Equitable Equations: *The central limit theorem*

When using R, remember to include both code and output with your work.

Problem 1

A certain population has mean $\mu = 27.3$ and standard deviation $\sigma = 4.2$. For a sample of size $n = 36$,

(a) Determine the mean, variance, and standard deviation of the sample mean $\overline{x}$.

(b) Determine the probability that $\overline{x} \leq 26.0$.

(c) Determine the probability that $\overline{x} \geq 25.9$.

Problem 2

At a local grocery, apples have mean weight .620 pounds with standard deviation .165 pounds. The distribution is approximately normal.

(a) What is the probability that a randomly-selected apple weighs more than .650 pounds?

(b) What is the probability that 10 randomly-selected apples weigh more than .650 pounds, on average?

(c) What is the probability that 50 randomly-selected apples weigh more than .650 pounds, on average?

(d) What is the probability that 500 randomly-selected apples weigh more than .650 pounds, on average?

Problem 3

Lengths of eruptions of the Old Faithful geyser are approximately normally distributed with mean 3.49 minutes and standard deviation 1.14 minutes.

(a) Which is more likely, a single eruption longer than 3.20 minutes or 20 eruptions with mean greater than 3.20 minutes? Compute both probabilities.

(b) Which is more likely, a single eruption longer than 3.60 minutes or 20 eruptions with mean greater than 3.60 minutes? Justify your answer without computing probabilities.

1

a) $\mu_{\bar{x}} = 27.3 = \mu$

$\sigma_{\bar{x}} = \frac{4.2}{\sqrt{36}} = 0.7 = \frac{\sigma}{\sqrt{n}}$

$\sigma_{\bar{x}}^2 = 0.7^2 = 0.49$

b) pnorm(26.0, 27.3, 0.7) = 3.2%

c) 1 − pnorm(25.9, 27.3, 0.7) = 98%

2

a) $1 - pnorm(.650, .620, .165) = 43\%$

b) $1 - pnorm\left(.650, .620, \frac{.165}{\sqrt{10}}\right) = 28\%$

c) $1 - pnorm\left(.650, .620, \frac{.165}{\sqrt{50}}\right) = 10\%$

d) $1 - pnorm\left(.650, .620, \frac{.165}{\sqrt{100}}\right) = 3\%$

3

a) $1 - pnorm(3.20, 3.49, 1.14) = 60\%$

$1 - pnorm\left(3.20, 3.49, \frac{1.14}{\sqrt{20}}\right) = 87\%$

20 eruptions w/ a mean greater than 3.20 min is more likely

b) 20 eruptions with mean greater than 3.60 is more likely because it is within one standard deviation so as the bell curve compresses it will be more likely.