



## Equitable Equations: *Calculating in the $\chi^2$ -distribution*

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Use R for all calculations. Include both code and a numerical answer.

### Problem 1

Compute  $P(\chi^2 < 7.1)$  in  $\chi^2(8)$ .

### Problem 2

Compute  $P(\chi^2 > 12)$  in  $\chi^2(15)$ .

$$1) \text{pchisq}(7.1, 8) = 0.474$$

$$2) 1 - \text{pchisq}(12, 15) = 0.679$$



## Equitable Equations: *Goodness of fit testing*

For each of the following problems,

- Write hypotheses appropriate to the research question.
- Compute expected counts for the specified categorical variable.
- Compute the  $\chi^2$  test statistic for the given sample.
- Identify the number of degrees of freedom of the  $\chi^2$  distribution.
- Compute the p-value of the test.
- Make a decision at significance level  $\alpha = .05$ . State your conclusion in non-technical language.
- Confirm your results with the `chisq.test` function.

### Problem 1

Problem #14 from chapter 10.1 of Larson & Farber's *Elementary Statistics* (attached).

### Problem 2

Problem #16 from chapter 10.1 of Larson & Farber's *Elementary Statistics* (attached).

1)  $H_0$  = women trust their spouses with finances at the same ratios that men do.

	Complete trust	Certain trust	Not trust	Not sure
ratios ←	65.6%	27.8%	5.7%	0.9%

$H_a$  = the frequency at which men and women trust their spouses with finance management is different.

b)	Complete trust	Certain trust	Not trust	Not sure

expectec(262.4	111.2	22.8	3.6)
observec(243	108	36	13)

$$d) \chi^2 = \sum ((\text{observ} - \text{expec})^2 / \text{expec}) = 33.7$$

$$df = k - 1 = 3$$

$$e) p = 1 - \text{pchisq}(\chi^2, df) = 1 - \text{pchisq}(33.7, 3)$$

$$= 2.28 \times 10^{-7}$$

f)  $p < \alpha$  therefore results are not statistically significant. The data does not support the idea that men and women trust spouses with money management the same amount.

$$g) \text{chisq.test}(\text{observ}, p = \text{ratios})$$

$$\chi^2 = 33.7 \quad df = 3 \quad p = 2.28 \times 10^{-7}$$

2)  $H_0$ : Births are evenly distributed across the days of the week

ratios	Su	M	T	w	R	F	Sa
ec(	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

$H_a$  = Births are not evenly distributed across the days of the week

b)

expec	Su	M	T	w	R	F	Sa
ec(	100	100	100	100	100	100	100

observ <- c(65 107 117 115 114 109 73)

$$c) \chi^2 = \text{sum}((\text{observ} - \text{expect})^2 / \text{expect}) = 27.94$$

$$d) df = k - 1 = 6$$

$$e) p = 1 - \text{pchisq}(\chi^2, df) = 9.64 \times 10^{-5}$$

f)  $p < \alpha$  therefore the results are not statistically significant. The data does not support the idea that births are evenly distributed across days of the week.

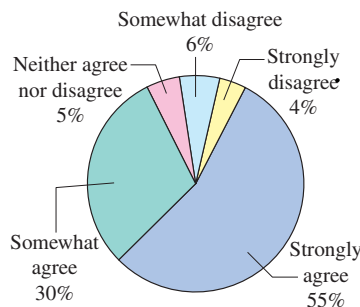
g) `chisq.test(observ, p = ratios)`

$$\chi^2 = 27.94 \quad df = 6 \quad p = 9.64 \times 10^{-5}$$

- 12. Homicides by Month** A researcher claims that the number of homicide crimes in California by month is uniformly distributed. To test this claim, you randomly select 1200 homicides from a recent year and record the month when each happened. The table shows the results. At  $\alpha = 0.10$ , test the researcher's claim. (*Adapted from California Department of Justice*)

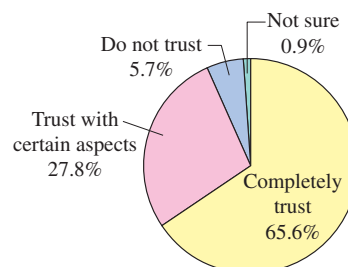
Month	Frequency, $f$	Month	Frequency, $f$
January	115	July	115
February	75	August	98
March	90	September	92
April	98	October	108
May	121	November	90
June	99	December	99

- 13. College Education** The pie chart shows the distribution of the opinions of U.S. parents on whether a college education is worth the expense. An economist claims that the distribution of the opinions of U.S. teenagers is different from the distribution for U.S. parents. To test this claim, you randomly select 200 U.S. teenagers and ask each whether a college education is worth the expense. The table shows the results. At  $\alpha = 0.05$ , test the economist's claim. (*Adapted from Upromise, Inc.*)



Survey results	
Response	Frequency, $f$
Strongly agree	86
Somewhat agree	62
Neither agree nor disagree	34
Somewhat disagree	14
Strongly disagree	4

- 14. Money Management** The pie chart shows the distribution of how much married U.S. male adults trust their spouses to manage their finances. A financial services company claims that the distribution of how much married U.S. female adults trust their spouses to manage their finances is the same as the distribution for married U.S. male adults. To test this claim, you randomly select 400 married U.S. female adults and ask each how much she trusts her spouse to manage their finances. The table shows the results. At  $\alpha = 0.10$ , test the company's claim. (*Adapted from Country Financial*)



Survey results	
Response	Frequency, $f$
Completely trust	243
Trust with certain aspects	108
Do not trust	36
Not sure	13

Male

Female

are they equal

Response	Frequency, $f$
Larger	285
Same size	224
Smaller	291

TABLE FOR EXERCISE 15

- 15. Home Sizes** An organization claims that the number of prospective home buyers who want their next house to be larger, smaller, or the same size as their current house is not uniformly distributed. To test this claim, you randomly select 800 prospective home buyers and ask them what size they want their next house to be. The table at the left shows the results. At  $\alpha = 0.05$ , test the organization's claim. (*Adapted from Better Homes and Gardens*)
- 16. Births by Day of the Week** A doctor claims that the number of births by day of the week is uniformly distributed. To test this claim, you randomly select 700 births from a recent year and record the day of the week on which each takes place. The table shows the results. At  $\alpha = 0.10$ , test the doctor's claim. (*Adapted from National Center for Health Statistics*)

Day	Frequency, $f$
Sunday	65
Monday	107
Tuesday	117
Wednesday	115
Thursday	114
Friday	109
Saturday	73

## EXTENDING CONCEPTS

**Testing for Normality** Using a chi-square goodness-of-fit test, you can decide, with some degree of certainty, whether a variable is normally distributed. In all chi-square tests for normality, the null and alternative hypotheses are as follows.

$H_0$ : The variable has a normal distribution.

$H_a$ : The variable does not have a normal distribution.

To determine the expected frequencies when performing a chi-square test for normality, first find the mean and standard deviation of the frequency distribution. Then, use the mean and standard deviation to compute the z-score for each class boundary. Then, use the z-scores to calculate the area under the standard normal curve for each class. Multiplying the resulting class areas by the sample size yields the expected frequency for each class.

In Exercises 17 and 18, (a) find the expected frequencies, (b) find the critical value and identify the rejection region, (c) find the chi-square test statistic, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

- 17. Test Scores** At  $\alpha = 0.01$ , test the claim that the 200 test scores shown in the frequency distribution are normally distributed.

Class boundaries	49.5–58.5	58.5–67.5	67.5–76.5	76.5–85.5	85.5–94.5
Frequency, $f$	19	61	82	34	4

- 18. Test Scores** At  $\alpha = 0.05$ , test the claim that the 400 test scores shown in the frequency distribution are normally distributed.

Class boundaries	50.5–60.5	60.5–70.5	70.5–80.5	80.5–90.5	90.5–100.5
Frequency, $f$	28	106	151	97	18