

Instructions

- Write and submit source files with the exact names specified in each exercise.
- Do not submit any file, folder, or archive, other than what is required.
- Your code must work with Python 3.
- You may only use the following, limited subset of the Python language and libraries.
You may only use the following built-in types:
 - numeric types, such as `int`
 - sequence types, such as arrays, tuples, and strings

With arrays or other sequence types, you may only use the following operations:

- direct access to an element by index, as in `print(A[7])` or `A[i+1] = A[i]`
- append an element, as in `A.append(10)`
- delete the last element, as in `del A[-1]` or `del A[len(A)-1]`
- read the length, as in `n = len(A)`
- shrink to a given length, as in `del A[length:]`
- sort in-place as in `A.sort()`
- sort with the `sorted()` function, as in `B = sorted(A)`

You may use for iterations as follows:

- iteration over the elements in a sequence, as in `for a in A:`
- range iteration, as in `for i in range(10):`

You may define classes but only with a single, constructor method `__init__(self, ...)`

You may not use any function or object or method or module except for the types and methods and functions from the standard library or built-in types listed above, namely `append()`, `len()`, `print()`, `range()`, `sort()`, `sorted()`, `__init__()`.

- If an exercise requires you to analyze the complexity of an algorithm, write your analysis as a code comment either at the beginning of the source file or anyway near the corresponding Python function.
 - Document any known issue using comments in the code.
 - Submit each file through the iCorsi system.
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- **Exercise 1.** Given a number k , a step- k sequence of length ℓ is a sequence of ℓ numbers a_1, a_2, \dots, a_ℓ such that either $a_i = a_{i+1} + k$ for all pairs of adjacent elements a_i, a_{i+1} , or $a_i + k = a_{i+1}$ for all pairs of adjacent elements a_i, a_{i+1} . For example, the sequence 2, 3.5, 5, 6.5, 8 is a step-1.5 sequence, and 7, 4, 1, -2 is a step-3 sequence. (20')

In a source file `ex1.py` write a python function called `maximal_step_k_length(A,k)` that takes a sequence of numbers A , and a number k , and returns the maximal length ℓ such that there is at least one contiguous sequence of elements in A that form a step- k sequence. Your solution must have a time complexity $O(n)$, where n is the length of A .

For example, `maximal_step_k_length([2,4,5,6,8,6,4,2,0,2,4,6,10,3,1],2)` must return 5.

- **Exercise 2.** Your sport watch is equipped with an altitude sensor that, every second, measures your altitude in meters. Given an array $A = [a_1, a_2, \dots, a_n]$ of n consecutive altitude measurements, you want to determine whether you had a high-power run. A high-power run occurs when there is a certain total altitude gain over a period of time, where the total altitude gain is the sum of all altitude gains (positive altitude variations) over that period. For example, the sequence of measurements 10, 10, 12, 11, 10, 11, 12 corresponds to a total altitude gain of 4 meters (10, 12 and then 10, 11, 12). (30')

In a source file `ex2.py` write a Python function called `high_power_run(A,h,t)` that takes a vector A of altitude measurements (measured consecutively every second), an altitude gain h , and a time limit t , and returns True if A indicates a steep climb of at least h meters in at most t seconds, or False otherwise. Your solution must have a complexity $O(n)$.

For example, `high_power_run([10,6,1,3,2,1,3,4,6,5,6,4,3,4],6,5)` must return True, because the measurements 1, 3, 4, 6, 5, 6 indicate a total gain of 6 meters in 5 seconds. However, `high_power_run([10,6,1,3,2,1,3,4,6,5,6,4,3,4],6,4)` must return False, because there is no total gain of at least 6 meters in 4 seconds.

- **Exercise 3.** An array $A = [a_1, a_2, \dots, a_n]$ of numbers is said to be in “peak” order if $a_i \geq a_{i-1}$ for all $1 < i \leq (n+1)/2$, and $a_j \geq a_{j+1}$ for all $(n+1)/2 \leq j < n$. In essence, A is in peak order when its first half is in ascending order while the second half is in descending order. In a source file `ex3.py`, write a Python function called `peak_order(A)` that takes an array of numbers A and reorders its elements into a peak order. `peak_order(A)` must change the array A *in-place*, and must run in $O(n \log n)$ time. (20')

- **Exercise 4.** A *left-rotation* of an array A is defined as a permutation of A such that every element is shifted by one position to the left except for the first element that is moved to the last position. For example, with $A = [1, 2, 3, 4, 5, 6, 7, 8, 9]$, a *left-rotation* would change A into $A = [2, 3, 4, 5, 6, 7, 8, 9, 1]$.

Question 1: In a source file `ex4.py` write an algorithm `rotate(A,k)` that takes an array A and performs k left-rotations on A . The complexity of your algorithm must be $O(n)$, which means that the complexity must not depend on k . (10')

Question 2: In the same source file `ex4.py` write a function `rotate_inplace(A,k)` that takes an array A and, in $O(n)$ steps, performs k left-rotations *in-place*. In-place means that `rotate_inplace(A,k)` may not use more than a constant amount of extra memory. If your implementation of `rotate(A,k)` is already in-place, then you may use it directly to implement `rotate_inplace(A,k)`. (30')

- **Exercise 5.** In a source file `ex5.py` write a function `is_sorted(A)` that returns True if A is sorted in either ascending or descending order. Analyze the complexity of `is_sorted(A)`. (10')