Phys 110

1.0 MAGNETISM

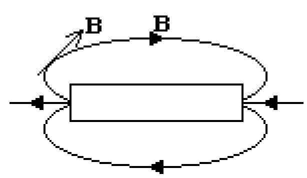
1.10 Magnetic Properties of Materials and Their Uses

We begin the discussion of magnetic materials by defining the different terms used. It is necessary to investigate the magnetic properties of various materials because it leads us to decide whether they are suitable for permanent magnets such as are used in loudspeakers or temporary magnets as are used in transformers (as cores). The magnetic properties of materials are attributed to the motion of electrons inside atoms.

1.11 Flux Density in the Magnetic material

The magnetic field intensity B

We define the space around a magnet or a current carrying conductor as the site of a magnetic field. The magnetic field is represented by the field vector **H** characterizing the force due to this field. Its direction at a point is the direction of the force experienced by a north seeking pole at a point. The other basic magnetic field vector **B**, is called the



magnetic field induction or the ***magnetic flux density***. It is represented by *lines of induction*. The tangent to a line of induction at any point gives the direction of **B** at that point.

The lines of induction are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of **B**. Where the lines are close together **B** is large and where they are far apart then **B** is small.
In order to define **B** and its units consider a positive charge q_o moving with velocity **v** through a point and it experiences a force **F** due to the field. Then we say that the magnetic field is present at the point, and **B** is the vector whose magnitude is given by the relation

$$F = q_o vB\sin\theta$$

Where **F** is in a direction perpendicular to both **v** and **B** and θ is the angle between **v** and **B**. The units for **B** from the above relation is thus Newton/(metre/second) which is referred to as *tesla*, T. **B** may also be defined from this relation as the force per unit current length and at right angles to the magnetic field.

Note: The product qv is a current so any assembly of moving charges or current will experience a sideways force when in a magnetic field

In general B is proportional to H and the constant of proportionality μ (called the permeability) depends on the medium of the space where **H** is present

Thus $\mathbf{B} = \mu\mathbf{H}$

In a vacuum $\mu = \mu_o$ and is called the *permeability of free space*. The ratio of μ to μ_o is called the *relative permeability* and is thus the ratio of the value of **B** when there is material medium to the value when there is only vacuum i. e.

$$\mu_r = \frac{\mu}{\mu_o} = \frac{B}{B_o}$$

Q: What is the relative permeability of vacuum

What are the units of μ_r

Magnetic flux φ

The product of intensity **B** normal to an area and the cross-sectional area A through which **B** passes is called the flux. Flux is thus the number of field lines crossing an area A and is usually denoted by φ and its SI unit is the *Weber* (Wb). Thus we can write

$$\varphi = Ba\cos\theta$$

where θ is the angle between the lines and the area.

Magnetic moment

The orientation of a magnet is specified by means of a vector **μ**(called the ***magnetic moment***) along the axis of the magnet pointing in the direction from the south seeking end toward the North seeking pole. The orientation of the magnetic moment due to a coil is specified by the vector **μ** lying along the axis of the coil and pointing in the direction related to the current by the right hand rule. The magnetic moment of a small plane coil is a vector whose magnitude is the product of the number of turns, N, the current in each turn, I, and the area A of the circuit. The direction of the vector is perpendicular to the plane of the coil in the same sense given by the right hand rule, that is

$$\boldsymbol{\mu} = NAI$$

And is measured in Am^2

A magnet in a uniform magnetic field experiences a couple which gives it an angular acceleration and provided there is damping ultimately comes to rest with its axis parallel to the field and so the magnetic moment vector and the field vector align. The direction of the magnetic intensity at a point is the direction into which the magnetic moment vector of either a small plane coil or a small magnet tends to turn when the small coil or magnet is placed at that point in space.

1.12: Magnetization

In materials the electrons moving round the nucleus constitute current loops and so the atoms may have resultant magnetic moments. This may happen in the presence or absence of a magnetic field. The physical quantity used to describe the magnetic state of a material is the magnetic moment per unit volume. This is called the magnetization **M** i. e.

$$\mathbf{M} = \frac{\text{Total magnetic dipole moment}}{\text{Volume}}$$

Q: What are the units of M

Let us now evaluate the expressions for **B** and **M** and their relationship for a point in a material placed in a magnetic field. Consider a toroid of length L, as shown:

EMBED PBrush

Let the total number of turns = N, mean radius = r, and circumference = L. The total flux density B depends on the following factors:

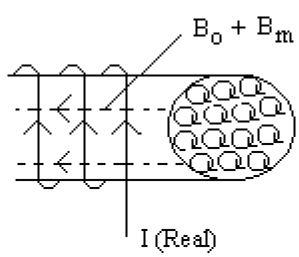
(i) the current flowing in the wire
(ii) the magnetization of the material.

Thus we have

$$B = B_o + B_m$$

where B_o is the flux density due to the current I in the wire, and B_m is the flux density due to the magnetization of the material. Usually $B_o >> B_m$.

Magnetization is related to the average magnet dipole moment for many molecules. The magnetization flux density B_m is produced by many small circulating currents inside the magnetic material, due to the circulating and spinning electrons in the atoms. This is shown in the diagram below:



In the same way, the small circulating currents inside the magnetic materials add up to a single current I_m flowing in the coil wound round the core. This current is called Surface or **Magnetization current**, which adds up to the actual current *I* flowing in the coil. The actual current produces a flux density B_o.
Let *n* be the turns per unit length ($n = N/L$), where $L = 2\pi r$. Then

EMBED Equation.3

Surface current I_m produces a flux density B_m, which is given by the equation below

EMBED Equation.3

Therefore, the total flux density B is given by

EMBED Equation.3

1.13 Intensity of Magnetization

The magnetic moment of each current turn due to the surface current I_m is given as

EMBED Equation.3

where A is the cross-sectional area of each turn. For the whole toroid, the magnetic moment is given by

EMBED Equation.3

Magnetic moment per unit volume, i.e. intensity of magnetization *M*, is then

EMBED Equation.3

Similarly, the magnetic field density or magnetic intensity due to the actual(applied) current *I* is given by

$B = \mu_o nI$

Hence the total flux density *B* in the material is given by

EMBED Equation.3

Q: 1. What are the units for H

2. What are the units for μ. Also find out the value for μ_o

1.14 Relative Permeability and magnetic susceptibility

From the foregoing discussion

EMBED Equation.3

And μ_r =

The ratio *M/H* is called the *magnetic susceptibility*, χ_M, of the material. Hence

` $\mu_r = 1 + \chi_M$

Thus we see that **M** is also proportional to **H** and the constant of proportionality is the magnetic susceptibility i. e.

$$\mathbf{M} = \chi_M \mathbf{H}$$

When a magnetic material, originally unmagnetized, is subjected to an increasing field, the intensity of magnetization M increases until it reaches a maximum value, as shown below.

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Then the material is saturated, i.e. it's magnetic domains are completely aligned with the field H. But B can still continue to increase with H.

1.15 Types of Magnetic Materials

Various materials respond differently to changes in the field H. The magnetic materials are put into three main categories of magnetic behaviour, mainly;

(i) Diamagnetism
(ii) Paramagnetism
(iii) Ferromagnetism

(i) **Diamagnets**

These are materials whose induced current gives rise to a magnetic field which opposes the applied magnetic field H. Thus magnetization, **M** will be in the opposite direction to **H** and therefore H/M is negative, and hence susceptibility χ is negative. Example is Bismuth with $\chi = -0.000015$.

(ii) **Paramagnets**

In these materials that atoms are always in thermal motion which causes the magnetic moments to be oriented purely at random and therefore no resultant magnetization. But if a field is applied, each atomic moment will try to align in the direction of the field even though thermal motion will prevent complete alignment. In this case there will be weak magnetization in the direction of the applied field. Hence susceptibility χ of a paramagnet substance is very small and positive. Example is Platinum with $\chi = +0.0001$.

(iii) **Ferromagnets**

In a ferromagnetic material the magnetization due to orbital electrons is in the same direction as the applied field. Thus a ferromagnetic material aligns itself with an applied field. Hence it has high positive susceptibilityχ. Ferromagnetic materials are further grouped into two:

(a) Soft Magnetic Materials

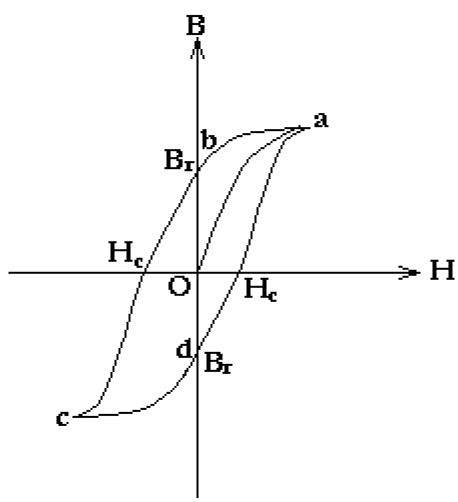
These magnetise easily and demagnetise when the applied field is removed. Example is Iron used in transformers.

(b) Hard Magnetic Materials

These are hard to magnetise but retain their magnetism when the applied field is removed. Example is Steel used in making permanent magnets.

1.16 Hysteresis Loop

The variation of *B* with applied field *H* when magnetic material specimen is taken through a complete circle is as shown below



oa $\rightarrow$ the field *B* increases with increase in applied field *H*.
ab $\rightarrow$ the applied field *H* is reduced to zero and the field *B* follows the path ab.
bc $\rightarrow$ the applied field *H* is increased in the opposite direction.
The same process is done with cd and da.

→ The point ‘a’ is the saturation point for the material. When the applied field H is reduced to zero, there is still some magnetic field remaining in the specimen as indicated at b. This is because the specimen is strongly magnetised, setting up a flux density B_r. This flux density is called the remanance; it is due to the tendency of groups of molecules, or domains to stay put once they have been aligned. At the point b, the reverse applied H is applied to the specimen. Each increment of applied field causes a decrease in flux density. Eventually, the flux density is reduced to zero, when the opposing field H has a value H_c. H_c is called the coercive force of the specimen. This is a measure of the difficulty of breaking up the alignment of the domains. We observe that once a specimen is magnetised, its magnetisation curve never passes through the origin again. Instead it forms a closed loop called hysteresis loop. Hysteresis is defined as the lagging of the magnetic induction B behind the applied field H when the specimen is taken through a magnetic cycle.

Self Test Questions

1. (a) Give an account of the domain theory of magnetisation
   (b) How does the theory explain the process of magnetization and demagnetization?
2. (a) What is meant by magnetic hysteresis? Sketch a typical hysteresis curve and explain. What can be deduced from this about the magnetic properties of the material?
   (b) What are the desirable magnetic properties for the material of (i) the core of an electromagnet and (ii) a permanent magnet?
3. A toroid core has N = 1200 turns, length L = 80cm, cross-sectional area A = 60cm^2, current I = 1.5A. Compute B and H. Assume an empty core.
4. A cast iron ring has a mean diameter of 0.2m and an area of cross-section of 5x10^{-4}m^2. It is uniformly wound with 2000 turns carrying a current of 2.0A and the magnetic flux in the iron is 8x10^{-3}Wb. What is the relative permeability of the iron?
5. The current in the windings on a toroid is 2.0A. There are 400 turns and the mean circumferential length is 40cm. With the aid of a search coil and charge measuring instrument, the magnetic field is found to be 1.0T. Calculate
   (a) the magnetic intensity
   (b) the magnetization
   (c) the magnetic susceptibility
   (d) the equivalent surface current
   (e) the relative permeability

TOPIC 2: CURRENT ELECTRICITY

1.2 DIRECT AND ALTERNATING CURRENT CIRCUITS

Most electrical circuits comprise a number of sources, resistors or other elements such as capacitors, motors, etc., interconnected in a more or less complicated manner. The general term applied to such a circuit is a network.

Relation between Q and I

The quantity of electricity is measured in coulombs. Electric current I, is the drift of free electrons and is given by

$$I = \frac{Q}{t}$$

Therefore $\quad Q = It$ (t in seconds)

Current and drift velocity

Suppose there are n 'free' electrons per unit volume in a conductor of cross-section area A drifting with velocity v in the same direction. The number of electrons passing a given section is nx(volume occupied in 1sec)

$=n\mathit{v}A$

I = quantify of electricity in 1 sec

$neA\mathit{v}$

$$\text{Current density} = \frac{I}{CSA} = ne\mathit{v}$$

To maintain a current I, a source of EMF (Electromotive force) V must furnish power at a given rate. A source of EMF of one volt is a source that does one joule of work on each coulomb of charge that passes through it from the low potential side to the high.

Ohms law

Ohm's law states for a given conductor at a given temperature, the current is directly proportional to the difference of potential between the ends of the conductor. That is $I \propto V$. The constant of proportionality is called the conductance G, and so we may write I=GV. More frequently used is the reciprocal of conductance, R=1/G which is called the resistance. In terms of the resistance we have:

$$I = \frac{V}{R} \quad \text{or } V=IR$$

The unit of resistance is the Ohm, usually denoted by Ω

Resistivity (r)

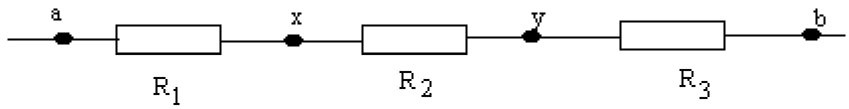
The resistance of a wire at constant temperature is found to be proportional to its length l and inversely proportional to its corresponding area A that is

$$R \propto \frac{l}{A} \Rightarrow R = \rho \frac{l}{A}$$

r is called the resistivity of the wire.

1.21 Direct current circuits

(i) Resistors in series



Consider the resistors R_1, R_2 and R_3 connected as shown above. If the current I is flowing through the circuit between points a and b then

$$V_{ax} = IR_1,\ V_{xy} = IR_2,\ V_{yb} = IR_3$$

The total voltage between points a and b is

$$\begin{aligned} V_{ab} &= V_{ax} + V_{xy} + V_{yb} \\ &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

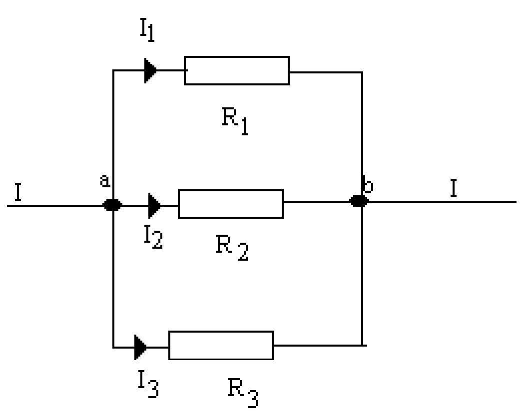
Let R be the total resistance (equivalent). Then

$$V_{ab} = IR$$

$$\Rightarrow R = R_1 + R_2 + R_3$$

Thus the equivalent resistance of any number of resistors in series equals the sum of the individual resistances.

(ii) Resistors in parallel.



Let the currents flowing in the resistors R_1, R_2 and R_3 be I_1, I_2 and I_3. Then

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Let the equivalent current be I. Then

EMBED Equation.3

Let the equivalent resistor be R. Then

EMBED Equation.3

EMBED Equation.3

$$\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If a network has only two resistors in parallel, then

EMBED Equation.3

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Thus in General, for any number of resistors in parallel, the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances.

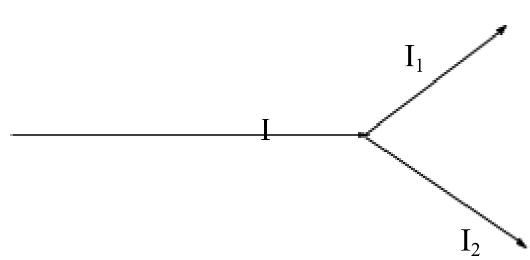
Kirchoff's Laws

When a network is complex, there are various methods of analysing it. One of them is the method developed by Kirchoff after he extended Ohm's laws to networks and came up with two laws.

(i) Kirchoff's Current Law (KCL)

This states that the algebraic sum of the current flowing into a junction is zero.
Kirchhoff’s first law is a statement of conservation of charge. According to this law, the sum of all currents going into a point in a circuit is equal to the sum of all currents going out of that point. In other words, current cannot be lost so whatever goes in, comes out.

EMBED Equation.3



(ii) Kirchoff's Voltage Law (KVL)

States that the algebraic sum of all potential differences in a closed loop (or a closed circuit) is equal to zero.

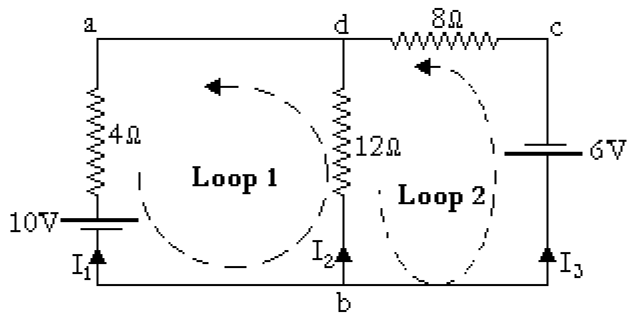
Kirchhoff's second law is a result of conservation of energy. It states that the sum of all e.m.f's in any loop in a circuit is equal to the sum of all p.d's in that loop. A point to note here is that the law is valid for any loop in the circuit

EMBED Equation.3

Where V_s is the voltage source

Example

Consider the following circuit. Calculate the current I_1, I_2 and I_3 in the above circuit.



Note:

1. The emf is counted as positive when it is traversed from -ve to +ve and negative when it is traversed from +ve to -ve.
2. An IR term (p.d) is counted as negative if the resistor is traversed in the same direction as the assumed current and positive if in the opposite direction.

__**Solution.**__

Using KCL at node b

EMBED Equation.3

Using KVL at:

loop abda

EMBED Equation.3

loop abcda

EMBED Equation.3

Make I_3 the subject in eqn(i) and substitute it in (iii).

EMBED Equation.3

Multiply eqn(iv) by 3 and add it to eqn(ii)

EMBED
Equation.3

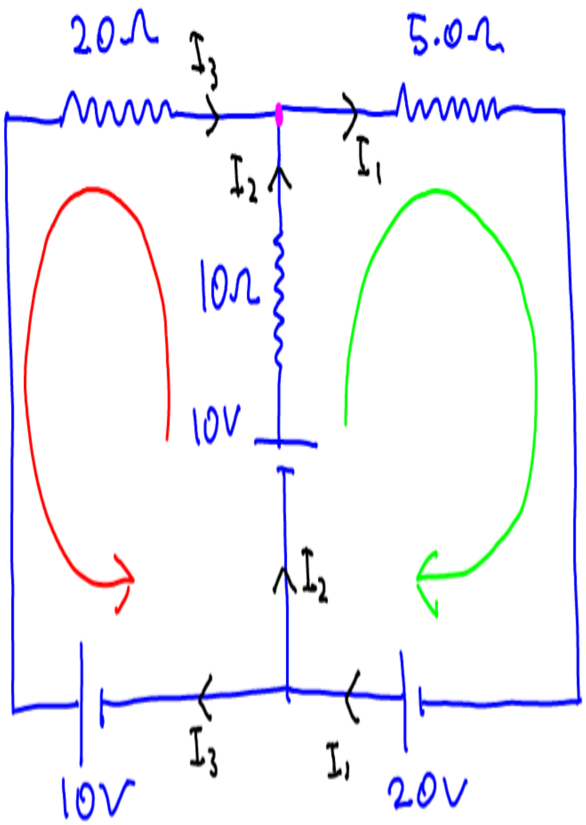
Now substitute I_1 in (iv)

$$\frac{24}{11} - 2I_2 = 1$$

$$\frac{13}{11} = 2I_2$$

$$I_2 = {}^{13}\!/_{22}\,\text{A}$$

$$\therefore I_3 = \frac{8}{11} - \frac{13}{22} = \frac{16-13}{22} = {}^{3}\!/_{22}\,\text{A}$$



KI: $I_1 = I_2 + I_3$ [1]

KII: $10 - 10I_2 - 5I_1 + 20 = 0$

$6 - 2I_2 - I_1 = 0$ [2]

KII: $10 - 10I_2 + 20I_3 - 10 = 0$

$I_2 = 2I_3$ [3]

[1] in [2]: $6 - 2I_2 - I_2 - I_3 = 0$

$6 - 3I_2 - I_3 = 0$

Subs. [3]: $6 - 6I_3 - I_3 = 0$

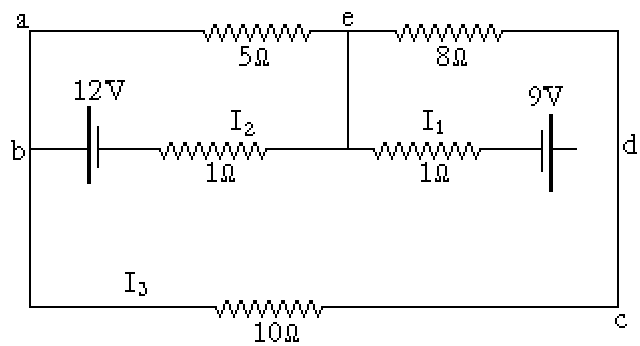
$I_3 = 0.86\,\text{A}$

[3]: $I_2 = 2(0.86) = 1.7\,\text{A}$

[1]: $I_1 = 0.86 + 1.7 = 2.6\,\text{A}$

Assignment

Calculate the three currents indicated in the circuit diagram below.



Energy and power in DC circuits

When a steady current I flows through a load (e.g resistor, electric motor, accumulator on charge etc) it dissipates energy in it which is equal to the potential energy lost by the charge as it moves through the potential difference that exists between the input and output terminals of the device. The energy is given by

$$W = QV$$

Where Q is the charge that flows in 1 sec and V the potential difference across the load.
Since $Q = It$

Then $W = Vit = I^2Rt = (\frac{V^2}{R})t$

Power is the rate of dissipation of energy

$$P = \frac{dW}{dT} = VI = I^2R = \frac{V^2}{R}$$

1.22 R-C Circuits

Capacitors

A capacitor is a device which is used to store electric charge. Consider two conductors *a* and *b* put at a reasonable distance from each other. When a positive voltage is applied on *a* and a negative voltage of the same magnitude is applied on *b*, the positive charges *(+Q)* and the negative charges *(-Q)* accumulate on plates a and b. These charges cause an electric field which creates a potential difference V_{ab}. Hence the magnitude of the charges is directly proportional to the potential difference between the conductors.

$$Q \propto V_{ab}$$
$$Q = CV_{ab}$$

C is called the capacitance (or charge storing ability) of the conductors.

$$C = Q/V_{ab}$$

The two conductors are mostly called the capacitor plates.

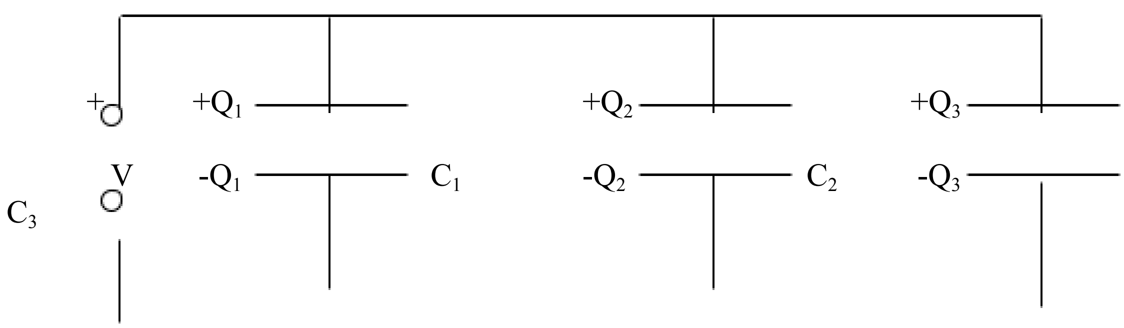
Symbol: 

Capacitance units: CV^{-1} or Farads (F)

Defn: **Capacitance is defined as the ratio of the magnitude of the charge *Q* on either conductor to the magnitude of the potential difference V_{ab} between the conductors.**

Capacitor Networks

(a) Capacitors in Parallel



Consider capacitors C_1, C_2 and C_3 arranged in parallel as shown above. The applied p.d *V* is the same across each but the charges are different. And therefore

$$Q_1 = VC_1,\ Q_2 = VC_2,\ Q_3 = VC_3$$

Total charge *Q* on the three capacitors is

$$\mathbf{Q = Q_1 + Q_2 + Q_2}$$
$$= V(C_1 + C_2 + C_3)$$

Let *C* be the equivalent capacitance. Then

$$C = C_1 + C_2 + C_3 = Q/V$$

The equivalent capacitance equals the sum of the individual capacitances.

(b) Capacitors in Series

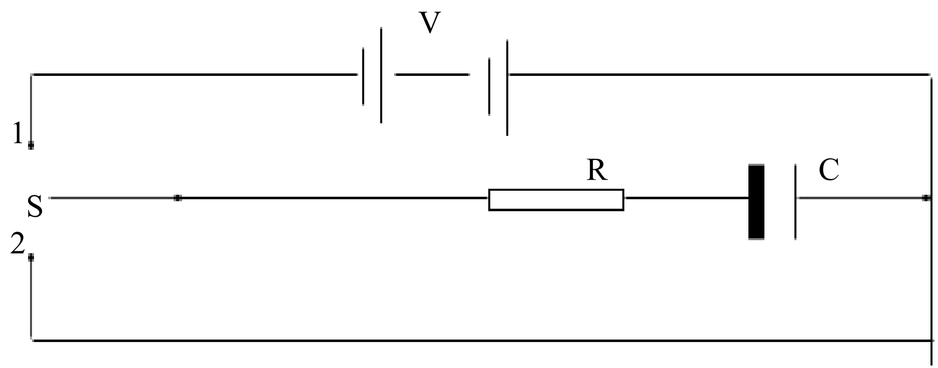
Assignment:

Show that the reciprocal of equivalent capacitances in series is given by

EMBED Equation.3

Charging and discharging a capacitor

Consider the circuit below



Consider the switch in position 1. Initially the capacitor is uncharged, $Q = 0$, hence the p.d V_c across it is zero, $V_c = 0$. The potential difference across the resistor R, V_R is equal to the source voltage V, $V_R = V$. When capacitor C starts charging the charge starts increasing, the p.d V_c across the capacitor plates increases. But the p.d across the resistor decreases and thus the current also decreases.

Let the instantaneous charge on the capacitor be Q. The current in the circuit at any time t is given by

EMBED Equation.3

But

$$V_C = Q/C$$

Therefore

EMBED Equation.3

But

EMBED Equation.3

Let $U = VC - Q \Rightarrow dU = -dQ$

EMBED Equation.3

But EMBED Equation.3

$$\Rightarrow \ln U = -\frac{t}{RC} + \text{constant}$$

$$\Rightarrow \ln(VC - Q) = -\frac{t}{RC} + \text{constant}$$

at $t = 0$, $Q = 0$, hence

$$\ln(VC - 0) = -{}^{0}\!/_{RC} + \text{constant}$$

Hence $\Rightarrow \ln(VC) = \text{constant}$

$$\ln(VC - Q) = -\frac{t}{RC} + \ln(VC)$$

$$\Rightarrow \ln\left(\frac{VC - Q}{VC}\right) = -\frac{t}{RC}$$

$$\Rightarrow \frac{VC - Q}{VC} = \mathrm{e}^{-t/RC}$$

$$\Rightarrow 1 - \frac{Q}{VC} = \mathrm{e}^{-t/RC}$$

$$\Rightarrow \frac{Q}{VC} = 1 - \mathrm{e}^{-t/RC} \quad \text{or } Q = CV\left(1 - \mathrm{e}^{-t/RC}\right)$$

Let

$$CV = Q_o \Rightarrow \textit{maximum charge}$$

Hence

$$Q = Q_o\left(1 - \mathrm{e}^{-t/RC}\right)$$

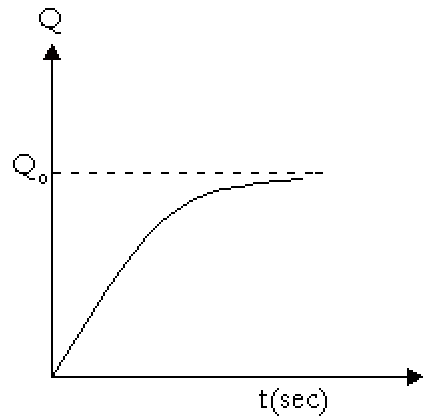
<u>Half-Life($t_{1/2}$)</u>

This is the time it takes the capacitor to obtain a charge of $\frac{1}{2}Q_o$.

$$Q = \frac{1}{2}Q_o = Q_o\left(1 - e^{-t_{1/2}/RC}\right)$$

$$\Rightarrow \frac{1}{2} = \left(1 - e^{-t_{1/2}/RC}\right)$$

$$\Rightarrow t_{\frac{1}{2}} = RC\ln 2 = 0.693RC$$



Discharging a capacitor

Now consider the switch is put in position 2. The capacitor will be discharged through resistor R. Suppose the capacitor was charged to a p.d V so that the charge is $Q = CV$. At a time t after the discharge through R has begun, the current I flowing = V_R/R where $V_R = V_C$ which is the p.d across C. Now $V_R = V_C = Q/C$ and $I = -dQ/dT$ (-ve indicates Q decreases with increasing t). Hence from

$$I = \frac{V_C}{R},\ V_C = \frac{Q}{C}$$

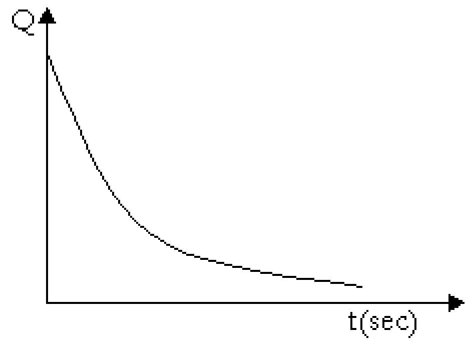
then $$-\left(\frac{dQ}{dT}\right) = \frac{1}{CR}Q$$

$$\Rightarrow \int_{Q_o}^{Q}\frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dT$$

$$\therefore \ln\left(\frac{Q}{Q_o}\right) = -\frac{t}{RC}$$

$$\Rightarrow Q = Q_o e^{-t/RC}$$

Hence Q decreases exponentially with time t.



<u>Time Constant (T)</u>

The time constant T of the discharge circuit is defined by CR seconds where C is in farads and R is in Ohms.

At time $t = RC$ then $Q = Q_o e^{-1}$

Thus the time constant is defined as the time for the charge to decay to *1/e* times the initial value.

$$\Rightarrow \ T = RC$$

Energy of a charged capacitor

A charged capacitor is a store of electrical energy. Assume a capacitor of capacitance C charged with a charge Q and to a potential difference V. Consider the charge is increased by a small value δQ, then this will cause a change in p.d by a value δV.

$$\delta V = \frac{\delta Q}{C}$$

If δQ is very small then p.d will almost be constant at the value V. Then the work done in displaying the charge δQ will be

$$\delta W = V\delta Q$$

(since work done =energy stored =charge x p.d)

But

$$V = Q/C$$

$$\therefore \delta W = \frac{Q}{C}\delta Q$$

The total work W, in increasing the charge from 0 to Q_o is therefore given by

$$W = \int_0^{Q_o} \frac{Q}{C} dQ$$

$$= \frac{1}{2}\frac{Q_o}{C^2}$$

Types of Capacitors

For a parallel plate capacitor large capacitances can be achieved if

(i) The plates are of large area
(ii) The plates are close together
(iii) A dielectric material with large permitivity is used.

Different capacitors used to achieve a desired need include paper capacitors, electrolytic capacitors and variable air capacitors

HomeWork 2

1. In the circuit below, $C_1 = 2\mu F$, $C_2 = C_3 = 0.5\mu F$ and $V = 6V$. For each capacitor, calculate

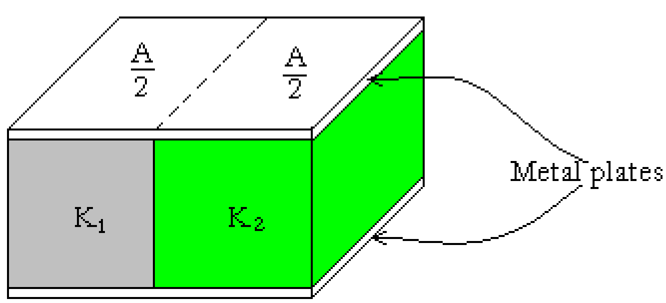
   (a) the charge on it
   (b) the p.d across it.

2. A 10μF capacitor is charged from a 30V supply and then connected across an uncharged 50μF capacitor. Calculate the
   (a) the final p.d across the combination
   (b) the initial and final energies.
3. (a) A capacitor of capacitance 10μF is fully charged from a 20V d.c supply.
   (i) Calculate the charge stored by the capacitor.
   (ii) Calculate the energy delivered by the 20V supply.
   (iii) Calculate the energy stored by the capacitor.
   (iv) Account for the difference between the answers for (ii) and (iii).

   (b) The 10μF capacitor in part (a) above was charged from the supply through a resistor of resistance 2.0KΩ.
   (i) Calculate the time constant for this circuit
   (ii) When the capacitor was charged from zero charge, how long did it take for V, the potential difference across the capacitor to reach 99% of its final value?
4. A parallel plate capacitor is completely filled with two dielectrics of equal volume as shown below. If the dielectric constants are K1 and K2 show that the capacitance is

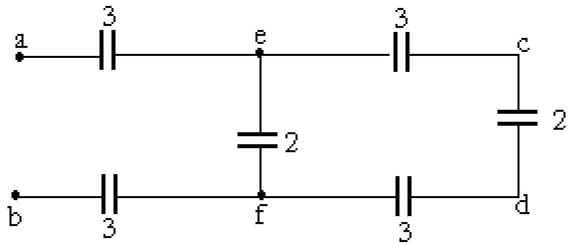
$$C = C_o \frac{K_1 + K_2}{2}$$

Where C_o is the value when there is vacuum between the plates.



5. Consider the diagram below. If all capacitances are in μF
   (i) Find the equivalent capacitance between a and b
   (ii) Find the charge on each capacitor nearest a and b if $V_{ab} = 900V$

(iii) Find V_{eff} if V_{ab} = 900V



TOPIC 3: AC Circuits

Many electric circuits of practical importance including nearly all large-scale electric-power distribution systems and much electronic equipment use alternating current (a.c) in which the voltages and currents vary with time, often in a sinusoidal manner i.e.

$$I = I_m \sin\omega t$$

$$V = V_m \sin\omega t$$

In commercial practice, a.c is always expressed in terms of the root mean square (r.m.s) value. The r.m.s of an alternating current is defined as that value of steady current which would dissipate heat at the same rate in a given resistance.
Power dissipated by a d.c current was found to be given by

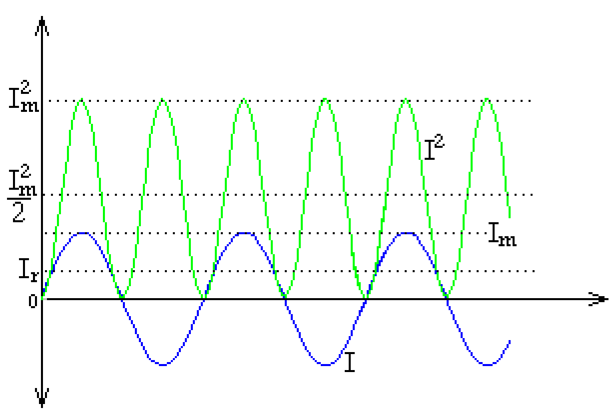
$$P=I^2R$$

The. For a.c

$$P= I_r^2 R$$

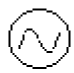
Average power = Average value of (I^2R)
= average of(I^2)xR (because R is constant

Taking the average value of I over a cycle I_r^2 = average value of I^2 called the mean square current. The variation of I^2 as seen below is symmetrical and so the mean square current is I_m^2 /2 where I_m is the peak current.

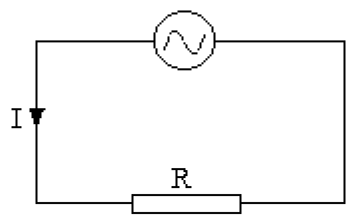


Thus	$I_r = \dfrac{I_m}{\sqrt{2}} = 0.707I_m$
Similarly	$V_r = 0.707V_m$

We shall now study the varying voltages and currents in relation to the circuit behaviour of inductors and capacitors.
The symbol for a.c is



AC and Resistor.



When a current I passes through a resistor R, we obtain the instantaneous voltage V_R from Ohms law

$$V_R = IR = RI_m\sin(2\pi ft)$$

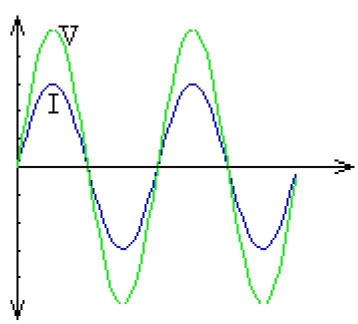
If we write this relation in the form

$$V_R = V_{Rm}\sin(2\pi ft)$$

We see that

$$V_{Rm} = RI_m$$

And so the current and voltage are *in phase*



$$\sqrt{2v_r} = R\sqrt{2I_r}$$

or $V_r = RI_r$

Thus the a.c. expression for the relationship between current and voltage hold if we use r.m.s values or if we use peak values.

a.c. Power in resistor

Power is supplied when current flows in either direction through the resistor. The instantaneous power supplied to the resistance is given by

$$IV_R = R\, I_M^2 \sin(2\pi ft)$$

Average power $\quad P = \dfrac{I_M V_M}{2}$

$$= \frac{I_M}{\sqrt{2}}\frac{V_M}{\sqrt{2}}$$

$$= I_r V_r = I_r^2 R$$

Thus r.m.s values must be used for a.c. power.

AC through a Capacitor

Consider capacitor plates being continually charged, discharged and charged the other way round by the alternating voltage of the mains. Let a p.d V be applied across a capacitor of capacitance C and let its value at time t be given by

$$V = V_M \sin 2\pi ft,$$

V_M is the peak voltage and f is the frequency of the supply.
The charge Q on the capacitor at time t is

$$Q = CV$$

The current I flowing in the capacitor is then given by

$$I = \text{rate of change of charge}$$

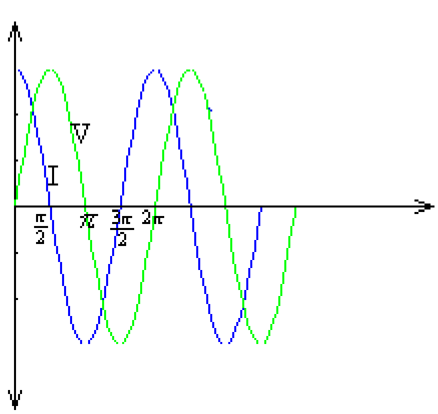
$$= \frac{dQ}{dt} = \frac{d(CV)}{dt}$$

$$= C\frac{dV}{dt} = C\frac{d}{dt}(V_M \sin(2\pi\, ft))$$

$$= 2\pi fCV_M \cos(2\pi ft)$$

where $2\pi fCV_M = I_M$ the peak value

The current flowing "through" the capacitor (cosine function) leads the applied p.d (a sine function) by one quarter of a cycle.



Consider the ratio $\frac{V_M}{I_M} = \frac{V_M}{2\pi\, fCV_M} = \frac{1}{2\pi\, fC} = \frac{1}{\omega\, C}$

This expression resembles V/I=R which defines resistance with $\frac{1}{2\pi\, fC}$ replacing R. This quantity is taken as a measure of the opposition of a capacitor to a. c. and is called capacitive reactance X_c

That is $X_c = \frac{1}{2\pi\, fC} = \frac{1}{\omega\, C}$

AC through an Inductor

Let us suppose that the resistance of the coil is negligible. An inductor in an a.c circuit behaves like a capacitor in that it causes a phase difference between the applied p.d and the current. In this case the current lags behind the voltage by π/2

Consider an inductor L through which current I flows at time t where

$$I = I_M \sin 2\pi ft$$

I_M is the peak current.

An alternating current flows through the the inductor and sets up a changing magnetic flux. This induces a back e.m.f given by

$$E = -L\frac{dI}{dT}$$

at some time t. L is the inductance of the coil

Suppose that the value of the applied p.d at time t is V. By Kirchoff's Voltage Law,

$\sum E = \sum IR$ and therefore, since R=0,

The p.d V in the inductor due to the changing current is

$$V - L\frac{dI}{dT} = 0$$

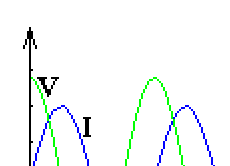
$$\text{i.e.,}\ V - L\frac{d}{dt}\left(I_M \sin\omega t\right) = 0$$

$$\Rightarrow\ V - \omega LI_M \cos\omega t = 0$$

Writing $V_M = \omega LI_M = 2\pi fLI_M$ gives

$$V = V_M \cos\, \omega t = V_M \cos 2\pi ft$$

Thus V is a cosine function whereas I is a sine function and therefore there is a phase difference of p/2 radians between the current and the applied p.d. The voltage reaches its maximum value before the current i. e. voltage leads the current.



$$\frac{V_M}{I_M} = \frac{2\pi f L I_M}{I_M} = 2\pi\ fL = \omega\ L$$

The quantity 2πfL is called the inductive reactance χ_L of the inductor

$\chi_L = 2\pi fL = \omega L$

Note: If ω is in radians per second and L is in henrys, then χ_L is in ohms.

$$V_{RMS} = {}^{V_M}\!/\!_{\sqrt{2}} \text{ and } I_{RMS} = {}^{I_M}\!/\!_{\sqrt{2}}$$

$$\therefore \frac{V_{RMS}}{I_{RMS}} = \frac{V_M}{I_M} = \chi_L \neq \frac{V}{I}$$

Series Circuits

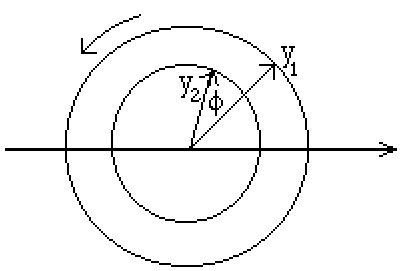
We are going to consider the network consisting of C, R and L together.

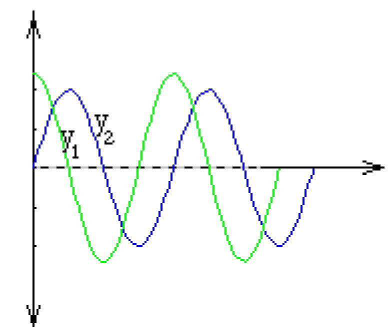
Vector Diagram

A sinusoidal alternating quantity can be represented by a rotating vector (often called a phase vector or a phasor). Suppose we have two waves

$y_1 = Y_1 \sin\omega t,$
$y_2 = Y_2 \sin(\omega t - \varphi)$

then it can be demonstrated as shown



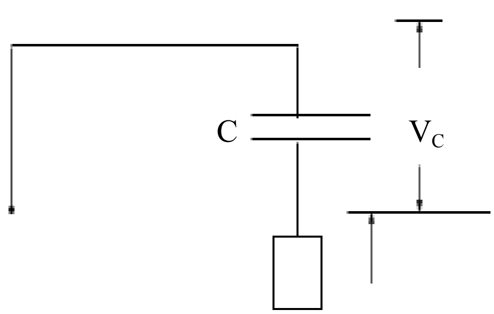


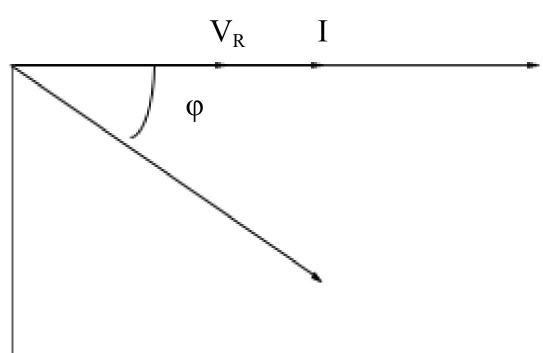
(a) **Resistor and Capacitor**

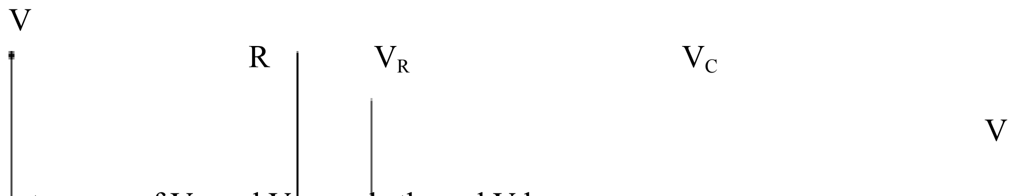
Suppose an alternating p.d V is applied across resistor R and capacitor C in series, as shown below. Our reference vector will be that representing I.

(1) The p.d V_R across R is always in phase with I

(2) The p.d V_c across C lags behind I by π/2







The vector sum of V_R and V_C equals the p.d V hence

$$V^2 = V_R^2 + V_C^2$$

But $V_R = IR$, and $V_C = I\chi_C$ $\chi_C = 2\pi fC$

$$\Rightarrow V^2 = I^2\left(R^2 + \chi_C^2\right)$$

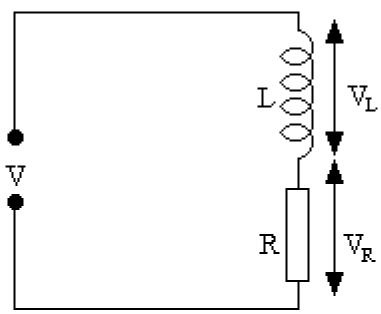
$$\therefore V = I\sqrt{\left(R^2 + \chi_C^2\right)}$$

The quantity $\sqrt{\left(R^2 + \chi_C^2\right)} = Z$, where Z is called the impendance of the circuit and measures its opposition to a.c.

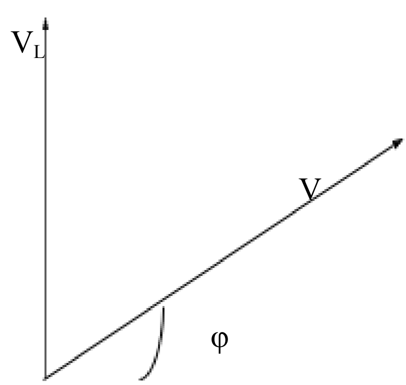
$$Z = \frac{V}{I} = \sqrt{\left(R^2 + \chi_C^2\right)}$$

Phase angle ϕ is given by $\tan\phi = \frac{V_C}{V_R} = \frac{\chi_C}{R}$

(b) Resistor and Inductor



The analysis is similar but in this case the p.d V_L across L leads on the current I and the p.d V_R across R is again in phase with I.



The p.d V equals the vector sum of V_L and V_R

$$V^2 = V_R^2 + V_L^2$$

But $V_R = IR$, and $V_L = I\chi_{LC}$, $\chi_L = \dfrac{1}{2\pi fL}$

$$\Rightarrow V^2 = I^2\left(R^2 + \chi_L^2\right)$$

$$\therefore V = I\sqrt{\left(R^2 + \chi_L^2\right)}$$

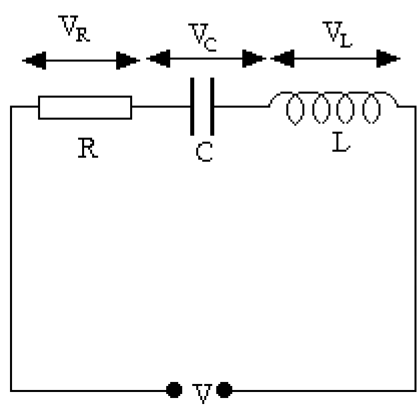
The quantity $\sqrt{\left(R^2 + \chi_L^2\right)} = Z$, where Z is called the impendance of the circuit and measures its opposition to a.c.

$$Z = \frac{V}{I} = \sqrt{\left(R^2 + \chi_L^2\right)}$$

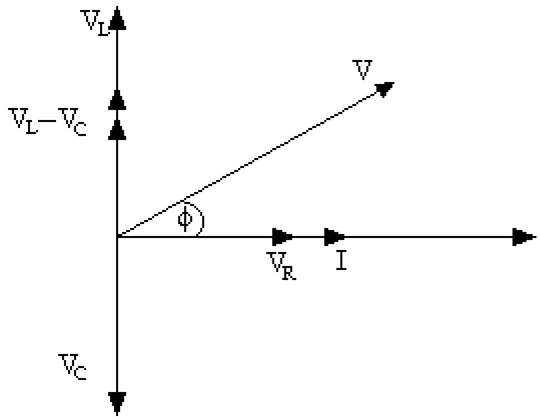
Phase angle ϕ is given by $\tan\phi = \dfrac{V_L}{V_R} = \dfrac{\chi_L}{R}$

(c) Resistor, Capacitor and Inductor in Series

An R-C-L circuit is shown below



Since V_L leads the current I by 90^0 and V_c lags I by 90_0 then V_L and V_c are 180^0 (half-cycle) out of phase, i.e. antiphase. If V_L is greater than V_c, then the resultant is $V_L - V_C$



The p.d V is given by

$$V^2 = V_R{}^2 + \left(V_L - V_C{}^2\right)$$

But V_R=IR, V_L=$I\chi_L$ and V_C=$I\chi_C$
hence

$$V^2 = I^2[R^2+(\chi_L+\chi_C)^2]$$

$$\therefore \quad V = I\sqrt{\left[R^2 + \left(\chi_L - \chi_C\right)^2\right]}$$

The impedance Z is given by

$$Z = \frac{V}{I} = \sqrt{R^2 + \left(\chi_L - \chi_C\right)^2}$$

The phase angle φ is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{\chi_L - \chi_C}{R}$$

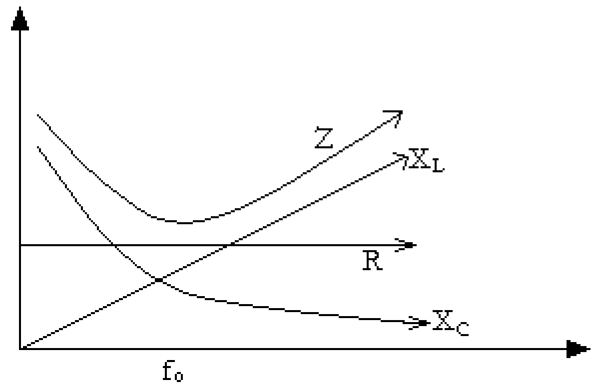
$$\therefore \tan\phi = \frac{\chi_L - \chi_C}{R}$$

Electrical Resonance in the L-C-R Series Circuit

The expression just derived for the impedance of an R-C-L circuit show that Z varies with the frequency f of the applied p.d, since both χ_L and χ_C depend on f.

- $\chi_L \alpha$ f and the variation of χ_L with frequency is a straight line passing through the origin.
- $\chi_C \alpha$ 1/f and the variation of χ_C with frequency is a curve approaching through the two axes.
- The resistance R is independent of frequency and thus is represented by a straight line.

This is summarised on the diagram shown below



At a certain frequency, $\chi_L = \chi_C$. This frequency is called the resonant frequency (f_o) and Z is a minimum. Then

$$\chi_C = \chi_L \Rightarrow \frac{1}{2\pi f_o C} = 2\pi f_o L$$

$$\therefore f_o = \frac{1}{2\pi\sqrt{LC}}$$

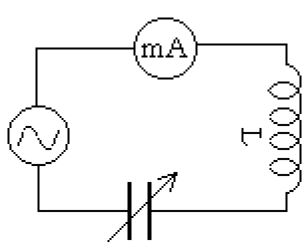
The phase angle φ is given by

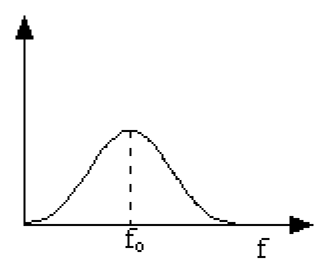
$$\tan\phi = \frac{\chi_L - \chi_C}{R} = 0$$

implying that the applied p.d V and the current are in phase.

How to measure f_o

The circuit shown above is used to measure either L or C. As the frequency increases the milliameter reading rises to a maximum and then falls. When various values of I and f are collected the graph of I vs. f is drawn, the peak is the resonance frequency.





H/Work: Read about L-R-C in parallel

Homework 3

1. A sinusoidal voltage of frequency f=60Hz and peak value 150V is applied to a series R-L circuit where R=20Ω and L=40mH.

(a) Compute the period T, ω, χ_L, Z and φ.

(b) Compute the amplitudes I, V_R, V_L and the instantaneous values i, V_R and V_L at t=T/6

(c) Compute the I_{RMS}, V_{RMS} and the average power into the circuit.

2. A 1000μF capacitor is joined in series with a 2.5V, 0.3A lamp and a 50Hz supply. Calculate
   (a) the p.d of the supply to light the lamp
   (b) the p.ds across the capacitor and the resistor respectively.
3. A 2.0H inductor of resistance 80Ω is connected in series with a 420Ω resistor and a 240V, 50Hz supply. Find
   (a) the current in the circuit and
   (b) the phase angle between the applied voltage and the current.
4. A circuit consists of an inductor of 200μH and resistance 10Ω in series with a variable capacitor and a 1.0V, 1MHz supply. Calculate
   (a) the capacitance to give resonance
   (b) the p.d's across the inductor and the capacitor at resonance.
5. A capacitor from a 50V DC supply is discharged across a charge-measuring instrument and found to have carried a charge of 10μC. What was the capacitance of the capacitor and how much energy was stored in it?
6. (a) Explain the meaning of the terms capacitance, relative permitivity and time constant.
   (b) Two capacitors C1 and C2 are connected in series and then charged with a battery. The battery is disconnected and C1 and C2, still in series, are discharged through an 80KΩ resistor. The time constant for the discharge is found to be 4.8 seconds. Calculate the capacitance of C1 and C2 in series and the capacitance of C1 if C2 has a capacitance of 100μF.