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Домашняя работа № 1

№ 1.1

$y(0)=2$ $\qquad \varphi(x) = a+bx+cx^2+dx^3$

$y(1)=3$

$y(2)=4$

$y(4)=6$

I способ

Интерполяционная формула Лагранжа

$$L_3(x) = \sum_{i=0}^{3} y_i \prod_{\substack{k=0\\k\neq i}}^{3} \frac{x-x_k}{x_i-x_k}$$

$$L_3(x) = \frac{2(x-1)(x-2)(x-4)}{(0-1)(0-2)(0-4)} + \frac{3(x-0)(x-2)(x-4)}{(1-0)(1-2)(1-4)} +$$

$$+\frac{4(x-0)(x-1)(x-4)}{(2-0)(2-1)(2-4)} + \frac{6(x-0)(x-1)(x-2)}{(4-0)(4-1)(4-2)} = -\frac{1}{4}(x-1)(x-2)(x-4)+$$

$$+x(x-2)(x-4) - x(x-1)(x-4) + \frac{1}{4}x(x-1)(x-2) = (x-1)(x-2)-x(x-4)$$

$$= \underline{x+2}$$

II способ

Интерполяционная формула Ньютона

$$P_3(x) = y_0 + \sum_{k=1}^{3} (x-x_0)(x-x_1)\ldots(x-x_{k-1})\, y(x_0, x_1, \ldots, x_k)$$

$$y(x_0,x_1) = \frac{y(x_0)-y(x_1)}{x_0-x_1} = \frac{2-3}{0-1} = 1$$

$y(x_1,x_2)=1$

$y(x_2,x_3)=1$

$y(x_0, x_1, x_2) = \dfrac{y(x_0,x_1) - y(x_1,x_2)}{x_0 - x_2} = 0$

$y(x_1, x_2, x_3) = 0$

$y(x_0, x_1, x_2, x_3) = \dfrac{y(x_0, x_1, x_2) - y(x_1, x_2, x_3)}{x_0 - x_3} = 0$

$P_3 = 2 + (x - 0) \cdot 1 = \underline{2 + x}$

III способ

Метод неопределенных коэффициентов

$\varphi(0) = a = 2$

$\varphi(1) = a + b + c + d = 3 \Rightarrow b + c + d = 1 \quad (1)$

$\varphi(2) = a + 2b + 4c + 8d = 4 \Rightarrow b + 2c + 4d = 1 \quad (2)$

$\varphi(4) = a + 4b + 16c + 64d = 6 \Rightarrow b + 4c + 16d = 1 \quad (3)$

(1) $b = 1 - c - d$

(2) $\cancel{1} - c - d + 2c + 4d = \cancel{1}$

$c = -3d$

(3) $1 + 3d - d - 12d + 16d = 1$

$7d = 0$

$\underline{d = 0} \Rightarrow \underline{c = 0}$

$\underline{b = 1} \quad \underline{a = 2}$

$\varphi = a + bx + cx^2 + dx^3 = \underline{2 + x}$

№1.2

$f(x) = e^x, \quad x = 0,3$

$x_0 = 0$

$x_1 = 0,2$

$x_2 = 0,4$

$$|y(x)-L_2(x)| \le \frac{M_3}{3!}|\omega_3(x)|$$

$$M_3 = \sup|y'''(x)| = \sup|e^x| = e^{0,4}$$

$$\omega_3(x) = \prod_{i=0}^{2}(x-x_i) = x(x-0,2)(x-0,4)$$

$$|y(0,3)-L_2(0,3)| \le \underbrace{\frac{e^{0,4}}{6}|0,3\cdot 0,1\cdot 0,1|}_{\approx 0,0007459}$$

Ответ: 0,0007459

№ 1.3

$y(0)=0$

$y(\frac{\pi}{4})=1$

$y(\frac{\pi}{2})=1$

$y(\frac{3\pi}{4})=1$

$y(\pi)=1$

$$G_2(x) = \sum_{k=0}^{2} a_k \cos(kx) + \sum_{k=1}^{2} b_k \sin(kx)$$

$$a_0 = \frac{1}{N}\sum_{i=0}^{N-1} f_i$$

$$a_k = \frac{2}{N}\sum_{i=0}^{N-1} f_i \cos(kx_i), \quad k=1,2$$

$$b_k = \frac{2}{N}\sum_{i=0}^{N-1} f_i \sin(kx_i), \quad k=1,2$$

$5a_0 = 0+1+1+1+1=4$

$5a_1 = 2(0+\cos\frac{\pi}{4}+\cos\frac{\pi}{2}+\cos\frac{3\pi}{4}+\cos\pi) = 2(\cancel{\frac{\sqrt{2}}{2}}-\cancel{\frac{\sqrt{2}}{2}}-1) = -2$

$5a_2 = 2(0+\cos\frac{\pi}{2}+\cos\pi+\cos\frac{3\pi}{2}+\cos 2\pi) = -1+1=0$

$5b_1 = 2(\sin\frac{\pi}{4}+\sin\frac{\pi}{2}+\sin\frac{3\pi}{4}+\sin\pi) = 2(\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}) =$

$= 2+2\sqrt{2}$

$5b_2 = 2(\sin\frac{\pi}{2}+\sin\pi+\sin\frac{3\pi}{2}+\sin 2\pi) = 2(1-1)=0$

$G_2(x) = \frac{4}{5} - \frac{2}{5}\cos x + \frac{2(1+\sqrt{2})}{5}\sin x$: Ответ:

1.4

$$P_3(x) = a_3x^3 + 2x^2 + a_1x + a_0 \quad , \quad x \in [3,5]$$

$$\overline{T}_n^{[3,5]}(x) = (b-a)^n 2^{1-2n} T_n\left(\frac{2x-b-a}{b-a}\right) \quad , \quad x \in [a,b]$$

$$T_0(x) = 1$$

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$\overline{T}_3^{[3,5]}(x) = 2^3 2^{-5} T_3\left(\frac{2x-8}{2}\right) = \frac{1}{4} T_3(x-4)$$

$$T_3(x-4) = 4(x-4)^3 - 3(x-4) = 4x^3 - 48x^2 + 189x - 244$$

$$T_3^{(2)}(x-4) = 24x - 96$$

$$P_3(x) = 4\frac{T_3(x-4)}{T_3^{(2)}(0)} = -\frac{1}{6}x^3 + 2x^2 - \frac{189}{24}x + \frac{61}{6}$$

Ответ: $-\frac{1}{6}x^3 + 2x^2 - \frac{189}{24}x + \frac{61}{6}$

1.5

$$y(x) = \sin x \quad , \quad L_2[0,\pi]$$

$$\varphi(x) = C_1 + C_2x + C_3x^2$$

$$(f_1, f_1) = \int_0^\pi dx = \pi$$

$$(f_1, f_2) = \int_0^\pi x\,dx = \frac{\pi^2}{2}$$

$$(f_1, f_3) = \int_0^\pi x^2 dx = \frac{\pi^3}{3}$$

$$(f_2, f_2) = \int_0^\pi x^2 dx = \frac{\pi^3}{3}$$

$$(f_2, f_3) = \int_0^\pi x^3 dx = \frac{\pi^4}{4}$$

$$(f_3, f_3) = \int_0^\pi x^4 dx = \frac{\pi^5}{5}$$

~~$(y, f_1) = \int_0^\pi \sin$~~

$$\sum_{k=1}^{3} C_k (f_k, f_i) = (y, f_i)$$

$$(y, f_1) = \int_0^{\pi} \sin x\,dx = (-\cos x)\Big|_0^{\pi} = 2$$

$$(y, f_2) = \int_0^{\pi} x\sin x\,dx = (-x\cos x + \sin x)\Big|_0^{\pi} = \pi$$

$$(y, f_3) = \int_0^{\pi} x^2 \sin x\,dx = (-x^2\cos x + 2x\sin x + 2\cos x)\Big|_0^{\pi} = \pi^2 - 4$$

Получим систему:

$$C_1\pi + C_2\frac{\pi^2}{2} + C_3\frac{\pi^3}{3} = 2 \quad (1)$$

$$C_1\frac{\pi^2}{2} + C_2\frac{\pi^3}{3} + C_3\frac{\pi^4}{4} = \pi \quad (2)$$

$$C_1\frac{\pi^3}{3} + C_2\frac{\pi^4}{4} + C_3\frac{\pi^5}{5} = \pi^2 - 4 \quad (3)$$

$$C_1 = \frac{2 - C_2\frac{\pi^2}{2} - C_3\frac{\pi^3}{3}}{\pi} \quad (1) \quad = \frac{2}{\pi} - C_2\frac{\pi}{2} - C_3\frac{\pi^2}{3}$$

(2) $$\frac{\cancel{2\pi}}{2} - C_2\frac{\pi^3}{4} - C_3\frac{\pi^4}{6} + C_2\frac{\pi^3}{3} + C_3\frac{\pi^4}{4} = \cancel{\pi}$$

$$C_2\frac{\pi^3}{12} = -C_3\frac{\pi^4}{12}$$

$$C_2 = -C_3\pi$$

(3) $$\frac{2\pi^2}{3} - C_2\frac{\pi^4}{6} - C_3\frac{\pi^5}{9} + C_2\frac{\pi^4}{4} + C_3\frac{\pi^5}{5} = \pi^2 - 4$$

$$\frac{2\pi^2}{3} + C_3\frac{\pi^5}{6} - C_3\frac{\pi^5}{9} - C_3\frac{\pi^5}{4} + C_3\frac{\pi^5}{5} = \pi^2 - 4$$

$$\frac{2\pi^2}{3} + C_3\frac{30\pi^5}{180} - C_3\frac{20\pi^5}{180} - C_3\frac{45\pi^5}{180} + C_3\frac{36\pi^5}{180} = \pi^2 - 4$$

$$C_3\frac{\pi^5}{180} = \frac{\pi^2 - 12}{9} \Rightarrow C_3 = \frac{60\pi^2 - 720}{\pi^5}$$

$$C_2 = \frac{-60\pi^2 + 720}{\pi^4}$$

$$C_1 = \frac{2}{\pi} - \left(\frac{-60\pi^2 + 720}{2\pi^3}\right) - \left(\frac{60\pi^2 - 720}{3\pi^3}\right) =$$

$$= \frac{2}{\pi} + \frac{30}{\pi} \cancel{+720} - \frac{20}{\pi} \cancel{+720} - \frac{360}{\pi^3} + \frac{240}{\pi^3} =$$

$$= \frac{12}{\pi} - \frac{120}{\pi^3} = \frac{12\pi^2 - 120}{\pi^3}$$

$$\varphi(x) = \frac{120\pi^2 - 720}{\pi^3} + \frac{-60\pi^2 + 720}{\pi^4} x + \frac{60\pi^2 - 720}{\pi^5} x^2$$: Ответ

№ 1.6

$y(x) = x^3$, $x \in [0, 1]$

$y(x)$ – нечетная => МНРП не содержит четных степеней, а первая и вторая степени совпадают ($Q_2^0(x) = Q_1^0(x)$)

$$x_m = \frac{b+a}{2} + \frac{b-a}{2} \cos\left(\frac{\pi(2m+1)}{2(n+1)}\right), \quad m = 0, 1 \ldots, n$$

$$x_0 = \frac{1}{2} + \frac{1}{2}\cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{4}$$

$$x_1 = \frac{1}{2} + \frac{1}{2}\cos\frac{3\pi}{6} = \frac{1}{2}$$

$$x_2 = \frac{1}{2} + \frac{1}{2}\cos\frac{5\pi}{6} = \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$y(x_0) = \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)^3 = \frac{1}{64}(2 - \sqrt{3})^3$$

$$P_2(x) = y_0 + \sum_{k=1}^{2} (x - x_0) \ldots (x - x_{k-x})\, y(x_0, x_1, \ldots, x_k)$$

$$y(x_0, x_1) = \frac{15 + 6\sqrt{3}}{16}$$

$$y(x_1, x_2) = \frac{15 - \cancel{3}6\sqrt{3}}{16}$$

$$y(x_0, x_1, x_2) = \frac{3}{2}$$

$$P_2(x) = \frac{1}{64}(2+\sqrt{3})^3 + \left(x - \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)\right)\frac{15+6\sqrt{3}}{16} + \left(x - \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)\right) \cdot \left(x - \frac{1}{8}\right)\frac{3}{2} = \frac{3}{2}x^2 - \frac{9}{16}x + \frac{1}{32}$$

Ответ: $\frac{3}{2}x^2 - \frac{9}{16}x + \frac{1}{32}$