Exercise : part - I :

sigmoid neurons simulating perceptrons - 1 :

$w, b \Rightarrow$ multiply by Constant +ve , "$c > 0$."

To show behaviour does not change after multiplying

$$f(z) = \begin{cases} wx + b > 0 & - 1 \\ wx + b \leq 0 & - 0 \end{cases}$$

multiply by c :

$$f(z') = \begin{cases} c[wx + b] > 0 & - 1 \\ c[wx + b] \leq 0 & - 0 \end{cases}$$

$$w' = cw, \quad b' = cb$$

$$z' = w'x + b'$$

$$z' = cwx + cb = c[wx + b]$$

As $c > 0$,

if $z \leq 0$, $cz \leq 0$

if $z > 0$, $cz > 0$

$$\boxed{\text{sign}(z') = \text{sign}(z)}$$

② Sigmoid

$$f(z) = \begin{cases} wx+b > 0 = 1 \\ wx+b \le 0 = 0 \end{cases}$$

$z' = c[wx+b]$, where $c > 0$

case 1: $wx+b > 0$:

$$z' = c[wx+b]$$

$$f(z') = \frac{1}{1+e^{-z'}}$$

$\lim c \to \infty$

$$f(z') = \frac{1}{1+e^{-[c(wx+b)]}}$$

$\to c > 0$

$$f(z') = \frac{1}{1+e^{-(\text{greater value})}}$$

$$= \frac{1}{1+0}$$

$$\boxed{f(z') = 1}$$ ✓

case 2: $wx+b < 0$:

$$f(z') = \frac{1}{1+e^{-z'}}$$

$\lim c \to \infty$

$$f(z') = \frac{1}{1+e^{-(c[wx+b])}}$$

where $c > 0$ and $(wx+b) < 0$

$$f(z') = \frac{1}{1 + e^{+\text{value}}}$$

$$= \frac{1}{1+\infty}$$

$$\boxed{f(z') = 0}$$ ✓

Case 3:

$$wx + b = 0 :$$

$$\lim_{c \to \infty} f(z') = \frac{1}{1 + e^{-[c(0)]}}$$

$$f(z') = \frac{1}{1 + e^{0}}$$

$$= \frac{1}{2}$$

$$\boxed{f(z') = 0.5}$$ ✗

Sigmoid act as step function same as perceptron on case 1 and case 2, But in case 3 it violated having o/p in b/w 0 to 1., but not as perceptron.