LAKE FOREST COLLEGE
Department of Physics

PHYS 114 **Experiment 12: Simple Harmonic Motion v2** Fall 2024

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Preliminary Instructions

Create a folder for you and your lab partner. Save a copy of these instructions for each student to that folder. Include your name in the filename. Save one copy of the Excel template with both your and your partner's name included in the filename.

Experimental purpose of today's experiment

Measure the force constant of a spring and investigate how the period of oscillation of a mass-spring system depends on mass and amplitude.

Pedagogical purpose of today's experiment

Study Hooke's law for springs and the periodic oscillations of a mass on a spring.

Background

A spring exerts a variable force that depends on how much the spring is compressed or stretched. For a limited range of displacements, the magnitude of the spring force is $\left|F_{spring}\right| = k\left|y\right|$, where k is the force constant and y is the stretch or compression of the of the spring from its original equilibrium length. The force constant (spring constant) k is measured in N/m or kg/s^2.

In the first part of the experiment, you will hang an object from a vertically mounted spring. The object is in equilibrium when the net force on it is zero; in this case, equilibrium occurs when the spring force pulling upward balances the gravitational force pulling downward: $F_{spring} - mg = 0 \Rightarrow mg = k\left|y\right|$.

If we move a mass hanging from a spring above (or below) its new equilibrium position it will experience a net force back toward its equilibrium position. If we release it from this non-equilibrium position, it will overshoot equilibrium and the result is a smooth, repetitive sinusoidal motion called *simple harmonic motion*.

The time required for a complete cycle of the motion is called the period. The period of the system is related to the mass of the object and the spring constant. You will learn in lecture that the period (T) of the mass-spring system is given by the following formula:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{1}$$

Here $m = m_S + m_A$, where m_S is the effective mass of the spring and m_A is the mass of the object added to the end of the spring. Therefore

$$T = 2\pi\sqrt{\frac{m_S + m_A}{k}} \tag{2}$$

In this experiment you will measure the force constant k and study how the period varies with the mass of the added object and the amplitude.

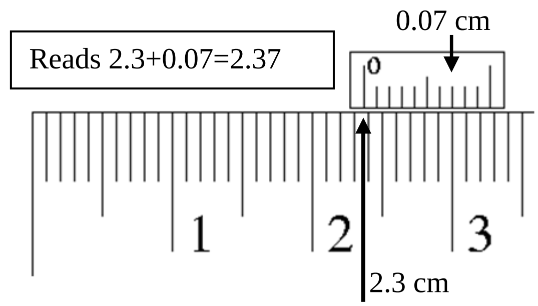
Procedure

<u>Part I: Static measurement of the spring constant</u>

1. Use the adjustable feet on the stand and the angle indicator to ensure that the stand is vertical.

2. Hang the spring (wide end down) and mass holder from the adjustable height stand (you will have to unscrew the base to fit the holder through the reference tube). Adjust the stand using the knob at the bottom so that the Vernier scale reads zero, then adjust the height of the reference tube arm so that the mark on the mass holder aligns with the line on the tube. For this part we will not include the mass of the holder. Call the length of the spring with only the holder attached zero. *DO NOT MOVE THE REFERENCE TUBE DURING THIS PART*.

3. Place a measured mass on the holder. Use the knob at the bottom to raise the stand height to align the groove on the mass holder with the mark on the reference tube. Use the Vernier scale on the stand to measure how much the spring has stretched. Repeat this procedure for these nominal masses: 50, 100, 150, 200, and 250 g. You should measure each added mass and measure the resulting stretch of the spring. Note that some of the stands measure in centimeters and others in millimeters.



4. You should check that your data seems reasonable. In this case, you should check that your measured masses are close to their nominal values and that the measured stretch of the spring seems close to how far you saw the spring stretch by eye. You should check that the data is monotonic – larger masses should result in more stretching. If anything seems suspicious, then you should investigate it more carefully.

5. **Paste a picture of your apparatus here. Label the important pieces**.

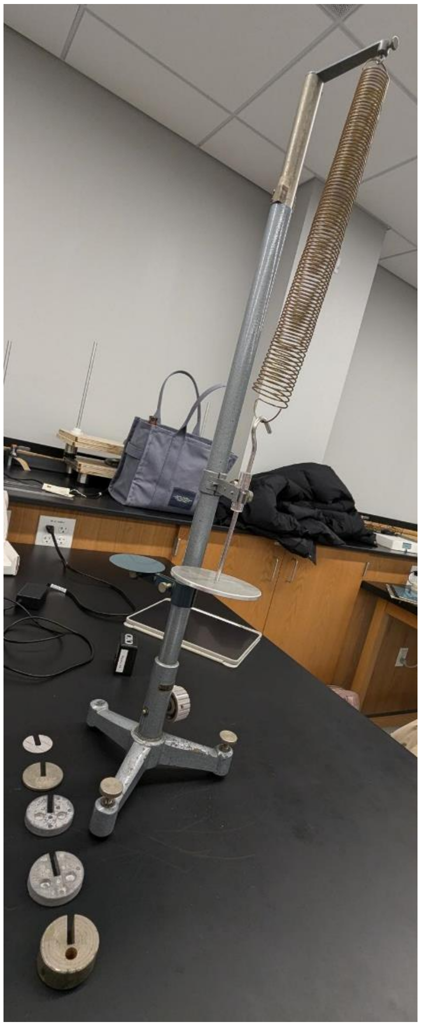


Figure 1. Static Measurement Apparatus. The metal weight holder is held by a plastic tube. A divot on the tube lined up with a divot on the holder which serves as a reference line. Before weight is added the zeros of the Central Scientific scale were equal, when weight was added the zero aligned with the stretch distance (cm). The knob on the bottom of the scale was used to adjust the height. The w50g, 100g, and 200g weights that were used are next to the scale.

6. Open Logger Pro (Open a blank file, not the part 2 file from Teams). Make a plot of the weight (in N) of the added mass as a function of the stretch of the spring (in m): weight on the vertical axis and stretch on the horizontal axis. Include a data point at (0,0). Fit a straight line to your data (curve fit and $y = mx+b$) and determine the slope and its uncertainty.

7. Does the straight-line fit match your data well? If any data points seem far off, then you should investigate them further.

8. Determine k and its uncertainty from your slope. **Paste a picture of your plot here**.

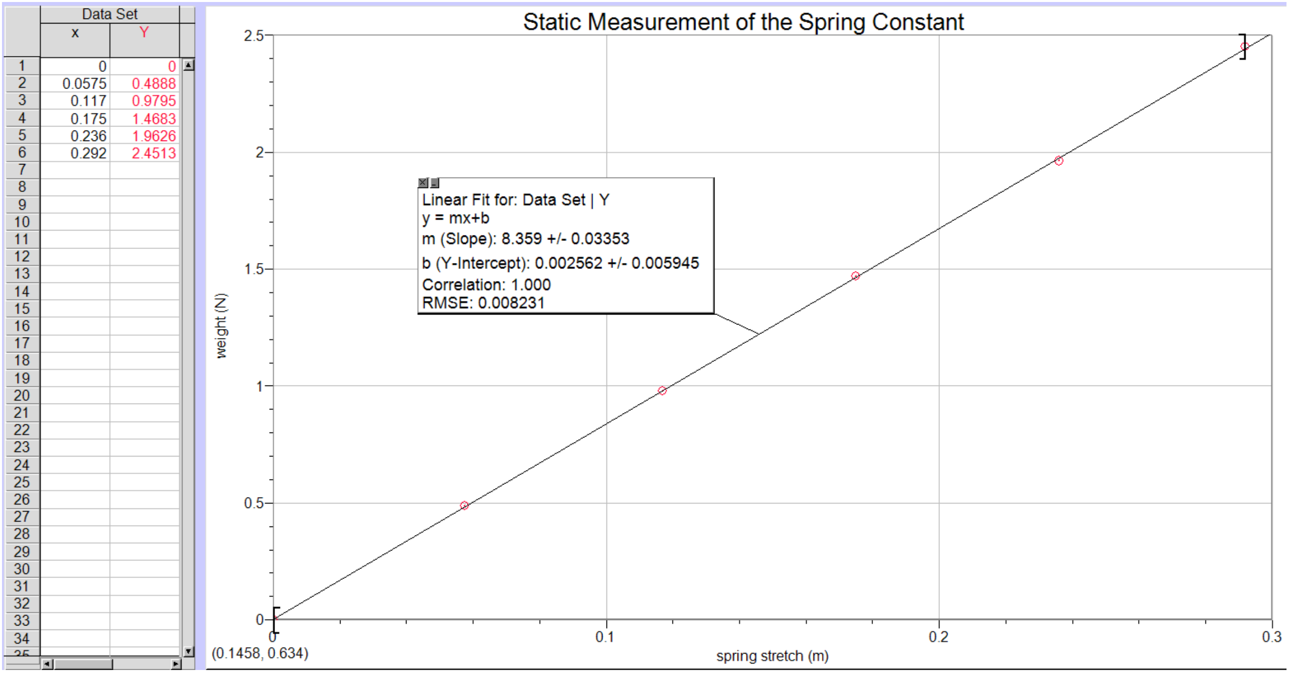


Figure 2. Static Measurement LoggerPro Graph. The x axis is the stretch of the spring in m and the y axis is the force of the added weights in N. Each measured point is marked by a red circle. The slope of the fitted line represents the spring constant and the +/- represents the uncertainty of the spring constant.

9. **Format part I of your Excel sheet and paste a picture of it here.**

g (m/s^2)
9.803

added mass (g)	added weight (N)	spring stretch (cm)	spring stretch (m)
0	0	0	0
49.86	0.4888	5.75	0.0575
99.92	0.9795	11.7	0.117
149.78	1.4683	17.5	0.175
200.2	1.9626	23.6	0.236
250.06	2.4513	29.2	0.292

Measured static spring constant (N/m)	8.359
Abs. unc. in spring constant (N/m)	0.034

Table 1. Static Measurement Excel Table. The stretch of the spring was observed using 5 different weights, 50g, 100g, 150g, 200g, and 250g as well as a neutral with no weight and therefore no stretch. The force of the weight (N) was calculated by multiplying the mass of the weight (kg) by the gravitational acceleration constant ($9.803 m/s^2$). The measured static spring constant and uncertainty comes from a line fitted to the measured points.

10. **Write an analysis for part I here**.

The spring constant (k) represents how stiff a given spring is. It would take 8.359N to stretch the spring out a meter. One of the ways it can be calculated is using the equation, $k|y| = mg = w$.

<u>Part II: Dynamic measurement of the spring constant</u>

11. Use the same spring that you used in part I. Move the reference tube out of the way. Hang the mass holder plus a nominally 50 g mass from the spring. *Now we will include the mass of the holder*: let m_A be the sum of the holder mass plus the added masses. Measure the mass of the holder and the added mass and record it in your Excel table.

12. Gently pull the mass downward a few centimeters and release so that the mass oscillates up and down. You must ensure that the oscillations are vertical with minimal swinging or shaking. The mass should stay at least 20 cm away from the motion sensor throughout the oscillation, so you may have to raise the height of hanger arm.

13. **Paste a picture of your apparatus here. Label the important components**.

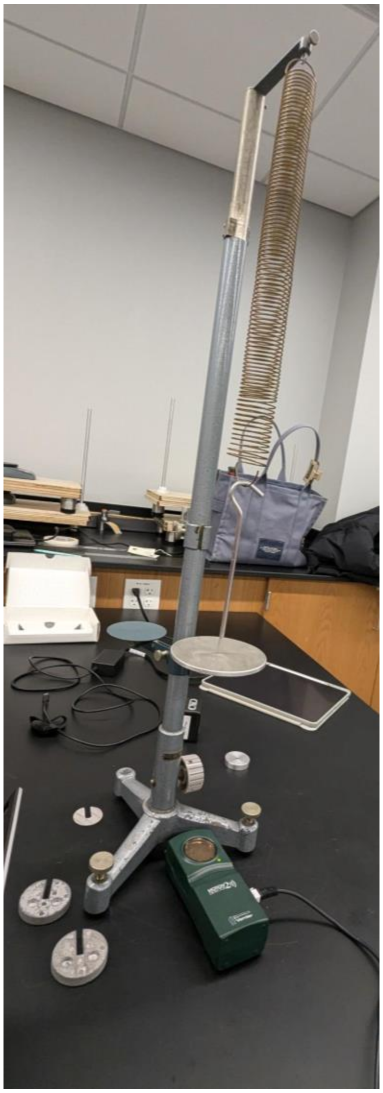


Figure 3. Dynamic Measurement Apparatus. The metal weight holder hangs freely from the spring. A Vernier Motion Detector 2 sits underneath the holder, measuring its motion and velocity as it oscillates up and down. The Central Scientific scale was adjusted for each added weight to ensure the base of the weight holder

remained 20cm or more away from the motion detector. The 50g, 100g, and 200g weights used in the experiment are on the lab bench beside the apparatus.

14. Download the file "Exp 12 Logger Pro file.cmbl" from Teams to your computer desktop. Use this file to acquire displacement (y) versus time data with the motion detector while the mass is oscillating up and down. The data should be a sinusoidal oscillation of the form:

$$y = A\sin\left(\frac{2\pi}{T}t + C\right) + D. \qquad (3)$$

15. We will use the curve-fitting capabilities of the Logger Pro software to determine the period, T, accurately. Under the analyze menu, choose curve fit. Your fitting equation will be $y = A\sin(Bt + C) + D$, where A = amplitude, $B = \frac{2\pi}{T}$, C is the phase (C shift the curve forward or backward in time), and D is the offset (D shifts the curve up or down).
16. Check that the fit curve matches your data well. Record the fit parameters A and B in your Excel table (the uncertainties are unnecessary). You do not need to record the C and D fit parameters.

17. Use Excel formulas to calculate the period, T, from the fit parameter B. Check that the result seems reasonable based on your observation of how long it takes to complete one full oscillation.

18. Use an Excel formula to calculate the period squared, T^2.

19. Repeat steps 11-18 for 5 additional added masses: 50, 100, 150, 200, 250, and 300 g. Try to start the oscillations such that they have similar amplitudes. *Save each of your Logger Pro files to your folder*. **Paste a picture of one representative graph here. Use Word's drawing capabilities to draw lines on top of the graph showing the distance of one amplitude and showing the time of one period.**

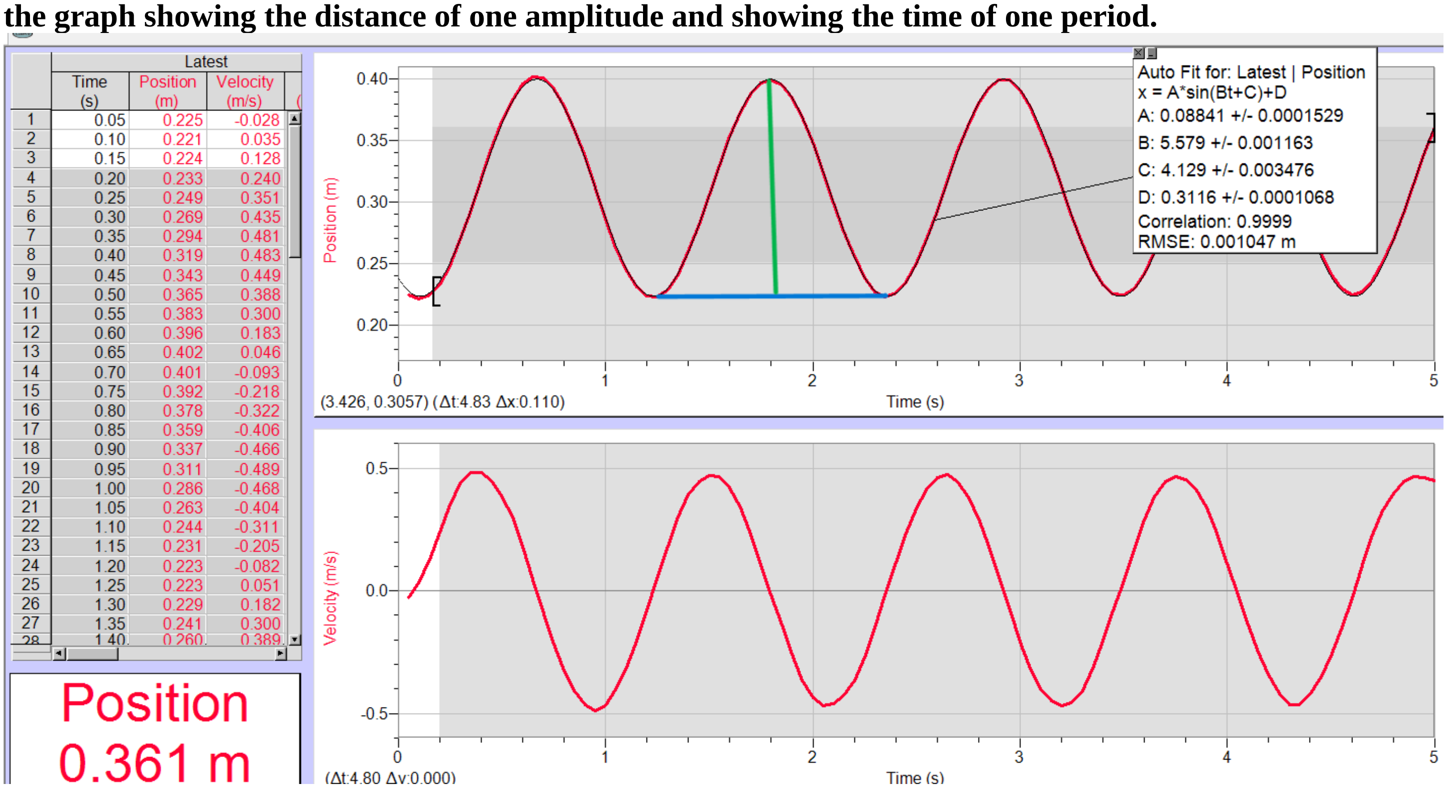


Figure 4. Dynamic Motion with 100g Weight LoggerPro Graph. The graphs comes from the oscillation of the spring when a 100g weight was added to the weight holder. The position graph indicated how far the

base of the weight holder was from the Vernier motion detector. The distance between peaks, marked in blue, is the period and the height of the peaks, marked in green, is the amplitude. The velocity graph indicates how quickly the spring was oscillating. When the position reached a peak, the velocity was zero.

20. Check that your data seems reasonable. Your measured period should increase as the mass increases.

21. Use the blank *Logger Pro* program to plot the mass as a function of period squared**:** m_A on the vertical axis and T^2 on the horizontal axis**.** Do **not** include a (0,0) data point on this part. Fit the data to a straight line (curve fit and $y = mx+b$).

22. Check that the fit line matches your data well. **Paste a picture of your graph with the fit here.**

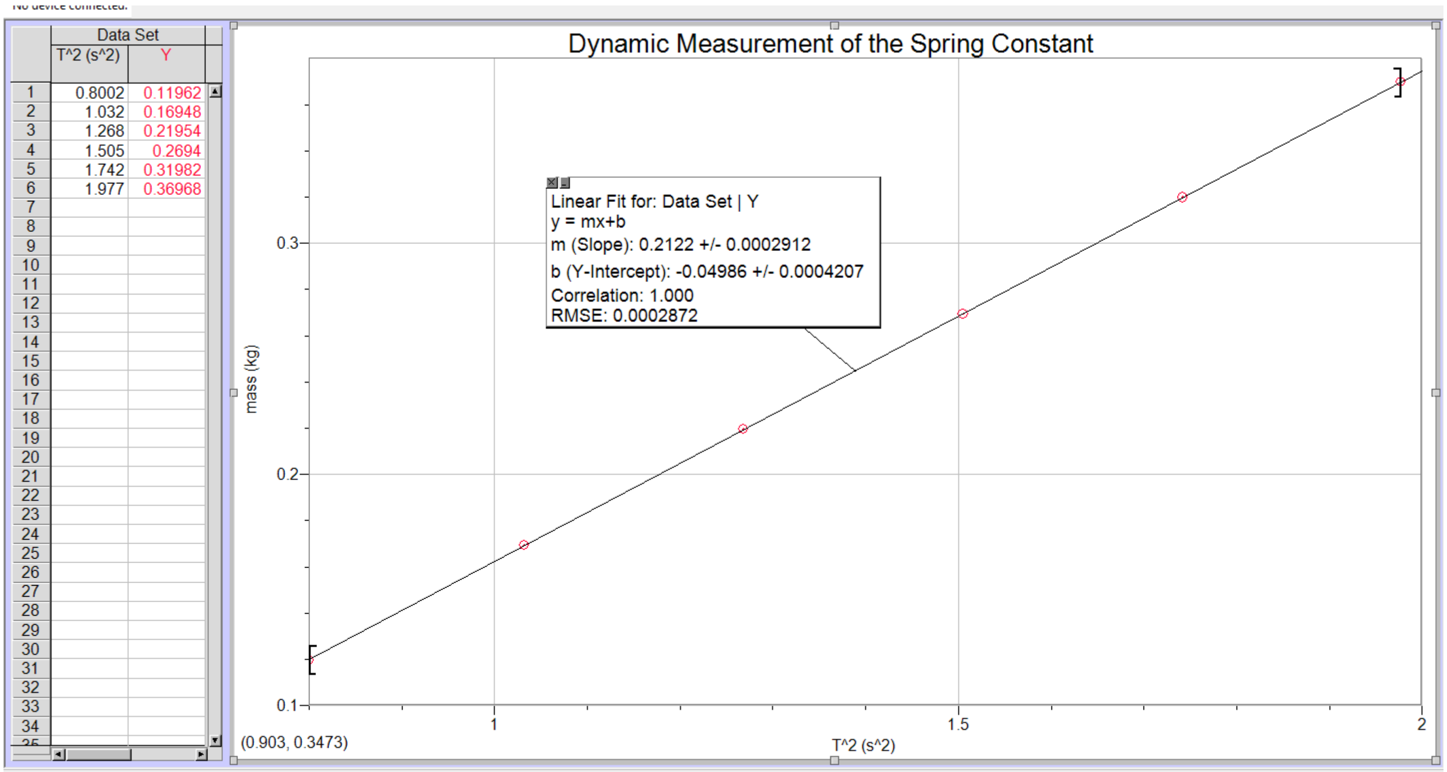


Figure 5. Dynamic Motion Measurements LoggerPro Graph. The x axis is the period squared (s^2) and the y axis is the mass of the weight holder and added weights (kg). The slope of the line is the spring constant (k) divided by $4\pi^2$. The y intercept is the mass of the spring.

23. **Starting with equation 2, explain why the slope of your plot should have the value** $\frac{k}{4\pi^2}$ **and why the intercept should have the value -m_s. Paste a picture of your derivation here.**

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_s + m_A}{k}}$$

$$T^2 = 4\pi^2\,\frac{m_s + m_A}{k}$$

$$\underbrace{m_A}_{y} = \underbrace{\frac{k}{4\pi^2}}_{m}\,\underbrace{T^2}_{x}\,\underbrace{-\,m_s}_{+\,b}$$

Figure 6. Dynamic Motion Equation Derivation. T represents the period of the spring's oscillation in s. m represents mass in kg with m_s being the mass of the spring and m_a being the mass of the weight holder and added weights. k is the string constant in N/m. The spring constant can be calculated by multiplying the slope of the line by $4\pi^2$ because the slope of equal to $k/4\pi^2$ with m_a as y and T^2 as x. The x intercept, b, is equivalent to the weight to the spring m_s.

24. Copy the value of the fitted slope and its uncertainty into your Excel table. Calculate the spring constant, *k* and its uncertainty from the slope.

25. Perform a ratio test to compare this dynamic measurement of the spring constant with the static measurement from part I.

26. Copy your fitting intercept and its uncertainty into your Excel table. Note that your intercept should be negative.

27. Use an Excel formula to calculate the effective mass of the spring, m_s, from the intercept.

28. Measure the actual mass of the spring using the balance. Record this value in your Excel table.

29. Your effective spring mass should be much less than the actual spring mass. Use an Excel formula to calculate the ratio of these.

30. **Paste a picture of part II of your Excel table here.**

Mass of holder and added mass m_A (kg)	*A* fit parameter (m)	*B* Fit parameter (rad/s)	T (s)	T^2 (s^2)
0.11962	0.07675	7.024	0.8945	0.8002
0.16948	0.06549	6.185	1.016	1.032
0.21954	0.08841	5.579	1.126	1.268
0.2694	0.1198	5.122	1.227	1.505
0.31982	0.1242	4.761	1.320	1.742
0.36968	0.1202	4.469	1.406	1.977

Fitted slope (kg/s^2)	abs unc. in slope (kg/s^2)	spring constant, *k* (N/m)	abs unc in *k* (N/m)	Ratio test between static and dynamic *k*'s
0.2122	0.0002912	8.377	0.011	0.407

Fitted intercept (kg)	abs unc. In intercept (kg)	effective spring mass, m_s (kg)	actual spring mass (kg)	(m_s) / (actual spring mass)
-0.04986	0.0004207	0.04986	0.16092	0.3098

Table 2. Dynamix Motion Excel Table 6 total trails were conducted, 5 with added weights and 1 with just the holder. The A and B fit parameters were observed from a sine graph fitted to the motion of the spring, using these values T and T^2 were calculated using the equations shown above. A line fitted to m_A and T^2 was used to calculate the spring constant and the effective spring mass.

31. **Write an analysis of part II here.** Include a discussion of the comparison of the spring constants between parts I and II.

Only around 30% of the total mass of the spring was used. Only the bottom part of the spring is being stretched so not all of the spring mass is being accounted for in the calculation of effective spring mass. This is why the calculated mass is so far from the actual mass of the spring.

The spring constant calculated through dynamic motion agrees with the constant calulcated through static motion in part one.

Part III: Dependence of the period on amplitude

32. Put a total mass (including the holder) of approximately 200 g on the spring and measure the period for three noticeably different amplitudes. For each trial record the oscillation and fit to a sinusoidal curve as before. Save all three in your folder. Record the *A* and *B* fit parameters and calculate the period.

33. **Paste a picture of part III of your Excel table here.**

trial	*A* fit parameter (m)	*B* fit parameter (rad/s)	*T* (s)
1	0.1855	4.760	1.319996913
2	0.08651	4.764	1.318888604
3	0.2197	4.757	1.320829369

Table 3. Dependance Excel Table. Three trials were conducted with the spring being initially stretched different amounts but with the same 200g weight added. The amplitude (A fit) differs for each trail. The B fit parameter is similar for each trail. A fit does not affect the period so the period remains consistent for each trail

34. **Qualitatively compare the periods for your three trials. Does the period depend significantly on the amplitude? Write a brief analysis for part III here.**

The period of the spring's oscillation is not affected by the amplitude, it depends on the external forces acting on the spring. In this experiment the only external force acting on the spring is the weight force of the weight holder and the added weights.

35. **Write a brief conclusion for all three parts here. Report numerical results for the static and dynamic measurements of *k* and their uncertainties. State whether they agreed or not and state your qualitative results from part III.**

The static spring constant was 8.359 ± 0.034 N/m and the dynamic spring constant was 8.377 ± 0.011 N/m. These values agree through the ratio test.

The periods in part 2 differ and part 3 shows that they do not differ because of the amplitude of the vertical oscillation but because of the different values of added mass.

This was a very fun experiment and it may also be interesting to compare spring constants of different materials or different length springs.

Conclusion

36. Adjust the formatting (pagination, margins, size of figures, etc.) of this report to make it easy to read. Save this report as a PDF document. Upload it to the course Moodle page.

37. Clean up your lab station. Put the equipment back where you found it. Remove any temporary files from the computer desktop. Make sure that you logout of Moodle and your email, then log out of the computers. Ensure that the laptop is plugged in.

38. Check with the lab instructor to make sure that they received your submission before you leave.