KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

SECOND SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

MATH 111: VECTOR GEOMETRY

STREAM: Y1S1 **TIME: 11.00-1.00P.M**

EXAMINATION SESSION: APRIL **DATE: 19/04/2018**

INSTRUCTIONS

Instructions to candidates:

- **QUESTION ONE** is compulsory.
- Answer **QUESTION ONE** and any other **TWO** questions

QUESTION 1 (30 Marks)

a) Distinguish between the following [2 Marks]

i. Scalar field and Vector field

ii. Free vector and base vector

b) Find the distance of the point $(-1,2,2)$ from the plane $\underline{r} \bullet \left(6\hat{i} + \hat{j} - 7\hat{k}\right) = 46$

[4 Marks]

c) Graph the vector fields defined by $\vec{V}(x, y) = x\hat{i} + y\hat{j}$ and $\vec{V}(x, y) = -x\hat{i} - y\hat{j}$

[4 Marks]

d) Find the area of a triangle whose vertices are $P(1,3,2)$, $Q(2,-1,1)$ and $R(-1,2,3)$

[3 Marks]

e) Determine a unit vector that is perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. [4 marks]

f) Find the angles which the vector $\underline{a} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ makes with the coordinate axis.

[3 Marks]

g) Show that the line $\underline{r} = 3\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(\hat{i} + \hat{j} - \hat{k})$ lies on the plane $\underline{r} \bullet (3\hat{i} + 3\hat{j} - 2\hat{k}) = 1$

[3 Marks]

h) The centroid of triangle OAB is denoted by G. If O is the origin and $\overline{OA} = 4\hat{i} + 3\hat{j}$ $\overline{OB} = 6\hat{i} - \hat{j}$ find $\overline{OG}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$ [3 Marks]

i) Given that A is the point $(1,-1,2)$, B is the point $(-1,2,2)$ and C is the point $(4,3,0)$, show that the $ABC = 69^0 14'$ [4 marks]

QUESTION 2 (20 Marks)

a) Evaluate $(2\hat{i} - 3\hat{j}) \bullet (\hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} - \hat{k})$. [3 Marks]

b) Find the resultant of the following displacements **A**, 40km 30^0 south of east; **B**, 90km 20^0 north of east; **C**, 140km due west; [4 marks]

c) Given that A is the point (1, 3) and that $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ respectively, find the coordinates of the vertices B, C and D of the parallelogram ABCD.

[3 Marks]

d) the vector equations of three lines are given below

L_1 : $\underline{r} = 17\hat{i} + 2\hat{j} - 6\hat{k} + \lambda(-9\hat{i} + 3\hat{j} + 9\hat{k})$

L_2 : $\underline{r} = 2\hat{i} - 3\hat{j} + 4\hat{k} + \mu(6\hat{i} + 7\hat{j} - \hat{k})$

L_3 : $\underline{r} = 2\hat{i} - 12\hat{j} - \hat{k} + \eta(-3\hat{i} + \hat{j} + 3\hat{k})$

State which of the lines are [6 marks]

i. Parallel
ii. Intersect
iii. Are skewed

e) Find a vector equation of the line which passes through the point $A(1,-1,0)$ and is parallel to the line $\overrightarrow{BC}$ where B and C are the points with coordinates $(-3,2,1)$ and $(2,1,0)$ hence show that the point $D(-14,2,3)$ lies on line. [4 marks]

QUESTION 3 (20 Marks)

a) Find the work done in moving an object along a vector $\underline{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$ [3 Marks]

b) The angular velocity of a rotating rigid body about an axis of rotation is given by $\underline{\omega} = 4\hat{i} + \hat{j} - 2\hat{k}$. Find the linear velocity of a point P on the body whose position vector relative to a point on the axis of rotation is $2\hat{i} - 3\hat{j} + \hat{k}$ [3 Marks]

c) Find the perpendicular distance between the point $A(4,-3,10)$ and the line L whose vector equation is $\underline{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ [3 Marks]

d) Find the acute angle between the line $\frac{x+1}{4} = y - 2 = \frac{z-3}{-1}$ and the plane $3x - 5y + 4z = 5$

[3 Marks]

e) At noon two boats P and Q are at points where position vectors are $4\hat{i} + 8\hat{j}$ and $4\hat{i} + 3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i} + \hat{j}$ and the velocity of Q is $2\hat{i} + 5\hat{j}$ where all distances are in kilometres and time is measured in hours.

i. Find the position vectors of P and Q and $\overrightarrow{PQ}$ after t hours. [4 Marks]
ii. Express the distance PQ between the boats in terms of t [2 Marks]

iii. Show that the least distance between the boats is $\sqrt{5}$ km [2 Marks]

QUESTION 4(20 Marks)

a) Determine the angles α β and γ which the vector makes with the positive directions of the coordinate axes and show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [4 Marks]

b) A scalar field is defined by $\Phi(x, y, z) = 4yz^3 + 3xyz - z^2 + 2$ find Φ at the points $\Phi(1,-2,-2)$ and $\Phi(0,-3,1)$ [3 Marks]

c) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find the angle between the two vectors $\underline{a}$ and $\underline{b}$ [3 Marks]

d) Three points *A, B* and *C* are given as $(1,1,1)$, $(5,0,0)$ and $(3,2,1)$ respectively find the equation of the plane *ABC*. [5 Marks]

e) Find the point of intersection of the lines $\frac{x-3}{-1} = \frac{y-2}{2} = \frac{z+3}{4}$ and $3x - y + 2z = 8$

[5 Marks]

QUESTION 5(20 Marks)

a) Find the vector equation and the corresponding Cartesian equation of the plane that passes through the points $(2,2,0)$ $(0,2,2)$ and $(2,0,2)$ and hence sketch it [4 Marks]

b) Points $A,\ B\ and\ C$ have the position vector $\hat{i} + 2\hat{j} - 3\hat{k}$, $\hat{i} + 5\hat{j}$ and $5\hat{i} + 6\hat{j} - \hat{k}$ respectively, relative to an origin

i. Show that $\vec{AB}$ is perpendicular to $\vec{BC}$ and find the area of the triangle ABC

[3 Marks]

ii. Find the vector product $\vec{AB} \times \vec{BC}$. [3 Marks]

iii. The point D has position vector. Show that the volume of the tetrahedron ABCD is equal to 21 [3 Marks]

c) Find an equation for the plane perpendicular to the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 16\hat{k}$ and passing through the terminal point of the vector $\vec{B} = \hat{i} + 5\hat{j} + 13\hat{k}$. Hence find the distance from the origin to the plane. [4 marks]

d) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 2\hat{i} - \hat{j} + 2\hat{k}$ [3 Marks]

KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

FIRST SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND MATHEMATICS, BACHELOR IN EDUCATION (SCIENCE AND ARTS) BACHELOR OF SCIENCE IN COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN TELECOMMUNICATION

MATH 111: VECTOR GEOMETRY

STREAM: [YEAR 1] **TIME: 11.00-1.00 P.M**

EXAMINATION SESSION: -DECEMBER **DATE: 14/12/2017**

INSTRUCTIONS:

1.Question ONE is compulsory.

2. Attempt question ONE and any other TWO

Question One [30 Marks]

a) Show that addition of vectors is associative. [4 Marks]

b) Distinguish between a scalar triple product and a vector triple product [2 Marks]

c) Given that $\underline{a} = 2i - \hat{j} + \hat{k}$, $\underline{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\underline{c} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ find

i) $(\underline{a} \bullet \underline{b})\underline{c}$ [2 Marks]

ii) $\underline{a} \bullet (\underline{b} \times \underline{c})$ [2 Marks]

iii) $\underline{a} \times (\underline{b} \times \underline{c})$ [2 Marks]

d) Find the magnitude and direction of the displacement vector $\overrightarrow{PQ}$ where P and Q are points $(9,7)$ and $(12,4)$ respectively. [3 Marks]

e) Given the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ find the direction cosines and hence the angles it makes with coordinate axis. [5 marks]

f) Points L, M, N are the mid-points of the sides AB, BC, CA of the triangle ABC.

Show that $2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{LC}$ [4 Marks]

g) Find the vector equation of the straight line that passes through the points $A(1,0,1)$ and $B(0,1,3)$. [2 Marks]

h) Given that $\underset{\sim}{a} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\underset{\sim}{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, find the projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. [4 Marks]

Question Two [20 Marks]

a) Find the area of a triangle having vertices at $P(1,3,2)$ $Q(2,-1,1)$ $R(-1,2,3)$

[4 marks]

b) Find a vector equation of the line which passes through the point $A(1,-1,0)$ and is parallel to the line $\vec{BC}$ where B and C are the points with the coordinates $(-3, 2, 1)$ and $(2,1,0)$. Hence show that the point $D(-14,2,3)$ lies on the line. [6 marks]

c) The vector equations of two lines are given below [6 marks]

$L: \quad \underset{\sim}{r} = 2\hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j})$

$M: \quad \underset{\sim}{r} = \hat{i} + \hat{j} - 2\hat{k} + \mu(\hat{j} - \hat{k})$

i) Show that the two lines intersect and their point of intersection

ii) Determine the Cartesian equation of the plane π containing L and M

[4 Marks]

Question Three [20 Marks]

a) Find the cosine of the acute angle between the two planes $\underline{r} \bullet \left(\hat{i} - \hat{j} + 5\hat{k}\right) = 2$ and $\underline{r} \bullet \left(3\hat{i} + 2\hat{j} - \hat{k}\right) = 5$ [4 marks]

b) Find the distance of the point $(4, 2, 3)$ from the plane $\underline{r} \bullet \left(6\hat{i} + 2\hat{j} - 9\hat{k}\right) = 46$ [4 Marks]

c) Show that the two planes $\underline{r} \bullet \left(\hat{i} + \hat{j} + \hat{k}\right) = 2$ and $\underline{r} \bullet \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3$ intersect in a line hence or otherwise find the cartesian equation of this line. [5 Marks]

d) Find the vector equation of a plane which passes through the point $(0, 1, 1)$ and has normal vector $\underline{n} = \hat{i} + \hat{j} + \hat{k}$. Hence

   i. Find its Cartesian equation

   ii. Show that the points $(1, 0, 1)$ and $(1, 1, 0)$ lie on the plane

   iii. Sketch the plane [7 marks]

Question Four [20 Marks]

a) Show that $\underline{a} \times (\underline{b} \times \underline{c}) \neq (\underline{a} \times \underline{b}) \times \underline{c}$ given that $\underline{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\underline{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\underline{c} = \hat{i} - 2\hat{j} + 2\hat{k}$ [5 Marks]

b) Find the resultant of the following displacements **A**, 10km north west; **B**, 20km 30^0 north of east; **C**, 35km due south. [4 marks]

c) Find the point of intersection of the lines $\frac{x-3}{-1} = \frac{y-2}{2} = \frac{z+3}{4}$ and $3x - y + 2z = 8$ [5 Marks]

d) Given that $\underline{v} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\underline{u} = 4\hat{i} - 3\hat{k}$ find

   i. the component of $\underline{v}$ in the direction of $\underline{u}$, [3 marks]

   ii. the projection of $\underline{v}$ in the direction of $\underline{u}$, [1 mark]

   iii. the resolution of $\underline{v}$ into components parallel and perpendicular to $\underline{u}$ [2 marks]

Question Five [20 Marks]

a) Given the scalar field defined by $\Phi(x, y, z) = 4yz^3 + 3xyz - z^2 + 2$ find Φ at the points $\Phi(1, -2, -2)$ and $\Phi(0, -3, 1)$ [4 Marks]

b) Find an equation for the plane perpendicular to the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and passing through the terminal point of the vector $\vec{B} = \hat{i} + 5\hat{j} + 3\hat{k}$. Hence find the distance from the origin to the plane. [6 marks]

c) Given that A is the point $(1, -1, 2)$, B is the point $(-1, 2, 2)$ and C is the point $(4, 3, 0)$,

   i. find the direction cosine of $\overrightarrow{BA}$ and $\overrightarrow{BC}$ [4 marks]

   ii. show that the $ABC = 69^0 14'$ [2 marks]

d) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 2\hat{i} - \hat{j} + 2\hat{k}$ [4 Marks]

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UNIVERSITY EXAMINATIONS

MAIN CAMPUS

SECOND SEMESTER, 2017 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

MATH 111: VECTOR GEOMETRY

STREAM: YEAR ONE SEMESTER ONE **TIME: 2 Hours**

EXAMINATION SESSION: JULY **DATE:** 2017

INSTRUCTIONS

Instructions to candidates:

- **QUESTION ONE** is compulsory.
- Attempt **QUESTION ONE** and any other **TWO** questions

Question One [30 Marks]

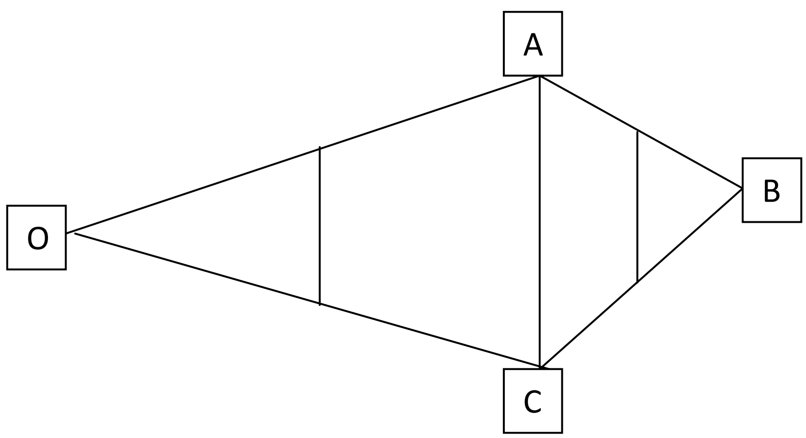
a) Find the magnitude and direction of the displacement vector $\overrightarrow{AB}$ where A and B are the points $(5,\ 7)$ and $(3,\ -4)$ respectively. [3 Marks]

b) Given that A is the point (1, 3) and that $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ respectively, find the coordinates of the vertices B, C and D of the parallelogram ABCD. [3 Marks]

c) In the figure below OABC is a quadrilateral and P, Q, R and S are the mid points of the sides OA, AB, BC and CO respectively. Given that $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, $\overrightarrow{OC} = \underline{c}$ express the following vectors in terms of $\underline{a}$ $\underline{b}$ and $\underline{c}$

i. $\overrightarrow{PS}$ [1 Mark]

ii. $\overrightarrow{QR}$ [2 Marks]



d) Find the centroid of the triangle whose vertices are A(1, 2), B(3, 7) and C(2, 3) [3 Marks]

e) Given that points A(1, 1), B(5, 4), C(8, 9) and D(0, 3). Show that ABCD is a trapezium. [4Marks]

f) Show that $\underline{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ is perpendicular to $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$ [2 Marks]

g) Find the vector product of $\underline{p}$ and $\underline{q}$ where $\underline{p} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\underline{q} = 2\hat{i} + 5\hat{j} - \hat{k}$ [3 Marks]

h) At noon two boats P and Q are at points where position vectors are $4\hat{i} + 8\hat{j}$ and $4\hat{i} + 3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i} + \hat{j}$ and the velocity of Q is $2\hat{i} + 5\hat{j}$ where all distances are in kilometres and time measured in hours.

i. Find the position vectors of P and Q and $\overrightarrow{PQ}$ after t hours. [4 Marks]

ii. Express the distance PQ between the boats in terms of t [2 Marks]

iii. Show that the least distance between the boats is $\sqrt{5}$ km [3 Marks]

Question Two [20 Marks]

b) Show that addition of vectors is associative. [4 marks]

c) Find a vector equation of the line which passes through the point $A(1,-1,0)$ and is parallel to the line $\overrightarrow{BC}$ where B and C are the points with the coordinates $(-3, 2, 1)$ and $(2,1,0)$. Hence show that the point $D(-14,2,3)$ lies on the line. [6 marks]

d) ABCD is a quadrilateral in which P and Q are the mid points of the diagonals AC and BD respectively. Show that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{PQ}$ [4 Marks]

e) the vector equations of three lines are given below

$L: \quad \underline{r} = (1+t)\hat{i} + (1-t)\hat{j} - 2\hat{k}$

$M: \quad \underline{r} = (3-\mu)\hat{i} - (1-\mu)\hat{j} - (\mu-2)\hat{k}$

$N: \quad \underline{r} = (1+s)\hat{i} - (1+3s)\hat{j} - s\hat{k}$

State which of the lines are [6 marks]

i) Parallel

ii) Intersect

iii) Are skewed

Question Three [20 Marks]

a) Find both the Cartesian and vector forms of the equation of the line of intersection of the two planes $7x - 4y + 3z = -3$ and $4x + 2y + z = 4$ [4 marks]

b) Find the distance of the point $(4,\ 2,\ 3)$ from the plane $\underline{r} \bullet \left(6\hat{i} + 2\hat{j} - 9\hat{k}\right) = 46$

[4 Marks]

c) Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the coordinate axis.

[4 Marks]

d) Find the resultant of the following displacements **A**, 10m northwest; **B**, 20m 30^0 north of east; **C**, 35m due south. [4 marks]

e) A stationery observer O observes a ship S at noon at a point whose coordinates relative to O are $(20,15)$; units are in kilometers. The ship is moving at a steady speed of 10 km/h on a bearing 150^0.

i) Express its velocity as a column vector. [2 Marks]

ii) Write down in terms of *t*, its position after *t* hours [2 Marks]

Question Four [20 Marks]

a) Show that $\underline{a} \times \underline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ given that $\underline{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\underline{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

[4 Marks]

b) Given that $\underline{a} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ and $\underline{b} = 8\hat{i} - 6\hat{j}$ find

i) $\underline{a} \bullet \underline{b}$ [2 Marks]

ii) The angle between the two vectors $\underline{a}$ and $\underline{b}$ [3 Marks]

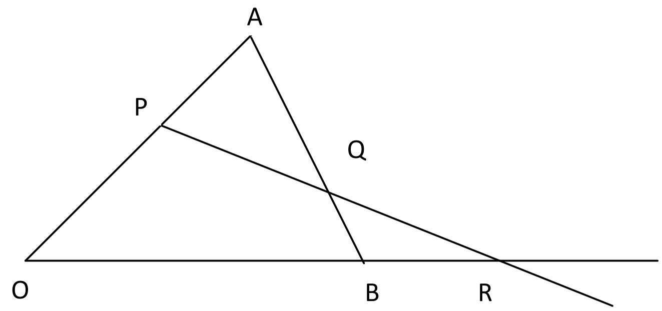
c) Given that *A, B* and *C* are the points $(1,1,1)$, $(5,0,0)$ and $(3,2,1)$ respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point *P* in the plane *ABC*. [6 Marks]

d) Find the point of intersection of the lines $\dfrac{x-3}{-1} = \dfrac{y-2}{2} = \dfrac{z+3}{4}$ and $3x - y + 2z = 8$

[5 Marks]

Question Five [20 Marks]

a) Given the triangle below prove Menelaus theorem which states that if $\overrightarrow{OP} = \alpha\overrightarrow{PA}$, $\overrightarrow{AQ} = \beta\overrightarrow{QB}$ and $\overrightarrow{BR} = \gamma\overrightarrow{RO}$ then $\alpha\beta\gamma = -1$ [4 marks]



b) Points $A,\ B\ and\ C$ have the position vector $\hat{i} + 2\hat{j} - 3\hat{k}$, $\hat{i} + 5\hat{j}$ and $5\hat{i} + 6\hat{j} - \hat{k}$ respectively, relative to an origin. Show that $\vec{AB}$ is perpendicular to $\vec{BC}$ and find the area of the triangle ABC [4 Marks]

c) Given the scalar field defined by $\Phi(x, y, z) = 4yz^3 + 3xyz - z^2 + 2$ find Φ at the points $\Phi(1, -2, -2)$ and $\Phi(0, -3, 1)$ [2 Marks]

d) Given that A is the point $(1,-1,2)$, B is the point $(-1,2,2)$ and C is the point $(4,3,0)$,

i. find the direction cosine of $\overrightarrow{BA}$ and $\overrightarrow{BC}$

ii. show that the $ABC = 69^0 14'$ [6 marks]

e) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 2\hat{i} - \hat{j} + 2\hat{k}$ [4 Marks]

KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN CAMPUS

FIRST SEMESTER, 2018/2019 ACADEMIC YEAR

FOR THE DEGREE OF (BSC) IN COMPUTER

MATH 111 VECTOR GEOMETRY

STREAM: **TIME: 11:00-1:00PM**

EXAMINATION SESSION: DEC **DATE 9:00-11:00AM**

VENUE:LI **COPIES:70**

INSTRUCTIONS:

1.Question ONE is compulsory.

2. Attempt question ONE and any other TWO

Question One [30 Marks]

a) Determine the angle between the following: $a = (9, -2)$ $b = (4, 1, 8)$ (4mks)

b) Find the equation of the line of intersection of the two planes -4x + y + z = -2 and 3x – y + 2z = -1. (4mks)

c) Find the magnitude and direction of the displacement vector $\overrightarrow{AB}$ where A and B are points $(-9,7)$ and $(12,-5)$ respectively. [4 Marks]

d) Find the area of a triangle whose vertices are $P(1,3,2)\ Q(2,-1,1)\ R(-1,2,3)$

[4 Marks]

e) Determine a unit vector that is perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. [4 marks]

f) Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the coordinate axis.

[4 Marks]

g) f). Given that $z = -5 + 5i$

a. Determine;

h) i). $|z|$ (ii) Arg z (iii) z in polar form (6marks)

g). Determine the value of a such that the vectors $\underline{p} = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$ are orthogonal. [3 Marks]

Question Two [20 Marks]

a) Find an equation of the plane containing the two lines

$$x - 1 = \frac{1-y}{2} = z - 2 \text{ and } \frac{x+1}{3} = \frac{2-y}{3} = \frac{z+2}{5}$$ [4 marks]

b) Find the angle between the vectors v = i+j+2k and w = 2i+3k+k. Give your answer in radians [4 marks]

c) Show that addition of vectors is associative. [4 marks]

d) Evaluate $(2\hat{i} - 3\hat{j}) \bullet (\hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} - \hat{k})$. [4 Marks]

e) Find the projection of v onto u if v = $\langle -3, 4 \rangle$ and u $\langle -5, 7 \rangle$. [4 marks]

Question Three [20 Marks]

a) Determine the value of $z = \frac{7}{3-i} - \frac{1-2i}{3+4i}$ (4mks)

b) Prove $|Z1 + Z_2| \leq |Z_1| + |Z_2|$ (4mks)

c) Show that $1 + i$ is a solution for the equation $x^2 - 2x + 2 = 0$. (4mks)

e). Simplify each of the following expressions: $\left(\dfrac{1+i}{1-i}\right)^2$ (4mks)

f). Show that $\frac{1}{\sqrt{2}}(1 + i) = \sqrt{i}$ (4mks)

Question Four [20 Marks]

a) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 2\hat{i} - \hat{j} + 2\hat{k}$ [4 Marks]

b). Find the perpendicular distance between the point $A(4, -3, 10)$ and the line L whose vector equation is $\underline{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ [6 Marks]

c). Find the Cartesian form of the equation of the line of intersection of the two planes $7x - 4y + 3z = -3$ and $4x + 2y + z = 4$ [4 marks]

d). Find the acute angle between the line $\dfrac{x+1}{4} = y - 2 = \dfrac{z-3}{-1}$ and the plane $3x - 5y + 4z = 5$ [6 Marks

Question Five [20 Marks]

a) Given that *A, B* and *C* are the points $(1,1,1)$, $(5,0,0)$ and $(3,2,1)$ respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point *P* in the plane *ABC*. [6 Marks]

b) Find the point of intersection of the lines $\frac{x-3}{-1} = \frac{y-2}{2} = \frac{z+3}{4}$ and $3x - y + 2z = 8$

[6 Marks]

c). Points A, B *and* C have the position vector $\hat{i} + 2\hat{j} - 3\hat{k}$, $\hat{i} + 5\hat{j}$ and $5\hat{i} + 6\hat{j} - \hat{k}$ respectively, relative to an origin

i. Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$ and find the area of the triangle ABC

[4 Marks]

ii. Find the vector product $\overrightarrow{AB} \times \overrightarrow{BC}$. [4 Marks]

KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER, 2018/2019 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BSC IN COMPUTER SCIENCE

MATH 111: VECTOR GEOMETRY

STREAM: [Y1S1] **TIME: 9.00-11.00 AM**

EXAMINATION SESSION: JAN-APRIL **DATE: 3/04/2019**

INSTRUCTIONS:

1.Question ONE is compulsory.

2. Attempt question ONE and any other TWO

Question One [30 Marks]

(a) Determine the angle between the following pairs

(i) $a = (9,-2)$ $b = (4,1,8)$

(ii) $u = (3,-1,6) V = (4,2,0)$ [6mks]

a) Find the magnitude and direction of the displacement vector $\overrightarrow{AB}$ where A and B are points $(-9,7)$ and $(12,-5)$ respectively. [3 Marks]

b) Points L, M, N are the mid-points of the sides AB, BC, CA of the triangle ABC. Show that $2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{LC}$ [4 Marks]

c) Find the area of a triangle whose vertices are $P(1,3,2)\ Q(2,-1,1)\ R(-1,2,3)$

[3 Marks]

d) Determine a unit vector that is perpendicular to the plane of $\vec{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$. [4 marks]

e) Find the angles which the vector $\vec{A}=3\hat{i}-6\hat{j}+2\hat{k}$ makes with the coordinate axis.

[4 Marks]

f). Given that $z=-5+5i$

Determine;

i). $|z|$ (ii) Arg z (iii) z in polar form [3marks]

g). Determine the value of a such that the vectors $\underline{p}=2\hat{i}+a\hat{j}+4\hat{k}$ and $q=5\hat{i}+2\hat{j}-4\hat{k}$ are orthogonal. [3 Marks]

Question Two [20 Marks]

a) Show that addition of vectors is associative. [3 marks]

b) Evaluate $(2\hat{i}-3\hat{j})\bullet\left(\hat{i}+\hat{j}-\hat{k}\right)\times\left(3\hat{i}-\hat{k}\right)$. [3 Marks]

c) Find the projection of v onto u if v = $\langle -3, 4\rangle$ and u $\langle -5, 7\rangle$. [4 marks]

d) Find the Cartesian form of the equation of the line of intersection of the two planes $7x-4y+3z=-3$ and $4x+2y+z=4$ [3 marks]

e) At noon two boats P and Q are at points where position vectors are $4\hat{i}+8\hat{j}$ and $4\hat{i}+3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i}+\hat{j}$ and the velocity of Q is $2\hat{i}+5\hat{j}$ where all distances are in kilometers and time measured in hours.

i. Find the position vectors of P and Q and $\overrightarrow{PQ}$ after *t* hours. [3 Marks]

ii. Express the distance PQ between the boats in terms of *t* [2 Marks]

iii. Show that the least distance between the boats is $\sqrt{5}$ km [2 Marks]

Question Three [20 Marks]

a). Determine the value of $z = \dfrac{7}{3-i} - \dfrac{1-2i}{3+4i}$ [4mks]

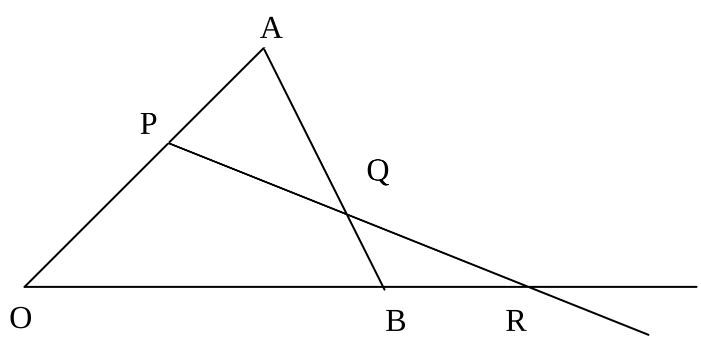
b). Find the perpendicular distance between the point $A(4,-3,10)$ and the line L whose vector equation is $\underline{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$ [6 Marks]

c) Show that $Z_1 + Z_2 = Z_1 + Z_2$ [4mks]

d) Prove $|Z1 + Z_2| \leq |Z_1| + |Z_2|$ [4mks]

Question Four [20 Marks]

a) Given the triangle below prove Menelaus theorem which states that if $\overrightarrow{OP} = \alpha\overrightarrow{PA}$, $\overrightarrow{AQ} = \beta\overrightarrow{QB}$ and $\overrightarrow{BR} = \gamma\overrightarrow{RO}$ then $\alpha\beta\gamma = -1$ [6 marks]



b) Find the acute angle between the line $\dfrac{x+1}{4} = y-2 = \dfrac{z-3}{-1}$ and the plane

$3x-5y+4z=5$ [3 Marks]

c) Given that A, B and C are the points $(1,1,1)$, $(5,0,0)$ and $(3,2,1)$ respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point P in the plane ABC. [6 Marks]

d) Find the point of intersection of the lines $\dfrac{x-3}{-1} = \dfrac{y-2}{2} = \dfrac{z+3}{4}$ and $3x-y+2z=8$

[5 Marks]

Question Five [20 Marks]

a) Points $A,\ B\ and\ C$ have the position vector $\hat{i}+2\hat{j}-3\hat{k}$, $\hat{i}+5\hat{j}$ and $5\hat{i}+6\hat{j}-\hat{k}$ respectively, relative to an origin

i. Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$ and find the area of the triangle ABC

[4 Marks]

ii. Find the vector product $\overrightarrow{AB} \times \overrightarrow{BC}$. [3 Marks]

iii. Find the equation of the plane ABC in the form $\underset{\sim}{r} \bullet \underset{\sim}{n} = p$ [3 marks]

iv. The point D has position vector $4\hat{i}-\hat{j}+3\hat{k}$ find the distance of the point D from the plane ABC [3 marks]

v. Show that the volume of the tetrahedron $ABCD$ is equal to 21 [3 Marks]

b) Find the volume of a parallelepiped whose edges are represented by $\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$

$\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=2\hat{i}-\hat{j}+2\hat{k}$ [4 Marks]