Problem 1

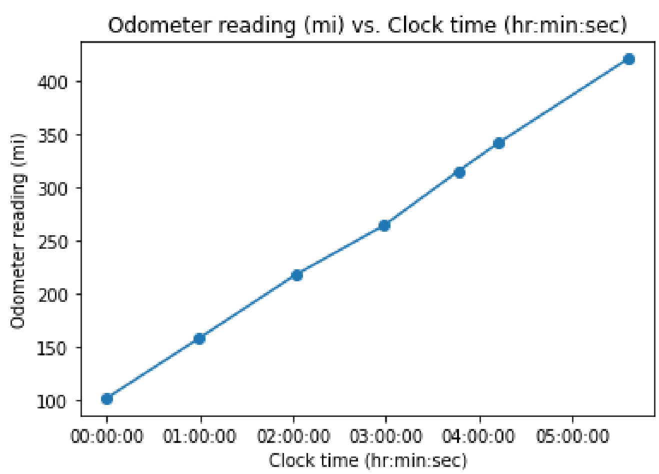
Given:

Clock time (hr:min:sec)	00:00:00	0:59:12	2:01:46	2:58:55	3:47:01	4:13:00	5:36:17
Odometer reading (mi)	102.0	157.8	217.6	264.1	315.2	341.7	420.3
Time interval (hr)							
Distance (mi)							
Average speed (mph)							

Find:

- Fill out the table using finite difference equations.
- Only show work for one time interval, distance, and average speed.
- Find the overall average speed of the car.

Diagram:



Theory:

$Time\ interval\ =\ \Delta t$, where t is the clock time.

$Distance\ (mi)\ =\ \Delta x$, where x is the odometer reading.

$Average\ speed\ (mph)\ =\ \Delta x / \Delta t.$

$Average\ = \frac{x_1 + ... + x_n}{n}$

Assumption:

- The experiment was run during the day. If not, the car had energy stored.
- ΔT is changing.

Ilan Sela
232007587
ENGR/PHYS 216-505

Solution:

For first time interval:

$\Delta t = t_2 - t_1$ = 0:59:12 - 00:00:00 = 59 minutes and 12 seconds · $\frac{1\ minute}{60\ seconds}$

= 59.20 minutes · $\frac{1\ hour}{60\ minutes}$ = 0.9867 hr

For first distance:

$\Delta x = x_2 - x_1$ = 157.8 mi - 102.0 mi = 55.80 mi

For first average speed:

$v = \Delta x / \Delta t$ = 55.80 mi / 0.9867 hr = 56.55 mph

For overall average speed:

$\bar{x} = \frac{56.55 + 57.33 + 48.82 + 63.74 + 61.19 + 56.63}{6}$ = $\boxed{57.38 \text{ mph}}$

Clock time (hr:min:sec)	00:00:00	0:59:12	2:01:46	2:58:55	3:47:01	4:13:00	5:36:17
Odometer reading (mi)	102.0	157.8	217.6	264.1	315.2	341.7	420.3
Time interval (hr)	0.9867	1.043	0.9525	0.8017	0.4331	1.388	
Distance (mi)	55.80	59.80	46.50	51.10	26.50	78.60	
Average speed (mph)	56.55	57.33	48.82	63.74	61.19	56.63	

Rest of answer highlighted in above table

Ilan Sela
232007587
ENGR/PHYS 216-505

Problem 2

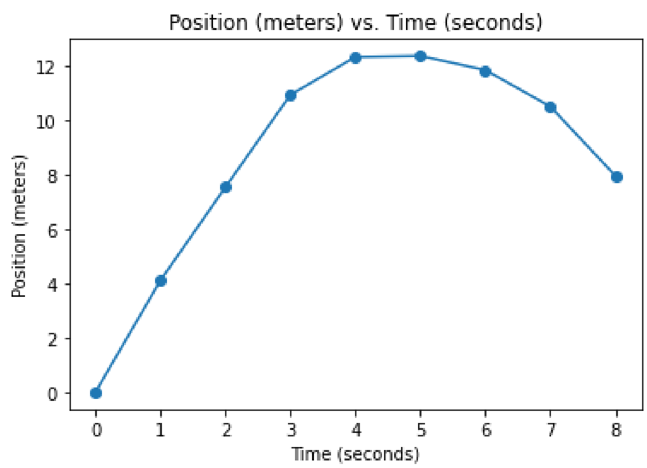
Given:

Time t, seconds	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
Position x, meters	0.00	4.10	7.53	10.92	12.31	12.35	11.83	10.49	7.95

Find:

- Velocity and acceleration of the ball at t = 2, 3, 4, 5, 6 using
    a) Forward finite difference
    b) Backward finite difference
    c) Centered finite difference
- Only show work for one velocity and one acceleration.

Diagram:



Theory:

For v_5 (velocity at position 5 given position):

Forward: $v_5 = \frac{x_6 - x_5}{t_6 - t_5}$

Backward: $v_5 = \frac{x_5 - x_4}{t_5 - t_4}$

Centered: $v_5 = \frac{x_6 - x_4}{t_6 - t_4}$

For a_5 use velocity instead of position:

Forward: $a_5 = \frac{v_6 - v_5}{t_6 - t_5}$

Backward: $v_5 = \frac{v_5 - v_4}{t_5 - t_4}$

Ilan Sela
232007587
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Centered: $v_5 = \frac{v_6 - v_4}{t_6 - t_4}$

Assumption:
- $\Delta t\ is\ constant.$
- Ball is rolling in a straight line.

Solution:

a) For forward finite difference at t = 3:

$$v_3 = \frac{x_4 - x_3}{t_4 - t_3} = \frac{12.31 - 10.92}{4.0 - 3.0} = 1.4\ m/s$$

$$a_3 = \frac{v_4 - v_3}{t_4 - t_3} = \frac{0.040 - 1.39}{4.0 - 3.0} = -1.4\ m/s^2$$

b) For backward finite difference at t = 3:

$$v_3 = \frac{x_3 - x_2}{t_3 - t_2} = \frac{10.92 - 7.53}{3.0 - 2.0} = 3.4\ m/s$$

$$a_3 = \frac{v_3 - v_2}{t_3 - t_2} = \frac{3.39 - 3.43}{3.0 - 2.0} = -0.040\ m/s^2$$

c) For centered finite difference at t = 3:

$$v_3 = \frac{x_4 - x_2}{t_4 - t_2} = \frac{12.31 - 7.53}{4.0 - 2.0} = 2.4\ m/s$$

$$a_3 = \frac{v_4 - v_2}{t_4 - t_2} = \frac{0.715 - 3.41}{4.0 - 2.0} = -1.3\ m/s^2$$

Time (seconds)	2.0	3.0	4.0	5.0	6.0
Position (meters)	7.53	10.92	12.31	12.35	11.83
Velocity (m/s) (forward)	3.4	1.4	0.040	-0.52	-1.3
Velocity (m/s) (backward)	3.4	3.4	1.4	0.040	-0.52
Velocity (m/s) (centered)	3.4	2.4	0.72	-0.24	-0.93
Acceleration (m/s^2) (forward)	-2.0	-1.4	-0.56	-0.82	-1.2
Acceleration (m/s^2) (backward)	-0.67	-0.040	-2.0	-1.4	-0.56
Acceleration (m/s^2) (centered)	-0.69	-1.3	-1.3	-0.82	-0.85

Ilan Sela
232007587
ENGR/PHYS 216-505

Problem 3

Given:

$$\tau = \mu \frac{du}{dy}$$

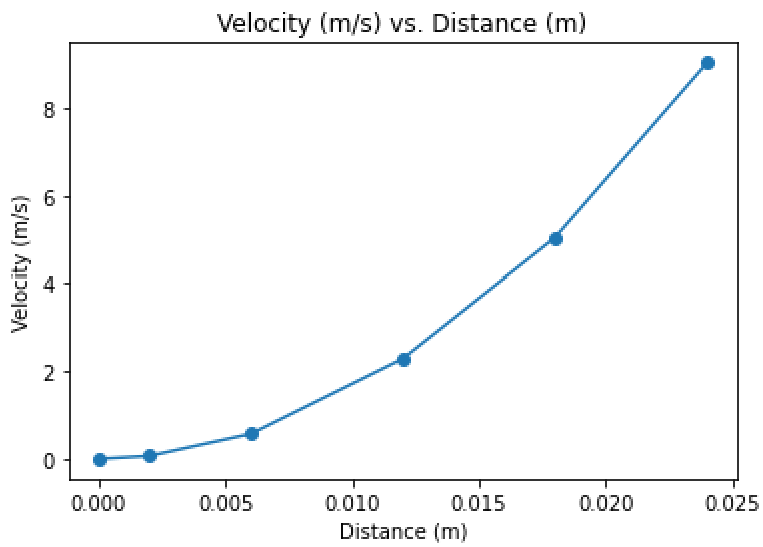
$$\mu = 1.8 \times 10^{-5} \, Ns/m^2$$

y (m)	0.000	0.00200	0.00600	0.0120	0.0180	0.0240
u (m/s)	0.000	0.067	0.572	2.291	5.047	9.041

Find:

- Use second order centered first finite difference method to find shear stress, $\tau\ (N/m^2)$, at y = 0.006, 0.012, and 0.018 m
- Answer: How do you think your answers would change if you use higher order finite differences?

Diagram:



Theory:

$$\frac{du}{dy} = \frac{u_2 - u_1}{y_2 - y_1}$$

$$\tau = \mu \frac{du}{dy}$$

Assumption:

- Constant dynamic viscosity
- Non-constant Δx

Ilan Sela
232007587
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Solution:

For y = 0.0120 m:

$$du/dy = \frac{5.047 - 0.572}{0.0180 - 0.00600}\ 1/s = 373\ 1/s$$

$$\tau = \mu\frac{du}{dy} = (1.8 \times 10^{-5}\ Ns/m^2) \cdot (373\ 1/s) = 0.00671\ N/m^2$$

y (m)	0.000	0.00200	0.00600	0.0120	0.0180	0.0240
u (m/s)	0.000	0.067	0.572	2.291	5.047	9.041
τ (N/m^2)		0.00172	0.0040	0.00671	0.0101	

First order assumes that $\Delta x < 1$. A higher order means $(\Delta x)^n$, which approaches zero as n gets larger. Thus, as the order increases, τ will become more accurate.

Ilan Sela
232007587
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Problem 4

Given:

$$f(x) = \frac{ln(x) \cdot sinh(x)}{e^x}$$

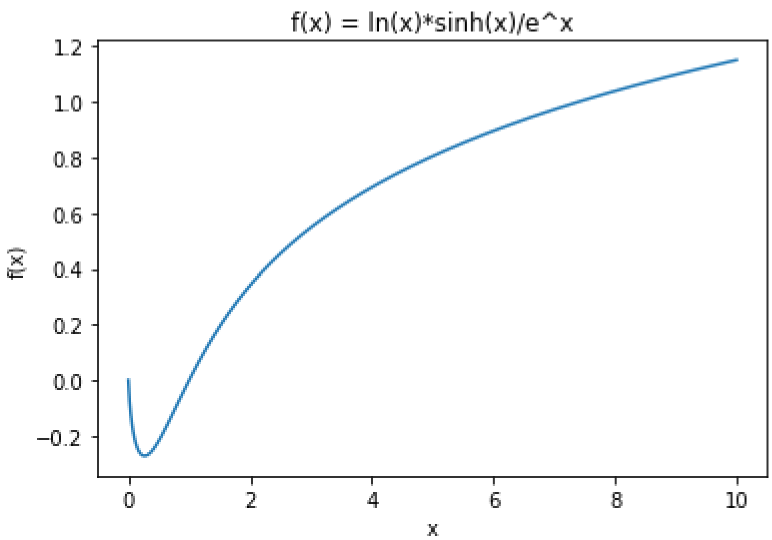
$$f'(1.5) = 0.336925$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\%$$

Find:

- Estimate the derivative numerically for the step size $\Delta x = 0.25$ using forward, backward, and centered first finite differences.
- Determine percent error between true and estimated values.
- Answer: what value of Δx would have to be used for the backward and forward finite differences to get the same percent error as the centered finite difference using $\Delta x = 0.25$?

Diagram:



Theory:

Forward first finite difference:

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Backward first finite difference:

$$f'(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

Centered first finite difference:

$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

Ilan Sela
232007587
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Assumption:

- A smaller Δx will give a more accurate result.
- Sinh(x) is hyperbolic sine in f(x)

Solution:

At x = 1.5 and Δx = 0.25 for forward first finite difference:

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(1.5) = \frac{f(1.5+0.25) - f(1.5)}{0.25} = \frac{f(1.75) - f(1.5)}{0.25} = \frac{0.27135 - 0.19263}{0.25} = \boxed{0.31488}$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\% = \left|\frac{0.336925 - 0.31488}{0.336925}\right| \times 100\% = \boxed{6.54299\%}$$

At x = 1.5 and Δx = 0.25 for backward first finite difference:

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

$$f'(1.5) = \frac{f(1.5) - f(1.5-0.25)}{0.25} = \frac{f(1.5) - f(1.25)}{0.25} = \frac{0.19263 - 0.10241}{0.25} = \boxed{0.36088}$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\% = \left|\frac{0.336925 - 0.36088}{0.336925}\right| \times 100\% = \boxed{7.10989\%}$$

At x = 1.5 and Δx = 0.25 for forward first finite difference:

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$$f'(1.5) = \frac{f(1.5+0.25) - f(1.5-0.25)}{2(0.25)} = \frac{f(1.75) - f(1.25)}{0.5} = \frac{0.27135 - 0.10241}{0.5} = \boxed{0.33788}$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\% = \left|\frac{0.336925 - 0.33788}{0.336925}\right| \times 100\% = \boxed{0.28344\%}$$

What value of Δx would have to be used for the backward and forward finite differences to get the same (or smaller) percent error as the centered finite difference using $\Delta x = 0.25$?

Answer: $\boxed{\Delta x = 0.0078125}$ gives a smaller percent error for forward and backward finite differences.

Process: I kept diving Δx by 2 until both the forward and backward percent errors were the same or smaller than the centered finite difference at $\Delta x = 0.25$.

Forward:

$$f'(1.5) = \frac{f(1.5+0.0078125) - f(1.5)}{0.0078125} = \frac{f(1.5078125) - f(1.5)}{0.0078125} = \frac{0.19526 - 0.19263}{0.0078125} = 0.33664$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\% = \left|\frac{0.336925 - 0.33664}{0.336925}\right| \times 100\% = 0.08458\%$$

Ilan Sela
232007587
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Backward:

$$f'(1.5) = \frac{f(1.5) - f(1.5-0.0078125)}{0.0078125} = \frac{f(1.5) - f(1.4921875)}{0.0078125} = \frac{0.19263 - 0.19000}{0.0078125} = 0.33664$$

$$\varepsilon = \left|\frac{true\ value - estimated\ value}{true\ value}\right| \times 100\% = \left|\frac{0.336925 - 0.33664}{0.336925}\right| \times 100\% = 0.08458\%$$

Do note: $\Delta x = 0.015625$ was the first value that gave only the forward finite difference a percent error below the centered finite difference.