now isolated from our difference amplifier by the buffer impedances, which can be $\sim$GΩ. However, the common-mode issue (problem (1) above) remains. The reason is that the difference amplifier is trying to do two jobs: both reject common-mode signals and amplify differential mode signals. We can improve this situation by moving some of the gain into the input stage, which relaxes a bit the requirement to match the resistors. This design is known as the *instrumentation amplifier*.

10.2. Instrumentation amplifier

The design of the classic *instrumentation amplifier* is shown in Figure 10.6. The circuit analysis for this is covered in Diefenderfer & Holton Section 9.5. For the purposes of this course though, it is not necessary to know the detail of the analysis. The output is given by

$$V_{out} = (V_2 - V_1)\,\frac{R_4}{R_3}\left(1 + 2\frac{R_1}{R_2}\right).$$

So, the amplifier produces a voltage output which is proportional to the difference between the two input voltages, and with a very high impedance at the inputs.

Applications. The instrumentation amplifier can be used wherever we want to measure the voltage across any circuit element without connecting either side to ground. For example, we might imagine that the input of a multimeter looks like the instrumentation amplifier because we can connect across any two points in a circuit without drawing any current from the circuit or shorting the connection point to ground.

The analogue inputs on the ELVIS units we will use in lab are instrumentation amplifiers. When we want to use them to make a single-ended measurement (e.g. making the Bode plot of the RC filter), we just connect the "$-$ve" input to ground.

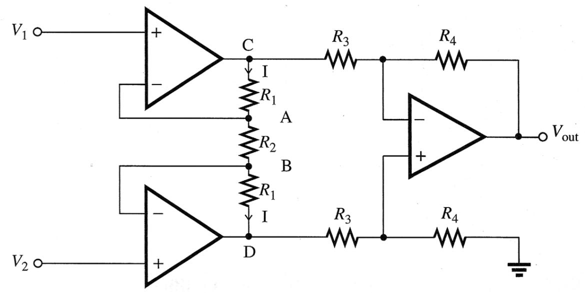

Figure 10.6: The instrumentation amplifier (Diefenderfer Chapter 9).

Integrated circuit instrumentation amplifiers. These days the semiconductor industry provides a 'ready-made' implementation on a single chip. One such device is the Analog Devices AD620 (Figure 10.7).[18] This has a CMRR (Common Mode Rejection Ratio) of 100 dB and a differential gain adjustable up to 10,000 set by an external gain resistor connected across pins 1 and 8 (this is R_2 in our circuit). All the other resistors are inside. The advantage of these devices is that the internal resistors have been matched to be as close as possible. This is done by laser-trimming the individual resistors for each part. Because they are manufactured in quantity, this process becomes cost effective and the AD620 retails for around £10.

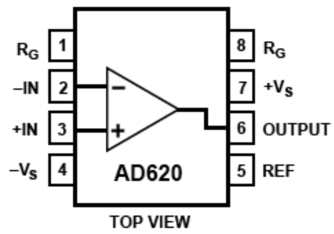

Figure 10.7: Pin connections for the Analog Devices AD620 instrumentation amplifier.

[18] See product datasheet and some standard circuit designs at https://www.analog.com/en/products/ad620.html.

11. Real-world op-amps

The ideal amplifier is a zero-order system with infinite gain, bandwidth and slew-rate. We saw in Section 10 that a "real" difference amp has some common-mode gain which is an imperfection due to non-matched resistors. For practical applications the real op-amp has the following characteristics:

Finite gain and input impedance. These non-ideal parameters result in the golden rules being only approximations, of course: in reality, the op-amp inputs will draw some non-zero current i_{in} as a result of the finite input resistance R_{in} of the device. It is easy to show (and you are encouraged to try) that the input impedance of the non-inverting input in a feedback design with closed-loop gain $1/\beta$ is $Z_i = V_{in}/i_{in} \approx R_{in}\beta A$. In older op-amp designs an extra resistance would often be added between the non-inverting input and ground, chosen such that the two inputs saw the same equivalent impedance – hence cancelling out any voltages between them to first order; modern op-amps have really very high input impedance so this is rarely needed. Since specialist op-amps with very high A and R_{in} are available, it is simply a matter of checking values of βA and i_{in}. For the latter, this simply means we should not use very large feedback resistors; for the finite gain, see the problem sheet question on this topic.

Input offset voltage. Connecting the inverting and non-inverting inputs of an op-amp to ground should result in zero output; however, manufacturing imperfections lead to what we can think of as a small potential difference between the inputs. This can be $\sim$mV, and so feedback will automatically correct for this by adjusting the output, with the result that the closed-loop amp shows a zero-offset. Most op-amps have a couple of pins across which a "trim" resistance can be placed to compensate for this effect. The datasheet will give the details about how this is done and also the maximum limits and stability expected for the input-offset voltage.

Finite bandwidth and slew rate. These parameters are intimately linked, and very important to a full understanding of op-amp behaviour. The open-loop op-amp has a surprisingly limited bandwidth, which ultimately limits the speed with which it can respond to a fast-changing input.

11.1. Gain-bandwidth product

The Bode plot for an op-amp such as the 741 (red line in Figure 11.1) shows that A is only "very large" for the range of frequencies up to $\sim$10 Hz. Above 10 Hz the amp shows a gain fall-off $G \propto 1/f$ like a low-pass filter. By 1 MHz we have $A \sim 1$. Here we define the *bandwidth B* as the range of frequencies from 0 to the -3 dB point. Under feedback we throw away the excess gain, so we find

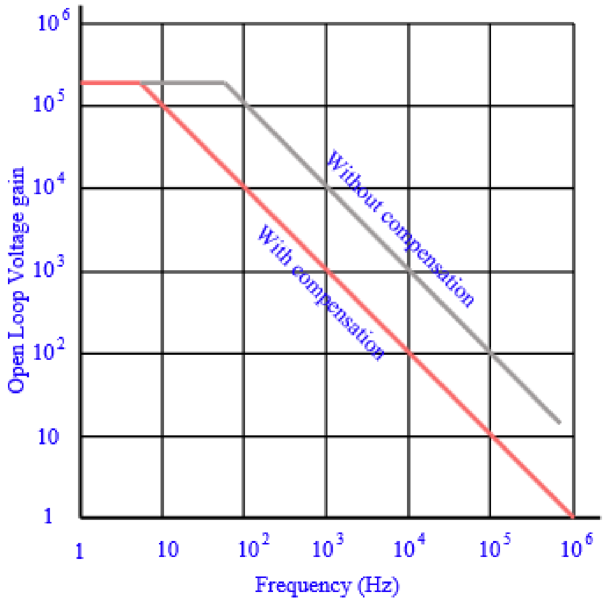

Figure 11.1: Open-loop gain response for the 741 op-amp (red); a similar device (748) without internal stability compensation is also shown (grey) (from www.electronics-notes.com).

that feedback widens the bandwidth of the amp and, above 10 Hz or so:

$$G \times B = \text{constant}\,.$$

This is illustrated by the red line in Figure 11.2. For example, if we have a 741 with $A = 1$ at 1 MHz, then $G{\cdot}B = 10^6$ and if we apply feedback with $\beta = 0.01$ then the closed-loop gain is 100, and hence $B = 10^4$ Hz. This is indeed the frequency where the closed- and open-loop gain curves intersect. The $G{\cdot}B$ product is a useful figure-of-merit for an op-amp.

11.2. Phase response

Accordingly, the phase shift at the output is about $-45°$ at 10 Hz (i.e. output lags input) and continues to fall to about $-180°$ at about 1 MHz. Imagine a 10 Hz sine wave going into a unity-gain buffer. The output has a phase shift which looks like a time-delay. This is very important because we will use the op-amp with feedback.

11.3. Stability under feedback

This requires a thorough understanding of how feedback really works, for which it is easiest to refer back to the block-diagram in Figure 9.4. Negative feedback works by subtracting some output from the input. This is easy to see when we think about DC signals going into the input. If we put 1 V on the input then the output reacts so as to feed back 1 V into the summing junction to counteract this, with the net result of 0 V at the input to the high-gain stage. However, consider a signal at 1 MHz where the phase change is $-180°$. When the input voltage is at its *positive* peak, the output is at its *negative* peak, so we actually *add* extra signal at the summing junction. This is

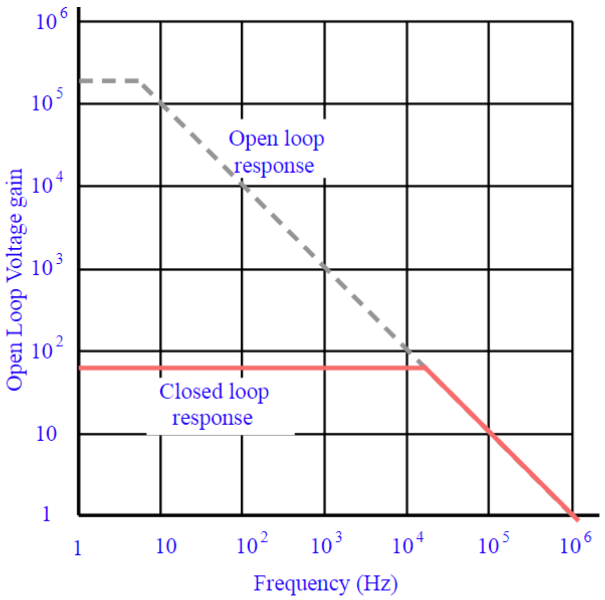

Figure 11.2: Gain-bandwidth product for the 741 op-amp (from www.electronics-notes.com).

positive feedback – which is disastrous. The enhanced signal at the amp input will just generate a bigger inverted output which just makes the problem worse, and soon the output will go to the supply rails. In practice, this usually results in the amp oscillating between $\pm V_{ss}$. Mathematically, for negative feedback we already saw in Section 9 that:

$$G = \frac{A}{1+\beta A}\,,$$

where we thought of A as a constant; however, our understanding of real op-amps is that at high frequency A appears to be negative (due to the phase shift).

Stability criterion. We must avoid at all costs the situation where the denominator goes to zero, i.e.

$$\beta A = -1\,,$$

otherwise $G \to \infty$. This is unstable, as well as being of no practical value (unless you want to build an oscillator!).

Gain and phase margins. The designer of the op-amp does not know what value of β you will use. Hence, most amps have a *built-in gain roll-off* to ensure that $A < 1$ for $\phi = -180°$. This is the reason for the rather sudden gain reduction with frequency: to ensure stability with respect to high-frequency inputs. In practice, any such an *internally compensated op-amp* will be designed to ensure $A \ll 1$ way before $\phi = -180°$, just to be on the safe side – see gray curve in Figure 11.1. So, we can define the *gain margin* as the gain at $\phi = -180°$, while the *phase margin* is how far we get from $\phi = -180°$ when $A = 1$. These are best visualised graphically – see Bode plot in Figure 11.3. Typical desirable margins are 6 dB and 45° (although both >0 is in theory sufficient).

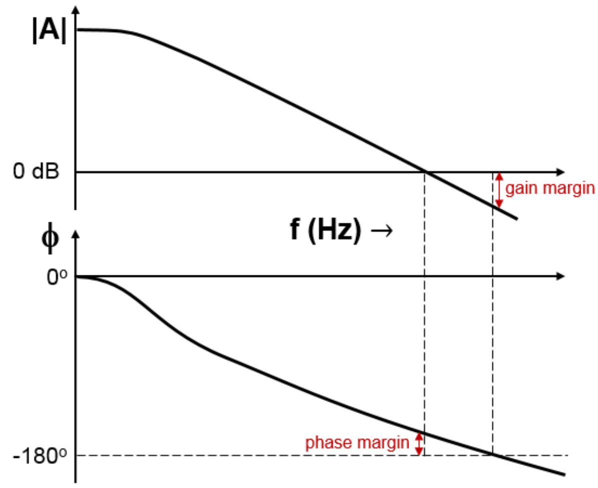

Figure 11.3: Gain and phase margins for stable operation of a negative feedback amplifier.

11.4. Final remarks

These considerations of stability under feedback apply to all situations where feedback is applied to a system or process, and we will encounter this again in a more mathematical and rigorous form later in the course. Since any feedback system has the potential to become unstable or oscillate, the question of stability becomes central to control theory. For feedback amplifiers we can assess the degree of stability we have through the stability criterion and the phase/gain margins. Note that many op-amps are sold without internal compensation; it is left to the user to ensure stability. For example the 748 is the *un-compensated* version of the 741 – shown by the grey line in Figure 11.1. With compensation we sacrifice gain for stability, so if we know what we are doing we can safely use the un-compensated device for greater performance at lower frequencies.

12. Analogue to digital conversion

We looked at sampling and digitisation of analogue signals in Section 5, but only now possess the required understanding of amplifiers to examine some concrete circuit implementations. These are to be understood rather than memorised. There are many implementations of these circuits, we look at one (the simplest) example of each.

12.1. Sample-and-hold circuit

We saw that converting a continuous signal into a digital form was a two-stage process: sampling and digitisation. Since the input signal can vary faster than our sampling speed, we need a circuit to hold the voltage while we digitise each sample – this is the job of the *sample & hold* circuit, such as the one shown in Figure 12.1.

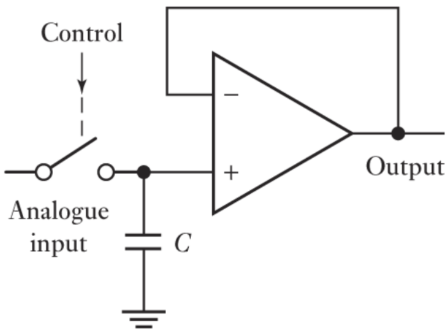

Figure 12.1: Concept for the sample & hold circuit.

The control signal comes from a 'sampling clock', here represented by opening the switch on the rising edge. The output will be held at the value seen by the input the instant the switch is opened, and it will remain there until we have measured it. The circuit is essentially a "switched" voltage buffer with a small capacitance C.

When the switch is closed then V_+ tracks the input whatever this is doing, and the output is equal to V_+. When the switch opens, the buffer's high input-resistance ensures that the output remains equal to V_+. We still need to figure out how to implement the switches. This we can be done with a pair of field effect transistors (FETs), which are readily available in integrated-circuit design (not covered in this course).

12.2. Analogue-to-digital converter

Now we are ready to convert the voltage to a number, using an *analogue-to-digital converter* (ADC or A/D). There are many IC implementations which are relatively inexpensive (a few pounds).

The simplest circuit is the *flash ADC* exemplified in Figure 12.2, where each comparator compares the input voltage with successively smaller reference voltages generated across the 'resistor ladder' (essentially a voltage divider with equal resistances). The conversion is near-instantaneous (hence the name), but clearly we would need

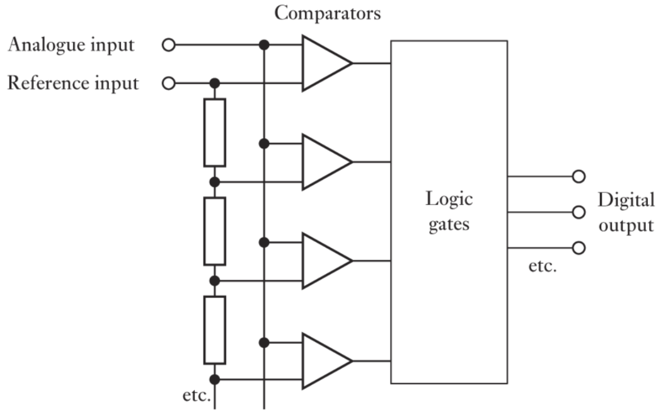

Figure 12.2: A 4-bit flash ADC.

pretty much one comparator for each digital value (e.g. $2^8-1 = 255$ comparators for an 8-bit device...). Another disadvantage is that all resistors need to be pretty equal or the device will be slightly non-linear. Having said that, this is still a popular design for low-resolution (e.g. 8-bit) ADCs, especially when speed is the key consideration. Other designs exist, such as the "successive approximation" ADC, the "sigma-delta" design, the "dual-slope" ADC, etc.

12.3. Digital-to-analogue converter

Frequently we need to convert digital signals back to the real world – and so we need a *digital-to-analogue converter* (DAC or D/A). For example, we may want to drive an output transducer such as a microphone. Some ADC technologies (e.g. successive approximation) also rely on a DAC to generate the intermediate stages of the ADC conversion.

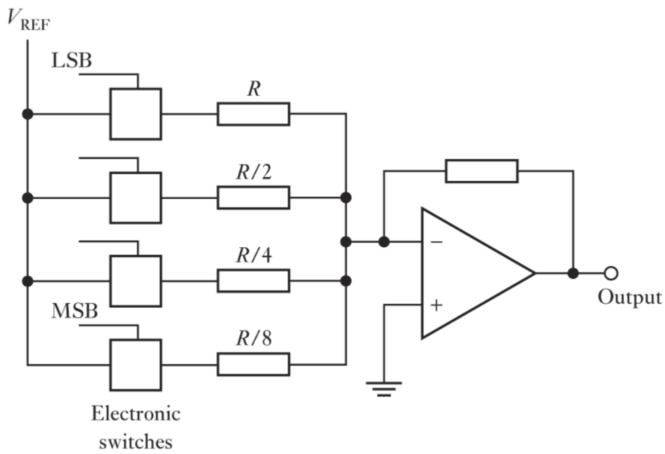

Figure 12.3: A 4-bit DAC.

A simple DAC circuit is that shown in Figure 12.3, at the core of which is a summing amplifier like the ones discussed in Section 9. Here we weight the 'bits' according to their significant-position in the binary scheme. Note the negative output: we could follow this with an inverting amplifier. This circuit has some advantages and disadvantages which are similar to those of the flash ADC design: it delivers near-instant conversion to an analogue voltage, but here, too, the resistors must be very accurate (exact power-of-2 ratio) or we will get non-linearity. As with ADC, many types exist and we usually buy an integrated circuit for the job.

As a final comment, these are unipolar designs – they operate with input/output voltages in the range $0 - V$. For bipolar input/output voltages in the range $\pm V$ these designs can easily be extended to allow bipolar signals – we just have to be careful how negative numbers are encoded in our binary scheme (we will not go into this in detail).