LAKE FOREST COLLEGE
Department of Physics

Physics 114 Experiment 11: Rotational Motion-v2 Fall 2024

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Date: 11/14/2024
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Preliminary Instructions
Create a folder for you and your lab partner. Save a copy of these instructions for each student to that folder. Include your name in the filename. Save one copy of the Excel template with both your and your partner's name included in the filename.

Experimental purpose of today's experiment
Measure the angular velocity and acceleration of spinning disks, test the equation $\tau = I\alpha$ and investigate perfectly inelastic angular collisions.

Pedagogical purpose of today's experiment
Study rotational motion, torque, and angular momentum

Background
Quantities that describe pure rotational motion are analogous to quantities that describe linear translational motion in one dimension. These are summarized in the following table. **Except for rotational inertia, all rotational quantities can be positive (for counterclockwise) or negative (for clockwise).** This assumes that the axis of rotation is directed upward.

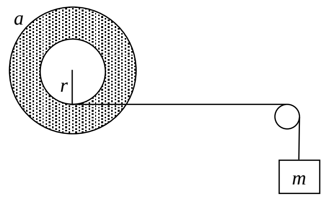
Quantity	**Linear (unit)**	**Rotational (unit)**
Kinematic	position x (m)	angular position θ (rad)
	velocity v_x (m/s)	angular velocity ω (rad/s)
	acceleration a_x (m/s^2)	angular acceleration α (rad/s^2)
Dynamic	force F_x (N)	torque τ (N·m)
	mass m (kg)	rotational inertia I (kg·m^2)
	momentum p_x (kg·m/s)	angular momentum L (kg·m^2/s)
Dynamical Equation	$F_x = ma_x$	$\tau = I\alpha$

Angles are commonly measured in degrees, radians, or revolutions. In most cases, radian is the unit of choice. The conversion between these units is given by $360° = 2\pi$ rad = 1 revolution.

The rotational quantities are defined similarly to the linear quantities. Angular velocity is the rate of change of angular position; angular acceleration is the rate of change of angular velocity. Torque is the amount of "twist" that a force applies to an object. The rotational inertia is a measure of the difficulty of changing the angular velocity of an object.

You will use a spinning disk in this experiment. The disk spins on an air bearing, so there is little friction. The apparatus is the rotational counterpart of an air track. There are two disks in the apparatus; the disks can either spin together or independently.

In the first part of the experiment, you will impart an angular acceleration to the disks with a string that runs over a pulley to a hanging mass. The other end of the string is wrapped around a spool, with radius *r*, attached to the disk. This is shown schematically in the figure.



The tension in the string, *T*, is related to the acceleration of the hanging mass, *m*. Choose up to be positive.

$$T - mg = ma_y \qquad (1)$$

The tension of the string exerts a torque on the disk. As drawn, the torque tends to cause counterclockwise rotations, so the torque is positive by convention

$$\tau = rT \qquad (2)$$

where *r* is the radius of the spool.

The relationship between the torque and the angular acceleration of the disk is

$$\tau = I\alpha\ . \qquad (3)$$

When released, the hanging mass accelerates downward (negative a_y) and the disk has a positive (CCW) angular acceleration. The angular acceleration of the disk is related to the linear acceleration of the hanging mass.

$$a_y = -r\alpha \qquad (4)$$

Combining equations (1), (2), (3), and (4) together, later you will show that the angular acceleration of the disk is given by

$$\alpha = \frac{mgr}{I + mr^2} \qquad (5)$$

We will treat the spool as a solid, uniform disk rotating about its axis of symmetry, which has a rotational inertia of

$$I_{spool} = \tfrac{1}{2} m_{spool} r^2\ , \qquad (6)$$

where m_{spool} and *r* are the mass and radius of the spool, respectively. The large disks have an outer radius, a, a total mass, m_{disk}, and a hole of radius *b* in the center. Their rotational inertia is

$$I_{disk} = \tfrac{1}{2} m_{disk} \left(a^2 + b^2\right). \qquad (7)$$

In the second part of the experiment, you measure the angular velocities of two disks before and after a collision in which the disks stick together: a perfectly inelastic angular collision. Objects that rotate possess angular momentum (*L*). This is the rotational analogue of (linear) momentum.

For one object, the rotational quantity is given by the following formula: $L = I\omega$, where ω is the angular velocity. The unit of angular momentum has no special name, it is just kg·m²/s.

If two rotating disks constitute a system, then the total angular momentum of the system is

$$L = L_{top} + L_{bottom} = I_{top}\omega_{top} + I_{bottom}\omega_{bottom} \qquad (8)\ .$$

The angular momentum depends on the direction of rotation. The convention is that angular velocity and angular momentum are positive for counterclockwise rotations and negative for clockwise rotations.

In this experiment, you will drop one spinning disk onto another spinning disk. The disks soon begin to spin with the same angular velocity. This is the analogue to the perfectly inelastic collision done with air track cars last week. The goal is to determine the total angular momentum before and after the collision and determine if this quantity is conserved.

Procedure

Part I. Setup

1. Place the two steel disks on the apparatus. Level the rotational dynamics apparatus using the two-dimensional bubble level on top of the disks.

2. Download the "Exp 11 Rotations" Logger Pro file from Teams onto the computer desktop and then open the file.

3. Set the air pressure to approximately 8 psi and turn on the power supply for the frequency to voltage converter (the aluminum box).

4. Calibrate the rotational motion sensors.
   - i. Remove all valve pins so that both disks are at rest.
   - ii. Under the "experiment" menu of Logger Pro, select "Calibrate"-> "Ch 1." Then choose both sensors.
   - iii. Click "Calibrate Now."
   - iv. Type "0" into the reading 1 dialog box and click "keep."
   - v. Type "6.2832" into the reading 2 dialog box, but **do not** yet click "keep." This will correspond to an angular speed of 2π radians (1 rev) per second.
   - vi. Insert the valve pin into the base (not the pin into the axle) so that the disks spin together.
   - vii. Spin the disks by hand and watch the digital readout of the counter. Check that the upper disk and lower disk are rotating at the same angular velocity using the counters. Adjust the rotational speed of the disks to be slightly more than 1 revolution per second (200 counts per second). Watch the counter as the angular speeds slowly decrease. When the counter reads 200, click "keep."

viii. Click “Done” and observe the angular speed readouts of Logger Pro. Spin the disks at ½ rev/s (100 counts per second) and check that the reading is appropriate (about 3.14 rad/s).

ix. Do not close Logger Pro or your calibration may be lost.

Part II. Angular acceleration due to applied torque

5. Set the air pressure to approximately 10 PSI. Check that the two disks stick together and spin freely.

6. The moment of inertia for the objects rotated by the falling mass can be approximated as $I = I_{disks} + I_{spool}$. We will approximate the two steel disks together as a single disk with a hole that has a rotational inertia given by equation (7). *The total mass of both steel disks together is 2.703 kg*. Measure the outer radius, *a*, and the inner radius, *b*. Likewise, we will consider the spool to be a solid disk with mass $m_{\text{spool}} = 0.038\ \text{kg}$ and radius *r* (measured where the string is wrapped). Use equation (6) to calculate the moment of inertia for the spool. Add these together to calculate the total moment of inertia.

7. Select a nominally 50 g mass. Measure the mass and hang it on the string wound around the spool and passing over the pulley. Make sure that the string is horizontal and that the pulley is aligned with the string.

8. **Paste a picture of the setup here. Label the important pieces.**

Figure 1. Angular Acceleration Apparatus. The air compressor pumps air to the rotational dynamics apparatus which simulates no friction between the ground and the top and bottom wheels. Both wheels spin freely when there is a pin in the bottom position. The wheels spin

independently when there is a pin in the top position. When released, the weight exerts force that spins the wheels.

9. Combine equations (1), (2), (3) and (4) to derive equation (5). **Paste a picture of your derivation here.** Use equation (5) and the measured values of *m*, *r*, and *I* to predict the angular acceleration of the disk.

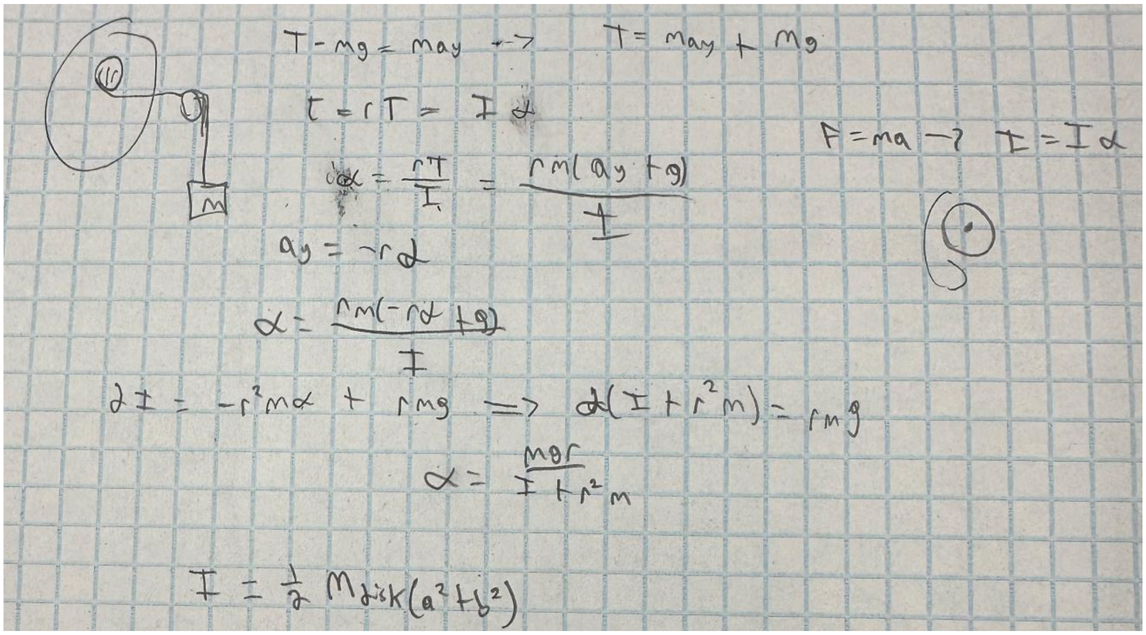


Figure 2. Angular Acceleration Equation Derivation. T represents the tension force of the string, m represents the mass of the hanging weight, r represents the inner radius or the radius of the spool, and g represents the gravitation force. α represents the acceleration, I represents inertia, a represents the about radius, and b represents the inner radius. L represents the angular momentum and ω represents the angular velocity.

10. Assume the relative uncertainty in the predicted angular acceleration is 2.0%. Use this to calculate the absolute uncertainty in the predicted acceleration.

11. Collect the angular speeds of the top and bottom disks as functions of time using Logger Pro. Begin with the string wrapped tightly around the spool and the hanging mass just below the pulley. Release from rest, so that the hanging mass accelerates downward and the disks begin turning.

12. Fit the angular speeds of the disks versus time to straight lines. The slope of these lines is the angular acceleration. Record the angular accelerations in the spreadsheet. Average the accelerations of the top and bottom disks
13. Repeat the measurement a total of five times. Find the average and standard deviation of the angular acceleration. Save all your Logger Pro files to your folder. **Paste one representative graph here**. Highlight or describe the important features of the graph.

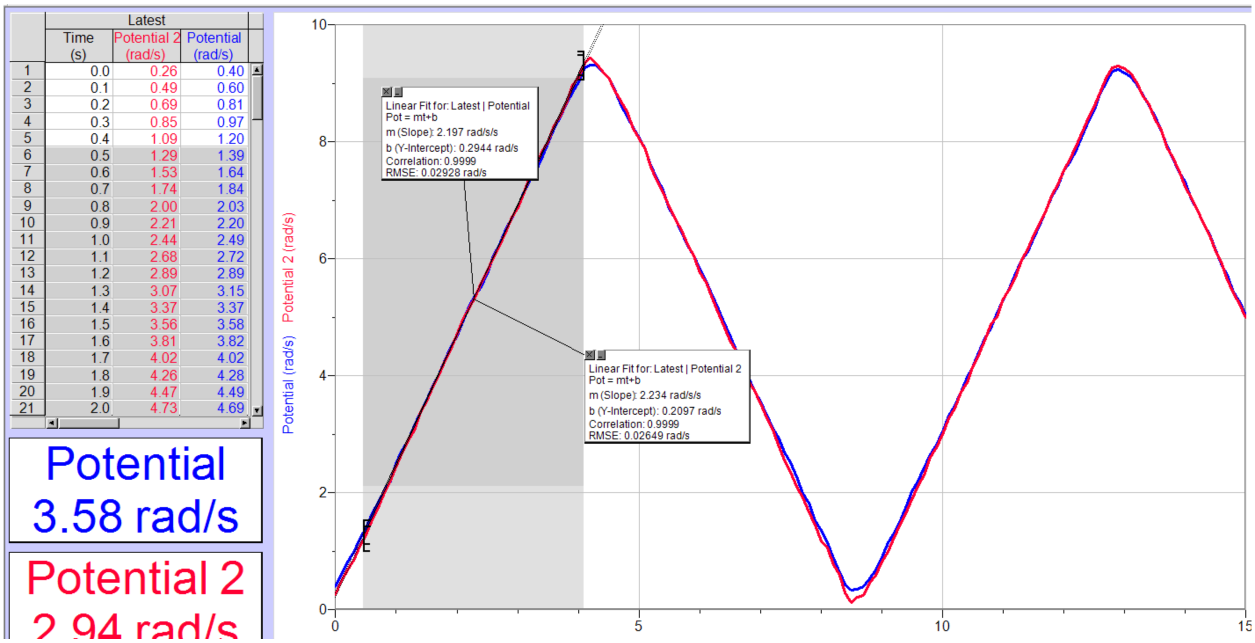


Figure 3. Angular Acceleration LoggerPro Graph. This graph comes from the first out of 5 trials. The red line represents the top wheel and the blue line represent the bottom wheel. Both wheels move together in this trail so they have very similar velocities at any given time, in theory they would have the exact same velocity at any given time. The slope of the lines represents the angular acceleration of the wheels.

14. Compare the experimental and predicted values of the angular acceleration using both a percent difference and a ratio test.
15. **Paste part II of the Excel table here. Also paste a picture of the equations that you used.**

	Total Mass of Disks	Outer Radius of Disks	Inner Radius of Disks	Moment of Inertia of Disks
g (m/s^2)	m_{disks} (kg)	a (m)	b (m)	I_{disks} ($kg \cdot m^2$)
9.803	2.703	0.06133	0.027	0.0061

Mass of spool	Radius of Spool	Moment of Inertia of Spool	Total Moment of Inertia	Hanging Mass	Predicted Angular Accel	Rel Unc Angular Accel	Unc Angular Accel
m_{spool} (kg)	r (m)	I_{spool} ($kg \cdot m^2$)	I ($kg \cdot m^2$)	m (kg)	α (rad/s^2)	$\delta\alpha/\alpha$	$\delta\alpha$ (rad/s^2)
0.038	0.027	1.4E-05	0.0061	0.050	2.16	2.0%	0.043

Trial	Angular Acceleration of top disk $\alpha 1$(rad/s^2)	Angular Acceleration of bottom disk $\alpha 2$ (rad/s^2)	Average angular acceleration (rad/s^2)
1	2.197	2.234	2.216
2	2.205	2.238	2.222
3	2.204	2.241	2.223
4	2.197	2.244	2.221
5	2.203	2.233	2.218
		Average	2.220
		Stnd Dev	0.003
		Rel Unc	0.13%

Compare predicted and measured angular accelerations	
Percent Diff	Ratio Test
2.7%	1.300

Table 1. Angular Acceleration Excel Table. The calculated acceleration comes from the equation derived above. The observed acceleration comes from the average of the acceleration of the wheels over 5 trials. The equation $I = 0.5m(a^2+b^2)$ was used to calculate the disk's inertia. The

equation I = 0.5mr^2 was used to calculate the spool's inertia. The excel functions AVERAGE, STDEV.S, and ABS were used for the average, standard deviation, and absolute value respectively.

16. **Write an analysis of part II here.**

Both wheels move together so they are considered one wheel for the sake of this experiment. The acceleration is averaged because both wheels have the slightly different velocities. The calculated acceleration agrees with the observed acceleration through the ration test and a low percent difference. The tension force from the string is the only external force on the system so the acceleration of the wheels is constant.

Part III. Angular collision

17. Set the air pressure to approximately 8 psi.

18. Replace the top steel disk and spool with the aluminum disk.

19. Approximate the disks as uniform cylinders and calculate the moment of inertia of each disk separately using $I_{disk} = \frac{1}{2} m_{disk} \left(a^2 + b^2\right)$, where m_{disk} is the mass of the disk, a is the outer radius and b is the radius of the hole. The mass of the steel disk is 1.3425 kg and the mass of the aluminum disk is 0.4657 kg.

20. Insert the valve pin into the center hole, so that the disks rotate independently. Spin the top and bottom disks both counter-clockwise but at very different angular speeds. Keep each angular speed less than about 10 rad/s. Use Logger Pro to collect the angular speeds of the two disks. While data is being collected, slowly remove the valve pin so that the two disks collide. They will start to rotate together. Ensure that you have several seconds worth of data both before and after the collision.

21. Use the statistics function of Logger Pro to record the mean of the angular speed of each disk before and after the collision. Ideally, the disks have the same angular velocity after the collision, so $\omega_{top,f} = \omega_{bottom,f}$.

22. Calculate the total angular momentum before and after the collision. Assume that the relative uncertainty of the total angular momentum (both before and after the collision) is 3.0%. Use this to calculate the absolute uncertainties in the total angular momentums.

23. Perform these measurements four total times: two times with the disks initially rotating in the same direction and two times with the disks rotating in opposite directions. Note that the computer does not "know" the direction of the rotation—it always reports a positive value for ω. You will have to manually record the directions of the disks before and after

the collision and assign a ± sign according to our convention. Save all your collision graphs to your folder. **Paste one example graph for a collision with both disks initially spinning in the same direction and one example with the disks initially spinning in opposite directions.**

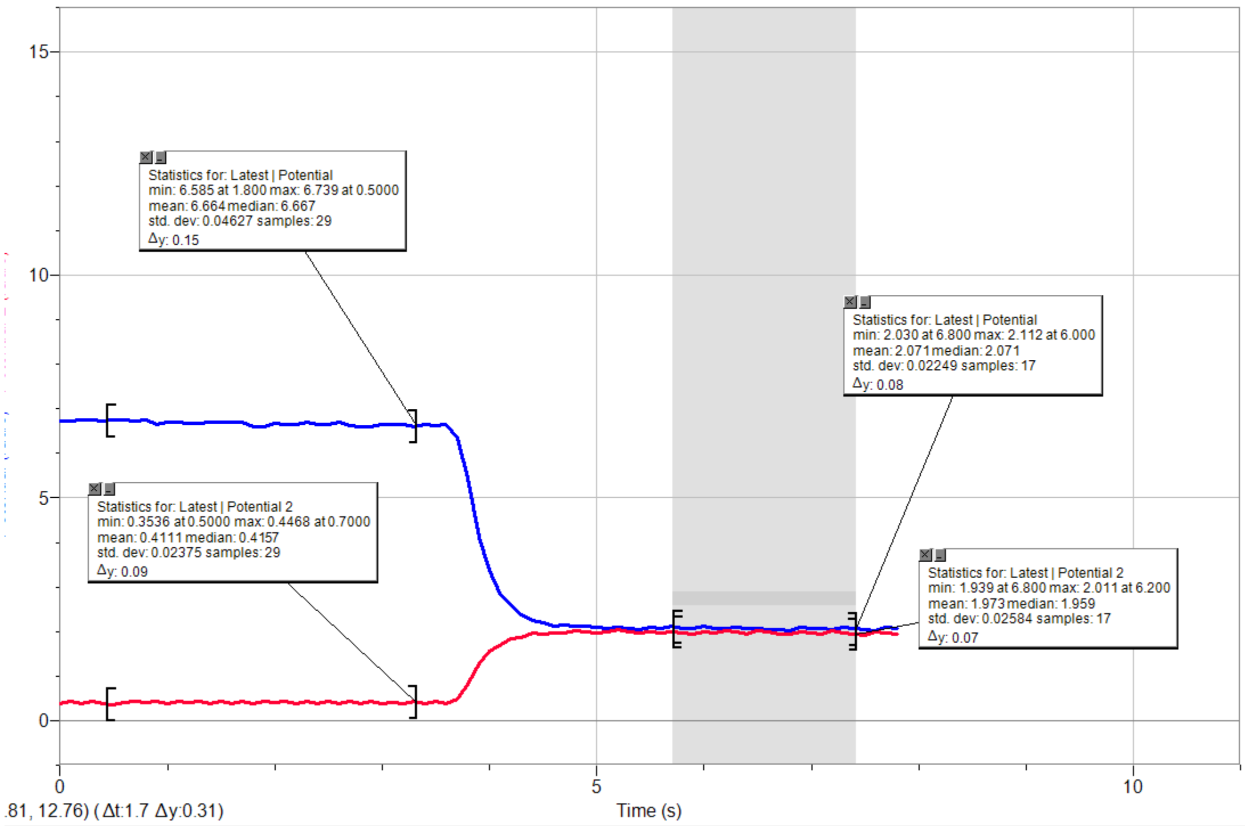


Figure 4. Same Direction Angular Collision LoggerPro Graph. The wheels were spun in the same direction with different speeds. The initial means represents the initial velocity that the wheels were spinning at. The top aluminum wheel started with a higher velocity than the bottom steel wheel. The pin in the center axles was then removed and the wheel collided. The wheels began spinning at the same speed, still in the same direction. The top wheel decreased in speed and the bottom wheel increased in speed. The final means represents the final velocity of the wheels.

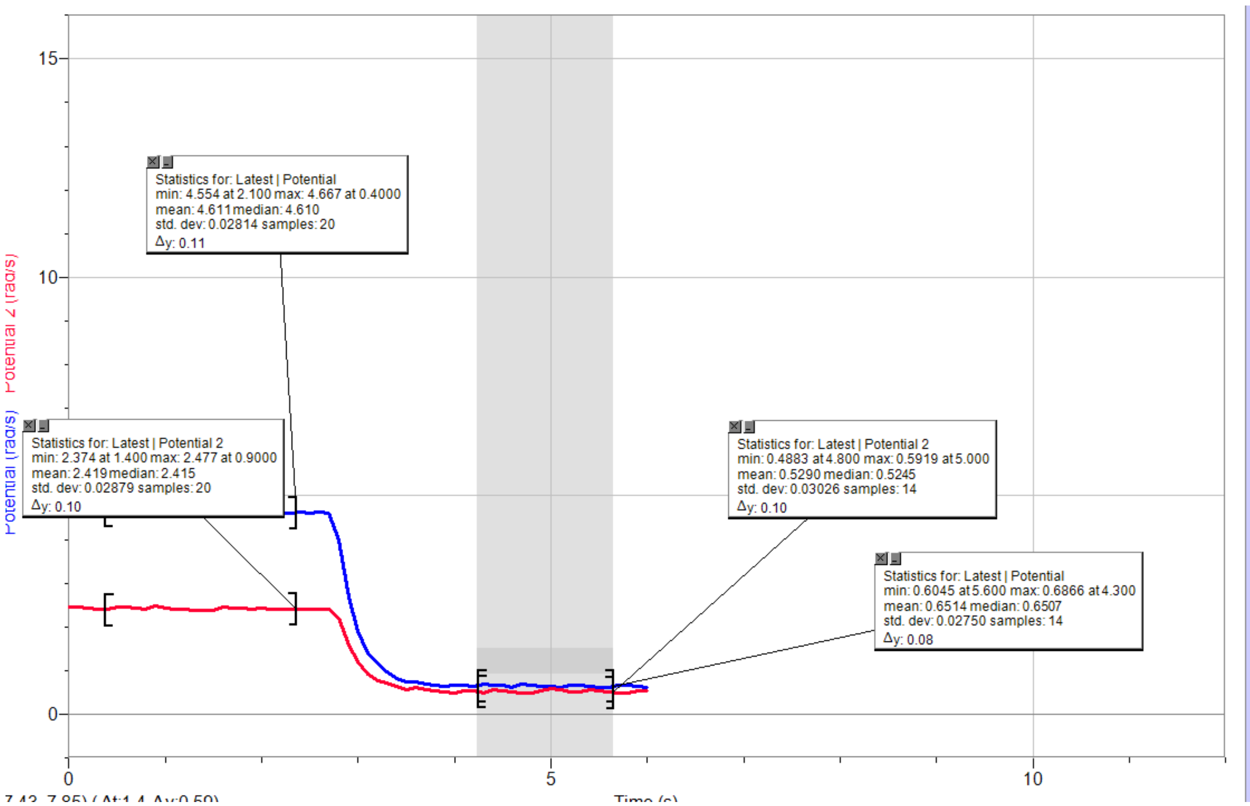


Figure 5. Opposite Direction Angular Collision LoggerPro Graph. The wheels were spun in opposite directiond with different speeds. The initial means represents the initial velocity that the wheels were spinning at. The top aluminum wheel started with a higher velocity than the bottom steel wheel. The pin in the center axles was then removed and the wheel collided. The wheels began spinning at the same speed and in the same direction. Both wheels decreased in speed and the direction of the top wheel reversed. The final means represents the final velocity of the wheels.

24. Compare the initial and final total angular momentums using both a percent difference and a ratio test.

25. **Paste your part III Excel table here. Also paste a picture of the equations that you used.**

Top disk mass (Al)	Top Disk Outer Radius	Top Disk Inner Radius	Top Disk Rotational Inertia	Bottom Disk Mass (Steel)	Bottom Disk Outer Radius	Bottom Disk Inner Radius	Bottom Disk Rotational Inertia
m_{top} (kg)	a_{top} (m)	b_{top} (m)	I_{top} (kg·m^2)	m_{bot} (kg)	a_{bot} (m)	b_{bot} (m)	I_{bot} (kg·m^2)
0.4657	0.06133	0.027	0.00105	1.3425	0.06133	0.027	0.00301

		Top Disk Initial Angular Velocity	Bottom Disk Initial Angular Velocity	Top Disk Final Angular Velocity	Bottom Disk Final Angular Velocity
trial	Direction	$\omega_{top,i}$ (rad/s)	$\omega_{bot,i}$ (rad/s)	$\omega_{top,f}$ (rad/s)	$\omega_{bot,f}$ (rad/s)
1	Same	5.734	0.9223	2.205	2.053
2	Same	6.664	0.4111	2.071	1.97
3	Opposite	11.122	2.167	1.205	1.094
4	Opposite	4.608	2.421	0.6532	0.5239

		Total Initial Angular Momentum	Rel Unc Total Initial Angular Momentum	Unc Total Initial Angular Momentum	Total Final Angular Momentum	Rel Unc Total Final Angular Momentum	Unc Total Final Angular Momentum
trial	Direction	$L_{total,\, i}$ (kg·m^2/s)	$\delta L_{total,\, i}/L_{total,\, i}$	$\delta L_{total,\, i}$ (kg·m^2/s)	$L_{total,\, f}$ (kg·m^2/s)	$\delta L_{total,\, f}/L_{total,\, f}$	$\delta L_{total,\, f}$ (kg·m^2/s)
1	Same	0.0088	3.0%	0.0003	0.0085	3.0%	0.0003
2	Same	0.0082	3.0%	0.0002	0.0081	3.0%	0.0002
3	Opposite	0.0051	3.0%	0.0002	0.0046	3.0%	0.0001
4	Opposite	0.0025	3.0%	0.0001	0.0023	3.0%	0.0001

Compare Initial and Final Total Angular Momentum	
percent diff	Ratio Test
3.3%	0.54
1.2%	0.19
11%	1.86
9.2%	1.53

Table 2. Angular Collision Excel Table. Four trials were performed, two where the wheels were initially spun in the same direction and two where they were initially spun in opposite directions. Both disks had the same inner and outer radii. The equation $I = 0.5m(a^2+b^2)$ was used to calculate the disks' inertia. The equation $L = I\omega$ was used to calculate the angular momentum. The excel functions AVERAGE, STDEV.S, and ABS were used for the average, standard deviation, and absolute value respectively.

26. **Write an analysis of part III here.**

The initial and final momentum agree in all trails through the ratio test and percent difference. This agrees with the idea that the momentum is conserved in the collision. The initial speed of the top wheel was a little high for the 3rd trail so the results are a little more skewed but they still agree.

27. **Write a brief conclusion here.**

The angular acceleration is relative to the external forces acting on the wheels and momentum of a collision is conserved.

This experiment resulted in good data and running the trails went smoothly once we figured out that the blue voltage box needed to be plugged in and on. It is not mentioned in the instruction. The results also improved when we used the calipers to measure the radii instead of a ruler.

28. Adjust the formatting (pagination, margins, size of figures, etc.) of this report to make it easy to read. Save this report as a PDF document. Upload it to the course Moodle page.

29. Clean up your lab station. Put the equipment back where you found it. Remove any temporary files from the computer desktop. Make sure that you logout of Moodle and your email, then shut down the computers. Ensure that the laptop is plugged in.

30. Check with the lab instructor to make sure that they received your submission before you leave.