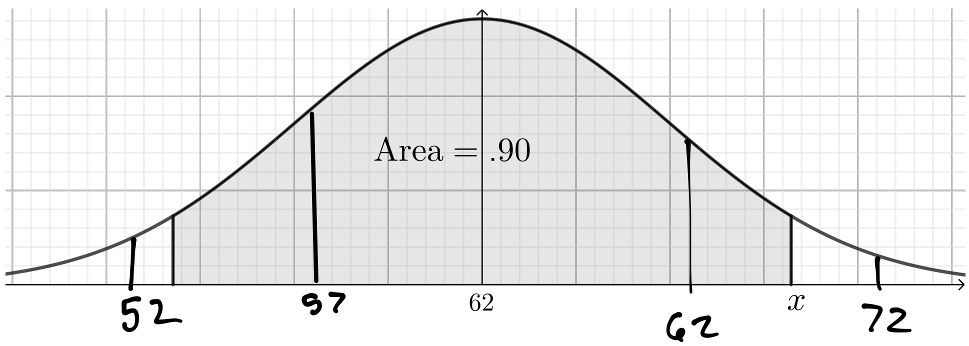
Problem 6 /12

The lengths of a certain breed of cat are normally-distributed with mean 62 cm and standard deviation 5 cm. $N(62, 5^2)$

(a) What is the probability that a randomly-selected cat is at least 60 cm long? Give both R code and output.

$P(x \geq 60) = 1 - \text{pnorm}(60, 62, 5) = 65.5\%$

(b) The graph below shows this distribution. Do **not** assume that it is drawn exactly to scale.



The shaded area is symmetric about the mean, has area .90, and ends at $\pm x$. What is x? Carefully explain your process.

$\text{pnorm}(62+x, 62, 5) - \text{pnorm}(62-x, 62, 5) = 0.90$

$67 < x < 72$ by empirical rule

$1 - 0.9 = 0.1/2 = 0.5$

$1 - 0.5 = .95$

$\text{qnorm}(.95, 62, 5) = \underline{70.22 = x}$

Check:

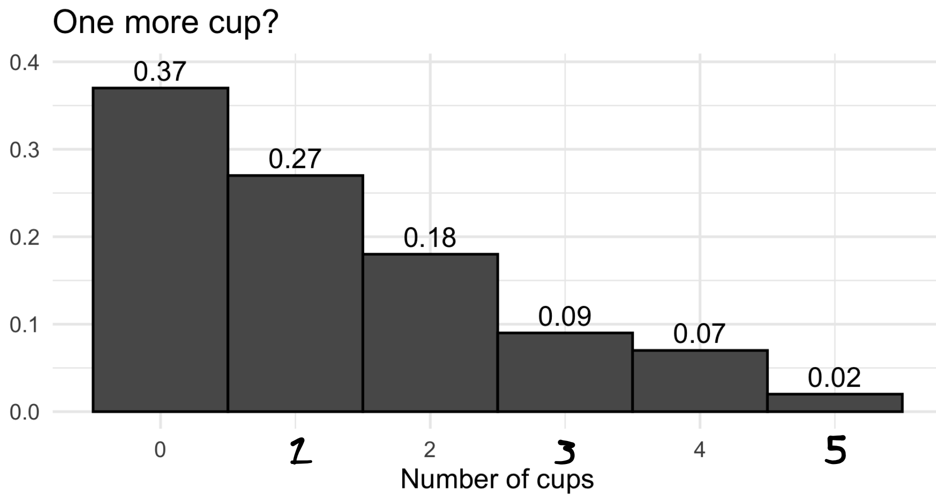
$70.22 - 62 = 8.22$

$\text{pnorm}(62+7.22, 62, 5) - \text{pnorm}(62-8.22, 62, 5) = .90$

$67 < 70.22 < 72$

Problem 7 /12

A study asks a random sample of Americans how many cups of coffee they drink on a typical weekday. The results are summarized in the following histogram, where the vertical axis represents proportions of respondents.



(a) What is the probability that a randomly-selected person drinks at least 2 cups of coffee per day?

$P(x \geq 2) = P(2) + P(3) + P(4) + P(5)$

$= 0.18 + 0.09 + 0.07 + 0.02$

$= 0.36 = 36\%$

(b) Let X be a random variable representing the number of cups reported by a randomly-selected individual in the sample. Use R to compute the expected value of X. Include all code used, making sure that your work is clear.

```
X <- c(0,1,2,3,4,5)
p <- c(0.37, 0.27, 0.18, 0.09, 0.07, 0.02)
```

sum(x * p) = 1.28 = expected value

(c) Use R to compute the variance and standard deviation of X. Include all code used, making sure that your work is clear.

μ = mean = 1.28

σ^2 = variance = $\Sigma(x - \mu)^2 \cdot p$ = sum((x -1.28)^2 * p) = 1.78

σ = standerd deviation = $\sqrt{\Sigma(x - \mu)^2 \cdot p}$ = sqrt(sum((x -1.28)^2 * p)) = 1.33