CE 562

TRANSIENT FLOW ANALYSIS

MINI PROJECT

PRACTICAL APPLICATION OF METHOD OF CHARACTERISTICS IN SOLVING PIPELINE NETWORKS

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ABSTRACT

Power failure of pumps, sudden valve actions and the operation of automatic control systems are all capable of generating high pressure waves in pipeline networks. These high pressure transient conditions can cause pipe failures by damaging valves and fittings. This project is aimed at analysing a pipe network using Method of Characteristics (MOC). The method of practically analysing a pipeline network using MOC is derived first and it is used to solve a linear pipe problem. After analysing the results, a study on how to apply MOC to a complex network is also carried out.

CONTENTS

INTRODUCTION

Due to sudden changes in the flow rate or pressure in the pipeline, waves or pressure surges propagating along the pipeline are produced. Analysing of these conditions is important to prevent damage to the pipeline system, such as pipe rupture or leakage.

The analysis can be done using various approaches. Eulerian and Lagrangian are the two most common analysis tools. Eulerian approach involves methods such as method of characteristics, finite difference method, finite element method, etc whereas Lagrangian approach consists of methods such as Wave Characteristic Method. The Eulerian approach explicitly solves the hyperbolic partial differential equations of continuity and momentum and updates the hydraulic state of the system in fixed grid points as time is advanced in uniform increments. Lagrangian approach tracks the movement and transformation of pressure waves and updates the hydraulic state of the system at fixed or variable time intervals at times when a change actually occurs.

In this study, an Eulerian approach, Method of Characteristics (MOC) is used to solve the partial differential equations. Firstly, how this method can be applied to a practical situation is studied. For this, the continuity and momentum equations are combined and simplified. This combined equation is then integrated along the positive and negative characteristic lines to derive two equations that can be used to solve a pipeline network. Application of this method is done next in a linear pipe – reservoir system. The results are then analysed and the steps that should be adopted for applying MOC to a complex pipeline network is explained.

OBJECTIVES

1. Derive practical equations for solving a pipeline network using method of characteristics from the governing and characteristic equations.
2. Apply these equations to solve a linear pipe network.
3. Analyse the results.
4. Study how the Method of Characteristics can be used to solve complex pipeline networks.

THEORY

Transient flow equations are hyperbolic partial differential equations. The significance of method of characteristics is the successful replacement of a pair of partial differential equations by an equivalent set of ordinary differential equations. The method of characteristics is developed from assuming that the continuity and the momentum equations can be replaced by a linear combination of themselves.

$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + \rho c^2 \frac{\partial v}{\partial x} = 0$ .... Continuity equation (C)

$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\sin\theta + \frac{fv|v|}{2D} = 0$ .... Momentum equation (M)

This combined equation is then simplified and integrated along the characteristic lines to find the flow conditions at an intermediate node at a time $t + \Delta t$ (assuming we know the flow values at time t in the boundaries).

DETAILED METHODOLOGY

1. **Conversion of governing and characteristic equations into practically applicable form.**

$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + \rho c^2\frac{\partial v}{\partial x} = 0$ .... Continuity equation (C)

$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\sin\theta + \frac{fv|v|}{2D} = 0$ .... Momentum equation (M)

where,

v → average velocity in the section

p → pressure at a certain point

ρ → density of the fluid

c → wave velocity

g → acceleration due to gravity

θ → angle the pipe section makes with the horizontal

f → friction factor

Neglect the convective acceleration terms $v\frac{\partial p}{\partial x}$ and $v\frac{\partial v}{\partial x}$ as they are very small.

Combining the momentum and continuity equations,

$S = M + \varphi C$, we get

$$S = \frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\sin\theta + \frac{fv|v|}{2D} + \varphi\left(\frac{\partial p}{\partial t} + \rho c^2\frac{\partial v}{\partial x}\right) = 0$$

Now, v and p are functions of x and t. So we can write

$\frac{dv}{dt} = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial t}$ and $\frac{dp}{dt} = \frac{\partial p}{\partial x}\frac{dx}{dt} + \frac{\partial p}{\partial t}$

$\Rightarrow \frac{\partial v}{\partial t} = \frac{dv}{dt} - \frac{\partial v}{\partial x}(\frac{dx}{dt})$ and $\frac{\partial p}{\partial t} = \frac{dp}{dt} - \frac{\partial p}{\partial x}(\frac{dx}{dt})$

Substituting this in the combined momentum and continuity equation,

$$\frac{dv}{dt} - \frac{\partial v}{\partial x}\left(\frac{dx}{dt}\right) + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\sin\theta + \frac{fv|v|}{2D} + \varphi(\frac{dp}{dt} - \frac{\partial p}{\partial x}\left(\frac{dx}{dt}\right) + \rho c^2\frac{\partial v}{\partial x}) = 0$$

Rearranging,

$$\frac{dv}{dt} + gsin\theta + \frac{fv|v|}{2D} + \varphi\frac{dp}{dt} - \frac{\partial v}{\partial x}\left(\frac{dx}{dt} - \varphi\rho c^2\right) - \frac{\partial p}{\partial x}\left(\varphi\frac{dx}{dt} - \frac{1}{\rho}\right) = 0 \text{ .... I}$$

On simplifying, $\frac{dx}{dt} - \varphi\rho c^2 = 0$ and $\varphi\frac{dx}{dt} - \frac{1}{\rho} = 0$

$\Rightarrow \frac{dx}{dt} = \varphi\rho c^2$ and $\frac{dx}{dt} = \frac{1}{\rho\varphi}$

On comparing, $\varphi = \pm\frac{1}{\rho c}$ and I becomes,

$\frac{dv}{dt} + \varphi\frac{dp}{dt} + gsin\theta + \frac{fv|v|}{2D} = 0$ II

and the characteristic equations are $\frac{dx}{dt} = \pm c$ III

Putting $\varphi = \frac{1}{\rho c}$ in II and multiplying it with(ρcdt),

$\rho cdv + dp + \rho cdtgsin\theta + \rho cdt\frac{fv|v|}{2D} = 0$ IV

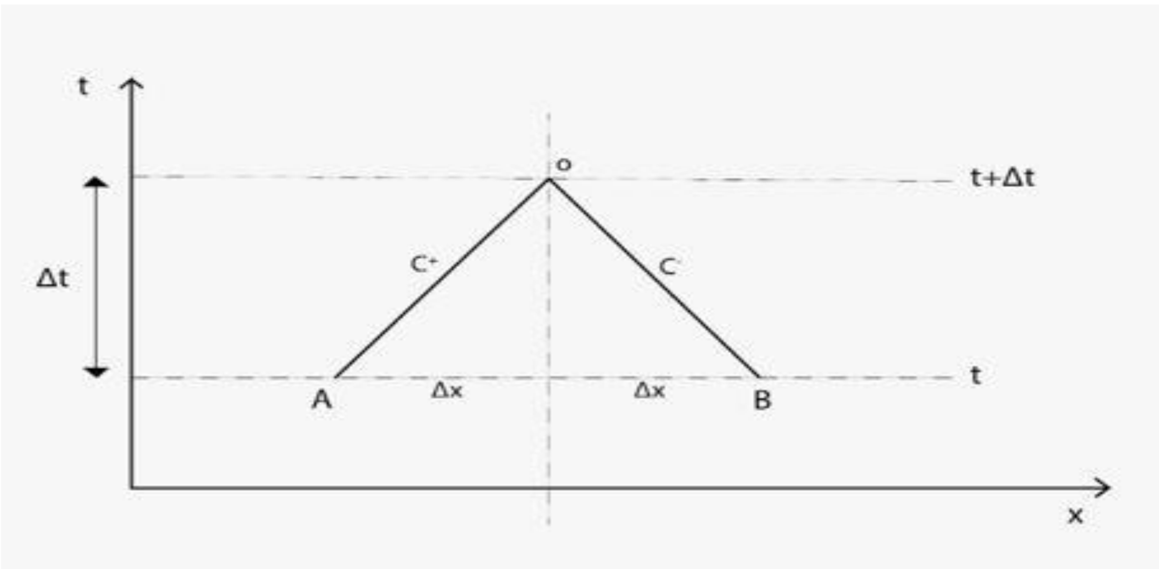


Figure 1

A and B are the boundaries and O is an intermediate node at a distance Δx from both A and B.

- Considering the positive characteristic equation $(\frac{dx}{dt} = c)$, $dx = cdt$

Substituting this value in IV,

$$\rho c dv + dp + \rho dx g sin\theta + \rho dx \frac{fv|v|}{2D} = 0 \quad \text{V}$$

Integrating V from A to O,

$$\rho c(v_o - v_A) + (p_o - p_A) + \rho g \Delta x sin\theta + \rho \Delta x \frac{fv_A|v_A|}{2D} = 0 \quad \text{VI}$$

- Considering the negative characteristic equation $(\frac{dx}{dt} = -c)$, $-dx = cdt$

$$\rho c dv + dp - \rho dx g sin\theta + \rho dx \frac{fv|v|}{2D} = 0 \quad \text{VII}$$

Integrating VII from O to B,

$$\rho c(v_o - v_B) - (p_o - p_B) + \rho g \Delta x sin\theta + \rho \Delta x \frac{fv_B|v_B|}{2D} = 0 \quad \text{VIII}$$

In equations VI and VIII, v_o and p_o are the unknown variables.

Now we convert these equations in terms of discharge and head.

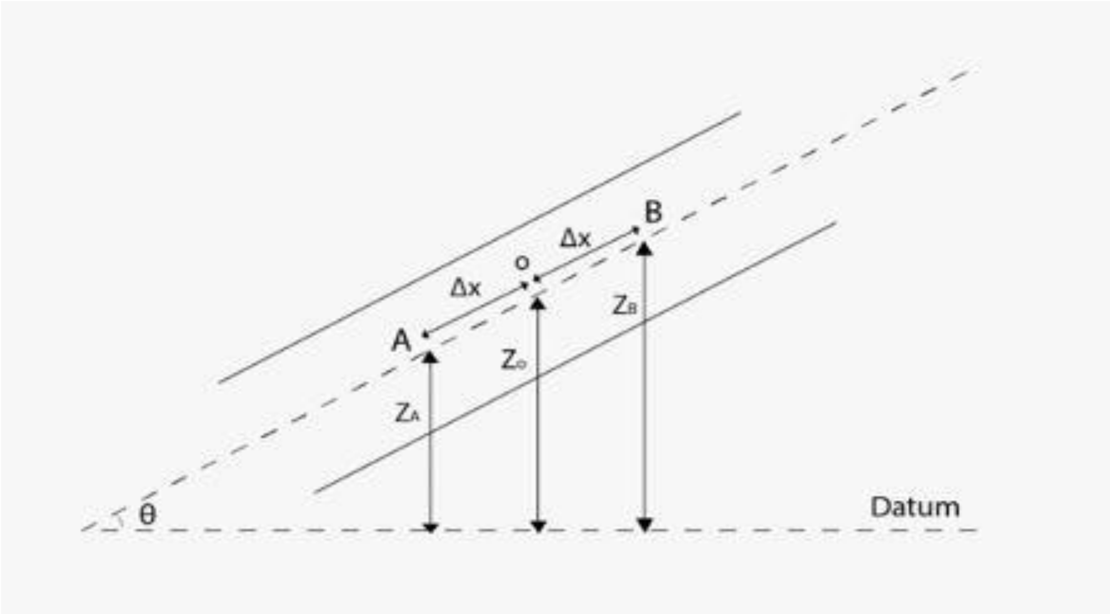


Figure 2

$p = \rho g(H - Z)$ where $H \rightarrow$ piezometric head and $Z \rightarrow$ datum head

$\Rightarrow p_o = \rho g(H_o - Z_o)$ and $p_A = \rho g(H_A - Z_A)$

$\Rightarrow p_o - p_A = \rho g(H_o - H_A) - \rho g(Z_o - Z_A)$

$sin\theta = \frac{Z_o - Z_A}{\Delta x} \qquad \Rightarrow \ Z_o - Z_A = sin\theta \Delta x$

$\Rightarrow p_o - p_A = \rho g(H_o - H_A) - \rho g \Delta x sin\theta$

Substituting this value in VI,

$$\rho c(v_o - v_A) + \rho g(H_o - H_A) - \rho g\Delta x sin\theta + \rho g\Delta x sin\theta + \rho\Delta x \frac{f v_A|v_A|}{2D} = 0$$

Putting $v = \frac{Q}{A}$ in the above equation

$c\frac{(Q_o - Q_A)}{A} + g(H_o - H_A) + \Delta x \frac{f Q_A|Q_A|}{2A^2 D} = 0$ (assume constant density)

$$\Rightarrow H_o = H_A - \frac{c}{gA}(Q_o - Q_A) - \Delta x \frac{f Q_A|Q_A|}{2gA^2 D}$$

Let $\xi = \frac{c}{gA}$ and $\lambda = \frac{\Delta x f}{2gA^2 D}$

$\Rightarrow H_o = H_A - \xi(Q_o - Q_A) - \lambda Q_A|Q_A|$ IX

From the figure,

$p_o - p_B = \rho g(H_o - H_B) + \rho g\Delta x sin\theta$

Substituting this value in VIII,

$$\rho c(v_o - v_B) - \rho g(H_o - H_B) - \rho g\Delta x sin\theta + \rho g\Delta x sin\theta + \rho\Delta x \frac{f v_B|v_B|}{2D} = 0$$

Putting $v = \frac{Q}{A}$ in the above equation and substituting the values of $\xi$and λ,

$H_o = H_B + \xi(Q_o - Q_B) + \lambda Q_B|Q_B|$ X

For solving a pipeline network, the value of head and discharge at the boundary A and B can be calculated using boundary conditions. Once those values are known, the values of head and discharge at the intermediate node can be found using the equations IX and X

ie, $H_o = H_A - \xi(Q_o - Q_A) - \lambda Q_A|Q_A|$

and $H_o = H_B + \xi(Q_o - Q_B) + \lambda Q_B|Q_B|$

2. Application of the above derived equations to solve a linear pipeline network.

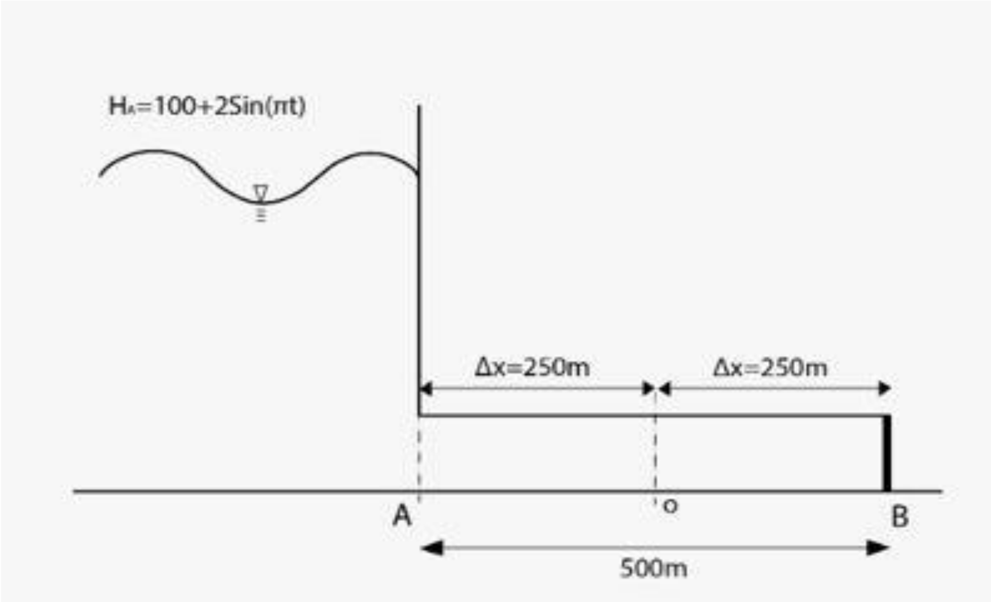


Figure 3

In the above problem, point B is a dead end. The HGL at the reservoir follows the equation $H_A = 100 + 2\sin(\pi t)$. Diameter of the pipe is 300 mm and the length of the pipe is 500m. Wave velocity in the pipe is 1000 m/s and consider a friction factor of 0.02. Find the variation in head and discharge at the middle point of the pipe for a period of 4 seconds.

Given,

$Q_{B=}$ 0 (because dead end)

$H_A = 100 + 2\sin(\pi t)$

$D = 0.3$ m

$L = 500$ m

$c = 1000$ m/s

$f = 0.02$

Since we need to find the variations in the middle point, consider two reaches $\Rightarrow \Delta x = 250\ m$

For stability, Courant's number ≤ 1. Assume CN = 1

$\Rightarrow \frac{\Delta t}{\Delta x} = \frac{1}{c} \quad \Rightarrow \Delta t = \frac{\Delta x}{c} = \frac{250}{1000} = 0.25$ seconds

Initially, ie , at time $t = 0$, we assume that the flow has not started. This makes all the flow values at the boundary as well as at the intermediate node (except for H_A) zero.

$A = \frac{\pi}{4}D^2 = 0.07068\ m^2$

$C_a = \frac{gA}{c}$ =6.934× 10^{-4}

$\xi = \frac{c}{gA} = 1442.23$

$R = \frac{f}{2DA} = 0.4716$

$\lambda = \frac{f\Delta x}{2gdA^2} = 170.04$

From the method of characteristics,

$H_o = H_A - \xi(Q_o - Q_A) - \lambda Q_A|Q_A|$ I

and $H_o = H_B + \xi(Q_o - Q_B) + \lambda Q_B|Q_B|$. But $Q_B = 0$

$\Rightarrow\ H_o = H_B + \xi Q_o$................... II

Comparing I and II,

$$H_B + \xi Q_o = H_A - \xi(Q_o - Q_A) - \lambda Q_A|Q_A|$$

On solving,

$Q_o = \frac{H_A - H_B + \xi Q_A - \lambda Q_A|Q_A|}{2\xi}$ III

From II and III, the values of head and discharge in the middle of the pipe can be calculated. Note that the boundary values used here are from the previous time step.

At point A, (neglect the velocity and entrance losses)

$H_A = 100 + 2\sin(\pi t)$

$Q_{A=C_n+C_aH_A}$ (reservoir in the upstream side)

$C_n = Q_B - C_aH_B - R\Delta t Q_B|Q_B|$

But $Q_B = 0 \Rightarrow C_n = -C_aH_B$

$\Rightarrow Q_A = -C_aH_B + C_aH_A$ IV

At point B,

$Q_B = 0$

$H_B = \frac{C_p}{C_a}$ (boundary condition for dead end)

$C_p = Q_A + C_aH_A - R\Delta t Q_A|Q_A|$

$$\Rightarrow H_B = \frac{Q_A + C_a H_A - R\Delta t Q_A |Q_A|}{C_a}$$

Substituting for Q_A in the above equation,

$$H_B = \frac{(-C_a H_B + C_a H_A) + C_a H_A - R\Delta t(-C_a H_B + C_a H_A)^2}{C_a}$$

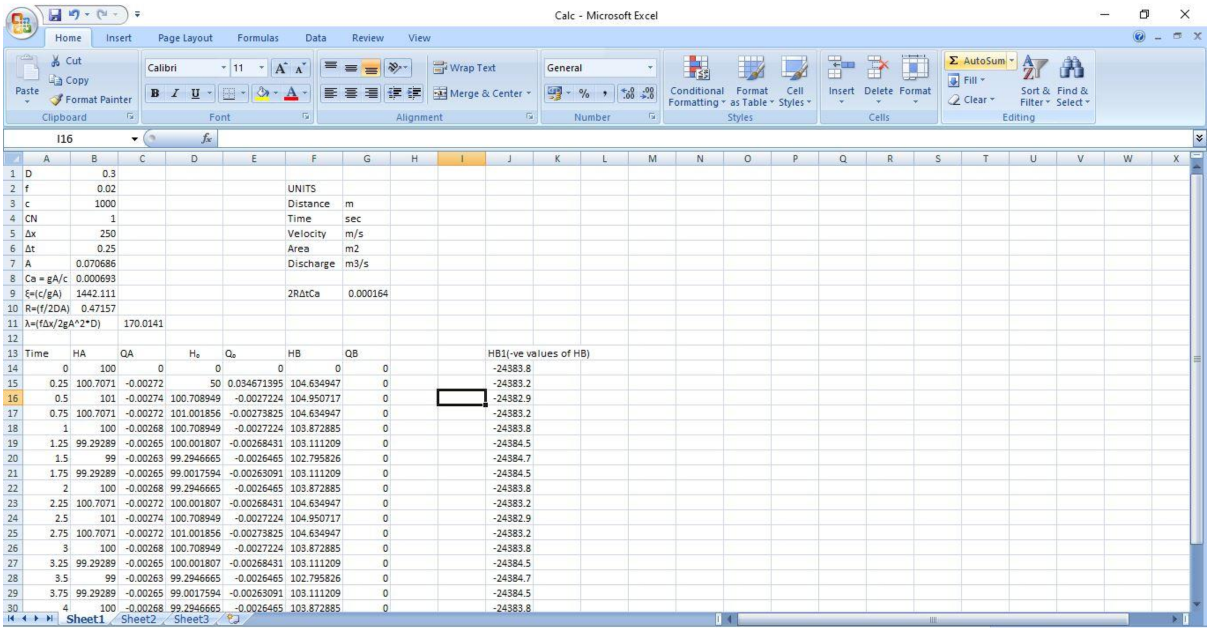
On solving, it becomes a quadratic equation of the form

$$R\Delta t C_a (H_B)^2 + H_B(2 - 2R\Delta t C_a H_A) - \left(2H_A + C_a {H_A}^2\right) = 0$$

$$\Rightarrow H_B = \frac{-(2-2R\Delta t C_a H_A) \pm \sqrt{(2-2R\Delta t C_a H_A)^2 + 4R\Delta t C_a (2H_A + C_a {H_A}^2)}}{2R\Delta t C_a} \quad \text{.................................. V}$$

From I and II, the boundary values for the current time step can be determined.

All the calculations are done in excel and graphs are plotted for variations of H_A, Q_A, H_B, H_o and Q_o with time and the results thus obtained are analysed.



	A	B	C	D	E	F	G	J
1	D	0.3						
2	f	0.02				UNITS		
3	c	1000				Distance	m	
4	CN	1				Time	sec	
5	Δx	250				Velocity	m/s	
6	Δt	0.25				Area	m2	
7	A	0.070686				Discharge	m3/s	
8	Ca = gA/c	0.000693						
9	ξ=(c/gA)	1442.111				2RΔtCa	0.000164	
10	R=(f/2DA)	0.47157						
11	λ=(fΔx/2gA^2*D)		170.0141					
12								
13	Time	HA	QA	H₀	Q₀	HB	QB	HB1(-ve values of HB)
14	0	100	0	0	0	0	0	-24383.8
15	0.25	100.7071	-0.00272	50	0.034671395	104.634947	0	-24383.2
16	0.5	101	-0.00274	100.708949	-0.0027224	104.950717	0	-24382.9
17	0.75	100.7071	-0.00272	101.001856	-0.00273825	104.634947	0	-24383.2
18	1	100	-0.00268	100.708949	-0.0027224	103.872885	0	-24383.8
19	1.25	99.29289	-0.00265	100.001807	-0.00268431	103.111209	0	-24384.5
20	1.5	99	-0.00263	99.2946665	-0.0026465	102.795826	0	-24384.7
21	1.75	99.29289	-0.00265	99.0017594	-0.00263091	103.111209	0	-24384.5
22	2	100	-0.00268	99.2946665	-0.0026465	103.872885	0	-24383.8
23	2.25	100.7071	-0.00272	100.001807	-0.00268431	104.634947	0	-24383.2
24	2.5	101	-0.00274	100.708949	-0.0027224	104.950717	0	-24382.9
25	2.75	100.7071	-0.00272	101.001856	-0.00273825	104.634947	0	-24383.2
26	3	100	-0.00268	100.708949	-0.0027224	103.872885	0	-24383.8
27	3.25	99.29289	-0.00265	100.001807	-0.00268431	103.111209	0	-24384.5
28	3.5	99	-0.00263	99.2946665	-0.0026465	102.795826	0	-24384.7
29	3.75	99.29289	-0.00265	99.0017594	-0.00263091	103.111209	0	-24384.5
30	4	100	-0.00268	99.2946665	-0.0026465	103.872885	0	-24383.8

Figure 4

3. Analysis of results obtained from the above problem

The analysis is done by plotting the variations of H_A, Q_A, H_B, H_o and Q_o with time.

4. Study on how method of characteristics can be applied to complex problems.

Upon understanding the underlying concepts, a methodology for solving a pipeline network using Method of Characteristics is developed. This method allows us to solve the network using excel without coding.

- Step 1: Divide the shortest pipe into 'n' number of reaches. $\Delta x = \frac{L}{n}$, where 'L' is the length of the pipe.
- Step 2: For stability, Courant Number (CN) $\leq 1 \Rightarrow \frac{\Delta t}{\Delta x} \leq \frac{1}{c}$, where 'c' is the wave velocity. Choose a value of Δt such that this condition is satisfied for all the pipelines in the network. According to Kaplan et al., for long conduits, Δt can be integral multiples of Δt for short conduits. This procedure is called 'zooming in'. Divide the other conduits also into reaches.
- Step 3: The initial condition of the flow in the pipes is identified and head and discharge values are computed at t=0.
- Step 4: Time is incremented. The value of head and discharge at the intermediate nodes are calculated from the known conditions in boundaries in the previous time step. The following formulae can be used for this:
  $H_o = H_A - \xi(Q_o - Q_A) - \lambda Q_A|Q_A|$
  and $H_o = H_B + \xi(Q_o - Q_B) + \lambda Q_B|Q_B|$ where A and B are the boundaries and O is the intermediate node.
- Step 5: According to the type of boundaries of each pipe, appropriate boundary condition equations can be used to find the values in boundaries for that time step.
- Step 6: Steps 4 and 5 are repeated until flow conditions for the required time are computed.

RESULTS AND ANALYSIS

From the linear pipe network that was solved using method of characteristics, the results obtained are plotted using Microsoft excel and the variations are given as:

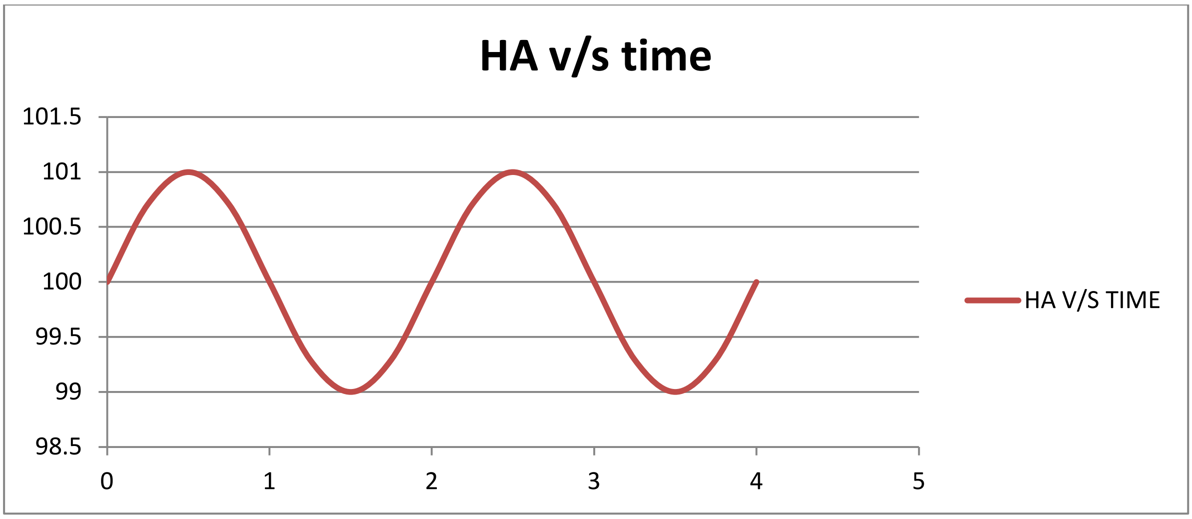


Figure 5

The head of the reservoir is changing according to $100 + 2\sin(\pi t)$ wrt t. We are neglecting the velocity head as well as entrance losses. So the head value at A will be changing periodically with a time period of 2 seconds as shown in figure 5.

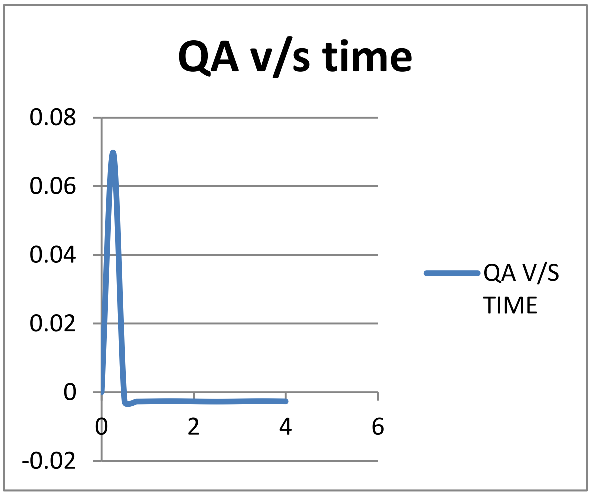


Figure 6

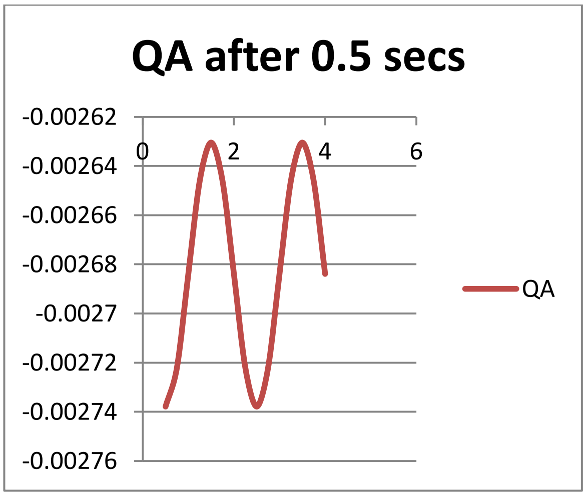
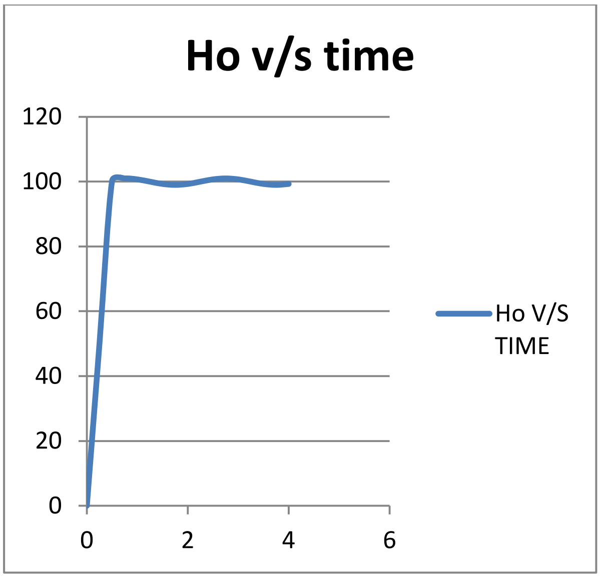


Figure 7

From figures 6 and 7: At time t=0, the Q value is zero and as the wave travels in the pipe from left to right, the discharge at A increases up to a maximum value of 0.06979 m^3/s at t=0.25 seconds. Once the wave is reflected back from B, the discharge at A fluctuates between 0.00263 m^3/s and 0.00274 m^3/s periodically with a time period of 2 seconds. The negative value indicates that the flow is from right to left.



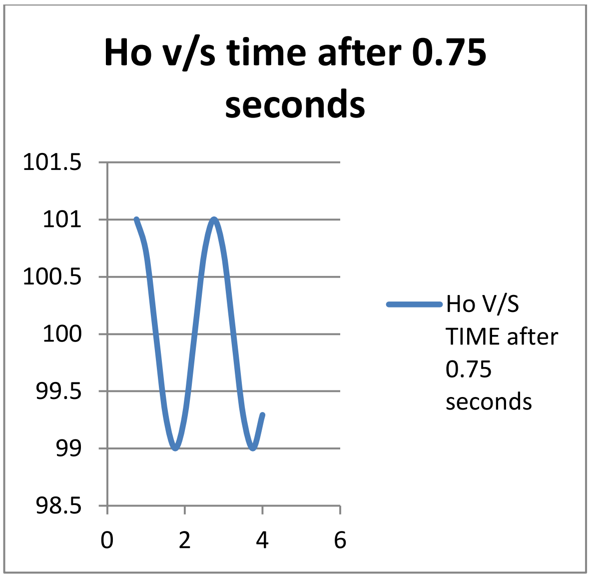
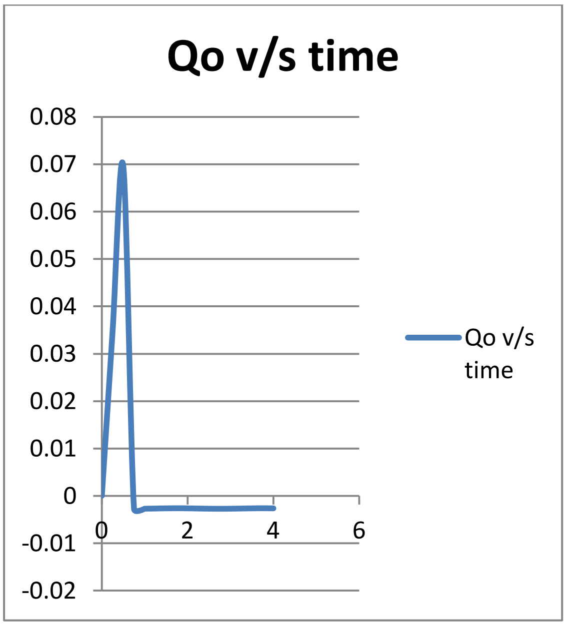


Figure 8 **Figure 9**

From figures 8 and 9: At time t=0, there is no flow. So the H_o value is zero. At time t=0.25 seconds, the head becomes 50 m and at t = 5 seconds, the head becomes 100.261995 m. After this the head starts changing periodically from 101.001856 m to 99.0017594 m with a time period of 2 seconds. So the maximum value of head in the middle section of the pipe is 101.001856 m.



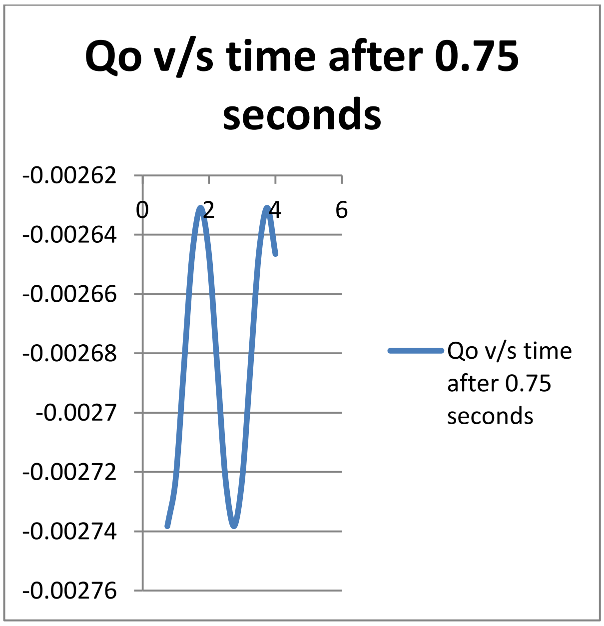


Figure 10 **Figure 11**

From figures 10 and 11: Initial value of Q_o is zero as there is no flow. Then it increases up to a maximum value of 0.06952 m^3/s at time t=0.5 seconds. Q_o then starts fluctuating from 0.00273825 m^3/s to 0.00263091m^3/s with a time period of 2 seconds. The negative sign indicates that the flow direction is from right to left.

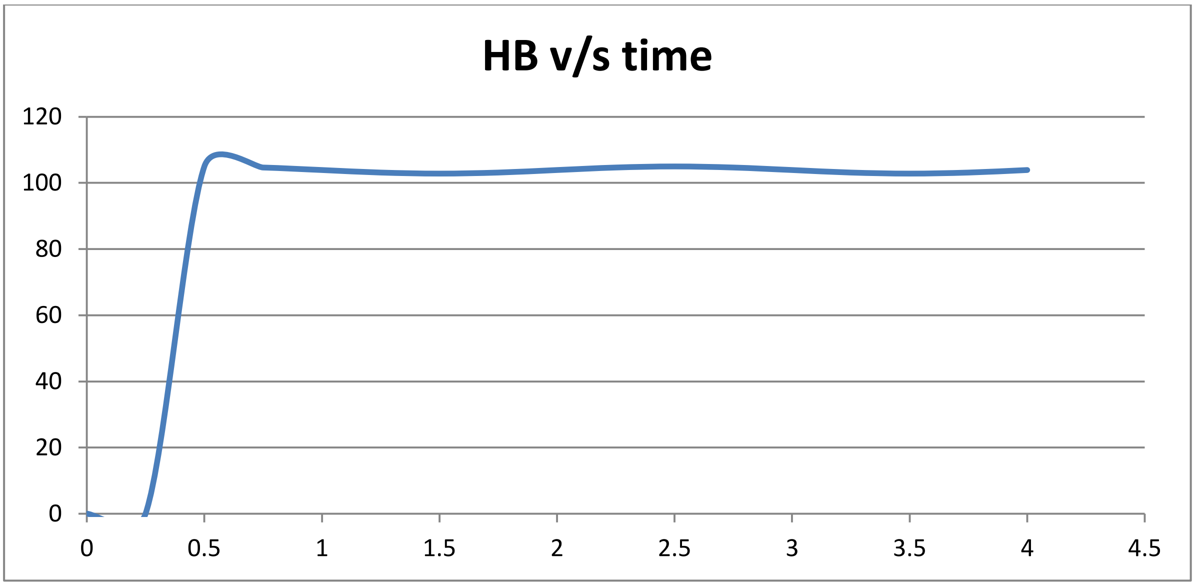


Figure 12

From figure 12: The head values at B at time t=0 and 0.25 seconds are zero. This is because the wave will reach B only at 0.5 seconds as the wave velocity is 1000 m/s and the length of the pipe is 500 m. At t=0.5 seconds, the head reaches a maximum value of 104.950717 m and starts fluctuating between this value and 102.795826 m. The time period is 2 seconds.

OBSERVATIONS:

1. The maximum value for discharge obtained is 0.06979 m^3/s and the maximum head value is 104.950717 m. So the pipe must be designed for these values.
2. The time period for change in discharge and head values at O and B is 2 seconds. This is because the head at A fluctuates with a time period of 2 seconds.
3. Water profiles at different time can be given as:

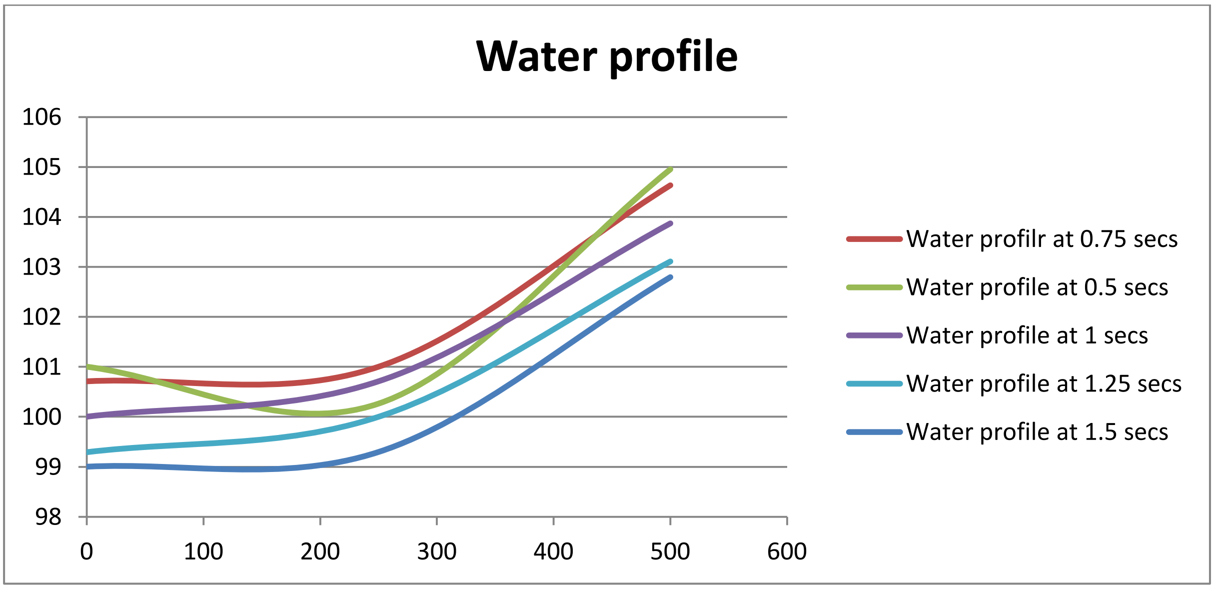


Figure 13

CONCLUSION

In method of characteristics, one sets out to find the two intersecting families of curves that are the positive and negative characteristic equations and integrates the combined form of continuity and momentum equations along these curves to produce the values of H and Q at the (x,t) coordinates which are the intersections of the curves. Even though it is one of the most accurate methods to analyse a pipeline network, applying this method to solve a complex network becomes cumbersome. For a complex network, the number of required calculations will be more and a computer program will be required for practical applications.

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