Algorithmic Methods for Mathematical Models
Course Project

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Formal Problem Definition

- n: number of products
- x: height of the suitcase in millimeters
- y: width of the suitcase in millimeters
- c: limit to the total weight of the suitcase in grams
- p_i: price of the i-th product in euros
- w_i: weight of the i-th product in grams
- s_i: side length of the i-th product's (square) box in millimeters

Decision Variables

- Chosen_i: binary variable that is 1 if object i is chosen, and 0 otherwise.
- PointsX_i: the x-coordinate of the bottom-left corner of object i.
- PointsY_i: the y-coordinate of the bottom-left corner of object i.
- $\text{Overlap}_{i,j,d}$: binary variable indicating if objects i and j do not overlap in direction d, where $d \in \{1, 2, 3, 4\}$.

Objective Function

Maximize the total price of the chosen objects:

$$\text{maximize} \sum_{i=1}^{n} p_i \cdot \text{Chosen}_i$$

Max Weight Constraint

Ensure the total weight of the chosen objects does not exceed the suitcase's capacity:

$$\sum_{i=1}^{n} w_i \cdot \text{Chosen}_i \leq c$$

Coordinate Bounds Constraints

Ensure each object lies entirely within the suitcase's boundaries:

$$\forall i \in \{1, \ldots, n\}, \quad \text{PointsX}_i \geq 1$$

$$\forall i \in \{1, \ldots, n\}, \quad \text{PointsY}_i \geq 1$$

$$\forall i \in \{1, \ldots, n\}, \quad \text{PointsX}_i + s_i - 1 \leq x$$

$$\forall i \in \{1, \ldots, n\}, \quad \text{PointsY}_i + s_i - 1 \leq y$$

Non-Overlapping Constraints

Left/Right/Up/Down non-overlapping:

$$\forall i, j \in \{1, \ldots, n\}, i \neq j, \quad \text{PointsX}_i - \text{PointsX}_j + s_i \leq -M \cdot (\text{Chosen}_i + \text{Chosen}_j + \text{Overlap}_{i,j,1} - 3)$$

$$\forall i, j \in \{1, \ldots, n\}, i \neq j, \quad \text{PointsX}_i - \text{PointsX}_j + s_i \leq -M \cdot (\text{Chosen}_i + \text{Chosen}_j + \text{Overlap}_{i,j,2} - 3)$$

$$\forall i, j \in \{1, \ldots, n\}, i \neq j, \quad \text{PointsX}_i - \text{PointsX}_j + s_i \leq -M \cdot (\text{Chosen}_i + \text{Chosen}_j + \text{Overlap}_{i,j,3} - 3)$$

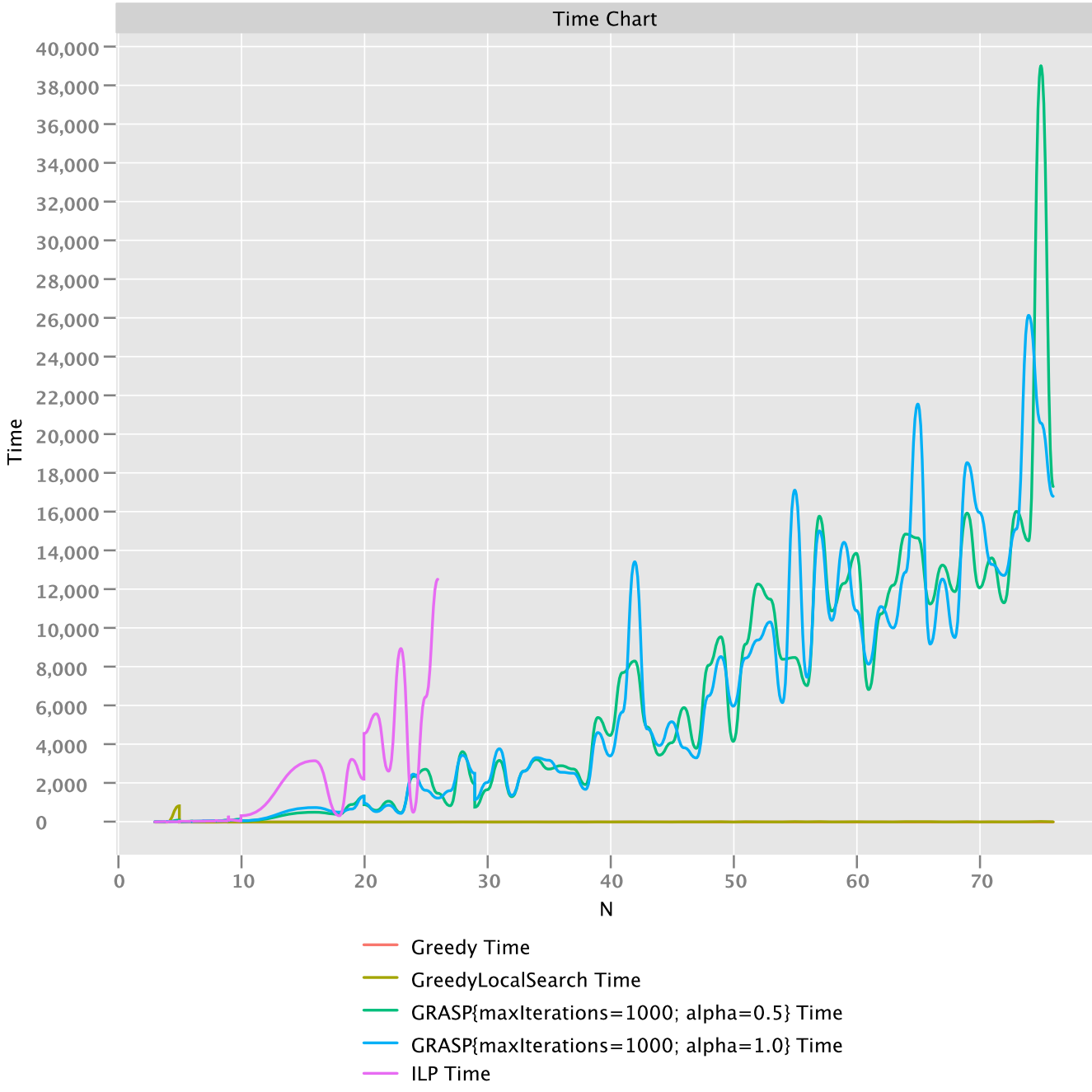
$$\forall i, j \in \{1, \ldots, n\}, i \neq j, \quad \text{PointsX}_i - \text{PointsX}_j + s_i \leq -M \cdot (\text{Chosen}_i + \text{Chosen}_j + \text{Overlap}_{i,j,4} - 3)$$

Non-Overlapping Constraints

At least one of the non-overlapping conditions is satisfied:

$$\forall i, j \in \{1, \ldots, n\}, i \neq j, \quad \sum_{d=1}^{4} \text{Overlap}_{i,j,d} \geq 1$$

Time chart


Time Chart
Time
40,000
38,000
36,000
34,000
32,000
30,000
28,000
26,000
24,000
22,000
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
0
10
20
30
40
50
60
70
N
Greedy Time
GreedyLocalSearch Time
GRASP{maxIterations=1000; alpha=0.5} Time
GRASP{maxIterations=1000; alpha=1.0} Time
ILP Time

Objective chart