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# Determination of a diffusion coefficient function for long rooms using a least square optimization approach

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**ABSTRACT**

*Over the last decades, considerable attention has been devoted to improving the diffusion equation model within the field of room acoustics. It has been found that this model is subject to certain limitations, with one constraint regarding the diffusion coefficient. The diffusion coefficient, a parameter integrated into the model, is a value that scales the sound intensity to the gradient of the sound energy density. Currently, this value is often treated as a constant, but research has shown that it is spatially dependent. This paper explores a method to optimise the diffusion coefficient for long rooms such as corridors and tunnels by comparing the results of the diffusion equation model with those computed with the radiosity model and using the least-square optimization method, minimising the difference between the results of both methods. The parameters of a diffusion coefficient function are optimised, targeting a function that can be applied to a range of elongated spaces.*

## 1. INTRODUCTION

In the last decades, sound field modelling in rooms has played a pivotal role in the analysis of a room's acoustic characteristics. These modelling approaches have traditionally relied on the widely accepted concept of the diffuse field which serves as the foundation for the reverberation time formula [1]. However, it has become evident that the theory of statistical room acoustics falls short in certain scenarios, as it has limited applicability and accuracy [2]. Multiple simulation techniques have therefore been defined to improve modelling prediction results. These include, amongst others, ray- and beam-tracing methods [2], image source method [2], radiosity method [2], different types of wave-based methods [3, 4] and diffusion equation method [5, 6]. The focus of this paper is an energy-based method using the diffusion equation.

Recently, the diffusion equation model has been extensively used for room acoustics calculations and has attracted attention due to its efficiency and flexibility. This diffusion equation model aims to find the temporal and spatial distribution of acoustic energy within a specific room, without solving the wave equation and without considering the acoustic phase. The modelling method is based on solving the partial differential diffusion equation, which is applicable in the high-frequency range and it assumes only diffusely reflecting boundaries according to Lambert's law [7].

Using the diffusion equation model, the purpose of this paper is to define a formula for one of the parameters of the diffusion equation model: the diffusion coefficient. Firstly, a comprehensive

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background of the diffusion equation model is presented in Section 2. To validate the diffusion equation model with its parameters, in Section 3, the results are compared with the well-known radiosity method [8]. The optimization method for extracting the diffusion coefficient is then explained in Section 4. Sections 5 and 6 present the results of the diffusion coefficient optimization for long rooms. Section 7 concludes the paper with a discussion of the results.

## 2. DIFFUSION EQUATION MODEL

The diffusion equation model is based on the concept of diffusion in fluid dynamics and thermodynamics. Diffusion is the process that involves the movement of particles within a system from one region to another due to random Brownian motion [9]. The acoustics diffusion equation model is based on an analogy between the motion of the particles in a fluid and the motion of a "sound particle" in a room as described by Ollendorff [5]. He demonstrates mathematically that "sound particles" (considered quasi-molecules) perform a Brownian motion colliding with the wall/boundaries around the room volume and reflecting in random directions. It has also been demonstrated that with certain specific assumptions, the diffusion equation model can be derived from the radiation transfer theory for the transport of photon energy in a scattering medium [10, 11]. Pioneer of the acoustics diffusion equation model, Picaut presented the diffusion equation model as a natural extension of the concept of the diffuse sound field, where a non-uniform sound energy density field, a net flow of energy within the domain, is considered [6]. In addition, he proposed sound absorption at the domain's boundaries, introducing the exchange coefficient [12]. This implies that the sound field does not need to be diffuse to be correctly computed by the diffusion equation model. Therefore, this model demonstrates efficacy in addressing non-uniform sound fields. The formulation by Picaut is based on the particle-tracing method [13].

Over the years, many studies have focused on this model, adding boundary conditions [12], improving the absorption term of the boundary conditions [14, 15], introducing the air absorption term [16], studying the diffusion coefficient [17] and introducing scattering in the model [18]. The objective of all these modifications was to make the method a valid energetic method for the computation of room acoustics.

Knowing that the diffusion equation derives from Fick's law of diffusion, the behaviour of the sound in a room can be described with the time-dependent diffusion equation model in Equation (1), for acoustic energy density $w(\mathbf{r}, t)$ in [kg/(m s$^2$)]:

$$\frac{\partial w(\mathbf{r}, t)}{\partial t} = \nabla \cdot (D(\mathbf{r}) \nabla w(\mathbf{r}, t)) - \sigma w(\mathbf{r}, t) + q(\mathbf{r}, t), \text{ in } V \tag{1}$$

where $\sigma w(\mathbf{r})$ is a term that accounts for absorption by the room boundaries, with $\sigma$ defined by a mean room-surface absorption coefficient $\alpha$ with the equation $\sigma = \frac{c\alpha}{\lambda}$ in [1/s], $\lambda$ being the mean free path of the room in [m] and $q$ is the energy per volume of an omnidirectional sound source. $D(\mathbf{r})$ is the diffusion coefficient, a proportionality factor with the units [m$^2$/s]. The diffusion coefficient as shown in Equation 1 is dependent on the distance $\mathbf{r}$ and defined as [19]:

$$D(\mathbf{r}) = \frac{c}{3 Q_{\text{sc}}(\mathbf{r}) n_{\text{sc}}(\mathbf{r})}, \tag{2}$$

where $Q_{\text{sc}}$ is the scattering cross-section of the "sound particles" and $n_{\text{sc}}$ is the total number of particles per unit volume. This is valid considering an integration over all propagation directions [10]. For rooms with proportionate geometry, the assumption is that $D(\mathbf{r})$ is taken as a constant value depending on the geometrical properties of the space as:

$$D_{\text{th}} = \frac{\lambda c}{3}, \tag{3}$$

where $c$ is the speed of sound and $\lambda$ is the mean free path of the room defined as $\frac{1}{Q_{sc}n_{sc}}$. Commonly, in classical acoustic theory, $\lambda$ is given by the formula $4V/S$, where $V$ is the volume of the room and $S$ is the total surface area of the room. This definition of the mean free path is valid for proportionate rooms and diffuse fields [10]. Numerous researchers have further defined that the homogeneous diffusion coefficient, as per the definition in Equation 3, is only valid for proportionate rooms and that for long rooms the diffusion coefficient depends on the distance from the source, the absorption of the surfaces and the scattering of the surfaces [17–19]. Since it has been demonstrated that the diffusion coefficient is spatially dependent, the diffusion equation is to be obtained including the original $D(\mathbf{r})$. In this study, the following diffusion equation is used, considering the first derivative of $D(\mathbf{r})$ small compared to its value.

$$\frac{\partial w(\mathbf{r},t)}{\partial t} = D(\mathbf{r})\nabla w(\mathbf{r},t) - \sigma w(\mathbf{r},t) + q(\mathbf{r},t), \text{ in } V \tag{4}$$

Currently in literature, no equation has been created to define a spatially-dependent diffusion coefficient for non-proportionate rooms. Therefore, the availability of appropriate diffusion coefficient values for spaces with non-uniform dimensions remains limited.

The diffusion equation defined in Equation (4) is valid for an evenly distributed absorption inside the room. Boundary conditions are added to understand the behaviour of sound at the surface boundaries of the room for localized absorption. The sound gets reflected and partly absorbed. Therefore, the $\sigma w(r)$ term of Equation (4) is removed and an equation on the room boundaries $\partial V$ is added, as follows:

$$D(\mathbf{r})\frac{\partial w(\mathbf{r},t)}{\partial n} + cA_x w(\mathbf{r},t) = 0 \text{ in } \partial V \tag{5}$$

where $n$ is the normal to the surface and $A_x$ is the absorption factor. The absorption factor depends on the absorption coefficient of the specific surface and it has been modified over the years to allow for the use of higher absorption coefficients in the model by Picaut [12] and Jing and Xiang [14, 15]. The absorption factor defined by Jing and Xiang as the Modified Absorption Term and derived from the theory of light propagation is used for the simulations of this paper [15].

In this paper, the objective is to determine how the diffusion coefficient changes over the distance when a sound source is introduced in a long room, maintaining the absorption coefficient uniform on all the surfaces of the room.

## 3. VERIFICATION BETWEEN DIFFUSION EQUATION MODEL AND RADIOSITY MODEL

To find a spatially dependent diffusion coefficient, the proposed approach is to determine the diffusion coefficients by comparing the results obtained from the diffusion equation model with those obtained using a well-defined method that includes these dependencies, the radiosity method [2, 8]. The radiosity method is an energy-based method for the calculation of energy inside a room. The radiosity method used for this study calculates the necessary acoustical parameters in a room based on purely diffusing boundaries. The comparison with the radiosity method is appropriate since the radiosity method assumes that the sound energy reflected from a boundary is dispersed over all directions according to Lambert's law, as also the diffusion equation model assumes. The radiosity method software has been generously provided by the Technical University of Denmark (DTU) for the completion of this task and, for the properties described above, it has been considered the reference solution.

The diffusion equation model used for the computation is based on the finite difference numerical method DuFort and Frankel [20] and uses an interrupted noise source which involves activating a noise source, maintaining it until the space is saturated with noise and then abruptly switching it off, causing a decay in sound energy in the room. The script has been written in the Python programming language.

It is important to note that the radiosity method does not use an interrupted noise source implementation of the source term but it uses an impulse source which entails generating an impulse inside the room, providing the initial energy value that then decays over time. Schroeder demonstrated the equivalence between the sound level decay of an impulse response and the average of a large number of decays of an interrupted stationary noise when his backward integration method is used to extract the decay curve from the impulse response. Therefore, the two source methods are interchangeable and the energy decay curve can be calculated with the integral of the impulse response of the room, while the impulse response of the room can be calculated by differentiating the energy decay curve [21]. Since the noise source in the radiosity method is an impulse source, the Sound Pressure Level is to be calculated after backward integration as described by Schroeder.

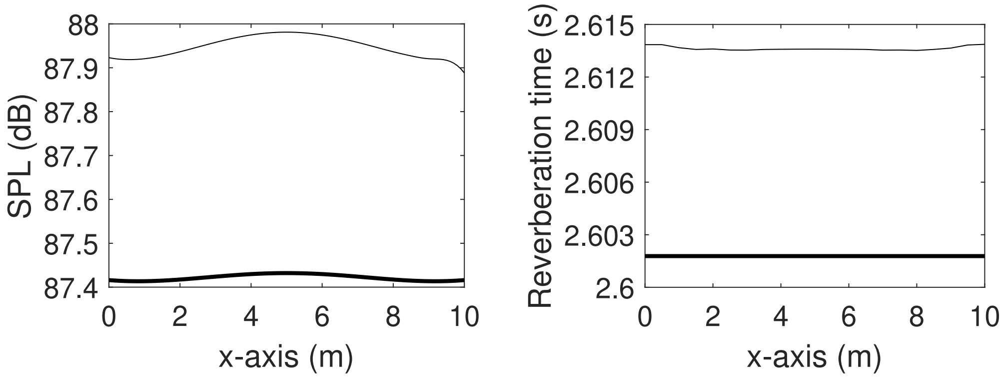
The computed acoustic parameters for the comparison are the sound pressure level (SPL) at the time step of t equal to zero (when the source gets switched off) and the reverberation time (RT). The SPL at the time t=0 is chosen because it corresponds to a stationary state sound behaviours of the room since the objective is to find the dependency of the diffusion coefficient over space (distance from the source position) rather than time. The room's longest dimension serves as the x-axis reference and it is assumed that the diffusion coefficient varies along the room's longest dimension (x-axis) while remaining constant over the other two axes.

The RT is computed from the energy decay curve at each receiver position in the x-axis line, considering the upper limit of -5 dB and a lower limit of -35 dB to extrapolate the $T_{30}$. The sound pressure level without direct field term is calculated for both the diffusion equation model and the radiosity method with the following formula:

$$\mathrm{SPL}(\mathbf{r}) = 10\log_{10}\left(\frac{\rho c^2 w(\mathbf{r})}{p_{\mathrm{ref}}^2}\right) \tag{6}$$

where $\rho$ is the density of air in [kg/m$^3$] and $p_{\mathrm{ref}}$ is equal to $2 \times 10^{-5}$ Pa. In the radiosity method calculation, the direct sound energy from source to receivers has been completely removed, since in the diffusion equation model the direct sound path is excluded intrinsically.

For geometrically proportionate shapes, the results obtained from the diffusion equation model align closely with those derived from the radiosity model. Figure 1 below shows the SPL and RT along the x-axis of a 10 x 10 x 10 m$^3$ room with a uniform absorption coefficient of 0.1. There is considerable agreement between the radiosity and the diffusion equation model, with SPL differences of up to 0.5 dB and RT differences of up to 0.012 s.




(a) (b)

Figure 1: SPL over the distance (a) and RT over the distance (b) for a 10 x 10 x 10 m$^3$ room with source position (5 m,5 m,5 m) and receiver position on the lines of y = 2.5 m and z = 2.5 m, absorption coefficient $\alpha$ = 0.1 and theoretical diffusion coefficient; —SPL radiosity method, ━SPL diffusion equation model.

For non-proportionate shapes, the results of the diffusion equation model are anticipated to be contingent upon the spatially dependent diffusion coefficients [17]. In fact, in the case of elongated rooms, differences emerge in the SPL across distance. The diffusion equation model tends to underestimate the SPL at distances far away from the source. These differences can be attributed to the assumption of a homogeneous diffusion coefficient. Figure 2 shows the SPL and RT from a distance of 3 m from the source of a 39 x 3 x 3 $m^3$ room. The distances close to the source have not been considered in the computation due to the intrinsic trait of the diffusion equation model of incorrectly computing the energy density at distances from the source position in the order of the mean free path [17]. A detailed explanation of these calculation outliers of the diffusion equation model is reported in Section 4 below.

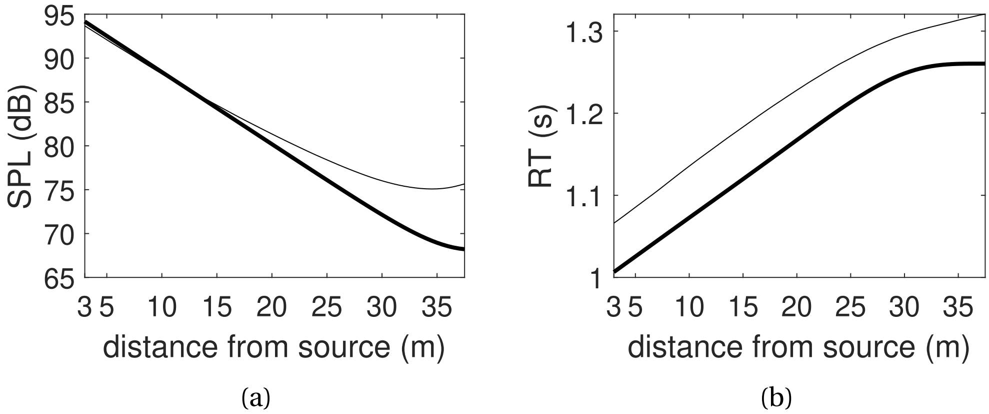



Figure 2: SPL over the distance (a) and RT over the distance (b) for a 39 x 3 x 3 $m^3$ room with source position (1.5 m,1.5 m,1.5 m) and receiver position on the lines of y = 1.5 m and z = 1.5 m, absorption coefficient $\alpha = 0.1$ and theoretical diffusion coefficient; —SPL radiosity method, ▬SPL diffusion equation model.

The findings of Figure 2 from the analysis of the 39 x 3 x 3 $m^3$ room indicate a deviation in the SPL values between the radiosity and diffusion equation models, notably observable beyond the 15-meter mark. Towards the room's extremity, a discrepancy of approximately 7 dB becomes apparent. This variation in SPL may be attributed to the assumption of a constant and uniform diffusion coefficient. It is visible that for both cases, the RT over the distance $x$ from the source calculated with both methods are similar, with a maximum difference of 0.07 s.

Since the RT calculated with both methods are quite similar to each other and do not change over the distance, the RT will not be considered a variable to be optimised. In the next sections, the objective will be to establish a non-homogeneous diffusion coefficient.

## 4. METHODOLOGY

The diffusion coefficient needs to be optimised to match the SPL results of the diffusion equation model with those from the radiosity method. The least-square optimization method is used to minimise the sum of the offsets of points from the plotted curves. The objective function of the least-square optimization is defined as follows:

$$f(D) = \frac{1}{2}\sum (\mathrm{SPL}_{\mathrm{diff}}(r_x) - \mathrm{SPL}_{\mathrm{rad}}(r_x))^2 \tag{7}$$

where $f(D)$ is the function of diffusion coefficient, $\mathrm{SPL}_{\mathrm{diff}}$ is the SPL computed with the diffusion equation model, $\mathrm{SPL}_{\mathrm{rad}}$ is the SPL computed with the radiosity method and $r_x$ is the distance between the receiver point and source $(x - x_s)$.

The diffusion coefficient function is defined using a polynomial function, as described by Munoz [19]. The polynomial function is defined as follows:

$$D_{\mathrm{opt}}(r_x) = C_2 r_x^2 + C_1 r_x + C_0 \tag{8}$$

where $r_x$ is the distance between the receiver points and the source $(x - x_s)$, $C_2$ and $C_1$ are polynomial coefficients for the determination of the curve fitting and $C_0$, the constant without any dependency on $x$, is considered equal to the theoretical value of $D_{\mathrm{th}}$ of the specific room. The function considers a spatial dependency of $D$ over the distance from the source in the x-axis, with a quadratic increase of the diffusion coefficient. The optimization algorithm depends on some initial values $C_2$ and $C_1$. The initial values of the constants $C_2$ and $C_1$ are set to zero. The constants $C_2$ and $C_1$ are all constrained to be positive.

The error has been calculated using the root-mean-square-deviation parameter (RMSD), which is the square root of the average of squared errors on the number of points optimised.

The function in Equation 8 should make sure that there are no fluctuations in the diffusion coefficient curves as found by Visentin et al. [17]. In fact, the end-wall instabilities found in this study, because of numerical errors in the computation, should be avoided using a polynomial function for the diffusion coefficient.

The procedure of the least-square optimization is suitable for the study since the objective is to minimise the error of the diffusion equation model with respect to a reference model, but also since the SPL curves of both methods are smooth and the penalizing outliers (mainly located at a close distance to the source) are not considered for this optimization. As shown in Figure 2, the distances smaller than the mean free path from the source have been removed from the optimization, since it has been demonstrated that the diffusion equation model has an inherent trait around the source position. In fact, the diffusion equation model computes the direct field around the source incorrectly because it assumes that even at small distances $r_x$ from the source, particles have already undergone reflection/scattering with other particles, while in reality, this is not true [17]. It has been discovered initially that the stationary diffusion equation solution is invalid for distances smaller than the mean free path [10].

In this study, the distances smaller than the mean free path from the source position have been discarded in the optimization process. The least-square optimization algorithm has been used for the mathematical optimization from the Scipy toolbox/package in Python [22, 23].

## 5. DIFFUSION COEFFICIENT DEPENDING ON LENGTH OF ROOM

The results are reported for long rooms with different room dimensions. Initially, a long room with dimensions 9 x 3 x 3 $m^3$ is considered, then an increase of 15 m in the length of the room is examined with a 24 x 3 x 3 $m^3$ room, and finally, a corridor of dimensions 39 x 3 x 3 $m^3$ is scrutinized. The cross-section 3 x 3 $m^2$ is maintained constant for all the room dimensions. These results have been obtained using the same source position for each room positioned at (1.5 m, 1.5 m, 1.5 m).

The optimised diffusion coefficient $D_{\mathrm{opt}}$ has been considered to change over the distance from the source on the x-axis but is kept equal on the y and z-axis.

In Figure 3, the predicted SPL over the distance $x$ of room 9 x 3 x 3 $m^3$ is shown, together with the optimized diffusion coefficient. The theoretical diffusion coefficient value of this room is 294.0 $m^2/s$.

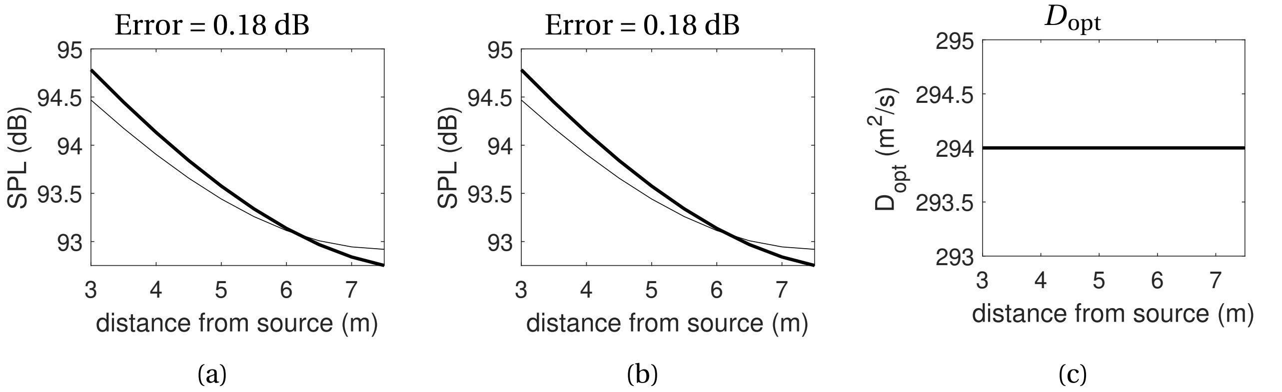



Figure 3: SPL over the distance from the source for a 9 x 3 x 3 $m^3$ room with source position (1.5 m,1.5 m,1.5 m) and receiver position on the lines of y = 1.5 m and z = 1.5 m, $\alpha$ = 0.1 before optimization (a) and after optimization (b) and optimised diffusion coefficient (c); —SPL radiosity method, ━SPL diffusion equation model.

Before optimization, the SPL computed with the diffusion equation model differs from the SPL computed with the radiosity method by a maximum of only 0.5 dB. For this specific room, the results before optimization and after the optimization do not change and the diffusion coefficient is confirmed to be constant over the room and equal to the theoretical diffusion coefficient. The RMSD error is equal before and after the optimization and is shown above the subfigures of Figure 3.

In Figure 4, the predicted SPL decay over the distance x of room 24 x 3 x 3 $m^3$ is shown. The theoretical diffusion coefficient value of this room is 322.8 $m^2$/s.

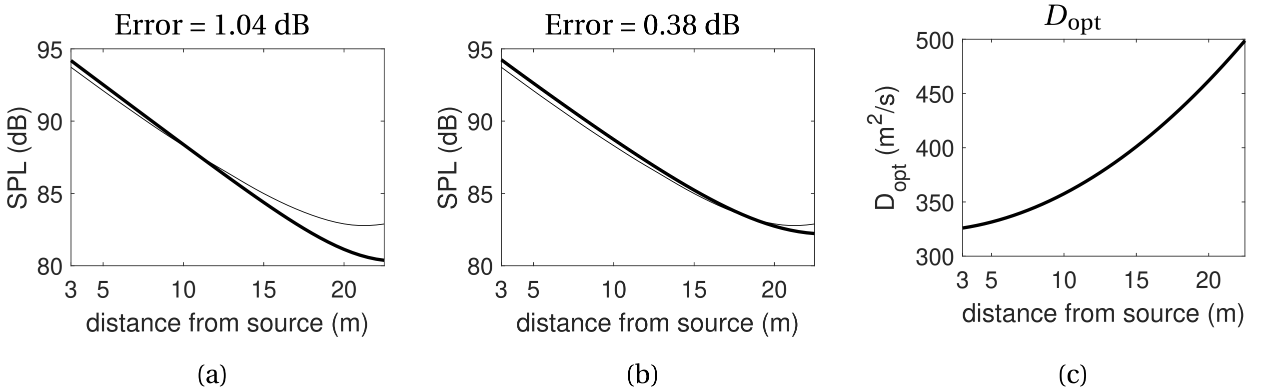



Figure 4: SPL over the distance from the source for a 24 x 3 x 3 $m^3$ room with source position (1.5 m,1.5 m,1.5 m) and receiver position on the lines of y = 1.5 m and z = 1.5 m, $\alpha$ = 0.1 before optimization (a) and after optimization (b) and optimised diffusion coefficient (c); —SPL radiosity method, ━SPL diffusion equation model.

The optimised diffusion coefficient for a 24 x 3 x 3 $m^3$ room increases quadratically with the distance from the source and gradually increases from the value of 325.95 $m^2$/s at 3 m from the source to the value of 498.70 $m^2$/s at 22.5 m from the source. Considering that 322.8 $m^2$/s is the value of the theoretical diffusion coefficient for a 24 x 3 x 3 $m^3$ room, it is concluded that a non-homogeneous diffusion governs the reverberant field. The diffusion coefficient is spatially dependent, as demonstrated by multiple researchers before [17, 18]. After the optimization of the 24 x 3 x 3 $m^3$ room, the constants $C_2$ and $C_1$ values are [0.35, 1.08e-11] and the RMSD error, shown above the subfigures of Figure 4, is considerably lower after optimization, indicating good results. The diffusion coefficient does not change linearly but parabolically, and the $C_1$ constant associated with the linear x term of the diffusion equation function can be approximated to zero. Therefore, the diffusion coefficient formula follows the parabolic equation $C_2 r_x^2 + C_0$.

If the room's length is increased to a corridor of 39 x 3 x 3 m$^3$, a diffusion coefficient increase can also be seen, as per Figure 5 below. The theoretical diffusion coefficient value of this room is 330.3 m$^2$/s.

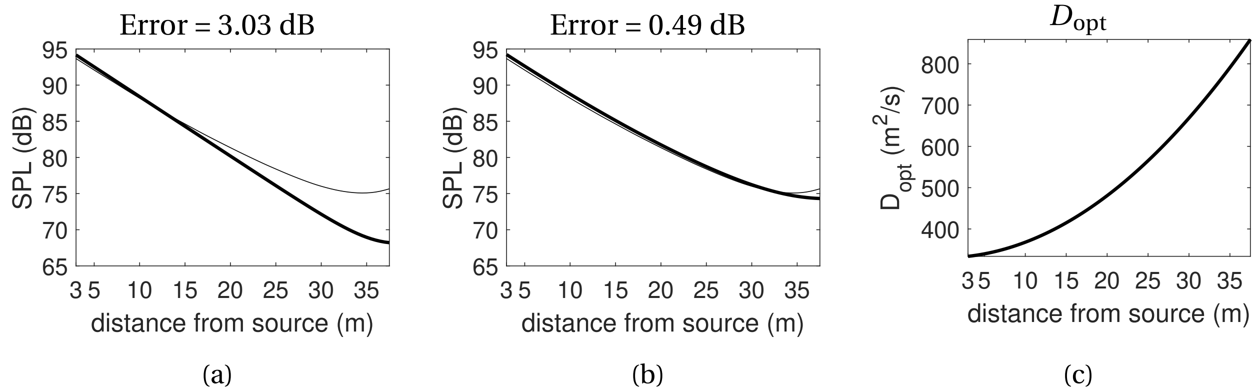



Figure 5: SPL over the distance from the source for a 39 x 3 x 3 m$^3$ room with source position (1.5 m,1.5 m,1.5 m) and receiver position on the lines of y = 1.5 m and z = 1.5 m, $\alpha$ = 0.1 before optimization (a) and after optimization (b) and optimised diffusion coefficient (c); —SPL radiosity method, ━SPL diffusion equation model.

The optimised diffusion coefficient of the 39 x 3 x 3 m$^3$ room increases parabolically from 333.68 m$^2$/s at 3 m from the source to 858.99 m$^2$/s at 37.5 m from the source. After the optimization, the constants $C_2$ and $C_1$ values are [0.38, 1.24e-14]. The RMSD error is also in this case considerably lower after optimization. The RMSD errors are shown above the subfigures of Figure 5. Again, in this case, the diffusion coefficient does not change linearly but parabolically.

The optimised diffusion coefficients of the analysed rooms above, estimated following the method explained in Section 4 can be observed in Figure 6.

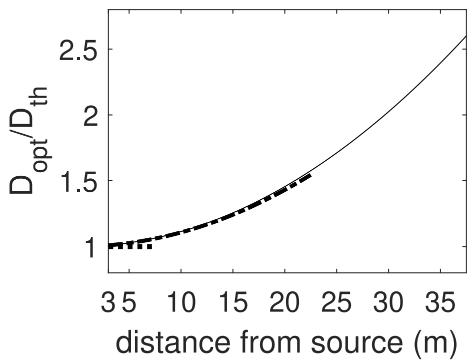



Figure 6: Ratio between the optimised diffusion coefficient ($D_{\text{opt}}$) and its theoretical value ($D_{\text{th}}$) within a long room with cross-sectional area 3 x 3 m$^2$ and absorption coefficient $\alpha$ = 0.1; ▪▪▪▪▪▪ $D_{\text{opt}}$ 9 x 3 x 3 m$^3$, ▪▪▪▪▪▪ $D_{\text{opt}}$ 24 x 3 x 3 m$^3$, —— $D_{\text{opt}}$ 39 x 3 x 3 m$^3$.

These results suggest that the diffusion coefficient increases parabolically with the distance from the source in elongated rooms.

## 6. DIFFUSION COEFFICIENT DEPENDING ON SOURCE POSITION

If for the 39 x 3 x 3 m$^3$ room, the source position is moved to position (15 m, 1.5 m, 1.5 m), the results are represented in Figure 7.

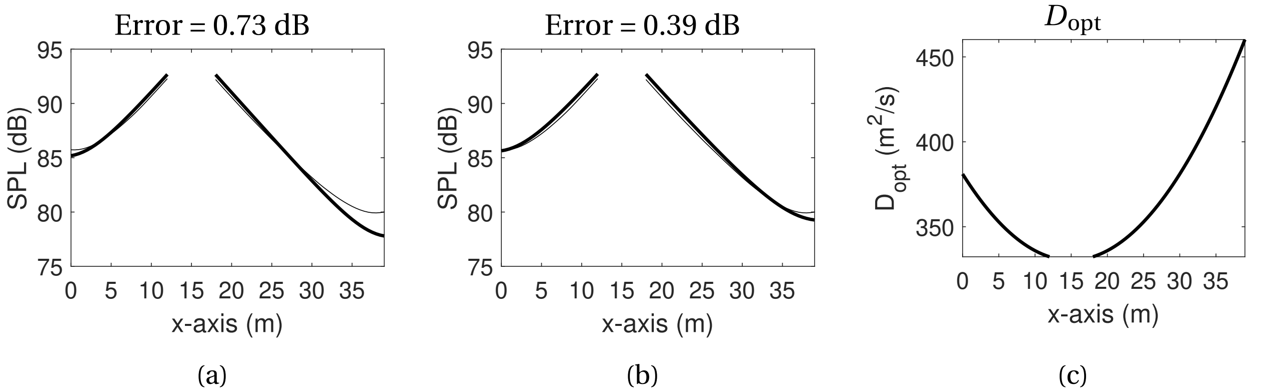



Figure 7: SPL over the distance from the source for a 39 x 3 x 3 $m^3$ room with source position (15 m,1.5 m,1.5 m) and receiver position on the lines of y = 1.5 m and z = 1.5 m, $\alpha = 0.1$ before optimization (a) and after optimization (b) and optimised diffusion coefficient (c); —SPL radiosity method, ━SPL diffusion equation model.

The optimised diffusion coefficient of a 39 x 3 x 3 $m^3$ room with source position (15 m,1.5 m,1.5 m) decreases parabolically from 381.03 $m^2$/s at 15 m from the source to 332.32 $m^2$/s at 3 m from the source and then increases back symmetrically from 332.32 $m^2$/s at 3 m from the source to 460.18 $m^2$/s at 24 m from the source. The constants $C_2$ and $C_1$ values are [0.23, 1.58e-11]. As $C_1$ is approximately equal to zero, the increase of diffusion coefficient is symmetrical with respect to the source position. The RMSD errors are shown above in the subfigures of Figure 7. The RMSD error is halved after optimization.

In Figure 8 the diffusion coefficient of a 39 x 3 x 3 $m^3$ room with a source position (1.5 m,1.5 m,1.5 m) is compared with the diffusion coefficient of the same room but with a source position (15 m,1.5 m,1.5 m).

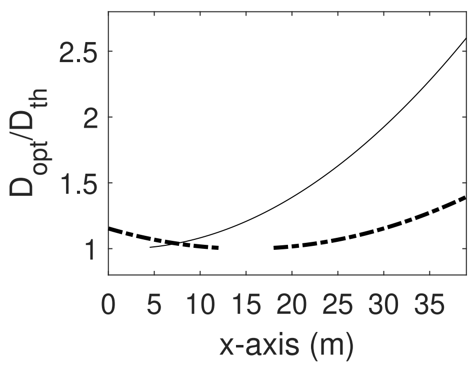



Figure 8: Ratio between the optimised diffusion coefficient ($D_{opt}$) and its theoretical value ($D_{th}$) within a long room with cross-sectional area 3 x 3 $m^2$ and absorption coefficient $\alpha = 0.1$; —— $D_{opt}$ of the 39 x 3 x 3 $m^3$ room with source position (1.5 m,1.5 m,1.5 m), ▪▪▪▪▪▪ $D_{opt}$ of the 39 x 3 x 3 $m^3$ room with source position (15 m,1.5 m,1.5 m).

The two curves of Figure 8 indicate that the diffusion coefficient depends not only on the length of the room but also on the source position. The constant $C_0$ for both cases is the same since it is assumed to be equal to the theoretical diffusion coefficient. However, the constant $C_2$ changes from 0.37 for a 39 x 3 x 3 $m^3$ room with source position (1.5 m,1.5 m,1.5 m) to 0.23 for a 39 x 3 x 3 $m^3$ room with source position (15 m,1.5 m,1.5 m). Therefore, this constant decreases with the positioning of the source more central to the room, meaning that the more the source is positioned central to the room, the smaller the constant $C_2$ is.

## 7. CONCLUSIONS

In this study, a formula for diffusion coefficients for long rooms is proposed considering the dependency on the theoretical diffusion coefficient. The comparison between the results of the diffusion equation method and the radiosity method has been reported for different room lengths and different source positions and has highlighted the inefficacy of the diffusion equation model when using the theoretical constant diffusion coefficient. Only the reverberant sound has been analysed in terms of sound pressure level. Due to the intrinsic problem of the diffusion equation model around the source position, the results within a mean free path from the source position have been removed for optimization. A mathematical optimization of the least square error function of the SPL has been used to derive the optimal diffusion coefficient for long rooms and changing source positions. The method seems to be suitable for the task as the sound pressure level obtained from the optimized diffusion coefficient has a better agreement with the reference solution especially at the end boundaries. For a long room, the diffusion coefficient increases parabolically, according to the formula $C_2 r_x^2 + C_0$, where $C_0$ is the theoretical diffusion coefficient. Currently, the simulations assume a constant diffusion coefficient over the cross-section and a low absorption coefficient $\alpha = 0.1$. The constant $C_2$ changes when the length of the room changes and when the position of the source changes.

In future works, the relation between the constant $C_2$ and the length of the room and source position will be found. In addition, the least-square function optimization method will be used for different absorption coefficients and different cross sections. To directly calculate a spatial-dependent diffusion coefficient for the specific room, the relation between the constant $C_2$ and the source position, the room's length, the room's cross-section, and the absorption coefficient will be assessed.

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