ФДП

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Интеграл

$$U(\bar{x}) = \frac{1}{2\pi}\int_{\Gamma^+} \alpha^2 P(\alpha, z)\, J_1(\alpha r) \cos\varphi \, d\alpha$$

$$V(\bar{x}) = \frac{1}{2\pi}\int_{\Gamma^+} \alpha^2 P(\alpha, z)\, J_1(\alpha r) \sin\varphi \, d\alpha \qquad \bar{x} = (x, y, z)$$

$$W(\bar{x}) = \frac{1}{2\pi}\int_{\Gamma^+} -\alpha\, R(\alpha, z)\, J_0(\alpha r)\, d\alpha$$

Γ^+ - почти совпадает с полуосью $\operatorname{Re} z \geqslant 0$ обходя снизу полюс $\xi : \Delta(\xi) = 0$.

$$P(\alpha, z) = P_1(\alpha) e^{\sigma_1 z} + P_2(\alpha) e^{\sigma_2 z}$$

$$R(\alpha, z) = R_1(\alpha) e^{\sigma_1 z} + R_2(\alpha) e^{\sigma_2 z}$$

$$P_1(\alpha) = -\left(\alpha^2 - \frac{1}{2}\varkappa_2^2\right) \cdot \frac{1}{\Delta(\alpha)}; \quad P_2(\alpha) = \frac{\sigma_1 \sigma_2}{\Delta(\alpha)};$$

$$R_1(\alpha) = -\left(\alpha^2 - \frac{1}{2}\varkappa_2^2\right) \frac{\sigma_1}{\Delta(\alpha)}; \quad R_2(\alpha) = \frac{\sigma_1 \alpha^2}{\Delta(\alpha)};$$

$$\Delta(\alpha) = 2\mu\left[-\left(\alpha^2 - \frac{1}{2}\varkappa_2^2\right)^2 + \alpha^2 \sigma_1 \sigma_2\right.$$

$$\varkappa_1^2 = \frac{\rho\omega^2}{\lambda + 2\mu}, \ \varkappa_2 = \frac{\rho\omega^2}{\mu}, \ \sigma_n = \sqrt{\alpha^2 - \varkappa_n^2}, n = 1, 2, \quad \operatorname{Re}\sigma_n \geqslant 0, \operatorname{Im}\sigma_n \leqslant 0$$

φ, r : $x = r\cos\varphi$, $y = r\sin\varphi$, $r = \sqrt{x^2 + y^2}$, $\varphi \in [0; 2\pi)$

Вычеты

$$u(\bar{x}) = \frac{ih}{2}(U^+ - U^-); \quad v(\bar{x}) = \frac{ih}{2}(V^+ - V^-); \quad w(\bar{x}) = \frac{ih}{2}(W^+ - W^-);$$

$$F^+ = F(\xi^+); \quad F^- = F(\xi^-); \quad \xi^+ = \xi + h; \quad \xi^- = \xi - h;$$

$$U(\alpha, r, \varphi, z) = \alpha^2 \cdot P(\alpha, z) \cdot \cos\varphi \cdot \frac{1}{2} H_1^1(\alpha r)$$

$$V(\alpha, r, \varphi, z) = \alpha^2 \cdot P(\alpha, z) \cdot \sin\varphi \cdot \frac{1}{2} H_1^1(\alpha r)$$

$$W(\alpha, r, \varphi, z) = -\alpha\, R(\alpha, z)\, H_0^1(\alpha r)$$

H_i^j – функция Ханкеля j рода, i-порядка, находится средствами NAG FORTRAN Library

Асимптотика, метод стационарной фазы

$$\bar{u} = (u, v, w); \quad \bar{u}(R, \varphi, \psi) = \bar{u}_p(R, \varphi, \psi) + \bar{u}_s(R, \varphi, \psi)$$

$$x = R\cos\varphi\sin\psi, \quad y = R\sin\varphi\sin\psi, \quad z = R\cos\psi,$$
$$0 \leqslant \varphi \leqslant 2\pi, \quad \pi/2 \leqslant \psi \leqslant \pi, \quad 0 \leqslant R \leqslant \infty, \quad R = \sqrt{x^2 + y^2 + z^2}.$$

$$\alpha_1^p = -\varkappa_1 \cdot \sin\psi\cos\varphi; \quad \alpha_2^p = -\varkappa_1 \cdot \sin\psi\sin\varphi; \quad \alpha^p = \varkappa_1 \sin\psi$$

$$\alpha_1^s = -\varkappa_2 \cdot \sin\psi\cos\varphi; \quad \alpha_2^s = -\varkappa_2 \cdot \sin\psi\sin\varphi; \quad \alpha^s = \varkappa_2 \sin\psi$$

$$u_p = \frac{i\cos\psi}{2\pi R}\left(-i\alpha_1^p \cdot P_1(\alpha^p)\right)\varkappa_1 \exp(iR\varkappa_1)$$

$$v_p = \frac{i\cos\psi}{2\pi R}\left(-i\alpha_2^p \cdot P_1(\alpha^p)\right)\varkappa_1 \exp(iR\varkappa_1)$$

$$w_p = \frac{i\cos\psi}{2\pi R} R_1(\alpha^s)\, \varkappa_1 \exp(iR\varkappa_1)$$

$$U_s = \frac{i\cos\psi}{2\pi R}\left(-i\alpha_1^s \cdot P_2(\alpha^s)\right)\varkappa_2 \exp(iR\varkappa_2)$$

$$V_s = \frac{i\cos\psi}{2\pi R}\left(-i\alpha_2^s \cdot P_2(\alpha^s)\right)\varkappa_2 \exp(iR\varkappa_2)$$

$$W_s = \frac{i\cos\psi}{2\pi R} R_2(\alpha^s)\, \varkappa_2 \exp(iR\varkappa_2)$$

Входные данные

Кирилл, напиши их!