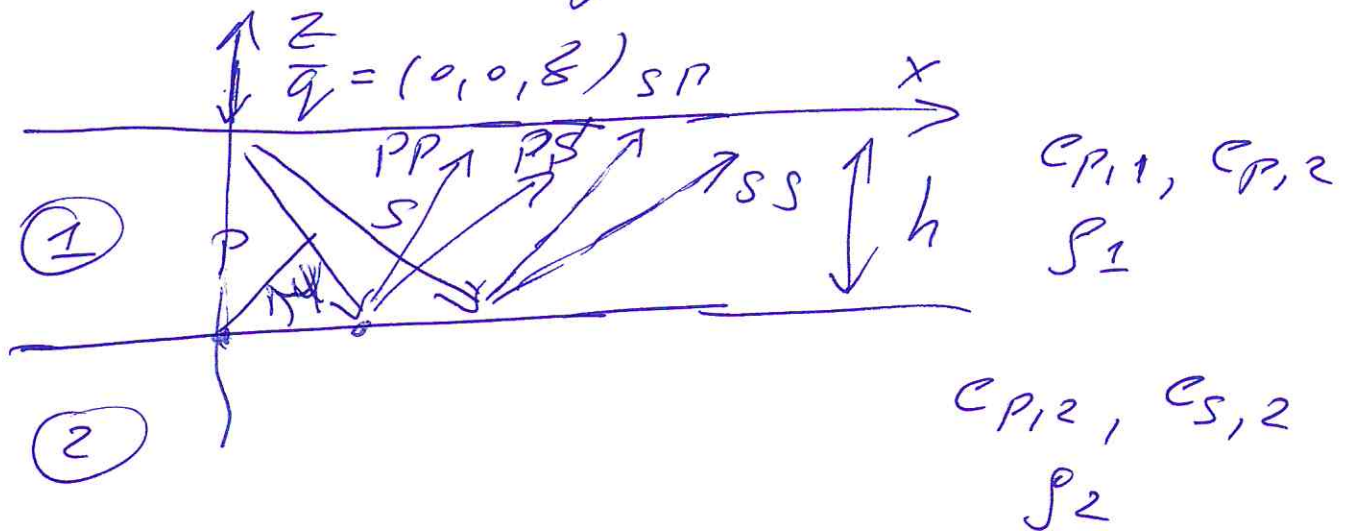
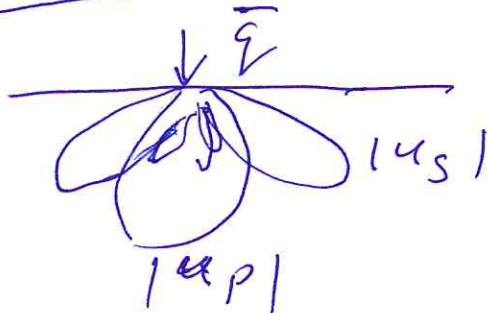


Асимптотическая орг. волн (2D)



1. Поле неоднородности $\bar{u}_0 = \bar{u}_P + \bar{u}_S =$



$$= \bar{u}_1 + \bar{u}_2$$

$$\mathbb{E} \parallel \bar{u}_0(\alpha, z) = K(\alpha, z) \bar{Q}(\alpha)$$

$$\text{где } \bar{Q} = (0, 0, 1)^T \bar{u}_0 = \begin{pmatrix} -i\alpha P \\ R \end{pmatrix}$$

$$P(\alpha, z) = P_1(\alpha) e^{b_1 z} + P_2(\alpha) e^{b_2 z}$$

$$R(\alpha, z) = R_1(\alpha) e^{b_1 z} + R_2(\alpha) e^{b_2 z}$$

$$\Rightarrow \bar{U}_0 = \begin{pmatrix} -i\alpha P_1 \\ R_1 \end{pmatrix} e^{\epsilon_1 z} + \begin{pmatrix} -i\alpha P_2 \\ R_2 \end{pmatrix} e^{\epsilon_2 z}$$

$$\begin{cases} P_1 = -\gamma^2 / \Delta \\ R_1 = -\epsilon_1 \gamma^2 / \Delta \end{cases} \quad \begin{cases} P_2 = \epsilon_1 \epsilon_2 / \Delta \\ R_2 = \alpha^2 \epsilon_1 / \Delta \end{cases} \quad \begin{matrix} (NVC-2000 \\ \text{quest} \\ \text{eqn. 41} \end{matrix}$$

$$\gamma^2 = \alpha^2 - x_2^2 / 2$$

$$\Delta = 2\mu [-\gamma^4 + \alpha^2 \epsilon_1 \epsilon_2]$$

Асимптотика \bar{U}_0

Метод стая. гради [Фогорик]

$$\| F(R) = \int_D f(\bar{x}) e^{iR\theta(\bar{x})} d\bar{x} \quad \text{— m-кратный интегр.}$$

$$R = |\bar{x}|$$

$$\text{— стая. точка } \bar{x}_0: \nabla \theta(\bar{x}_0) = 0$$

$$\text{— невырожденное ст. б., если } |\theta''(\bar{x}_0)| \neq 0$$

$$\text{где } \theta'' = \left[\frac{\partial^2 \theta}{\partial x_i \partial x_j} \right]_{i,j=1}^m \quad \text{— матрица Якоби } m \times m$$

$$\text{— } \theta_0'' = \theta''(\bar{x}_0), \quad \text{sgn } \theta_0'' = N^+ - N^-$$

$$\text{— } N^\pm \text{ — число собств. значений } \theta_0'' \text{ с положительными/отрицательными значениями}$$

Введем непериодическую част. \bar{x}_0 :

$$\| F_0(R) = \left(\frac{2\bar{x}}{R}\right)^{m/2} f(\bar{x}_0) e^{iR\theta_0} e^{i\frac{\pi}{4} \text{sgn} \theta_0''} \times |\det \theta_0''|^{-\frac{1}{2}} (1 + O(R^{-1})),$$

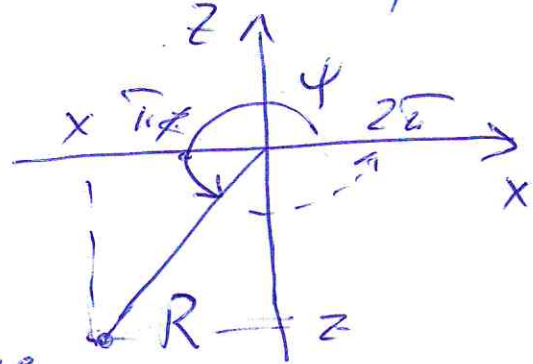
$R \rightarrow \infty$

У нас, $m=1$

$$\| \bar{u}_0 = \sum_{n=1}^2 \bar{A}_n(x) e^{b_n z}, \quad \bar{A}_n = \begin{pmatrix} -i d P_n \\ R_n \end{pmatrix}$$

$$\| \bar{u}_n = \frac{1}{2\pi} \int_{\Gamma} \bar{A}_n(x) e^{b_n z - i d x} d\alpha, \quad n=1, 2$$

$$\begin{cases} x = R \cos \psi \\ z = R \sin \psi \leq 0 \end{cases}$$



$0 < \psi < 2\pi$ - направление
по часовой стрелке

$$b_n = \begin{cases} -i \sqrt{x_n^2 - d^2}, & d^2 < x_n^2 \\ \sqrt{d^2 - x_n^2}, & d^2 > x_n^2 \end{cases}$$

$$\| \theta_n = -(\sqrt{x_n^2 - d^2} \cdot \sin \psi + d \cos \psi) -$$

$n=1, 2$

графике при
где $d^2 < x_n^2$

-4-

$$\theta_n' = \frac{x \sin \psi}{\sqrt{x_n^2 - x^2}} - \cos \psi = 0 \Rightarrow$$

$$\Rightarrow x_0 = \pm x_n |\cos \psi|$$

Второй случай:

$$\sqrt{x_n^2 - x_0^2} = x_n |\sin \psi| = -x_n \sin \psi > 0,$$

т.е. $\sin \psi < 0$ где $\forall \psi \in [\pi, 2\pi]$

тогда

$$\theta_n' = \frac{\pm x_n \cancel{\sin \psi} |\cos \psi|}{-x_n \cancel{\sin \psi}} - \cos \psi =$$

$$= \mp |\cos \psi| - \cos \psi; \quad \cos \psi = \begin{cases} < 0, & \psi < \frac{3\pi}{2} \\ > 0, & \psi > \frac{3\pi}{2} \end{cases}$$

$$\Rightarrow \theta_n' = \begin{cases} -|\cos \psi| - \cos \psi = 0, & \pi < \psi < \frac{3\pi}{2} \\ |\cos \psi| - \cos \psi = 0, & \frac{3\pi}{2} < \psi < 2\pi \end{cases}$$

т.е. верхний случай в I кв. и нижний в II.

$$x_0 = \begin{cases} x_0 |\cos \psi| = -x_n \cos \psi, & \pi < \psi < \frac{3\pi}{2} \\ -x_0 |\cos \psi| = -x_n \cos \psi, & \frac{3\pi}{2} < \psi < 2\pi \end{cases}$$

т.е. где $\forall \psi$

$$x_0 = -x_n \cos \psi, \quad \pi < \psi < 2\pi$$

$$\theta_n(\alpha_0) = -(-x_n \sin^2 \psi + x_n \cos^2 \psi) = x_n$$

$$\begin{aligned} \theta_n''(\alpha_0) &= \sin \psi \left[\frac{1}{\sqrt{x_n^2 - \alpha_0^2}} - \frac{\alpha_0}{x_n^2 - \alpha_0^2} \left(-\frac{\alpha_0}{\sqrt{x_n^2 - \alpha_0^2}} \right) \right] \\ &= \cancel{x_n} \frac{\sin \psi}{x_n} \left[\frac{1}{-\sin \psi} - \frac{-\cos \psi}{\sin^2 \psi} \left(\frac{\cos \psi}{-\sin \psi} \right) \right] \\ &= -\frac{\sin \psi}{x_n} \cdot \frac{\sin^2 \psi + \cos^2 \psi}{\sin^2 \psi} = -\frac{1}{x_n \sin^2 \psi} < 0 \end{aligned}$$

$$\Rightarrow \underline{\text{sgn} \theta_n''(\alpha_0) = -1}$$

Таким образом в формуле выше с.с.

$$\parallel \left(\frac{2\bar{u}}{R} \right)^{m/2} \Leftrightarrow \sqrt{\frac{2\bar{u}}{2}}, R \Leftrightarrow r = |\bar{x}| = \sqrt{x^2 + z^2}$$

$$\parallel f(\bar{x}_0) \Leftrightarrow A_n(\alpha_0) = \bar{a}_n(\psi)$$

$$\parallel e^{iR\theta_0} \Leftrightarrow e^{ix_n z} ; e^{i\frac{\bar{u}}{4} \text{sgn} \theta_0''} = e^{-i\frac{\pi}{4}} = \sqrt{-i} =$$

$$\parallel |\det \theta_0''|^{-1/2} \Leftrightarrow \frac{1}{\sqrt{x_n \sin^2 \psi}} = \frac{1}{\sqrt{x_n} (-\sin \psi)} = 1/\sqrt{i}$$

$$\Rightarrow \underline{-\sqrt{x_n} \sin \psi \neq 0}$$

Для \bar{u}_n получаем

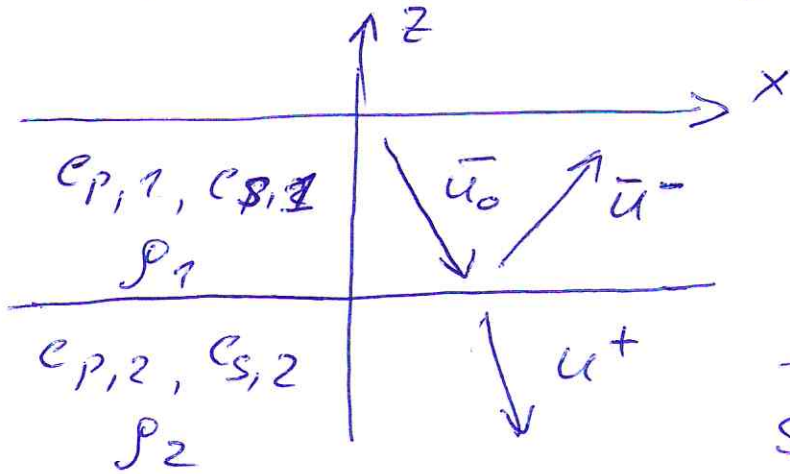
$$\parallel \bar{u}_n = \frac{x_n |\sin \psi|}{\sqrt{2\pi i}} \bar{a}_n(\psi) e^{i x_n z} \sqrt{x_n z} \left[1 + O\left(\frac{1}{x_n z}\right) \right]$$

$x_n z \rightarrow \infty, n=1,2.$

$$\bar{a}_n(\psi) = \bar{A}_n(\alpha_n),$$

$$\alpha_n = -x_n \cos \psi, \quad \pi_n < \psi < 2\pi$$

2. Ограничение \bar{u}_0 от границы $z = -h$



$$\alpha_n = \sqrt{\alpha_n^2 - \alpha^2}, \quad |\alpha| < \alpha_n$$

$$\alpha_1 = \sqrt{\omega / c_{p,1}}$$

$$\alpha_2 = \sqrt{\omega / c_{s,1}}$$

$$s_n = \sqrt{k_n^2 - \alpha^2}, \quad |\alpha| < k_n$$

$$k_1 = \sqrt{\omega / c_{p,2}}$$

$$k_2 = \sqrt{\omega / c_{s,2}}$$

В Фурье символах

$$\bar{u}_0 = \sum_{n=1}^2 \bar{A}_n(\alpha) e^{\alpha_n z}$$

(см. сж. 2-3)

Для \bar{u}^- и \bar{u}^+ исходят из предполож.

Гельмгольца:

$$\bar{u} = \nabla \varphi + \text{rot } \bar{\psi}$$

В плоском сл.

$$\bar{u} = (u, w), \quad \varphi = \varphi(x, z)$$

$$\bar{\psi} = (0, \psi, 0), \quad \psi = \psi(x, z)$$

Тогда

$$\begin{cases} u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z} \\ w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \end{cases} \quad \left| \quad \begin{cases} \Delta \bar{\varphi} + \alpha_1^2 \bar{\varphi} = 0 \\ \Delta \bar{\varphi} + \alpha_2^2 \bar{\varphi} = 0, \quad \underline{z \geq -h} \\ \Delta \varphi^+ + k_1^2 \varphi^+ = 0 \\ \Delta \varphi^+ + k_2^2 \varphi^+ = 0, \quad z \leq -h \end{cases} \right.$$

Good evening

$$\begin{cases} u = -i\alpha\phi - \psi'_z \\ w = \phi'_z - i\alpha\psi \end{cases} \quad \begin{cases} \phi^- = t_1 e^{-\epsilon_1(z+h)} \\ \psi^- = t_2 e^{-\epsilon_2(z+h)}, \quad \underline{z \geq -h} \\ \phi^+ = t_3 e^{s_1(z+h)} \\ \psi^+ = t_4 e^{s_2(z+h)}, \quad \underline{z \leq -h} \end{cases}$$

$$\begin{cases} \bar{u} = t_1(-i\alpha)e^{-\epsilon_1(z+h)} - t_2(-\epsilon_2)e^{-\epsilon_2(z+h)} \\ \bar{w} = t_1(-\epsilon_1)e^{-\epsilon_1(z+h)} + t_2(-i\alpha)e^{-\epsilon_2(z+h)}, \quad z \geq 0 \end{cases}$$

$$\begin{cases} u^+ = t_3(-i\alpha)e^{s_1(z+h)} - t_4 s_2 e^{s_2(z+h)} \\ w^+ = t_3 s_1 e^{s_1(z+h)} + t_4(-i\alpha)e^{s_2(z+h)} \end{cases} \quad \underline{z \leq -}$$

Условие непрерывности \bar{u} на границе $z = -h$ выглядит так:

$$[(\bar{u}_0 + \bar{u}^-) - \bar{u}^+]|_{z=0} = 0 \quad \text{u.s.m.}$$

$$(\bar{u} - \bar{u}^+)|_{z=-h} = -\bar{u}_0|_{z=-h} \Rightarrow$$

$$\begin{cases} u^- - u^+ = -u_0 \\ w^- - w^+ = -w_0 \end{cases} \text{ upon } z = -h$$

revez ~~t₁~~ ~~t₂~~ ~~t₃~~ ~~t₄~~ ~~t₅~~ ~~t₆~~ ~~t₇~~ ~~t₈~~ ~~t₉~~ ~~t₁₀~~ ~~t₁₁~~ ~~t₁₂~~ ~~t₁₃~~ ~~t₁₄~~ ~~t₁₅~~ ~~t₁₆~~ ~~t₁₇~~ ~~t₁₈~~ ~~t₁₉~~ ~~t₂₀~~ ~~t₂₁~~ ~~t₂₂~~ ~~t₂₃~~ ~~t₂₄~~ ~~t₂₅~~ ~~t₂₆~~ ~~t₂₇~~ ~~t₂₈~~ ~~t₂₉~~ ~~t₃₀~~ ~~t₃₁~~ ~~t₃₂~~ ~~t₃₃~~ ~~t₃₄~~ ~~t₃₅~~ ~~t₃₆~~ ~~t₃₇~~ ~~t₃₈~~ ~~t₃₉~~ ~~t₄₀~~ ~~t₄₁~~ ~~t₄₂~~ ~~t₄₃~~ ~~t₄₄~~ ~~t₄₅~~ ~~t₄₆~~ ~~t₄₇~~ ~~t₄₈~~ ~~t₄₉~~ ~~t₅₀~~ ~~t₅₁~~ ~~t₅₂~~ ~~t₅₃~~ ~~t₅₄~~ ~~t₅₅~~ ~~t₅₆~~ ~~t₅₇~~ ~~t₅₈~~ ~~t₅₉~~ ~~t₆₀~~ ~~t₆₁~~ ~~t₆₂~~ ~~t₆₃~~ ~~t₆₄~~ ~~t₆₅~~ ~~t₆₆~~ ~~t₆₇~~ ~~t₆₈~~ ~~t₆₉~~ ~~t₇₀~~ ~~t₇₁~~ ~~t₇₂~~ ~~t₇₃~~ ~~t₇₄~~ ~~t₇₅~~ ~~t₇₆~~ ~~t₇₇~~ ~~t₇₈~~ ~~t₇₉~~ ~~t₈₀~~ ~~t₈₁~~ ~~t₈₂~~ ~~t₈₃~~ ~~t₈₄~~ ~~t₈₅~~ ~~t₈₆~~ ~~t₈₇~~ ~~t₈₈~~ ~~t₈₉~~ ~~t₉₀~~ ~~t₉₁~~ ~~t₉₂~~ ~~t₉₃~~ ~~t₉₄~~ ~~t₉₅~~ ~~t₉₆~~ ~~t₉₇~~ ~~t₉₈~~ ~~t₉₉~~ ~~t₁₀₀~~ ~~t₁₀₁~~ ~~t₁₀₂~~ ~~t₁₀₃~~ ~~t₁₀₄~~ ~~t₁₀₅~~ ~~t₁₀₆~~ ~~t₁₀₇~~ ~~t₁₀₈~~ ~~t₁₀₉~~ ~~t₁₁₀~~ ~~t₁₁₁~~ ~~t₁₁₂~~ ~~t₁₁₃~~ ~~t₁₁₄~~ ~~t₁₁₅~~ ~~t₁₁₆~~ ~~t₁₁₇~~ ~~t₁₁₈~~ ~~t₁₁₉~~ ~~t₁₂₀~~ ~~t₁₂₁~~ ~~t₁₂₂~~ ~~t₁₂₃~~ ~~t₁₂₄~~ ~~t₁₂₅~~ ~~t₁₂₆~~ ~~t₁₂₇~~ ~~t₁₂₈~~ ~~t₁₂₉~~ ~~t₁₃₀~~ ~~t₁₃₁~~ ~~t₁₃₂~~ ~~t₁₃₃~~ ~~t₁₃₄~~ ~~t₁₃₅~~ ~~t₁₃₆~~ ~~t₁₃₇~~ ~~t₁₃₈~~ ~~t₁₃₉~~ ~~t₁₄₀~~ ~~t₁₄₁~~ ~~t₁₄₂~~ ~~t₁₄₃~~ ~~t₁₄₄~~ ~~t₁₄₅~~ ~~t₁₄₆~~ ~~t₁₄₇~~ ~~t₁₄₈~~ ~~t₁₄₉~~ ~~t₁₅₀~~ ~~t₁₅₁~~ ~~t₁₅₂~~ ~~t₁₅₃~~ ~~t₁₅₄~~ ~~t₁₅₅~~ ~~t₁₅₆~~ ~~t₁₅₇~~ ~~t₁₅₈~~ ~~t₁₅₉~~ ~~t₁₆₀~~ ~~t₁₆₁~~ ~~t₁₆₂~~ ~~t₁₆₃~~ ~~t₁₆₄~~ ~~t₁₆₅~~ ~~t₁₆₆~~ ~~t₁₆₇~~ ~~t₁₆₈~~ ~~t₁₆₉~~ ~~t₁₇₀~~ ~~t₁₇₁~~ ~~t₁₇₂~~ ~~t₁₇₃~~ ~~t₁₇₄~~ ~~t₁₇₅~~ ~~t₁₇₆~~ ~~t₁₇₇~~ ~~t₁₇₈~~ ~~t₁₇₉~~ ~~t₁₈₀~~ ~~t₁₈₁~~ ~~t₁₈₂~~ ~~t₁₈₃~~ ~~t₁₈₄~~ ~~t₁₈₅~~ ~~t₁₈₆~~ ~~t₁₈₇~~ ~~t₁₈₈~~ ~~t₁₈₉~~ ~~t₁₉₀~~ ~~t₁₉₁~~ ~~t₁₉₂~~ ~~t₁₉₃~~ ~~t₁₉₄~~ ~~t₁₉₅~~ ~~t₁₉₆~~ ~~t₁₉₇~~ ~~t₁₉₈~~ ~~t₁₉₉~~ ~~t₂₀₀~~ ~~t₂₀₁~~ ~~t₂₀₂~~ ~~t₂₀₃~~ ~~t₂₀₄~~ ~~t₂₀₅~~ ~~t₂₀₆~~ ~~t₂₀₇~~ ~~t₂₀₈~~ ~~t₂₀₉~~ ~~t₂₁₀~~ ~~t₂₁₁~~ ~~t₂₁₂~~ ~~t₂₁₃~~ ~~t₂₁₄~~ ~~t₂₁₅~~ ~~t₂₁₆~~ ~~t₂₁₇~~ ~~t₂₁₈~~ ~~t₂₁₉~~ ~~t₂₂₀~~ ~~t₂₂₁~~ ~~t₂₂₂~~ ~~t₂₂₃~~ ~~t₂₂₄~~ ~~t₂₂₅~~ ~~t₂₂₆~~ ~~t₂₂₇~~ ~~t₂₂₈~~ ~~t₂₂₉~~ ~~t₂₃₀~~ ~~t₂₃₁~~ ~~t₂₃₂~~ ~~t₂₃₃~~ ~~t₂₃₄~~ ~~t₂₃₅~~ ~~t₂₃₆~~ ~~t₂₃₇~~ ~~t₂₃₈~~ ~~t₂₃₉~~ ~~t₂₄₀~~ ~~t₂₄₁~~ ~~t₂₄₂~~ ~~t₂₄₃~~ ~~t₂₄₄~~ ~~t₂₄₅~~ ~~t₂₄₆~~ ~~t₂₄₇~~ ~~t₂₄₈~~ ~~t₂₄₉~~ ~~t₂₅₀~~ ~~t₂₅₁~~ ~~t₂₅₂~~ ~~t₂₅₃~~ ~~t₂₅₄~~ ~~t₂₅₅~~ ~~t₂₅₆~~ ~~t₂₅₇~~ ~~t₂₅₈~~ ~~t₂₅₉~~ ~~t₂₆₀~~ ~~t₂₆₁~~ ~~t₂₆₂~~ ~~t₂₆₃~~ ~~t₂₆₄~~ ~~t₂₆₅~~ ~~t₂₆₆~~ ~~t₂₆₇~~ ~~t₂₆₈~~ ~~t₂₆₉~~ ~~t₂₇₀~~ ~~t₂₇₁~~ ~~t₂₇₂~~ ~~t₂₇₃~~ ~~t₂₇₄~~ ~~t₂₇₅~~ ~~t₂₇₆~~ ~~t₂₇₇~~ ~~t₂₇₈~~ ~~t₂₇₉~~ ~~t₂₈₀~~ ~~t₂₈₁~~ ~~t₂₈₂~~ ~~t₂₈₃~~ ~~t₂₈₄~~ ~~t₂₈₅~~ ~~t₂₈₆~~ ~~t₂₈₇~~ ~~t₂₈₈~~ ~~t₂₈₉~~ ~~t₂₉₀~~ ~~t₂₉₁~~ ~~t₂₉₂~~ ~~t₂₉₃~~ ~~t₂₉₄~~ ~~t₂₉₅~~ ~~t₂₉₆~~ ~~t₂₉₇~~ ~~t₂₉₈~~ ~~t₂₉₉~~ ~~t₃₀₀~~

$$\begin{cases} -i\alpha t_1 + \alpha_2 t_2 + i\alpha t_3 + S_2 t_4 = -u_0 \\ -\alpha_1 t_1 - i\alpha t_2 - S_1 t_3 + i\alpha t_4 = -w_0 \end{cases}$$

Если ещё два ур. равенства на произвольных

$$\| (\bar{\sigma} - \bar{\sigma}^+) |_{z=-h} = -\bar{\sigma}_0 |_{z=-h}$$

$$\bar{\sigma} = (\bar{\sigma}_x, \alpha_z)_{xz}, \begin{cases} \sigma_x = \mu \left(\frac{\partial^4}{\partial z^2} + \frac{\partial w}{\partial x} \right) \\ \alpha_z = \lambda \frac{\partial^4}{\partial x} + (\lambda + 2\mu) \frac{\partial w}{\partial z} \end{cases}$$

~~$$\bar{T}|_{z=-h} = \mu_1 [i\alpha \alpha_1 t_1 - \alpha_2^2 t_2]$$~~

В ур. Φ :

~~$$\frac{\partial^4}{\partial x} \rightarrow -i\alpha u$$~~

В фуре уравнениях при $z = -h$:

$$\left\{ \begin{aligned} (u^-)'_z &= i\alpha \alpha_1 t_1 - \alpha_2^2 t_2 + \\ -i\alpha w^- &= i\alpha \alpha_1 t_1 - \alpha^2 t_2 \\ -i\alpha u^- &= -\alpha^2 t_1 - i\alpha \alpha_2 t_2 \\ (w^-)'_z &= \alpha_1^2 t_1 + i\alpha \alpha_2 t_2 \end{aligned} \right. \begin{cases} \mu_1 \\ \text{где } \bar{\sigma}, \bar{\sigma}^- \\ \lambda_1 + (\lambda + 2\mu)_1 \end{cases}$$

Пусть $F[\bar{\sigma}] = \bar{T} = (T_x, S_z)$, тогда

$$\begin{cases} T_x^- = \mu_1 [2i\alpha c_1 t_1 - (c_2^2 + \alpha^2) t_2] \\ S_z^- = \underbrace{(-\lambda_1 \alpha^2 + (\lambda_1 + 2\mu_1) c_1^2)}_{\mu_1(\alpha^2 + c_2^2)} t_1 - \underbrace{2i\alpha c_2 (\lambda_1 + \mu_1)}_{2\mu_1 i\alpha c_2} t_2 \end{cases}$$

- give $\bar{\tau}^+$:

$$\begin{cases} (u^+)'_z = -i\alpha s_1 t_3 - s_2^2 t_4 & \mu_2 \\ -i\alpha w^+ = -i\alpha s_1 t_3 - \alpha^2 t_4 & \mu_2^+ \\ \begin{cases} -i\alpha u^+ = -\alpha^2 t_3 + i\alpha s_2 t_4 \\ (w^+)'_z = s_1^2 t_3 - i\alpha s_2 t_4 \end{cases} & \begin{cases} \lambda_2 \\ (\lambda_2 + 2\mu_2)^+ \end{cases} \end{cases}$$

$$\begin{cases} T_x^+ = \mu_2 [-2i\alpha s_1 t_3 - (s_2^2 + \alpha^2) t_4] \\ S_z^+ = \underbrace{(-\lambda_2 \alpha^2 + (\lambda_2 + 2\mu_2) s_1^2)}_{\mu_2(\alpha^2 + s_2^2)} t_3 + \underbrace{2i\alpha s_2 (\lambda_2 + \mu_2)}_{2\mu_2 i\alpha s_2} t_4 \end{cases}$$

- give $\bar{\tau}_0 i$

$$\begin{cases} \bar{u}_0(-h) = \bar{A}_1 e^{-c_1 h} + \bar{A}_2 e^{-c_2 h}, \quad \bar{A}_n = \begin{pmatrix} -i\alpha P_n \\ R_n \end{pmatrix} \\ (u_0^+)'_z = \underbrace{c_1 P_1}_{e_1} e^{-c_1 h} + \underbrace{c_2 P_2}_{e_2} e^{-c_2 h} \\ -i\alpha w_0 = -i\alpha R_1 e^{-c_1 h} - i\alpha R_2 e^{-c_2 h}, \quad \underline{e_n = e^{2f - c_n h}} \end{cases}$$

$$\begin{cases} -i\alpha \mathcal{U}_0 = -\alpha^2 P_1 e_1 - \alpha^2 P_2 e_2 \\ W'_0 = \epsilon_1 R_1 e_1 + \epsilon_2 R_2 e_2 \end{cases} \quad \begin{array}{l} \lambda_1 + \\ (\lambda_1 + 2\mu_1) \end{array}$$

$$\begin{cases} T_{x,0} = \mu_1 [-i\alpha (\epsilon_1 P_1 + R_1) e_1 - i\alpha (\epsilon_2 P_2 + R_2) e_2] \\ S_{z,0} = (-\lambda_1 \alpha^2 P_1 + (\lambda_1 + 2\mu_1) \epsilon_1 R_1) e_1 + \\ \quad + (-\lambda_1 \alpha^2 P_2 + (\lambda_1 + 2\mu_1) \epsilon_2 R_2) e_2 \end{cases}$$

Условие равенства напряжений при-
нимает вид:

$$\begin{aligned} & \parallel \quad 2\mu_1 i\alpha \epsilon_1 t_1 - \mu_1 (\epsilon_2^2 + \alpha^2) t_2 + \quad \left| \begin{array}{l} T_x^- \\ -T_x^+ \end{array} \right. \\ & + 2\mu_2 i\alpha s_1 t_3 + \mu_2 (s_2^2 + \alpha^2) t_4 = \\ & = +\mu_1 i\alpha (\epsilon_1 P_1 + R_1) e_1 + \mu_1 i\alpha (\epsilon_2 P_2 + R_2) e_2 \quad \left| \begin{array}{l} T_x^- \\ -T_x^+ \end{array} \right. \end{aligned}$$

$$\begin{aligned} & \parallel \quad \mu_1 (\alpha^2 + \epsilon_2^2) t_1 + 2\mu_1 i\alpha \epsilon_2 t_2 + \quad \left| \begin{array}{l} S_z^- \\ -S_z^+ \end{array} \right. \\ & - \mu_2 (\alpha^2 + s_2^2) t_3 + 2\mu_2 i\alpha s_2 t_4 = \\ & = [\lambda_1 \alpha^2 P_1 + (\lambda_1 + 2\mu_1) \epsilon_1 R_1] e_1 + \quad \left| \begin{array}{l} S_z^- \\ -S_z^+ \end{array} \right. \\ & + [\lambda_1 \alpha^2 P_2 + (\lambda_1 + 2\mu_1) \epsilon_2 R_2] e_2 \quad \left| \begin{array}{l} S_z^- \\ -S_z^+ \end{array} \right. \end{aligned}$$

$$\begin{aligned} & = [\lambda_1 \alpha^2 P_1 + (\lambda_1 + 2\mu_1) \epsilon_1 R_1] e_1 + \\ & + [\lambda_1 \alpha^2 P_2 + (\lambda_1 + 2\mu_1) \epsilon_2 R_2] e_2 \quad \left| \begin{array}{l} S_z^- \\ -S_z^+ \end{array} \right. \end{aligned}$$

-11.1-

Сделаем правую часть для произв. на-
дающего поля. Вобщем же.

$$\bar{u}_0 = K \cdot \bar{Q} = \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} e^{\epsilon_1 z} + \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} e^{\epsilon_2 z}$$

$$\| \bar{u}_0(-h) = \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} e_1 + \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} e_2 \quad \boxed{e_h = e^{\delta_h}}$$

$$T_{x,0} = \mu_1 \left[(\epsilon_1 u_1 - i\alpha w_1) e_1 + \right. \quad \left. \begin{array}{l} \text{при} \\ z = -h \end{array} \right. \\ \left. + (\epsilon_2 u_2 - i\alpha w_2) e_2 \right]$$

$$f_{z,0} = [\bar{\lambda}_1(-i\alpha)u_1 + (\lambda_1 + 2\mu_1)\epsilon_1 w_1] e_1 + \\ + [\bar{\lambda}_2(-i\alpha)u_2 + (\lambda_1 + 2\mu_1)\epsilon_2 w_2] e_2$$

$$u_3 \quad \bar{f} = \begin{pmatrix} -\bar{u}_0 \\ -\bar{w}_0 \\ -\bar{T}_0 \end{pmatrix} = \bar{f}_1 e_1 + \bar{f}_2 e_2 \text{ получаем}$$

$$\| \bar{f}_1 = \left\{ -u_1, -w_1, -\mu_1(\epsilon_1 u_1 - i\alpha w_1), \right. \\ \left. (\lambda_1 i\alpha u_1 - \epsilon \mu_1 \epsilon_1 w_1) \right\}^T$$

$$\| \bar{f}_2 = \left\{ -u_2, -w_2, -\mu_1(\epsilon_2 u_2 - i\alpha w_2), \right. \\ \left. \lambda_2 i\alpha u_2 - \epsilon \mu_1 \epsilon_2 w_2 \right\}^T$$

$$\| \text{здесь } \epsilon \mu_1 = \lambda_1 + 2\mu_1$$

В матричном виде

$$\| A \cdot \bar{f} = \bar{f}, \quad \bar{f} = \bar{f}_1 e_1 + \bar{f}_2 e_2 \quad (\text{см. 11.1})$$

$$A = \begin{bmatrix} -i\alpha & \epsilon_2 & i\alpha & S_2 \\ -\epsilon_1 & -i\alpha & -S_1 & i\alpha \\ 2\mu_1 i\alpha \epsilon_1 & -\mu_1(\alpha^2 + \epsilon_2^2) & 2\mu_2 i\alpha S_1 & \mu_2(\alpha^2 + S_2^2) \\ \mu_1(\alpha^2 + \epsilon_2^2) & -2(\lambda_1 + \mu_1)i\alpha \epsilon_2 & 2\mu_1 i\alpha \epsilon_2 & -\mu_2(\alpha^2 + S_2^2) \\ & & & 2(\lambda_2 + \mu_2)i\alpha S_2 \\ & & & 2\mu_2 i\alpha S_2 \end{bmatrix}$$

$$\bar{f} = \begin{pmatrix} t_1 & t_2 & t_3 & t_3 \end{pmatrix}$$

Соответственно структуре \bar{f} решение системы

$$\| \bar{f} = \bar{t}_1 e_1 + \bar{t}_2 e_2, \quad \text{где } A \bar{t}_n = \bar{f}_n, \quad n=1,2$$

т.е. каждая компонента $t_n^{(j)}$ раскладывается на два слагаемых:

$$\| \bar{t}_j^{(n)} = t_1^{(j)} e_1 + t_2^{(j)} e_2, \quad \text{где } t_n^{(j)}:$$

$$\bar{t}_n = (t_n^{(1)}, t_n^{(2)}, t_n^{(3)}, t_n^{(4)}), \quad n=1,2$$

3. Асимптотика ограниченного поля \bar{u}^-

Вернёмся к виду $\bar{u}(\alpha, z)$:

$$\| \bar{u}^- = \bar{V}_1 t_1 e^{-\phi_1(z+h)} + \bar{V}_2 t_2 e^{-\phi_2(z+h)}$$

$$\bar{V}_1 = \begin{pmatrix} -i\alpha \\ -\phi_1 \end{pmatrix}, \quad \bar{V}_2 = \begin{pmatrix} -\phi_2 \\ -i\alpha \end{pmatrix}$$

$$\mathcal{D}f \quad \phi_4 = -i\hat{\phi}_4, \quad \hat{\phi}_4 = \sqrt{x_4^2 - \alpha^2}$$

и устроим, что

$$\begin{cases} t_1 = t_1^{(1)} e^{-\phi_1 h} + t_2^{(1)} e^{-\phi_2 h} \\ t_2 = t_1^{(2)} e^{-\phi_1 h} + t_2^{(2)} e^{-\phi_2 h} \end{cases} \Rightarrow$$

$$\bar{u}^- = \bar{V}_1 \left[t_1^{(1)} e^{i\hat{\phi}_1(z+2h)} + t_2^{(1)} e^{i(\phi_2 h + \hat{\phi}_1(z+h))} \right] + \bar{V}_2 \left[t_1^{(2)} e^{i(\hat{\phi}_1 h + \phi_2(z+h))} + t_2^{(2)} e^{i\hat{\phi}_2(z+2h)} \right]$$

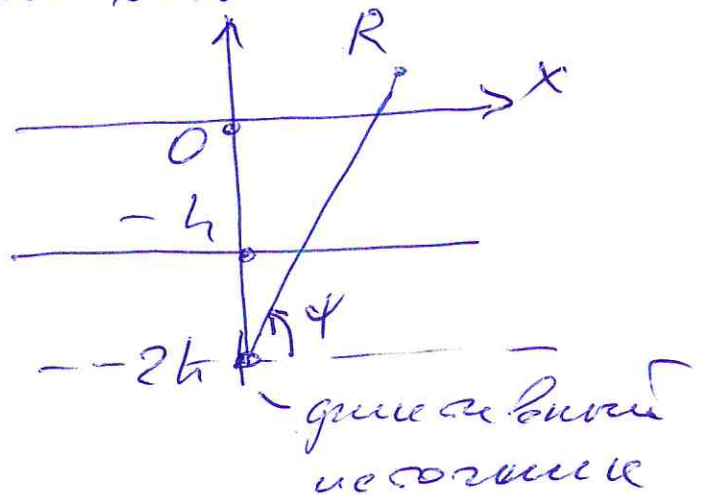
С учётом $e^{-i\alpha x}$ получим четыре вида фазовых ф-ий

$$\begin{cases} \theta_1 = \hat{\phi}_1(z+2h) - \alpha x \\ \theta_2 = \hat{\phi}_2 h + \hat{\phi}_1(z+h) - \alpha x \\ \theta_3 = \hat{\phi}_1 h + \hat{\phi}_2(z+h) - \alpha x \\ \theta_4 = \hat{\phi}_2(z+2h) - \alpha x \end{cases}$$

В полярных координатах

$$\begin{cases} x = R \cos \varphi \\ z+2h = R \sin \varphi \end{cases}$$

$$R = \sqrt{x^2 + (z+2h)^2}$$



$$\theta_1 = \hat{e}_1 \sin \varphi - \alpha \cos \varphi$$

$$\theta_2 = (-\hat{e}_1 h + \hat{e}_2 h) / R + \hat{e}_1 \sin \varphi - \alpha \cos \varphi$$

$$\theta_3 = (\hat{e}_1 h - \hat{e}_2 h) / R + \hat{e}_2 \sin \varphi - \alpha \cos \varphi$$

$$\theta_4 = \hat{e}_2 \sin \varphi - \alpha \cos \varphi$$

Составим уравнения на максимумы и минимумы

$$\left\| \theta'_m \right\|_2 = 0, \quad \left(\hat{e}_n \right)'_{\alpha} = - \frac{\alpha}{\hat{e}_n} \quad \underline{\underline{b = h/R}}$$

$$\theta'_1 = - \frac{\alpha}{\hat{e}_1} \sin \varphi - \cos \varphi = 0$$

$$\theta'_2 = \left(\frac{\alpha}{\hat{e}_1} - \frac{\alpha}{\hat{e}_2} \right) b - \frac{\alpha}{\hat{e}_1} \sin \varphi - \cos \varphi = 0$$

$$\theta'_3 = \alpha b \left(- \frac{1}{\hat{e}_1} + \frac{1}{\hat{e}_2} \right) - \frac{\alpha}{\hat{e}_2} \sin \varphi - \cos \varphi = 0$$

$$\theta'_4 = - \frac{\alpha}{\hat{e}_2} \sin \varphi - \cos \varphi = 0$$

Для θ_1 и θ_4 спец. случаи стандартиза, в яком виде:

$$\alpha_0 = \pm x_n |\cos \varphi|, \quad n = \begin{cases} 1 & \text{для } \theta_1 \\ 2 & \text{для } \theta_4 \end{cases}$$

осцирует нулевым корнем.

1) $n=1, \theta_1'=0$, $\alpha_0 = \pm x_1 |\cos \varphi|, 0 < \varphi < \pi$

$$\hat{\sigma}_1(\alpha_0) = \sqrt{x_1^2 - x_1^2 \cos^2 \varphi} = x_1 \sin \varphi > 0$$

$$\theta_1'(\alpha_0) = \frac{\mp x_1 |\cos \varphi| \cdot \sin \varphi - \cos \varphi}{x_1 \sin \varphi} = 0$$

\Rightarrow нижний знак при $0 < \varphi < \frac{\pi}{2}$
верхний - " при $\frac{\pi}{2} < \varphi < \pi$

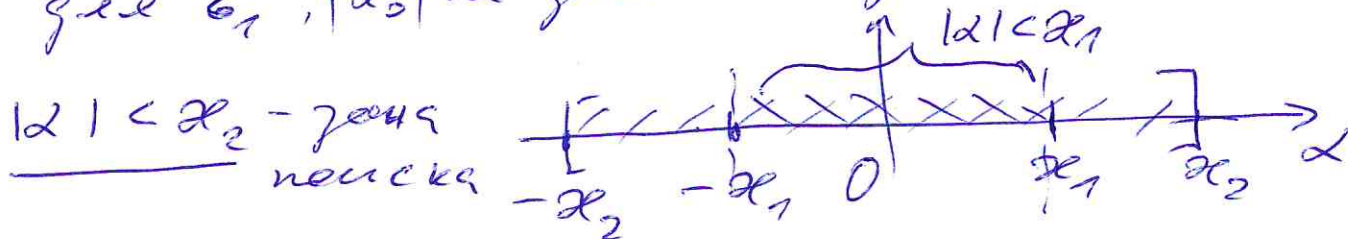
в целом $\alpha_0 = \begin{cases} -x_1 |\cos \varphi| = -x_1 \cos \varphi, & \text{I кв} \\ x_1 |\cos \varphi| = -x_1 \cos \varphi, & \text{II кв} \end{cases}$

$$\boxed{\alpha_0 = -x_1 \cos \varphi}$$

2) Аналогично для θ_4'

$$\boxed{\alpha_0 = -x_2 \cos \varphi}$$

3) θ_2', θ_3' - численно, учитывая, что
для $\hat{\sigma}_1, |\alpha_0|$ не должно превышать x_1 .



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$$\text{При } \alpha > x_1, \hat{c}_1 = \sqrt{x_1^2 - \alpha^2} = i\sqrt{\alpha^2 - x_1^2},$$

$$\text{т.е. } c_1 = -i\hat{c}_1 = \sqrt{\alpha^2 - x_1^2} > 0 \Rightarrow \text{уделяем} \\ \text{взв. гр. гр.}.$$

$$\theta_2 = \begin{cases} (-\hat{c}_1 + \hat{c}_2)b + \hat{c}_1 \sin \psi - \alpha \cos \psi, & |\alpha| < x_1 \\ \hat{c}_2 b - \alpha \cos \psi & x_1 < |\alpha| < x_2 \end{cases}$$

$$\theta_3 = \begin{cases} (\hat{c}_1 - \hat{c}_2)b + \hat{c}_2 \sin \psi - \alpha \cos \psi, & |\alpha| < x_1 \\ -\hat{c}_2 b + \hat{c}_2 \sin \psi - \alpha \cos \psi, & x_1 < |\alpha| < x_2 \end{cases}$$