

KABARAK



UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN
ECONOMICS AND MATHEMATICS**

MATH 211: LINEAR ALGEBRA I

Instructions:

- Answer question **ONE** and any other **TWO** questions
- Begin each question on a separate page
- Show your workings clearly

QUESTION ONE (30 MARKS)

a) Find k so that vector \mathbf{u} and \mathbf{v} are orthogonal

i) $\mathbf{U}=(12,k,-32)$ and $\mathbf{v} = (22,-52,42)$ (4 marks)

ii) $\mathbf{U}=(20,30k,-40,10,50)$ and $\mathbf{v} = (6,-1,3,7,2k)$ (4 marks)

b) Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and suppose $f(x) = 2x^2 - 3x + 5$, then find $f(A)$ (5 marks)

c) Solve for x for $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ (6 marks)

d) For the given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

Show that $\text{Det}(AB)=\text{det}(A)\text{det}(B)$ (6 marks)

- e). Given $u = (6i+, 4j, -2k)$, $v = (0i+, 4j, -6k)$ and $w = (4i+, 12j+, 14k)$
 $U \times (V \times W)$ (3 marks)

f). Given the matrix $\begin{bmatrix} 1 & 6 & 5 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ calculate the determinant (4mks)

QUESTION TWO (20 MARKS)

- a) Use Cramers Rule to solve for the unknown variables x_1 , x_2 and x_3 given that
 $12x_1 + 3x_2 + 6x_3 = 56$
 $2x_1 + 7x_3 = 41$
 $5x_1 + 2x_2 + 2x_3 = 34$ (10 marks)
- b) Verify that the Cauchy – Schwarz inequality holds for
- i) $u = (15, 10)$ and $v = (20, 5)$ (3 marks)
- ii) $U = (-20, 10, 5)$ and $v = (40, -20, -20)$ (3 marks)
- c) Show that $u = (75, 0, 25, 0, 100, -25)$ and $v = (-50, 125, 0, 50, -75, -450)$ are orthogonal and verify that the pythagorean Theorem holds. (4 marks)

QUESTION THREE (20 MARKS)

- a). Given the set of simultaneous equations
- $$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5 \end{aligned}$$
- i) Use Gaussian elimination method to solve for x_1 , x_2 and x_3 (13 marks)
- ii) Proceed to solve for x_1 , x_2 and x_3 using Gauss-Jordan Elimination method (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $u = (12, -32, 24)$ and $v = (32, 14, 17)$, Find
- i) $\cos \theta$, where θ is the angle between u and v (4 marks)
- ii) $\text{Proj}(u, v)$, projection of u onto v (3 marks)
- iii) $d(u, v)$, the distance between u and v (2 marks)

- b) Use cross product and dot product to find the angle between the vectors $u = (8, 12, -24)$ and $v = (8, 12, 24)$ (6 marks)

c) If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

Prove that $A(B+C) = AB + AC$

(5 marks)

QUESTION FIVE (20 MARKS)

- a). Find A^{-1} of the following matrix using determinant method and show that $A^{-1}A = AA^{-1} = I$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (20 \text{ marks})$$

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UNIVERSITY EXAMINATIONS

MAIN CAMPUS

FIRST SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE AND
BACHELOR OF EDUCATION ARTS

MATH 211: LINEAR ALGEBRA I

STREAM: Y2. S 1

TIME: 1.00-3.00 P.M

EXAMINATION SESSION: DECEMBER

DATE: 30/11/2017

INSTRUCTIONS

Instructions to candidates:

- **QUESTION ONE** is compulsory.
- Answer **QUESTION ONE** and any other **TWO** questions
- Begin each question on a separate page - Show your workings clearly

Question One [30 Marks]

a) Find k so that vector u and v are orthogonal

i. $u = (1, k, -3)$ and $v = (2, -5, 4)$ $v = (2, -5, 4)$ (3 marks)

ii. $u = (2, 3k, -4, 1, 5)$ and $v = (6, -1, 3, 7, 2k)$ (3 marks)

b) Find x and y given that $\begin{pmatrix} 1 & 2 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (5 marks)

As members of Kabarak University family, we purpose at all times and in all places, to set apart in one's heart,
Jesus as Lord. (1 Peter 3:15)

c) Solve for x for $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ (6 marks)

d) For the given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

Show that

i. $\text{Det}AB = \text{Det}A \cdot \text{Det}B$ (6 marks)

ii. $\text{Det}(A+B) \neq \text{Det}A + \text{Det}B$ (3 marks)

e) Given $u = (6, 4, -2)$, $v = (0, 4, -6)$ and $w = (4, 12, 14)$ find

i. $v \times w$ (3 marks)

ii. $u \times (v \times w)$ (3 marks)

iii. $(u \times v)(v \times w)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Use Cramer's Rule to solve for the unknown variables x , y and z given that

$$\begin{aligned} 10x + 3y + 6z &= 76 \\ 4x + \quad \quad 5z &= 41 \\ 5x + 2y + 2z &= 34 \end{aligned} \quad (10 \text{ marks})$$

b) Verify that the Cauchy – Schwarz inequality holds for

i) $u = (75, 50)$ and $v = (100, 25)$ (3 marks)

ii) $u = (-80, 40, 20)$ and $v = (160, -80, -80)$ (3 marks)

c) Show that $u = (75, 0, 25, 0, 100, -25)$ and $v = (-50, 125, 0, 50, -75, -450)$ are orthogonal and verify that the Pythagorean Theorem holds. (4 marks)

QUESTION THREE (20 MARKS)

Given the set of simultaneous equations

$$\begin{aligned} 3x + 4y + 9z &= 45 \\ 4x + 5y + 2z &= 32 \\ 4x + 2y + 4z &= 32 \end{aligned}$$

i. Use Gaussian elimination method to solve for x , y and z (13 marks)

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- ii. Proceed to solve for x , y and z using Gauss-Jordan Elimination method (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $a = (1, -3, 4)$ and $b = (3, 4, 7)$

Find

- i. $\cos \theta$, where θ is the angle between a and b (4 marks)
- ii. $\text{proj.}(a, b)$, projection of a onto b (3 marks)
- iii. $d(a, b)$, the distance between a and b (2 marks)

- b) Use cross product and dot product to find the sine of the angle between the vectors $u = (8, 12, -24)$ and $v = (8, 12, 24)$ (6 marks)

- c) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

Show that $A(B + C) = (A + B)C$ (5marks)

QUESTION FIVE (20 MARKS)

Find A^{-1} of the following matrix using determinant method and show that $A^{-1}A = AA^{-1} = I$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (20\text{Marks})$$



UNIVERSITY EXAMINATIONS
MAIN CAMPUS

FIRST SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE AND
BACHELOR OF EDUCATION ARTS

MATH 211: LINEAR ALGEBRA I

STREAM: Y2. S 1

TIME: 1.00-3.00 P.M

EXAMINATION SESSION: DECEMBER

DATE: 30/11/2017

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Question One [30 Marks]

a) Find k so that vector u and v are orthogonal

i. $u = (1, k, -3)$ and $v = (2, -5, 4)$ $v = (2, -5, 4)$ (3 marks)

ii. $u = (2, 3k, -4, 1, 5)$ and $v = (6, -1, 3, 7, 2k)$ (3 marks)

b) Find x and y given that $\begin{pmatrix} 1 & 2 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (5 marks)

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c) Solve for x for $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ (6 marks)

d) For the given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

Show that

i. $\text{Det}AB = \text{Det}A \cdot \text{Det}B$ (6 marks)

ii. $\text{Det}(A+B) \neq \text{Det}A + \text{Det}B$ (3 marks)

e) Given $u = (6, 4, -2)$, $v = (0, 4, -6)$ and $w = (4, 12, 14)$ find

i. $v \times w$ (3 marks)

ii. $u \times (v \times w)$ (3 marks)

iii. $(u \times v)(v \times w)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Use Cramer's Rule to solve for the unknown variables x , y and z given that

$$\begin{aligned} 10x + 3y + 6z &= 76 \\ 4x + \quad \quad 5z &= 41 \\ 5x + 2y + 2z &= 34 \end{aligned} \quad (10 \text{ marks})$$

b) Verify that the Cauchy – Schwarz inequality holds for

i) $u = (75, 50)$ and $v = (100, 25)$ (3 marks)

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c) Show that $u = (75, 0, 25, 0, 100, -25)$ and $v = (-50, 125, 0, 50, -75, -450)$ are orthogonal and verify that the Pythagorean Theorem holds. (4 marks)

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Given the set of simultaneous equations

$$\begin{aligned} 3x + 4y + 9z &= 45 \\ 4x + 5y + 2z &= 32 \\ 4x + 2y + 4z &= 32 \end{aligned}$$

i. Use Gaussian elimination method to solve for x , y and z (13 marks)

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- ii. Proceed to solve for x , y and z using Gauss-Jordan Elimination method (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $a = (1, -3, 4)$ and $b = (3, 4, 7)$

Find

- i. $\cos \theta$, where θ is the angle between a and b (4 marks)
- ii. $\text{proj.}(a, b)$, projection of a onto b (3 marks)
- iii. $d(a, b)$, the distance between a and b (2 marks)

- b) Use cross product and dot product to find the sine of the angle between the vectors $u = (8, 12, -24)$ and $v = (8, 12, 24)$ (6 marks)

- c) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

Show that $A(B + C) = (A + B)C$ (5marks)

QUESTION FIVE (20 MARKS)

Find A^{-1} of the following matrix using determinant method and show that $A^{-1}A = AA^{-1} = I$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (20\text{Marks})$$

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UNIVERSITY EXAMINATIONS

MAIN CAMPUS

SECOND SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN

ECONOMICS AND MATHEMATICS; ECONOMICS AND STATISTICS

AND ACTUARIAL SCIENCES

MATH 211: LINEAR ALGEBRA I

STREAM: [Y2S1 & Y3S1]

TIME: 2:00-4.00P.M

EXAMINATION SESSION: APRIL

DATE: 17/04/2018

INSTRUCTIONS

- Instructions to candidates: Answer **QUESTION ONE** and any other **TWO** questions
 - Do not write on the question paper
 - Follow instruction that are given on the answer sheet
-

QUESTION 1 (30 MARKS)

- a) For the 3×3 matrix $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, use row reduction to find A^{-1} (8 marks)
- b) Find the area of a parallelogram whose adjacent sides are the vectors $\underline{u} = (4, -1, 2)$ and $\underline{v} = (6, -1, 6)$. (2 marks)
Given the set $S = \{(1, 2, 3), (0, 1, 2), (-1, 0, 1)\}$, show that $V = (1, -2, 2)$ is not a linear combination of vectors in S . (5 marks)
- c) Given that $s = \{(-4, 3, 4), (1, -2, 3), (6, 0, 0)\}$. Find if s is linearly independent set of vectors in R^3 (5 marks)
- d) Define a linear dependent vector space (4 marks)
- e) Find the dimension of W $\dim(w)$ where $w = \{\alpha, \beta - \alpha, \beta; \beta, \alpha \in R^3\}$ (6 marks)

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QUESTION 2 (20 MARKS)

- a) Use Gauss–Jordan method elimination process to solve the following system of linear equations:

$$2x_1 + 3x_2 + x_3 = 6$$

$$3x_1 + 2x_2 - 4x_3 = 12 \quad (6 \text{ marks})$$

$$x_1 - 2x_2 + 3x_3 = -3$$

- b) Use the row echelon form to solve the following system of linear equations:

$$x_1 + 2x_2 - 7x_3 = -4$$

$$2x_1 + x_2 + x_3 = -13 \quad (8 \text{ marks})$$

$$3x_1 + 9x_2 - 36x_3 = -33$$

- c) For what values of k does the homogenous system below have non-trivial solutions

$$kx + y - 3z = 0$$

$$x + ky - 3z = 0$$

(6 marks)

QUESTION 3 (20 MARKS)

For the matrix $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

- i) Show that $AB \neq BA$ (4 marks)

- ii) Find $A + B, A - B$. (6 marks)

- b) Find the area of a triangle with vertices at points $A(1,0), B(2,2), C(4,3)$

(6 marks)

- c) Use the determinate on matrix method to get the equation a line that have collinear points given as $A(1,8), B(-2,1)$

(4 Marks)

QUESTION 4 (20 MARKS)

- a) Use the inverse method to solve the system below,

$$2x_1 + 2x_2 - x_3 = 1$$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$-x_1 + 2x_2 + 3x_3 = 1 \quad (9 \text{ marks})$$

- b) i) Define the term basis S of a vector space V . (1 mark)

- ii) Show that the set $S = \{\tilde{u}_1 = (1,0,-1), \tilde{u}_2 = (1,2,1), \tilde{u}_3 = (0,-3,2)\}$ forms a basis for the vector space \mathbb{R}^3 (5 marks)

As members of Kabarak University family, we purpose at all times and in all places, to set apart in one's heart, Jesus as Lord. (1 Peter 3:15)

- i) Express vector $\tilde{\mathbf{v}} = (3, -5, 7)$ as a linear combination of vectors in \mathcal{S} .
(5 marks)

QUESTION 5 (20 MARKS)

- a) Given the vectors $\tilde{\mathbf{u}} = (2, 1, -3)$ and $\tilde{\mathbf{v}} = (-1, 5, -4)$, find:
- i) the orthogonal projection of $\tilde{\mathbf{v}}$ on $\tilde{\mathbf{u}}$ (6 marks)
 - ii) the angle θ between $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$ (6 marks)
- b) If $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$ are vectors in R^3 , using the vector components of $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$, show that:
- i) $\tilde{\mathbf{v}} \cdot (\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) = 0$ (3 marks)
- c) Use Cramer's rule to solve the system below
- $$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + x_2 + 4x_3 &= 2 \\ 3x_1 + 3x_2 + 4x_3 &= 1 \end{aligned}$$
- (5 marks)

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UNIVERSITY EXAMINATIONS
MAIN CAMPUS

FIRST SEMESTER, 2017 ACADEMIC YEAR
EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE
IN EDUCATION

MATH 211: LINEAR ALGEBRA I

STREAM: (PART TIME)

TIME: 11-1PM

EXAMINATION SESSION: APRIL

DATE: 14/04/2017

Instructions:

- Answer question **ONE** and any other **TWO** questions
- Begin each question on a separate page
- Show your workings clearly 1

QUESTION ONE (30 MARKS)

a) $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$

Show that $(AB)^{-1} = A^{-1}B^{-1}$ (6 marks)

b) Given that $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$. Calculate $p(x) = 2x^2 - 3x + 4$ (5 marks)

c) Find the components of vector $v = p_1p_2$ with initial point $p_1(2, -1, 4)$ and terminal point $p_2(7, 5, -8)$ (3 marks)

d) Given $u = (3, 2, -1)$, $v = (0, 2, -3)$ and $w = (2, 6, 7)$

i) $V \times W$ (3 marks)

ii) $U \times (V \times W)$ (3 marks)

- iii) $(\mathbf{U} \times \mathbf{V}) \times (\mathbf{V} \times \mathbf{W})$ (3 marks)
- iv) $\mathbf{U} \times (\mathbf{V} - 2\mathbf{W})$ (3 marks)
- e) If $\mathbf{u} = (1, 3, -2, 7)$ and $\mathbf{v} = (0, 7, 2, 2)$. Find the Euclidean space R^4 (3 marks)

QUESTION TWO (20 MARKS)

Given the following system of equations

$$x + y - z - 4 = 0$$

$$2x - 3y + 4z - 33 = 0$$

$$3x - 2y - 2z - 2 = 0$$

Solve these systems of equations using

- i) Gaussian elimination and Gauss-Jordan Elimination methods (10 marks)
- ii) Cramers Rule (10 marks)

QUESTION THREE (20 MARKS)

- a) If \mathbf{u} and \mathbf{a} are vectors in 2 or 3 space and if $a \neq 0$. Proof that

i) $proj_a \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \cdot \mathbf{a}$ (Vector component of \mathbf{u} along \mathbf{a}) (4 marks)

ii) $\mathbf{u} - proj_a \mathbf{u} = \mathbf{u} - \frac{\mathbf{u} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \cdot \mathbf{a}$ (Vector component of \mathbf{u} orthogonal to \mathbf{a})
(4 marks)

- b) Let $\mathbf{u} = (2, -1, 3)$ and $\mathbf{a} = (4, -1, 2)$.

Find

- i) The vector component of \mathbf{u} along \mathbf{a} (3 marks)
- ii) The vector component of \mathbf{u} orthogonal to \mathbf{a} (3 marks)

- c) Use cross product to find the sine of the angle between the vectors $u = (2, 3, -6)$ and $v = (2, 3, 6)$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Obtain the inverse of the following matrix using row reduction method

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} \quad (10 \text{ marks})$$

- b) Obtain the inverse of the following matrix using determinant method

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix} \quad (10 \text{ marks})$$

QUESTION FIVE (20 MARKS)

- a) Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ be two nonzero vectors. If θ is the angle between u and v . Show that $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$ (6 marks)

- b) Let $u = (u_1, u_2, u_3, \dots, u_n)$ and $v = (v_1, v_2, v_3, \dots, v_n)$ are vectors in R^n . Show that $|u \cdot v| \leq \|u\| \|v\|$ (4 marks)

- c) Verify that the Cauchy – Schwarz inequality holds for

i) $u = (3, 2)$ and $v = (4, 1)$ (3 marks)

ii) $U = (-4, 2, 1)$ and $v = (8, -4, -4)$ (3 marks)

- d) Given $u = (-3, 2, 1, 0)$, $v = (4, 7, -3, 2)$ and $w = (5, -2, 8, 1)$. Find

i) $2u + 7v$ (2 marks)

ii) $(6v - w) - (4u + v)$ (2 marks)

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AND ACTUARIAL SCIENCES

MATH 211: LINEAR ALGEBRA I

STREAM: [Y2S1 & Y3S1]

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EXAMINATION SESSION: APRIL

DATE: 17/04/2018

INSTRUCTIONS

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QUESTION 1 (30 MARKS)

- a) For the 3×3 matrix $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, use row reduction to find A^{-1} (8 marks)
- b) Find the area of a parallelogram whose adjacent sides are the vectors $\underline{u} = (4, -1, 2)$ and $\underline{v} = (6, -1, 6)$. (2 marks)
Given the set $S = \{(1, 2, 3), (0, 1, 2), (-1, 0, 1)\}$, show that $V = (1, -2, 2)$ is not a linear combination of vectors in S . (5 marks)
- c) Given that $s = \{(-4, 3, 4), (1, -2, 3), (6, 0, 0)\}$. Find if s is linearly independent set of vectors in R^3 (5 marks)
- d) Define a linear dependent vector space (4 marks)
- e) Find the dimension of W $\dim(w)$ where $w = \{\alpha, \beta - \alpha, \beta; \beta, \alpha \in R^3\}$ (6 marks)

As members of Kabaraka University family, we purpose at all times and in all places, to set apart in one's heart, Jesus as Lord. (1 Peter 3:15)

QUESTION 2 (20 MARKS)

- a) Use Gauss–Jordan method elimination process to solve the following system of linear equations:

$$2x_1 + 3x_2 + x_3 = 6$$

$$3x_1 + 2x_2 - 4x_3 = 12 \quad (6 \text{ marks})$$

$$x_1 - 2x_2 + 3x_3 = -3$$

- b) Use the row echelon form to solve the following system of linear equations:

$$x_1 + 2x_2 - 7x_3 = -4$$

$$2x_1 + x_2 + x_3 = -13 \quad (8 \text{ marks})$$

$$3x_1 + 9x_2 - 36x_3 = -33$$

- c) For what values of k does the homogenous system below have non- trivial solutions

$$kx + y - 3x = 0$$

$$x + ky - 3y = 0$$

(6 marks)

QUESTION 3 (20 MARKS)

For the matrix $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

- i) Show that $AB \neq BA$ (4 marks)

- ii) Find $A + B, A - B$. (6 marks)

- b) Find the area of a triangle with vertices at points $A(1,0), B(2,2), C(4,3)$

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- c) Use the determinate on matrix method to get the equation a line that have collinear points given as $A(1,8), B(-2,1)$ (4 Marks)

QUESTION 4 (20 MARKS)

- a) Use the inverse method to solve the system below,

$$2x_1 + 2x_2 - x_3 = 1$$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$-x_1 + 2x_2 + 3x_3 = 1 \quad (9 \text{ marks})$$

- b) i) Define the term basis S of a vector space V . (1 mark)

- ii) Show that the set $S = \{\tilde{u}_1 = (1,0,-1), \tilde{u}_2 = (1,2,1), \tilde{u}_3 = (0,-3,2)\}$ forms a basis for the vector space \mathbb{R}^3 (5 marks)

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- i) Express vector $\tilde{\mathbf{v}} = (3, -5, 7)$ as a linear combination of vectors in \mathcal{S} .
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QUESTION 5 (20 MARKS)

- a) Given the vectors $\tilde{\mathbf{u}} = (2, 1, -3)$ and $\tilde{\mathbf{v}} = (-1, 5, -4)$, find:
- i) the orthogonal projection of $\tilde{\mathbf{v}}$ on $\tilde{\mathbf{u}}$ (6 marks)
 - ii) the angle θ between $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$ (6 marks)
- b) If $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$ are vectors in R^3 , using the vector components of $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$, show that:
- i) $\tilde{\mathbf{v}} \cdot (\tilde{\mathbf{u}} \times \tilde{\mathbf{v}}) = 0$ (3 marks)
- c) Use Cramer's rule to solve the system below
- $$\begin{aligned}x_1 + 2x_2 - x_3 &= 1 \\2x_1 + x_2 + 4x_3 &= 2 \\3x_1 + 3x_2 + 4x_3 &= 1\end{aligned}$$
- (5 marks)

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MAIN CAMPUS

FIRST SEMESTER, 2017/2018 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE AND
BACHELOR OF EDUCATION ARTS

MATH 211: LINEAR ALGEBRA I

STREAM: Y2. S 1

TIME: 1.00-3.00 P.M

EXAMINATION SESSION: DECEMBER

DATE: 30/11/2017

INSTRUCTIONS

Instructions to candidates:

- **QUESTION ONE** is compulsory.
- Answer **QUESTION ONE** and any other **TWO** questions
- Begin each question on a separate page - Show your workings clearly

Question One [30 Marks]

a) Find k so that vector u and v are orthogonal

i. $u = (1, k, -3)$ and $v = (2, -5, 4)$ $v = (2, -5, 4)$ (3 marks)

ii. $u = (2, 3k, -4, 1, 5)$ and $v = (6, -1, 3, 7, 2k)$ (3 marks)

b) Find x and y given that $\begin{pmatrix} 1 & 2 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (5 marks)

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Jesus as Lord. (1 Peter 3:15)

c) Solve for x for $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ (6 marks)

d) For the given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

Show that

i. $\text{Det}AB = \text{Det}A \cdot \text{Det}B$ (6 marks)

ii. $\text{Det}(A+B) \neq \text{Det}A + \text{Det}B$ (3 marks)

e) Given $u = (6, 4, -2)$, $v = (0, 4, -6)$ and $w = (4, 12, 14)$ find

i. $v \times w$ (3 marks)

ii. $u \times (v \times w)$ (3 marks)

iii. $(u \times v)(v \times w)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Use Cramer's Rule to solve for the unknown variables x , y and z given that

$$\begin{aligned} 10x + 3y + 6z &= 76 \\ 4x + \quad \quad 5z &= 41 \\ 5x + 2y + 2z &= 34 \end{aligned} \quad (10 \text{ marks})$$

b) Verify that the Cauchy – Schwarz inequality holds for

i) $u = (75, 50)$ and $v = (100, 25)$ (3 marks)

ii) $u = (-80, 40, 20)$ and $v = (160, -80, -80)$ (3 marks)

c) Show that $u = (75, 0, 25, 0, 100, -25)$ and $v = (-50, 125, 0, 50, -75, -450)$ are orthogonal and verify that the Pythagorean Theorem holds. (4 marks)

QUESTION THREE (20 MARKS)

Given the set of simultaneous equations

$$\begin{aligned} 3x + 4y + 9z &= 45 \\ 4x + 5y + 2z &= 32 \\ 4x + 2y + 4z &= 32 \end{aligned}$$

i. Use Gaussian elimination method to solve for x , y and z (13 marks)

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- ii. Proceed to solve for x , y and z using Gauss-Jordan Elimination method (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $a = (1, -3, 4)$ and $b = (3, 4, 7)$

Find

- i. $\cos \theta$, where θ is the angle between a and b (4 marks)
- ii. $\text{proj.}(a, b)$, projection of a onto b (3 marks)
- iii. $d(a, b)$, the distance between a and b (2 marks)

- b) Use cross product and dot product to find the sine of the angle between the vectors $u = (8, 12, -24)$ and $v = (8, 12, 24)$ (6 marks)

- c) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$

Show that $A(B + C) = (A + B)C$ (5marks)

QUESTION FIVE (20 MARKS)

Find A^{-1} of the following matrix using determinant method and show that $A^{-1}A = AA^{-1} = I$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (20\text{Marks})$$

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SECOND SEMESTER, 2018/2019 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF (BSC) IN COMPUTER SCIENCE,

ECONOMICS&MATHEMATICS AND EDUCATION SCIENCE

MATH 211: LINEAR ALGEBRA I

STREAM: Y2S1

TIME: 2.00-4.00 PM

EXAMINATION SESSION: JAN-APRIL

YEAR: 9/04/2019

VENUE:AUDIT

COPIES: 70

INSTRUCTIONS:

Attempt Question ONE and any other TWO Questions

QUESTION ONE (20 MARKS)

- a) Solve the following system.

$$-2x_1 + 3x_2 = 1$$

$$6x_1 - 9x_2 = 2$$

(4Mks)

- b) Evaluate each of the following for the given matrix

$$A = \begin{bmatrix} -7 & 3 \\ 5 & 1 \end{bmatrix}$$

i. A^2 **(2mks)**

ii. A^3 **(2mks)**

iii. $P(A)$ where $p(x) = -6x^3 + 10x - 9$ **(5mks)**

- c) Find the determinant and the inverses of the following matrices

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(i) $A = \begin{bmatrix} 3 & 9 & 2 \\ 0 & 0 & 0 \\ -4 & -5 & 1 \end{bmatrix}$ (7mks)

(d) Given the vectors $A=2i+3j+k$ and $B=i+2i+4k$ find

i) $A \cdot B$ (3mks)

ii) $A \times B$ (3mks)

e). Suppose $B = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric. Find x and B (4mks)

QUESTION TWO (20MARKS)

a) Define an orthogonal matrix. (2marks)

b) Find the diagonal and trace of the following square matrix. (4marks)

$$A = \begin{bmatrix} 2 & -5 & 8 \\ 3 & -6 & -7 \\ 4 & 0 & -1 \end{bmatrix}$$

c) Let $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and let $f(x) = 2x^3 - 4x + 5$ and $g(x) = x^2 + 2x + 11$.

Find, (i) A^2 (ii) A^3 (iii) $f(A)$ (iv) $g(A)$ (9marks)

c) Given that $B = \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$, find the inverse of matrix B and show that $B^{-1}B = BB^{-1} = I$ (5marks)

QUESTION THREE (20 MARKS)

a) Given the system of linear Equations below solve the system by

(i) Gauss-Jordan elimination method (10 mks)

(ii) Cramer's Rule (10 mks)

$$4x_1 - 8x_2 - 4x_3 = 4$$

$$x_1 + x_2 + 3x_3 = 3$$

$$2x_1 - 2x_2 + 2x_3 = 2$$

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Comment briefly on your answer

QUESTION FOUR (20 MARKS)

a) Let $A = \begin{bmatrix} 5 & 2 \\ 0 & k \end{bmatrix}$. Find all numbers k for which A is a root of the polynomial,

$$f(x) = x^2 - 7x + 10 \quad (4\text{marks})$$

a) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ determine $C = 2(B + A) - (2A + 2B + B)$ (6marks)

b) Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$ find

(i) $\mathbf{a} \cdot \mathbf{b}$ (4marks)

(ii) $\mathbf{a} \times \mathbf{b}$ (6marks)

QUESTION FIVE (20 MARKS)

a) What is a vector space?. State any of its two axioms. (3mks)

b) Given that $V = R^3$ and $w = (a, b, 0)$; a, b are real numbers. Determine if W is a subspace of R^3 (5mks)

c) Given that $V = P_3$ $w = (P(x) = a_1x + a_2x^2 + a_3x^3)$. Determine if W is a subspace of P_3 (5mks)

d). If $A = \begin{bmatrix} 4 & 0 \\ 1 & -2 \end{bmatrix}$ then prove that $A^2 - 2A - 8I = 0$ where zero is a null matrix (5mks)

e). Find k so that vector \mathbf{u} and \mathbf{v} are orthogonal for $\mathbf{u} = (9, 9k, -27)$ and $\mathbf{v} = (18, -45, 36)$ (2mks)

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MAIN CAMPUS

FIRST SEMESTER 2018/2019 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION
ARTS/SCIENCE, COMPUTER SCIENCE; MATHEMATICS AND ECONOMICS;
ECONOMICS AND STATISTICS; ACTUARIAL SCIENCES.

MATH 211: Linear algebra I

EXAMINATION SESSION:DEC

DATE:26/11/2018

YEAR: Y2S1

TIME 2:00-4:00

VENUE:SMHS

COPIES:210

Instructions to Candidates

1. Time allowed: 2 hours
 2. This paper consists of FIVE questions.
 3. Attempt QUESTION ONE and any other TWO
 4. Start each question on a fresh page.
 5. Indicate question numbers clearly at the top of each page.
 6. Observe further instructions from the booklet
-

QUESTION ONE (30MARKS)

- a) Find x and y given that,

$$\begin{pmatrix} 1 & 2 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (5\text{marks})$$

- b) Use Cramer's rule to solve for the unknown variables x , y and z given that,

$$10x + 3y + 6z = 76$$

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$$4x + 5z = 41$$

$$5x + 2y + 2z = 34 \quad (10\text{marks})$$

c) Compute A^{-1} using cofactor method given that

$$\begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix} \quad (8\text{marks})$$

d) Use Gauss – Jordan elimination process to solve the following systems of linear equations. (7marks)

$$2x_1 - x_3 = 3$$

$$x_1 + x_2 + 4x_3 = 0$$

$$3x_1 + 2x_2 - x_3 = 1$$

QUESTION TWO (20MARKS)

a) If $A = \begin{pmatrix} 2 & -5 & 7 \\ -3 & 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 7 & -2 \\ 3 & -2 & -4 \end{pmatrix}$, find

(i) $A + B$ (2marks)

(ii) $A - B$ (2marks)

b) Let $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

find (i) the magnitude of \mathbf{d} given that $\mathbf{d} = 2\mathbf{a} + 3\mathbf{b} - \mathbf{c}$ (3marks)

(ii) the dot and cross product of \mathbf{b} and \mathbf{c} . (8marks)

c) Given that $B = \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$, find the inverse of matrix B and show that $B^{-1}B = BB^{-1} = I$ (5marks)

QUESTION THREE (20MARKS)

a) Find k so that vectors \mathbf{u} and \mathbf{v} are orthogonal.

(i) $\mathbf{U} = (12, k, -32)$ and $\mathbf{V} = (22, -52, 42)$ (3marks)

(ii) $\mathbf{U} = (20, 30k, -40, 10, 50)$ and $\mathbf{V} = (6, -1, 3, 7, 2k)$ (3marks)



b) Suppose $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ and suppose ,

$$f(x) = 2x^2 - 3x + 5, \text{ then find } f(A) \quad (5\text{marks})$$

c) Solve for x for ,

$$\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0 \quad (6\text{marks})$$

d) For the given matrices,

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix} \text{ show that } \text{Det}(AB) = \det(A) \det(B) \quad (6\text{marks})$$

QUESTION FOUR (20MARKS)

a) (i) Define an orthogonal matrix A. (2marks)

(ii) Verify that $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is an orthogonal matrix. (3marks)

b) Given the following matrix $A = \begin{pmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 3 & 0 \end{pmatrix}$

Find (i) $\det A$ (3marks)

(ii) Inverse of A. (6marks)

c) Use Cramer's Rule to solve the following systems of equations, (6marks)

$$-x + 2y - 3z = 1$$

$$2x + z = 0$$

$$3x - 4y - 4z = 2$$



QUESTION FIVE (20MARKS)

a) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ determine $C = 2(B + A) - (2A + 2B + B)$ (6marks)

b) Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$ find

(i) $\mathbf{a} \cdot \mathbf{b}$ (4marks)

(ii) $\mathbf{a} \times \mathbf{b}$ (6marks)

c) Find the distance between the following pair of points $P(-3, 1, 2)$ and $Q(7, 5, 2)$ (4marks)



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2016/2017 ACADEMIC YEAR

**FOR THE DEGREE OF BACHELOR OF SCIENCE IN
ECONOMICS AND MATHEMATICS**

MATH 211: LINEAR ALGEBRA I

Instructions:

- Answer question **ONE** and any other **TWO** questions
- Begin each question on a separate page
- Show your workings clearly

QUESTION ONE (30 MARKS)

a) Find k so that vector \mathbf{u} and \mathbf{v} are orthogonal

i) $\mathbf{U}=(12,k,-32)$ and $\mathbf{v}=(22,-52,42)$ (4 marks)

ii) $\mathbf{U}=(20,30k,-40,10,50)$ and $\mathbf{v}=(6,-1,3,7,2k)$ (4 marks)

b) Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and suppose $f(x) = 2x^2 - 3x + 5$, then find $f(A)$ (5 marks)

c) Solve for x for $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$ (6 marks)

d) For the given matrices $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 7 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 & 8 \\ 4 & -1 & 1 \\ 0 & 3 & 3 \end{pmatrix}$

Show that $\text{Det}(AB)=\text{det}(A)\text{det}(B)$ (6 marks)

- e). Given $u = (6i+, 4j, -2k)$, $v = (0i+, 4j, -6k)$ and $w = (4i+, 12j+, 14k)$
 $U \times (V \times W)$ (3 marks)

f). Given the matrix $\begin{bmatrix} 1 & 6 & 5 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ calculate the determinant (4mks)

QUESTION TWO (20 MARKS)

- a) Use Cramers Rule to solve for the unknown variables x_1 , x_2 and x_3 given that
 $12x_1 + 3x_2 + 6x_3 = 56$
 $2x_1 + 7x_3 = 41$
 $5x_1 + 2x_2 + 2x_3 = 34$ (10 marks)
- b) Verify that the Cauchy – Schwarz inequality holds for
- i) $u = (15, 10)$ and $v = (20, 5)$ (3 marks)
- ii) $U = (-20, 10, 5)$ and $v = (40, -20, -20)$ (3 marks)
- c) Show that $u = (75, 0, 25, 0, 100, -25)$ and $v = (-50, 125, 0, 50, -75, -450)$ are orthogonal and verify that the pythagorean Theorem holds. (4 marks)

QUESTION THREE (20 MARKS)

- a). Given the set of simultaneous equations
- $$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5 \end{aligned}$$
- i) Use Gaussian elimination method to solve for x_1 , x_2 and x_3 (13 marks)
- ii) Proceed to solve for x_1 , x_2 and x_3 using Gauss-Jordan Elimination method (7 marks)

QUESTION FOUR (20 MARKS)

- a) Let $u = (12, -32, 24)$ and $v = (32, 14, 17)$, Find
- i) $\cos \theta$, where θ is the angle between u and v (4 marks)
- ii) $\text{Proj}(u, v)$, projection of u onto v (3 marks)
- iii) $d(u, v)$, the distance between u and v (2 marks)

- b) Use cross product and dot product to find the angle between the vectors $u = (8, 12, -24)$ and $v = (8, 12, 24)$ (6 marks)

c) If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

Prove that $A(B+C) = AB + AC$

(5 marks)

QUESTION FIVE (20 MARKS)

- a). Find A^{-1} of the following matrix using determinant method and show that $A^{-1}A = AA^{-1} = I$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (20 \text{ marks})$$

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MAIN CAMPUS

FIRST SEMESTER, 2017 ACADEMIC YEAR
EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE
IN EDUCATION

MATH 211: LINEAR ALGEBRA I

STREAM: (PART TIME)

TIME: 11-1PM

EXAMINATION SESSION: APRIL

DATE: 14/04/2017

Instructions:

- Answer question **ONE** and any other **TWO** questions
- Begin each question on a separate page
- Show your workings clearly 1

QUESTION ONE (30 MARKS)

a) $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$

Show that $(AB)^{-1} = A^{-1}B^{-1}$ (6 marks)

b) Given that $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$. Calculate $p(x) = 2x^2 - 3x + 4$ (5 marks)

c) Find the components of vector $v = p_1p_2$ with initial point $p_1(2, -1, 4)$ and terminal point $p_2(7, 5, -8)$ (3 marks)

d) Given $u = (3, 2, -1)$, $v = (0, 2, -3)$ and $w = (2, 6, 7)$

i) $V \times W$ (3 marks)

ii) $U \times (V \times W)$ (3 marks)

- iii) $(\mathbf{U} \times \mathbf{V}) \times (\mathbf{V} \times \mathbf{W})$ (3 marks)
- iv) $\mathbf{U} \times (\mathbf{V} - 2\mathbf{W})$ (3 marks)
- e) If $\mathbf{u} = (1, 3, -2, 7)$ and $\mathbf{v} = (0, 7, 2, 2)$. Find the Euclidean space R^4 (3 marks)

QUESTION TWO (20 MARKS)

Given the following system of equations

$$x + y - z - 4 = 0$$

$$2x - 3y + 4z - 33 = 0$$

$$3x - 2y - 2z - 2 = 0$$

Solve these systems of equations using

- i) Gaussian elimination and Gauss-Jordan Elimination methods (10 marks)
- ii) Cramers Rule (10 marks)

QUESTION THREE (20 MARKS)

- a) If \mathbf{u} and \mathbf{a} are vectors in 2 or 3 space and if $a \neq 0$. Proof that

i) $proj_a \mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \cdot \mathbf{a}$ (Vector component of \mathbf{u} along \mathbf{a}) (4 marks)

ii) $\mathbf{u} - proj_a \mathbf{u} = \mathbf{u} - \frac{\mathbf{u} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \cdot \mathbf{a}$ (Vector component of \mathbf{u} orthogonal to \mathbf{a})
(4 marks)

- b) Let $\mathbf{u} = (2, -1, 3)$ and $\mathbf{a} = (4, -1, 2)$.

Find

- i) The vector component of \mathbf{u} along \mathbf{a} (3 marks)
- ii) The vector component of \mathbf{u} orthogonal to \mathbf{a} (3 marks)

- c) Use cross product to find the sine of the angle between the vectors $u = (2, 3, -6)$ and $v = (2, 3, 6)$ (6 marks)

QUESTION FOUR (20 MARKS)

- a) Obtain the inverse of the following matrix using row reduction method

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} \quad (10 \text{ marks})$$

- b) Obtain the inverse of the following matrix using determinant method

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix} \quad (10 \text{ marks})$$

QUESTION FIVE (20 MARKS)

- a) Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ be two nonzero vectors. If θ is the angle between u and v . Show that $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$ (6 marks)

- b) Let $u = (u_1, u_2, u_3, \dots, u_n)$ and $v = (v_1, v_2, v_3, \dots, v_n)$ are vectors in R^n . Show that $|u \cdot v| \leq \|u\| \|v\|$ (4 marks)

- c) Verify that the Cauchy – Schwarz inequality holds for

i) $u = (3, 2)$ and $v = (4, 1)$ (3 marks)

ii) $U = (-4, 2, 1)$ and $v = (8, -4, -4)$ (3 marks)

- d) Given $u = (-3, 2, 1, 0)$, $v = (4, 7, -3, 2)$ and $w = (5, -2, 8, 1)$. Find

i) $2u + 7v$ (2 marks)

ii) $(6v - w) - (4u + v)$ (2 marks)