Домашняя работа № 4 Шамаев И.

(1) 1) $x^2 - 2\cos x = -1$ $\quad x_1 \in [-1;0]$;

2 $\quad x_2 \in [0;1]$;

2) $x^2 - 2\cos x + 1 \pm x = 0$

$x = x + x^2 - 2\cos x + 1 = \varphi(x)$

$\varphi'(x) = 1 + 2x + 2\sin x$

$\varphi(x) = x + \frac{1}{2}(x^2 - 2\cos x + 1) \Rightarrow \varphi'(x) = 1 + x + \sin x \Rightarrow$

$\Rightarrow |\varphi'(x)| < 1$ на $[-1;0)$

$|1 + x + \sin x| < 1$

$x^{k+1} = x^k + \frac{1}{2}((x^k)^2 - 2\cos x^k + 1)$

$x = -x^2 + 2\cos x - 1 + x \Rightarrow \varphi(x) = x + \frac{1}{2}(-x^2 + 2\cos x - 1)$

$\varphi'(x) = 1 - x - \sin x$

на $[0, 1]$ $|\varphi'(x)| < 1$

$x^{k+1} = x^k + \frac{1}{2}(-(x^k)^2 + 2\cos x^k - 1)$

Ответ: $x^{k+1} = x^k + \frac{1}{2}((x^k)^2 - 2\cos x^k + 1)$

$x^{k+1} = x^k + \frac{1}{2}(-(x^k)^2 + 2\cos x^k - 1)$

(2) $x^{k+1} = \dfrac{4 - e^{x^k}}{3}$

$x^{k+1} = \ln(4 - 3x^k)$

$x^{k+1} = 1 + \dfrac{x^k - e^{x^k}}{4}$

$e^x + 3x = 4 \quad - f(x)$

1) $\varphi(x) = \dfrac{4 - e^x}{3}$

$\varphi'(x) = \dfrac{-3e^x}{9} = \dfrac{e^x}{3}$; $|\varphi'(x)| < 1 \Rightarrow \dfrac{e^x}{3} < 1 \Rightarrow x < \ln 3 \Rightarrow$

$x \in (-\infty; \ln 3)$ Сходится
$\approx 1{,}097$

$f(x)$:

$e^x + 3x - 4 = 0$

$x \approx 0{,}677$

2) $\varphi(x) = \ln(4-3x)$

$\varphi'(x) = \frac{-3}{4-3x}$

$|\varphi'(x)| < 1 \Rightarrow \left.\begin{array}{l} \frac{-3}{4-3x} > 1 \\ \frac{3}{4-3x} < 1 \end{array}\right] \Rightarrow \left.\begin{array}{l} x > \frac{7}{3} \\ x < \frac{1}{3} \end{array}\right] \Rightarrow \left.\begin{array}{l} x > \frac{7}{3} \approx 2{,}3 \\ x < \frac{1}{3} \approx 0{,}33 \end{array}\right\} \Rightarrow$ расходит.

3) $\varphi(x) = 1 + \frac{x - e^x}{4}$

$\varphi'(x) = \frac{1-e^x}{4}$

$|\varphi'(x)| < 1 \Rightarrow -1 < \frac{1-e^x}{4} < 1$

$-5 < -e^x < 3$

$-3 < e^x < 5$

$x < \ln 5 \approx 1{,}61$ Сходится

③

$\begin{cases} 2x^2 + 3y^2 = 1 \\ 10xy(x+y) = 1 \end{cases} \Rightarrow \begin{cases} x = \sqrt{\frac{1-3y^2}{2}} = \varphi_1 \\ y = \frac{1}{10x(x+y)} = \varphi_2 \end{cases}$

$\frac{\partial \varphi_1}{\partial x} = 0$; $\frac{\partial \varphi_1}{\partial y} = \frac{-6y\sqrt{2}}{2\sqrt{1-3y^2}\cdot 2} = \frac{-3y}{\sqrt{2-6y^2}}$

$\frac{\partial \varphi_2}{\partial y} = \frac{-1}{10x(x+y)^2}$; $\frac{\partial \varphi_2}{\partial x} = \frac{(20x+y)(-1)}{10^2x^2(x+y)^2} = \frac{-2x-y}{10x^2(x+y)^2}$

$$\left|\frac{\partial \varphi_1}{\partial x}\right| + \left|\frac{\partial \varphi_1}{\partial y}\right| < 1 \qquad |0| + \left|\frac{-3y}{\sqrt{2-6y^2}}\right| < 1$$

$$\left|\frac{\partial \varphi_2}{\partial x}\right| + \left|\frac{\partial \varphi_2}{\partial y}\right| < 1 \qquad \left|\frac{-1}{10x(x+y)^2}\right| + \left|\frac{-2x-y}{10x^2(x+y)^2}\right| < 1$$

ИП: $\begin{cases} x_1^{k+1} = \sqrt{\dfrac{1-3y^{(k)}}{2}} \\ y_2^{k+1} = \dfrac{1}{10x^{(k)}(x^{(k)}+y^{(k)})} \end{cases}$

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$$tg x + 1 = x \ln x$$

$$\underbrace{tg x + 1 - x \ln x}_{f(x)} = 0 \qquad \text{на } [4; 5]$$

$$f'(x) = \frac{1}{\cos^2 x} - \ln x - 1$$

$$x^{(k+1)} = x^{(k)} - \frac{tg x^{(k)} - x^{(k)} \ln x^{(k)} + 1}{\frac{1}{\cos^2 x^{(k)}} - \ln x^{(k)} - 1}$$

$x_0 = 4{,}5$

$x_1 = 4{,}5 + \frac{1{,}151}{20} =$

$= 4{,}56$

$$x_2 = 4{,}56 - \frac{0{,}592}{40{,}879} = 4{,}545$$

$$x_3 = 4{,}545 - \frac{0{,}0369}{33{,}5112} = 4{,}5439$$

. . . .

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$$\begin{cases} 2xy = -1 \\ \cos(x+2y) + 5x = 2 \end{cases} \Rightarrow \begin{cases} x = -\dfrac{1}{2y} \\ y = \dfrac{\arccos(2-5x) - x}{2} \end{cases}$$

$\exists$ решение, при $x \in (0; 0{,}5]$

$x_0 = 0{,}25$

$y_0 = -1$

$$\begin{cases} 2xy + 1 = f_1(x,y) \\ \cos(x+2y) + 5x - 2 = f_2(x,y) \end{cases}$$

$$\frac{\partial f_1(x,y)}{\partial x} = 2y \qquad \frac{\partial f_1(x,y)}{\partial y} = 2x$$

$$\frac{\partial f_1(x,y)}{\partial x} = -\sin(x+2y)+5 \qquad \frac{\partial f_2(x,y)}{\partial y} = -2\sin(x+2y)$$

$$\begin{cases} x^{(k+1)} = x^k + g_k \\ y^{k+1} = y^k + h_k \end{cases}$$ где h_k и g_k - решение системы:

$$\begin{cases} \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial x} g_k + \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial y} h_k = -f_1(x^{(k)}, y^{(k)}) \\ \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial x} g_k + \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial y} h_k = -f_2(x^{(k)}, y^{(k)}) \end{cases}$$