

Stochastic Discounted Cash Flow

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- Le vendite in Nord America e in Europa costituiscono il 65% e il 21% dei ricavi, rispettivamente
- Ford è la seconda più grande prodruttrice di auto negli U.S.A (dopo General Motors) e la quinta nel mondo (dopo Toyota, Volkswagen, Hyundai-Kia and General Motors)

Anno	Free Cash Flow	Present Value
2019		
2020		
2021		
2022		
2023		
į		
∞		

Anno	Free Cash Flow	Present Value
2019	FCFF ₂₀₁₉	
2020	<i>FCFF</i> ₂₀₂₀	
2021	<i>FCFF</i> ₂₀₂₁	
2022	<i>FCFF</i> ₂₀₂₂	
2023	<i>FCFF</i> ₂₀₂₃	
:		
∞		

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2023	<i>FCFF</i> ₂₀₂₃	
:		
∞	TV	

Anno	Free Cash Flow		Present Value	
2019	FCFF ₂₀₁₉	\longmapsto	PV_{2019}	
2020	<i>FCFF</i> ₂₀₂₀		PV_{2020}	
2021	<i>FCFF</i> ₂₀₂₁		PV_{2021}	
2022	FCFF ₂₀₂₂		PV_{2022}	
2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	
÷				
∞	TV	\longmapsto	PV_{TV}	

Anno	Free Cash Flow		Present Value	
2019	FCFF ₂₀₁₉	\longmapsto	PV_{2019}	+
2020	<i>FCFF</i> ₂₀₂₀		PV_{2020}	+
2021	<i>FCFF</i> ₂₀₂₁		PV_{2021}	+
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2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	+
:				
∞	TV	\longmapsto	PV_{TV}	=

Firm Value

Anno	Free Cash Flow		Present Value	
2019	FCFF ₂₀₁₉	\longmapsto	PV_{2019}	+
2020	FCFF ₂₀₂₀		PV_{2020}	+
2021	<i>FCFF</i> ₂₀₂₁		PV_{2021}	+
2022	FCFF ₂₀₂₂		PV_{2022}	+
2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	+
:				
∞	TV	\longmapsto	PV_{TV}	=

Firm Value - Market Value of Debt

Anno	Free Cash Flow		Present Value	
2019	FCFF ₂₀₁₉	\longmapsto	PV_{2019}	+
2020	<i>FCFF</i> ₂₀₂₀		PV_{2020}	+
2021	<i>FCFF</i> ₂₀₂₁		PV_{2021}	+
2022	<i>FCFF</i> ₂₀₂₂		PV_{2022}	+
2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	+
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∞	TV	\longmapsto	PV_{TV}	=

Firm Value Market Value of Debt /
Number of Shares

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2019	FCFF ₂₀₁₉	\longmapsto	PV ₂₀₁₉	+
2020	<i>FCFF</i> ₂₀₂₀		PV_{2020}	+
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2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	+
:				
∞	TV	\longmapsto	PV_{TV}	=

Firm Value - Market Value of Debt / Number of Shares =

Value of Stock

	Anno	Free Cash Flow		Present Value	
	2019	FCFF ₂₀₁₉	\longmapsto	PV ₂₀₁₉	+
	2020	FCFF ₂₀₂₀		PV_{2020}	+
	2021	<i>FCFF</i> ₂₀₂₁		PV_{2021}	+
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	2023	<i>FCFF</i> ₂₀₂₃		PV_{2023}	+
	:				
	∞	TV	\longmapsto	PV_{TV}	=
				Firm Value	_
				Market Value of Debt	/
				Number of Shares	=
Droco	n+\/alua	Value			
rrese	ntValue :	$= \overline{(1 + \mathit{WACC})^{\Delta_{\mathit{ar}}}}$	nni	Value of Stock	

Discount Rate

Weighted Average Cost of Capital

$$WACC = rac{Value_{equity}}{Value_{capital}} Cost_{equity} + (1 - tax) rac{Value_{debt}}{Value_{capital}} Cost_{debt}$$

$$Value_{capital} = Value_{equity} + Value_{debt}$$

Valore

$$Value_{equity} = \underbrace{Number_{shares}}_{\text{seg.gov 10k}} \times \underbrace{Value_{shares}}_{\text{Yahoo Finance}}$$

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$$= 3'974'000'000 \times 7.65$$

= $30'401'100'000$ \$

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Costo

$$Cost_{equity} = \underbrace{Riskfree}_{\text{fred.stlouisfed.org}} + \underbrace{\beta}_{\text{hist_stock_data.m}} \underbrace{\underbrace{(MarketPremium - Riskfree})}_{\text{Damodaran ERP}}$$

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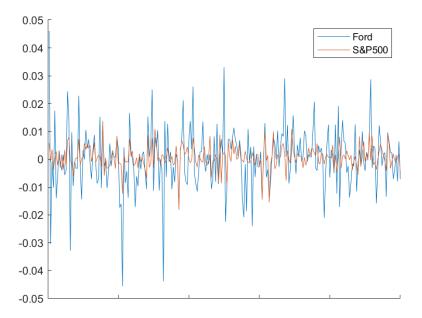
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$$= 2.69 + \beta \times 5.96$$

= $2.69 + 1.18 \times 5.96$
= 9.74%

$$\beta = \frac{\textit{Cov}(\textit{R}_{\textit{ford}}, \textit{R}_{\textit{market}})}{\textit{Var}(\textit{R}_{\textit{market}})}$$



Valore

$$Value_{debt} = \underbrace{Interest_{debt}}_{\text{sec.gov 10k}} \underbrace{\frac{1 - \frac{1}{(1 + Cost_{debt})^{Maturity}}}{Cost_{debt}}}_{\text{Sec.gov 10k}} + \underbrace{\underbrace{MarketVal_{debt}}_{(1 + Cost_{debt})^{Maturity}}}_{\text{Morningstar}} \underbrace{\frac{1}{(1 + Cost_{debt})^{Maturity}}}_{\text{Morningstar}}$$

```
Interest on Debt = 1'200'000'000 $
Market Value of Debt = 93'635'000'000 $
Maturity = 15 anni (Bond)
```

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$$= 1'200'000'000 \times 10.6734 + 93'635'000'000 \times 0.5101$$

= 60'570'440'820 \$

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Costo

$$Cost_{debt} = \underbrace{Riskfree}_{ ext{fred.stlouisfed.org}} + \underbrace{Spread}_{ ext{Moody's Baa3 moto.it}}$$

$$= 2.69 + 1.9$$

= 4.59%



Discount Rate

Weighted Average Cost of Capital

$$WACC = rac{Value_{equity}}{Value_{capital}} Cost_{equity} + (1 - tax) rac{Value_{debt}}{Value_{capital}} Cost_{debt}$$

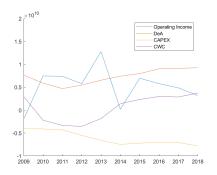
Discount Rate

Weighted Average Cost of Capital

$$W\!ACC = rac{Value_{equity}}{Value_{capital}} Cost_{equity} + (1 - tax) rac{Value_{debt}}{Value_{capital}} Cost_{debt}$$

=
$$0.334 \times 9.74 + (1 - 0.27)0.665 \times 4.59$$

= $\boxed{\mathbf{5.65\%}}$

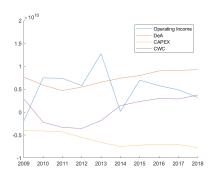


EBIT = Earnings Before Interest

D&A = Depreciation and Amortization

|CAPEX| = Capital Expenditures

 Δ NWC = Changes in Net Working Capital

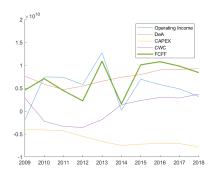


EBIT = Earnings Before Interest

 $\mathsf{CAPEX} = \mathsf{Capital} \; \mathsf{Expenditures}$

 Δ NWC = Changes in Net Working Capital

 $\textit{FreeCashFlowToFirm} = \textit{FCFF} = \textit{EBIT}(1-\textit{Tax}) + \textit{D&A} + \textit{CAPEX} + \Delta \textit{NWC}$



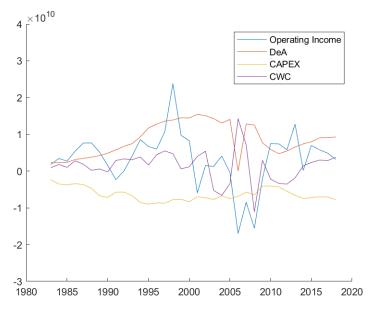
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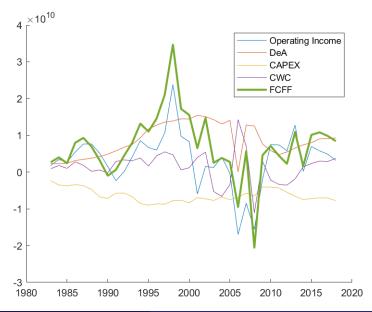
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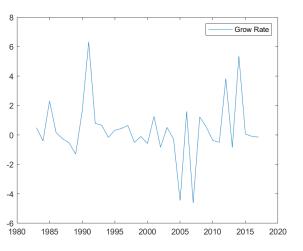


Free Cash Flow - Grow Rate

$$GrowRate_i = \frac{FCFF_{i+1} - FCFF_i}{|FCFF_i|}$$
 per $i = 1983, ..., 2017$

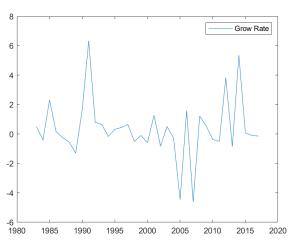
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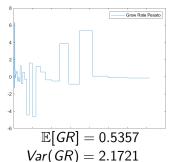


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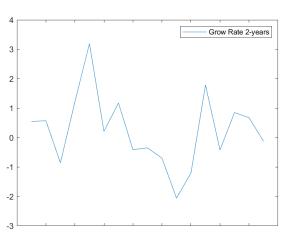


Dando maggiore importanza agli ultimi anni (fattore $\times 1.2$)

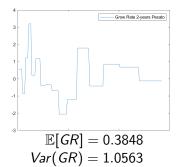


Free Cash Flow - Grow Rate Mediato

Invece mediando su coppie di anni consecutive si ottiene:

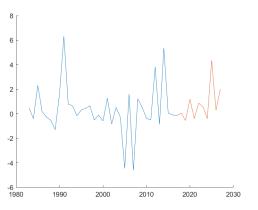


Dando maggiore importanza agli ultimi anni (fattore $\times 1.2$)



Free Cash Flow - Future Grow Rate

Ipotizziamo i Grow Rate nei prossimi 10 anni utilizzando una Gaussiana



 $N(\mathbb{E}[GR], Var(GR))$

Free Cash Flow - Future FCFF

Calcoliamo i Free Cash Flow to Firm nei prossimi 10 anni utilizzando

$$\textit{FCFF}_i = \textit{FCFF}_{i-1} + \textit{GrowRate}_{i-1} \left| \textit{FCFF}_{i-1} \right|$$

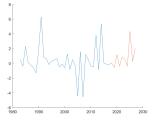
per i = 2019, ..., 2028

Free Cash Flow - Future FCFF

Calcoliamo i Free Cash Flow to Firm nei prossimi 10 anni utilizzando

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per i = 2019, ..., 2028

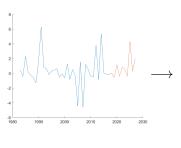


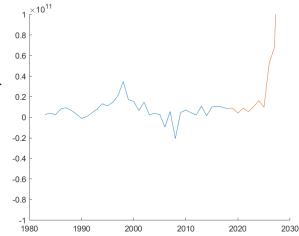
Free Cash Flow - Future FCFF

Calcoliamo i Free Cash Flow to Firm nei prossimi 10 anni utilizzando

$$FCFF_i = FCFF_{i-1} + GrowRate_{i-1} | FCFF_{i-1} |$$
 per $i = 20$

per i = 2019, ..., 2028





Free Cash Flow - Terminal Value

Terminal Value

$$TV_0 = \sum_{i=1}^{\infty} PV_0(FCFF_i) =$$

$$= \sum_{i=1}^{\infty} \frac{FCFF_0(1 + GR)^i}{(1 + WACC)^i} =$$

$$= FCFF_0 \sum_{i=1}^{\infty} \left(\frac{1 + GR}{1 + WACC}\right)^i =$$

$$= FCFF_0 \frac{1 + GR}{1 + WACC} \frac{1}{1 - \frac{1 + GR}{1 + WACC}} = FCFF_0 \frac{1 + GR}{WACC - GR}$$

$$\mathsf{WACC} = 5.65\% \qquad \qquad \mathsf{GR} = \mathsf{TerminalGrowRate} := \mathbb{E}[\mathit{FutureGrowRate}]$$

Results

Firm Value

$$\textit{FirmValue} = \sum_{i=1}^{10} \frac{\textit{FCFF}_{2018+i}}{(1 + \textit{WACC})^i} + \frac{\textit{TV}}{(1 + \textit{WACC})^{10}}$$

Results

Firm Value

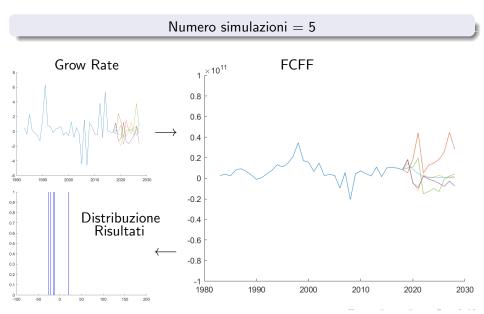
$$FirmValue = \sum_{i=1}^{10} \frac{FCFF_{2018+i}}{(1 + WACC)^i} + \frac{TV}{(1 + WACC)^{10}}$$

Stock Value

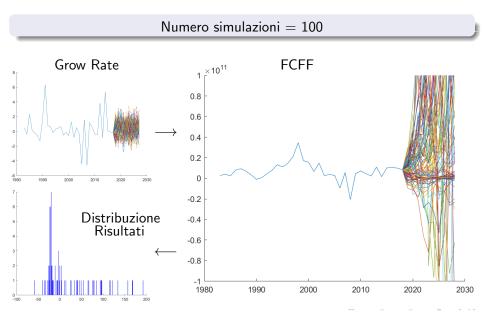
$$StockValue = \frac{FirmValue - MarketValueOfDebt}{NumberOfShares}$$



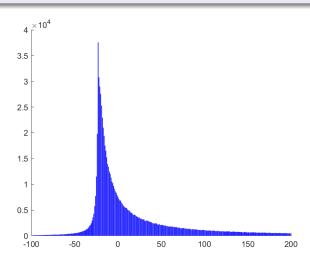
Montecarlo Simulation



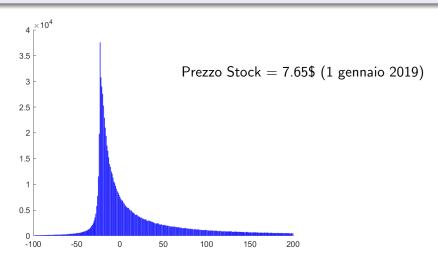
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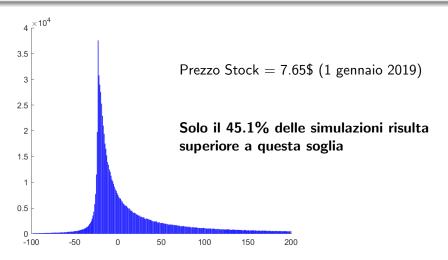
Numero simulazioni = 1'000'000



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