Tree-level consequences of proposed $O_{H\Box} \to O_{DH}$ replacement

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Contents

Executive summary

- O_{DH} renormalizes the Higgs kinetic term ($\Delta\kappa_H$) and affects input shift corrections giving a nonzero Δm_W^2
- For all EW input schemes containing 3 observables among $\alpha(m_Z), m_W, m_Z, G_F$ (plus m_h), C_{DH} ends up correcting only $\delta v/v$ and $\delta\lambda/\lambda$ in a way that is universal to the three schemes. Gauge coupling shifts do not receive new contributions.
- After renormalization and input scheme shifts, C_{DH} contributes only to $h^n VV$ ($n = 2, 3, 4$, $V = W, Z$) and h^3, h^4 couplings. The Higgs self-interactions are only modified in a momentum-dependent manner. The dependence on C_{DH} cancels in the Yukawas and in the scalar potential.
- The conversion between the Warsaw basis' $O_{H\Box}$ and O_{DH} involves $\lambda, O_H, O_{uH}, O_{dH}, O_{eH}$. Therefore, a priori it would require a non-trivial manipulation of Warsaw basis predictions.

  However, there aren't so many processes where more than one among those operators are simultaneously relevant. Moreover:

  - whenever $C_{H\Box}$ and $C_{\psi H}$ ($\psi = u, d, e$) appear together due to a Yukawa being present, the conversion can be simply obtained by removing $C_{H\Box}$
  - whenever $C_{H\Box}$ and C_H appear together due to a SM-like Higgs self-coupling present, the conversion can again be obtained by removing $C_{H\Box}$.
  - whenever $C_{H\Box}$ appears due to a VVh coupling present, the conversion can be obtained via the basis rotation $C_{H\Box} \to C_{DH}/2$.

  In practice, the only process where the conversion would not be completely trivial is double Higgs production. Even there, it turns out that, by some accident, the $gg \to hh$ channel can be translated simply, while VBF-hh seems to require a re-simulation.

These notes use `SMEFTsim 3.0` conventions. See [1] for all definitions.

1 Basis rotation

The proposal includes replacing

$$O_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \tag{1}$$

with

$$O_{DH} = (H^\dagger H)(D_\mu H^\dagger D^\mu H)\,. \tag{2}$$

The basis change can be derived via

$$\begin{aligned}
O_{H\Box} &= (H^\dagger H)\Box(H^\dagger H) & (3)\\
&= 2O_{DH} + \Big[(H^\dagger H)(H^\dagger \Box H) + \text{h.c.}\Big] & (4)\\
&= 2O_{DH} + \Big[(H^\dagger H)H_i^\dagger \Big(H_i m^2/2 - 2\lambda H_i(H^\dagger H) - \bar{e}Y_l l_i - \bar{d}Y_d q_i - (\bar{q}\varepsilon)_i Y_u^\dagger u\Big) + \text{h.c.}\Big] & (5)\\
&= 2O_{DH} + m^2(H^\dagger H)^2 - 4\lambda O_H - \Big[O_{\substack{eH\\pr}} Y_{\substack{l\\pr}} + O_{\substack{dH\\pr}} Y_{\substack{d\\pr}} + O_{\substack{uH\\pr}} Y_{\substack{u\\pr}} + \text{h.c.}\Big] & (6)
\end{aligned}$$

Denoting O_i^W the vector of Warsaw basis operators, O_i^M the vector of new basis operators, and C_i^W, C_i^M the corresponding columns of Wilson coefficients, the C rotation is determined by:

$$C_i^W O_i^W = C_i^W R_{ij} O_j^M \stackrel{!}{=} C_j^M O_j^M \qquad \Rightarrow \qquad R^T C^W = C^M \qquad \Rightarrow \qquad C^W = (R^T)^{-1} C^M \tag{7}$$

where R is the rotation matrix between the two operator vectors $O^W = RO^M$:

$$O^W = \begin{pmatrix} O_{H\Box} \\ O_H \\ O_{\substack{eH\\pr}} \\ O_{\substack{dH\\pr}} \\ O_{\substack{uH\\pr}} \\ (H^\dagger H)^2 \end{pmatrix} \qquad O^M = \begin{pmatrix} O_{DH} \\ O_H \\ O_{\substack{eH\\pr}} \\ O_{\substack{dH\\pr}} \\ O_{\substack{uH\\pr}} \\ (H^\dagger H)^2 \end{pmatrix} \qquad R = \begin{pmatrix} 2 & -4\lambda & -Y_{\substack{l\\pr}} & -Y_{\substack{d\\pr}} & -Y_{\substack{u\\pr}} & m^2 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{pmatrix}. \tag{8}$$

Taking

$$\begin{aligned}
C^W &= \Big(C_{H\Box} \quad C_H \quad C_{\substack{eH\\pr}} \quad C_{\substack{dH\\pr}} \quad C_{\substack{uH\\pr}} \quad -\lambda\Big)^T & (9)\\
C^M &= \Big(C_{DH} \quad C_H \quad C_{\substack{eH\\pr}} \quad C_{\substack{dH\\pr}} \quad C_{\substack{uH\\pr}} \quad -\lambda\Big)^T & (10)
\end{aligned}$$

we find that, to go from Warsaw to the new basis, the required replacements are

$$\begin{aligned}
C_{H\Box} &\to \frac{1}{2}C_{DH} & (11)\\
C_H &\to C_H + 2\lambda C_{DH} & (12)\\
C_{\substack{eH\\pr}} &\to C_{\substack{eH\\pr}} + \frac{1}{2}C_{DH} Y_{\substack{l\\pr}} & (13)\\
C_{\substack{dH\\pr}} &\to C_{\substack{dH\\pr}} + \frac{1}{2}C_{DH} Y_{\substack{d\\pr}} & (14)\\
C_{\substack{uH\\pr}} &\to C_{\substack{uH\\pr}} + \frac{1}{2}C_{DH} Y_{\substack{u\\pr}} & (15)\\
\lambda &\to \lambda + \frac{m^2}{2}C_{DH} & (16)
\end{aligned}$$

For economy of notation, we assume that the Wilson coefficients are dimensionful, ie. the Λ^{-2} powers are absorbed in their definition. In this way the λ redefinition has the correct dimensions.

Important note. The basis rotation derived here translates correctly to the new basis only when applied to on-shell scattering amplitudes. Intermediate results such as pure input shifts and individual Feynman rules need to be re-derived in the new basis and cannot be obtained by rotating their Warsaw basis expressions. This is what we do below.

2 Potential minimization, unitary gauge and h field redefinition

O_{DH} does not contribute to the scalar potential, so the true vacuum is the same as in the Warsaw basis

$$\frac{v_T^2}{2} = \langle H^\dagger H\rangle = \frac{m^2}{4\lambda}\left[1+\frac{3m^2}{8\lambda^2}C_H + \mathcal{O}(\Lambda^{-4})\right] = \frac{m^2}{4\lambda}\left[1+\frac{3v_T^2}{4\lambda}C_H + \mathcal{O}(\Lambda^{-4})\right] \tag{17}$$

The whole SMEFT Lagrangian is expanded in unitary gauge (neglecting Goldstones) using

$$H = \frac{v_T+h}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \tag{18}$$

which, for the relevant operators, leads to

$$O_{H\Box} = \frac{1}{4}(v_T+h)^2\Box(v_T+h)^2 = -\frac{1}{4}\partial_\mu(v_T+h)^2\partial^\mu(v_T+h)^2 = -(v_T+h)^2\partial_\mu h\partial^\mu h \tag{19}$$

$$O_{DH} = \frac{(v_T+h)^2}{2}\left[\frac{\partial_\mu h\partial^\mu h}{2} + \frac{(v_T+h)^2}{2}\left(\frac{g^2}{2}W_\mu^+W^{-\mu} + \frac{(g^2+g'^2)}{4}Z_\mu Z^\mu\right)\right] \tag{20}$$

$$O_{HD} = (H^\dagger D_\mu H)(D^\mu H^\dagger H) \tag{21}$$

$$= \frac{(v_T+h)^2}{2}\left[\frac{\partial_\mu h\partial^\mu h}{2} + \frac{(v_T+h)^2}{2}\frac{(g^2+g'^2)}{4}Z_\mu Z^\mu\right] \tag{22}$$

The kinetic term of the Higgs boson needs to be canonically normalized via

$$\mathcal{L} \supset \frac{\partial_\mu h\partial^\mu h}{2}\left[1-2\Delta\kappa_H\right] \qquad \Rightarrow \qquad h\to h\left[1+\Delta\kappa_H\right] \tag{23}$$

In the Warsaw basis

$$\Delta\kappa_{H,W} = v_T^2\left[C_{H\Box} - \frac{C_{HD}}{4}\right] \tag{24}$$

while in the new basis

$$\Delta\kappa_{H,M} = -\frac{v_T^2}{4}\left[C_{DH}+C_{HD}\right] \tag{25}$$

Regardless of the form, $\Delta\kappa_H$ ends up in the scalar potential for the physical Higgs as:

$$V(h) = \lambda v_T^2\,h^2\left[1+2\Delta\kappa_H - \frac{3v_T^2}{2\lambda}C_H\right] + \lambda v_T\,h^3\left[1+3\Delta\kappa_H - \frac{5v_T^2}{2\lambda}C_H\right] + \frac{\lambda}{4}\,h^4\left[1+4\Delta\kappa_H - \frac{15v_T^2}{2\lambda}C_H\right] - h^5\frac{3v_T}{4}C_H - h^6\frac{1}{8}C_H \tag{26}$$

and it enters all other SM Higgs couplings as well, ie. Yukawas and $hVV, hhVV$ vertices.

3 Electroweak inputs

The Lagrangian parameters of the EW sector are usually fixed by choosing three observables among $\alpha(m_Z), m_W, m_Z, G_F$ (with G_F measured in muon decay) as input quantities, in addition to m_h. A detailed explanation of the procedure is in [1]. The net result is that the 4 SM parameters g, g', v_T, λ can be expressed as

$$g = \hat{g}\left[1 + \frac{\delta g}{g}\right] \tag{27}$$

where $\hat{g}$ is the numerical value of the constant, computed as a function of the input measurements, while δg is a linear polynomial in the Wilson coefficients. The meaning of the equation is that the parameter g appearing in the SMEFT Lagrangian should be replaced everywhere with the expression $(\hat{g} + \delta g)$. In this way the corrections contained in δg get pushed to the predicted observables.

At tree level and for a general dim-6 SMEFT Lagrangian, the expressions of the candidate input observables are

$$\alpha(m_Z) = \frac{1}{4\pi}\frac{g^2 g'^2}{g^2 + g'^2}\left[1 + \Delta\alpha\right] \qquad G_F = \frac{1}{\sqrt{2}v_T^2}\left[1 + \Delta G_F\right] \tag{28}$$

$$m_W^2 = \frac{g^2 v_T^2}{4}\left[1 + \Delta m_W^2\right] \qquad m_Z^2 = \frac{(g^2 + g'^2)v_T^2}{4}\left[1 + \Delta m_Z^2\right] \qquad m_h^2 = 2\lambda v_T^2\left[1 + \Delta m_h^2\right] \tag{29}$$

From these expressions, one can derive[1]

	$\{m_W, m_Z, G_F\}$	$\{\alpha, m_Z, G_F\}$	$\{m_W, m_Z, \alpha\}$
$\dfrac{\delta g'^2}{g'^2}$	$-(\Delta G_F + \Delta m_W^2) - \dfrac{\Delta m_Z^2 - \Delta m_W^2}{s_\theta^2}$	$\dfrac{s_\theta^2}{c_{2\theta}}\left(\Delta m_Z^2 + \Delta G_F\right) - \dfrac{c_\theta^2}{c_{2\theta}}\Delta\alpha$	$-(\Delta m_Z^2 - \Delta m_W^2) - \Delta\alpha$
$\dfrac{\delta g^2}{g^2}$	$-(\Delta G_F + \Delta m_W^2)$	$-\dfrac{c_\theta^2}{c_{2\theta}}\left(\Delta m_Z^2 + \Delta G_F\right) + \dfrac{s_\theta^2}{c_{2\theta}}\Delta\alpha$	$\dfrac{\Delta m_Z^2 - \Delta m_W^2}{t_\theta^2} - \Delta\alpha$
$\dfrac{\delta v_T^2}{v_T^2}$	ΔG_F	ΔG_F	$-\dfrac{\Delta m_Z^2 - \Delta m_W^2}{t_\theta^2} - \Delta m_W^2 + \Delta\alpha$
$\dfrac{\delta\lambda}{\lambda}$	$-\Delta G_F - \Delta m_h^2$	$-\Delta G_F - \Delta m_h^2$	$\dfrac{\Delta m_Z^2 - \Delta m_W^2}{t_\theta^2} + \Delta m_W^2 - \Delta m_h^2 - \Delta\alpha$
s_θ^2	$1 - \dfrac{m_W^2}{m_Z^2}$	$\dfrac{1}{2}\left(1 - \sqrt{1 - \dfrac{2\sqrt{2}\pi\alpha}{G_F m_Z^2}}\right)$	$1 - \dfrac{m_W^2}{m_Z^2}$

In the Warsaw basis

$$\Delta\alpha = -\frac{2gg'v_T^2}{g^2 + g'^2}C_{HWB} \tag{30}$$

$$\Delta m_W^2 = 0 \tag{31}$$

$$\Delta m_Z^2 = v_T^2\left[\frac{1}{2}C_{HD} + \frac{2gg'}{g^2 + g'^2}C_{HWB}\right] \tag{32}$$

$$\Delta m_h^2 = 2\Delta\kappa_H - \frac{3v_T^2}{2\lambda}C_H = v_T^2\left[2C_{H\Box} - \frac{1}{2}C_{HD} - \frac{3}{2\lambda}C_H\right] \tag{33}$$

$$\Delta G_F = v_T^2\left[C^{(3)}_{\substack{Hl\\11}} + C^{(3)}_{\substack{Hl\\22}} - \frac{1}{2}C_{\substack{ll\\1221}}\right] \tag{34}$$

[1]Note that trigonometric functions of the weak mixing angle are used only as shorthand notation. Their definition changes in each scheme: the tree-level expression of s_θ^2 is reported at the bottom of the table.

While in the new basis

$$\Delta\alpha = -\frac{2gg'v_T^2}{g^2+g'^2}C_{HWB} \tag{35}$$

$$\Delta m_W^2 = \frac{v_T^2}{2}C_{DH} \tag{36}$$

$$\Delta m_Z^2 = v_T^2\left[\frac{1}{2}(C_{DH}+C_{HD}) + \frac{2gg'}{g^2+g'^2}C_{HWB}\right] \tag{37}$$

$$\Delta m_h^2 = 2\Delta\kappa_H - \frac{3v_T^2}{2\lambda}C_H = v_T^2\left[-\frac{1}{2}(C_{DH}+C_{HD}) - \frac{3}{2\lambda}C_H\right] \tag{38}$$

$$\Delta G_F = v_T^2\left[C^{(3)}_{\underset{11}{Hl}} + C^{(3)}_{\underset{22}{Hl}} - C_{\underset{1221}{ll}} - \frac{1}{2}C_{DH}\right] \tag{39}$$

where the C_{DH} contributions to $\Delta m_Z^2, \Delta m_W^2$ come directly from opening O_{DH} in unitary gauge while the contribution to ΔG_F enters via the correction to the W mass in the μ decay diagram.

Importantly, C_{DH} always **cancels off** in $(\Delta m_Z^2 - \Delta m_W^2)$, in $(\Delta G_F + \Delta m_W^2)$ and in $(\Delta G_F + \Delta m_Z^2)$. Therefore, for **all** the schemes considered:[2] C_{DH} never enters the corrections to the gauge couplings, and it enters always in the same way in those to v_T and λ:

$$\delta v_T^2/v_T^2 = -v_T^2 C_{DH}/2 + \dots \qquad\qquad \delta\lambda/\lambda = v_T^2 C_{DH} + \dots \tag{40}$$

It's interesting to check the impact of input corrrections on the scalar potential, which are universal for the 3 schemes considered: shifting λ and v_T we get

$$\begin{aligned} V(h) &= \hat\lambda\hat v^2\, h^2\left[1 + 2\Delta\kappa_H - \Delta m_h^2 - \frac{3v_T^2}{2\lambda}C_H\right] + \hat\lambda\hat v\, h^3\left[1 + 3\Delta\kappa_H - \frac{\Delta G_F}{2} - \Delta m_h^2 - \frac{5v_T^2}{2\lambda}C_H\right] \\ &+ \frac{\hat\lambda}{4}h^4\left[1 + 4\Delta\kappa_H - \Delta G_F - \Delta m_h^2 - \frac{15v_T^2}{2\lambda}C_H\right] - h^5\frac{3\hat v}{4}C_H - h^6\frac{1}{8}C_H \end{aligned} \tag{41}$$

$$\begin{aligned} &= \hat\lambda\hat v^2\, h^2 + \hat\lambda\hat v h^3\left[1 + \frac{\hat v^2}{2}\left(\frac{C_{HD}}{2} - C^{(3)}_{\underset{11}{Hl}} - C^{(3)}_{\underset{22}{Hl}} + C_{\underset{1221}{ll}} - \frac{2}{\lambda}C_H\right)\right] \\ &+ \frac{\hat\lambda}{4}h^4\left[1 + \hat v^2\left(-\frac{C_{HD}}{2} - C^{(3)}_{\underset{11}{Hl}} - C^{(3)}_{\underset{22}{Hl}} + C_{\underset{1221}{ll}} - \frac{6}{\lambda}C_H\right)\right] - h^5\frac{3\hat v}{4}C_H - h^6\frac{1}{8}C_H \end{aligned} \tag{42}$$

Corrections to the Higgs mass cancel by design. Moreover, the contributions of C_{DH} to the cubic and quartic interactions drop out, and only corrections from the other operators remain, which are identical to those in the Warsaw basis.

[2] I secretly checked that this is all true even for $\{m_W, \alpha, G_F\}$.

4 Feynman rules

In the end of the day, the set of vertices affected by C_{DH} is quite limited. Here we report the full set of pheno-ready Feynman rules affected by C_{DH}, obtained after renormalizing everything and implementing input shifts.

We only write down terms potentially containing C_{DH} and omit contributions by other operators, which remain unchanged compared to the Warsaw basis. We give first the results in arbitrary EW input scheme and then the explicit dependence on C_{DH} which holds for all the 3 schemes discussed above. It's important to note that the latter expressions might not be correct for alternative schemes, eg. employing $\sin^2\theta_\ell^{eff}$. For the input schemes considered here, the contribution of C_{DH} cancels in the Yukawa couplings[3] and only remains in Higgs couplings to itself and to W, Z. We have:

$$h\bar{b}b \qquad -\frac{i\hat{y}_b}{\sqrt{2}}\left[1+\Delta\kappa_H-\frac{\delta v_T}{v_T}\right] = -\frac{i\hat{y}_b}{\sqrt{2}} \tag{43}$$

$$h\bar{t}t \qquad -\frac{i\hat{y}_t}{\sqrt{2}}\left[1+\Delta\kappa_H-\frac{\delta v_T}{v_T}\right] = -\frac{i\hat{y}_t}{\sqrt{2}} \tag{44}$$

$$h\bar{\tau}\tau \qquad -\frac{i\hat{y}_\tau}{\sqrt{2}}\left[1+\Delta\kappa_H-\frac{\delta v_T}{v_T}\right] = -\frac{i\hat{y}_\tau}{\sqrt{2}} \tag{45}$$

$$hW^+_\mu W^-_\nu \qquad \frac{i\hat{g}^2\hat{v}}{2}\eta_{\mu\nu}\left[1+\Delta\kappa_H+\frac{\delta v_T}{v_T}+\hat{v}^2C_{DH}\right] = \frac{i\hat{g}^2\hat{v}}{2}\eta_{\mu\nu}\left[1+\frac{\hat{v}^2}{2}C_{DH}+\dots\right] \tag{46}$$

$$hZ_\mu Z_\nu \qquad \frac{i\hat{g}^2\hat{v}}{2c_\theta^2}\eta_{\mu\nu}\left[1+\Delta\kappa_H+\frac{\delta v_T}{v_T}+\hat{v}^2C_{DH}\right] = \frac{i\hat{g}^2\hat{v}}{2c_\theta^2}\eta_{\mu\nu}\left[1+\frac{\hat{v}^2}{2}C_{DH}+\dots\right] \tag{47}$$

$$h^2W^+_\mu W^-_\nu \qquad \frac{i\hat{g}^2}{2}\eta_{\mu\nu}\left[1+2\Delta\kappa_H+3\hat{v}^2C_{DH}\right] = \frac{i\hat{g}^2}{2}\eta_{\mu\nu}\left[1+\frac{5\hat{v}^2}{2}C_{DH}\right] \tag{48}$$

$$h^2Z_\mu Z_\nu \qquad \frac{i\hat{g}^2}{2c_\theta^2}\eta_{\mu\nu}\left[1+2\Delta\kappa_H+3\hat{v}^2C_{DH}\right] = \frac{i\hat{g}^2}{2c_\theta^2}\eta_{\mu\nu}\left[1+\frac{5\hat{v}^2}{2}C_{DH}\right] \tag{49}$$

$$h^3W^+_\mu W^-_\nu \qquad 3i\hat{g}^2\hat{v}\eta_{\mu\nu}C_{DH} \tag{50}$$

$$h^3Z_\mu Z_\nu \qquad \frac{3i\hat{g}^2\hat{v}}{c_\theta^2}\eta_{\mu\nu}C_{DH} \tag{51}$$

$$h^4W^+_\mu W^-_\nu \qquad 3i\hat{g}^2\eta_{\mu\nu}C_{DH} \tag{52}$$

$$h^4Z_\mu Z_\nu \qquad \frac{3i\hat{g}^2}{c_\theta^2}\eta_{\mu\nu}C_{DH} \tag{53}$$

and also

$$\begin{aligned} h^3 \qquad & -6i\hat{\lambda}\hat{v}\left[1+3\Delta\kappa_H+\frac{\delta v_T}{v_T}+\frac{\delta\lambda}{\lambda}\right]-i\hat{v}C_{DH}\left(p_1\cdot p_2+p_1\cdot p_3+p_2\cdot p_3\right) \\ & = -6i\hat{\lambda}\hat{v}-i\hat{v}C_{DH}\left(p_1\cdot p_2+p_1\cdot p_3+p_2\cdot p_3\right) \end{aligned} \tag{54}$$

$$\begin{aligned} h^4 \qquad & -6i\hat{\lambda}\left[1+4\Delta\kappa_H+\frac{\delta\lambda}{\lambda}\right]-iC_{DH}\left(p_1\cdot p_2+p_1\cdot p_3+p_1\cdot p_4+p_2\cdot p_3+p_2\cdot p_4+p_3\cdot p_4\right) \\ & = -6i\hat{\lambda}-iC_{DH}\left(p_1\cdot p_2+p_1\cdot p_3+p_1\cdot p_4+p_2\cdot p_3+p_2\cdot p_4+p_3\cdot p_4\right) \end{aligned} \tag{55}$$

As already observed above, the corrections by C_{DH} to SM-like vertices cancel. However, the operator gives genuine momentum-dependent corrections to the triple and quartic Higgs vertices, which should be accounted for.

[3]The relevant fermionic vertices are the SM Yukawas. We write them down in the mass eigenstate basis, with diagonal Yukawas, and only write the 3rd generation coupling for brevity. It is understood that all generations behave analogously.

5 Observables

We can consider some simple observables and check, in particular, that the result in the new basis can be obtained by rotating the one in the Warsaw basis.

Given the nature of the basis change, the processes where the basis change has a non-trivial impact are those that involve $C_{H\Box}, C_{dH}, C_{uH}, C_{eH}, C_H$ in the Warsaw basis, as well as the SM Higgs self-couplings. Anything else (eg. tree level diboson) is unaffected.

5.1 Tree-level $h \to \bar{b}b$

This is a very simple process because the amplitude is given by a single Feynman rule.

We have seen that C_{DH} does *not* enter $h\bar{\psi}\psi$ vertices after all manipulations, so, unlike $C_{H\Box}$, the new operator does not enter fermionic Higgs decays. It's easy to check that taking the FR in the new basis one gets the same result as by applying the basis rotation to the FR in the Warsaw basis. We impose the $U(3)^5$ flavor symmetry and use the $\{m_W, m_Z, G_F\}$ scheme for concreteness, but the conclusion holds in general:

$$\frac{\Gamma^W_{h\to\bar{b}b}}{\Gamma_{h\to\bar{b}b,SM}} = \left[1 + \hat{v}^2\left(-\Re C_{dH} + C_{H\Box} - \frac{C_{HD}}{4} - C^{(3)}_{Hl} + \frac{C'_{ll}}{2}\right)\right]^2 \tag{56}$$

$$\frac{\Gamma^M_{h\to\bar{b}b}}{\Gamma_{h\to\bar{b}b,SM}} = \left[1 + \hat{v}^2\left(-\Re C_{dH} - \frac{C_{HD}}{4} - C^{(3)}_{Hl} + \frac{C'_{ll}}{2}\right)\right]^2 \tag{57}$$

Any fermionic Higgs decay, tree-level $gg \to \bar{t}th$ and 1-loop $gg \to h$ production are modified analogously, as $C_{H\Box}$ only enters there via a single $h\bar{\psi}\psi$ vertex. 1-loop QCD corrections should not spoil the change (an explicit check wasn't done, but the condition is that no additional contribution by $C_H, C_{\psi H}, C_{H\Box}$ or λ is brought in by the loops, which seems safe to assume for QCD).

5.2 Tree-level $\bar{q}q \to ZH$

This process receives contributions by many operators which are however unaffected by the proposed basis change. The only modification we are interested in is the normalization scaling induced by $C_{H\Box}$ in the Warsaw basis, which enters via the ZZh interaction:

$$\frac{\sigma^W_{\bar{q}q\to ZH}}{\sigma_{\bar{q}q\to ZH,SM}} \sim \left(1 + \hat{v}^2 C_{H\Box} + \ldots\right) + \ldots \tag{58}$$

In the new basis we find instead

$$\frac{\sigma^M_{\bar{q}q\to ZH}}{\sigma_{\bar{q}q\to ZH,SM}} \sim \left(1 + \frac{\hat{v}^2}{2} C_{DH} + \ldots\right) + \ldots \tag{59}$$

while everything else is unchanged. Again, this can be trivially obtained with the basis rotation.

Single Higgs production in VBF and Higgs decays to 4 leptons are modified analogously.

5.3 One-loop $gg \to hh$

For this process we need to consider 1 loop diagrams (state of the art is 2 QCD-loops but we don't go there for this first assessment).

The box diagram contains two $\bar{t}th$ couplings. Therefore it's enough to remove $C_{H\Box}$ to obtain it in the new basis, while C_{DH} won't enter there.

There are two relevant triangle diagrams in SMEFT, shown in Fig. 1. Both are needed to understand the basis conversion.

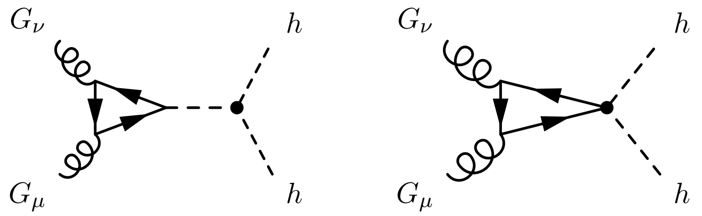

Figure 1: Feynman diagrams relevant for the basis conversion of $gg \to hh$ at 1 loop

Let us start from the Warsaw basis:

- the $\bar{t}th$ vertex receives corrections from C_{uH} and $C_{H\Box}$. We have established that the contribution from this particular vertex can be converted to the new basis by simply removing $C_{H\Box}$, so we won't consider this contribution further.

- the $\bar{t}thh$ vertex in the second diagram also receives a contribution from C_{uH}, which is[4]

$$\mathcal{A}^{W\mu\nu} \supset f^{\mu\nu}(s)\left(\frac{3i}{\sqrt{2}}C_{\substack{uH\\33}}\right) \tag{60}$$

  where $f^{\mu\nu}(s)$ encodes all the terms that come from integrating the loop function. For this operator, it's the same as for the SM loop.

- the hhh vertex in the first diagram receives contributions from $C_{H\Box}$ and C_H, such that

$$\mathcal{A}^{W\mu\nu} \supset f^{\mu\nu}(s)\left(-\frac{iy_t}{\sqrt{2}}\frac{i}{s-m_h^2}\right)\left[-3im_h^2 v\left(C_{H\Box}-\frac{2v^2C_H}{m_h^2}\right)-ivC_{H\Box}\left(3s+6m_h^2-s-2m_h^2\right)\right] \tag{61}$$

  where $f^{\mu\nu}(s)$ is the same as above, the first piece in the squared brackets comes from the rescaling of the SM-like h^3 vertex and the second one from the momentum-dependent correction to h^3 induced by $C_{H\Box}$. Note that we have also expressed the contributions in terms of m_h and v, rather than λ, to highlight that this is already expressed in terms of inputs and that there is no manifest λ dependence left. This is relevant for the basis rotation.

Summing the contributions from (60) and (61) and applying the basis rotation gives the result in the new basis:

$$\mathcal{A}^{M\mu\nu} \supset f^{\mu\nu}(s)\left(-\frac{iy_t}{\sqrt{2}}\frac{i}{s-m_h^2}\right)\left[-3im_h^2 v\left(-\frac{2v^2C_H}{m_h^2}\right)-\frac{iv}{2}C_{DH}\left(-s-2m_h^2\right)\right]+f^{\mu\nu}(s)\frac{3i}{\sqrt{2}}C_{\substack{uH\\33}} \tag{62}$$

$$= f^{\mu\nu}(s)\left(-\frac{iy_t}{\sqrt{2}}\frac{i}{s-m_h^2}\right)\left[6iv^3C_H+ivC_{DH}\left(\frac{s}{2}+m_h^2\right)\right]+f^{\mu\nu}(s)\frac{3i}{\sqrt{2}}C_{\substack{uH\\33}} \tag{63}$$

It's not hard to check that starting off with the new Feynman rules given above leads directly to the same expression.

In practice, to convert the Warsaw basis prediction to the new basis it's enough to

- remove the SM-like contribution from $C_{H\Box}$ (both from yukawas and h^3)

- rescale the $C_{H\Box}$ momentum-dependent contribution by a factor $-1/4$ and assign it to C_{DH}.

  In fact, in the Warsaw basis we had a term $-C_{H\Box}(4m_h^2+2s)$, which in the new one is replaced by $C_{DH}(m_h^2+s/2)$

[4]In this case we use a flavor-general formalism with no flavor symmetry, and only include the top loop.

In doing the calculation we have also learned that

- working after imposing an EW input scheme, the expressions do not manifestly depend on λ anymore, so we get the correct result by *not* applying the basis rotation to $\hat{\lambda}$
- when rotating the Warsaw basis result, the correct kinematic structure in front of C_{DH} emerges through a cancellation among the terms that arise from the $\bar{t}thh$ vertex, the SM-like h^3 vertex and the term proportional to $C_{H\Box}(2s+6m_h^2)$ in h^3.

  This cancellation only happens when all three pieces are included. In particular, the inclusion of the $\bar{t}thh$ diagram is required to remove the extra s dependence from the h^3 term.

5.4 Tree-level $\bar{q}q \to \bar{q}qhh$ (VBF-hh)

In practice we can limit ourselves to the $VV \to hh$ diagram, since none of the operators under study affects the connection to initial state quarks.

There are three relevant diagrams: the contact term $VVhh$, the h exchange in s channel and the V exchange in t channel.

In Warsaw basis:

$$\mathcal{A}_c^{W\mu\nu} = \frac{i2m_V^2}{v^2}\eta^{\mu\nu}\left[1+2v^2C_{H\Box}\right] \tag{64}$$

$$\mathcal{A}_s^{W\mu\nu} = \frac{i2m_V^2}{v}\frac{i}{s-m_h^2}\eta^{\mu\nu}\left[-3im_h^2v\left(1+2C_{H\Box}-\frac{2v^2C_H}{m_h^2}\right)-2ivC_{H\Box}\left(s+2m_h^2\right)\right] \tag{65}$$

$$\mathcal{A}_t^{W\mu\nu} = \left(\frac{i2m_V^2}{v}\right)^2\frac{i}{t-m_V^2}\left[-\eta^{\mu\nu}-\frac{q^\mu q^\nu}{m_V^2}\right]\left[1+2v^2C_{H\Box}\right] \tag{66}$$

In new basis:

$$\mathcal{A}_c^{M\mu\nu} = \frac{i2m_V^2}{v^2}\eta^{\mu\nu}\left[1+\frac{5v^2}{2}C_{DH}\right] \tag{67}$$

$$\mathcal{A}_s^{M\mu\nu} = \frac{i2m_V^2}{v}\frac{i}{s-m_h^2}\eta^{\mu\nu}\left[-3im_h^2v\left(1+\frac{C_{DH}}{2}-\frac{2v^2C_H}{m_h^2}\right)+ivC_{DH}\left(\frac{s}{2}+m_h^2\right)\right] \tag{68}$$

$$\mathcal{A}_t^{M\mu\nu} = \left(\frac{i2m_V^2}{v}\right)^2\frac{i}{t-m_V^2]}\left[-\eta^{\mu\nu}-\frac{q^\mu q^\nu}{m_V^2}\right]\left[1+v^2C_{DH}\right] \tag{69}$$

The t channel contribution can be rotated individually with $C_{H\Box} \to C_{DH}/2$, while the contact and s-channel diagrams need to be recombined together. Considering their sum:

$$\frac{\mathcal{A}_c^{W\mu\nu}+\mathcal{A}_s^{W\mu\nu}}{\mathcal{A}_c^{SM\mu\nu}+\mathcal{A}_s^{SM\mu\nu}} = 1+4v^2C_{H\Box}-6v^2C_H\frac{v^2}{s+2m_h^2} \tag{70}$$

$$\frac{\mathcal{A}_c^{M\mu\nu}+\mathcal{A}_s^{M\mu\nu}}{\mathcal{A}_c^{SM\mu\nu}+\mathcal{A}_s^{SM\mu\nu}} = 1+2v^2C_{DH}\frac{s-m_h^2}{s+2m_h^2}-6v^2C_H\frac{v^2}{s+2m_h^2} \tag{71}$$

so we see that in this case the kinematics recombination is non-trivial, and C_{DH} will have a different shape than $C_{H\Box}$. In principle this would require a repetition of the event simulation.

5.5 Tree-level $\bar{q}q \to \bar{q}qVV$ (VBS)

Again we can limit ourselves to the $VV \to VV$ diagrams. The contact terms are not relevant in this case, cause they are not affected by the operators considered. We only need to worry about the s and t channel h exchanges.

In the warsaw basis

$$\frac{\mathcal{A}_{s,t}^{W}}{\mathcal{A}_{s,t}^{SM}} = 1 + 2v^2 C_{H\Box} + \dots \tag{72}$$

while in the new basis

$$\frac{\mathcal{A}_{s,t}^{M}}{\mathcal{A}_{s,t}^{SM}} = 1 + v^2 C_{DH} + \dots \tag{73}$$

So this process can be trivially converted via the basis rotation.

References

[1] I. Brivio, *SMEFTsim 3.0 — a practical guide*, *JHEP* **04** (2021) 073, [2012.11343].