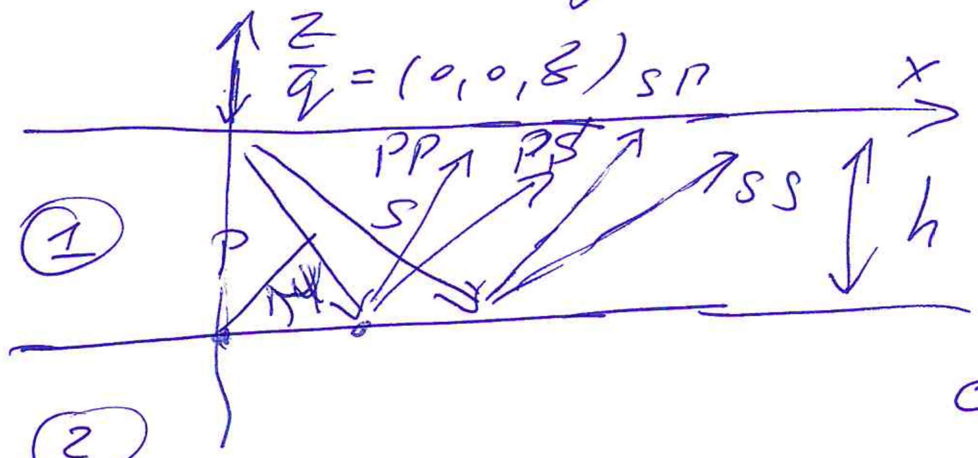
 2022-10-28

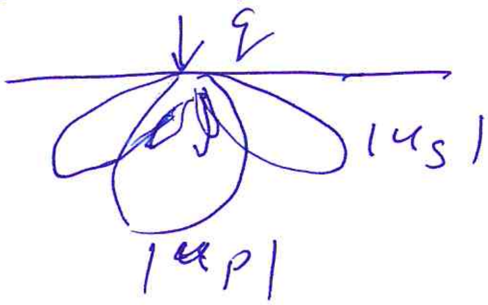
Асимптотика отр. волн (2D)



$c_{p,1}, c_{p,2}$
ρ_1

$c_{p,2}, c_{s,2}$
ρ_2

1. Поле источника $\bar{u}_0 = \bar{u}_p + \bar{u}_s =$
$= \bar{u}_1 + \bar{u}_2$



$$\bar{U}_0(\alpha, z) = K(\alpha, z)\,\bar{Q}(\alpha)$$

где $\bar{Q} = (0, 0, 1)^T$ $\bar{U}_0 = \begin{pmatrix} -i\alpha P \\ R \end{pmatrix}$

$$P(\alpha, z) = P_1(\alpha) e^{\sigma_1 z} + P_2(\alpha) e^{\sigma_2 z}$$

$$R(\alpha, z) = R_1(\alpha) e^{\sigma_1 z} + R_2(\alpha) e^{\sigma_2 z}$$

$$\Rightarrow \bar{u}_0 = \begin{pmatrix} -i\alpha P_1 \\ R_1 \end{pmatrix} e^{\sigma_1 z} + \begin{pmatrix} -i\alpha P_2 \\ R_2 \end{pmatrix} e^{\sigma_2 z}$$

(NVG-2000 дисс. стр. 41

$$\begin{cases} P_1 = -\gamma^2/\Delta \\ R_1 = -\sigma_1 \gamma^2/\Delta \end{cases} \qquad \begin{cases} P_2 = \sigma_1\sigma_2/\Delta \\ R_2 = \alpha^2 \sigma_1/\Delta \end{cases}$$

$$\gamma^2 = \alpha^2 - \varkappa_2^2/2$$

$$\Delta = 2\mu\left[-\gamma^4 + \alpha^2 \sigma_1 \sigma_2\right]$$

Асимптотика $\bar{u}_0$

Метод стац. фазы [Федорюк]

|| $F(R) = \int\limits_D f(\bar{x}) e^{iR\Theta(\bar{x})} d\bar{x}$ — m-кратный инт.

$R = |\bar{x}|$

- стац. точка $\bar{x}_0$: $\nabla\Theta(\bar{x}_0) = 0$
- невырожденная ст. т., если $|\Theta''(x_0)| \neq 0$

где $\Theta'' = \left[\dfrac{\partial^2\Theta}{\partial x_i \partial x_j}\right]_{i,j=1}^{m}$ — матр. Якоби $m \times m$

- $\Theta_0'' = \Theta_0''(\bar{x}_0)$, $\underline{\operatorname{sgn}\Theta_0'' = N^+ - N^-}$
- $N^\pm$ — число собств. знач. Θ_0'' с $\underline{\text{положит}}$/$\text{отрицат}$ вещ. частью

Вклад невырожденной стац. т. $\bar{x}_0$:

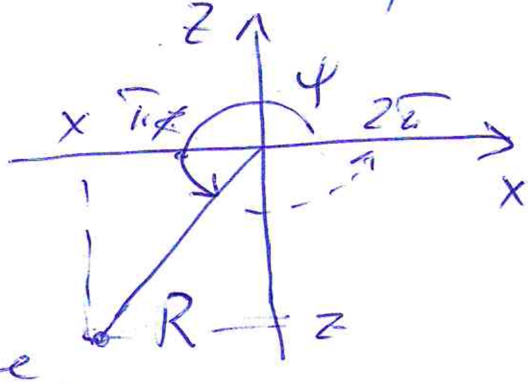
$$F_0(R) = \left(\frac{2\pi}{R}\right)^{m/2} f(\bar{x}_0)\, e^{iR\theta_0} \cdot e^{i\frac{\pi}{4}\operatorname{sgn}\theta''_{0x}} \times \left|\det\theta''_0\right|^{-\frac{1}{2}}\left(1+O(R^{-1})\right),$$

$$R\to\infty$$

<u>У нас</u>, $m = 1$

$$\bar{u}_0 = \sum_{n=1}^{2} \bar{A}_n(\alpha)\, e^{\gamma_n z}, \quad \bar{A}_n = \begin{pmatrix} -i\alpha P_n \\ R_n \end{pmatrix}$$

$$\bar{u}_n = \frac{1}{2\pi}\int_\Gamma \bar{A}_n(\alpha)\, e^{\gamma_n z} e^{-i\alpha x}\, d\alpha, \quad n = 1, 2$$



$$\begin{cases} x = R\cos\psi \\ z = R\sin\psi \le 0 \end{cases}$$

$\pi < \psi < 2\pi$ – нижнее полупр-во

$$\gamma_n = \begin{cases} -i\sqrt{\varkappa_n^2-\alpha^2}, & \alpha^2 < \varkappa_n^2 \\ \sqrt{\alpha^2-\varkappa_n^2}, & \alpha^2 > \varkappa_n^2 \end{cases}$$

$$\theta_n = -\left(\sqrt{\varkappa_n^2-\alpha^2}\cdot\sin\psi + \alpha\cos\psi\right) -$$

<u>$n = 1, 2$</u> фазовые ф-ии для <u>$\alpha^2 < \varkappa_n^2$</u>

$$\theta_u' = \frac{\alpha \sin\psi}{\sqrt{\varkappa_u^2-\alpha^2}} - \cos\psi = 0 \Rightarrow$$

$$\Rightarrow \alpha_0 = \pm \varkappa_u |\cos\psi|$$

Выбираем знак:

$$\sqrt{\varkappa_u^2-\alpha_0^2} = \varkappa_u |\sin\psi| = -\varkappa_u \sin\psi > 0,$$

т.к. $\sin\psi < 0$ для $\forall \psi \in [\pi, 2\pi]$

тогда

$$\theta_u' = \frac{\pm \cancel{\varkappa_u} \cancel{\sin}\psi |\cos\psi|}{-\cancel{\varkappa_u \sin\psi}} - \cos\psi =$$

$$= \mp |\cos\psi| - \cos\psi \;; \quad \cos\psi = \begin{cases} < 0, & \psi < \frac{3\pi}{2} \\ > 0, & \psi > \frac{3\pi}{2} \end{cases}$$

$$\Rightarrow \theta_u' = \begin{cases} -|\cos\psi| - \cos\psi = 0, & \pi < \psi < 3\pi/2 \\ |\cos\psi| - \cos\psi = 0, & \frac{3\pi}{2} < \psi < 2\pi \end{cases}$$

т.е. верхний знак в I сл. и нижний во II.

$$\alpha_0 = \begin{cases} \varkappa_0 |\cos\psi| = -\varkappa_u \cos\psi, & \pi < \psi < \frac{3\pi}{2} \\ -\varkappa_0 |\cos\psi| = -\varkappa_u \cos\psi, & \frac{3\pi}{2} < \psi\ 2\pi \end{cases}$$

т.е. для $\forall \psi$

$$\boxed{\alpha_0 = -\varkappa_u \cos\psi, \quad \pi < \psi < 2\pi}$$

$$\Theta_n(\alpha_0) = -(-\varkappa_n \sin^2\psi - \varkappa_n \cos^2\psi) = \varkappa_n$$

$$\Theta_n''(\alpha_0) = \sin\psi\left[\frac{1}{\sqrt{\varkappa_n^2-\alpha_0^2}} - \frac{\alpha_0}{\varkappa_n^2-\alpha_0^2}\left(-\frac{\alpha_0}{\sqrt{\varkappa_n^2-\alpha_0^2}}\right)\right]$$

$$= \cancel{\varkappa_n}\frac{\sin\psi}{\varkappa_n}\left[\frac{1}{-\sin\psi} - \frac{-\cos\psi}{\sin^2\psi}\left(\frac{\cos\psi}{-\sin\psi}\right)\right]$$

$$= -\frac{\sin\psi}{\varkappa_n}\cdot\frac{\sin^2\psi+\cos^2\psi}{\sin^2\psi} = -\frac{1}{\varkappa_n \sin^2\psi} < 0$$

$$\Rightarrow \operatorname{sgn}\Theta_n''(\alpha_0) = -1$$

Таким образом в формуле вклада ст. т.

$\left(\frac{2\pi}{R}\right)^{m/2} \Longleftrightarrow \sqrt{\frac{2\pi}{r}}$, $R \Longleftrightarrow r = |\bar{x}| = \sqrt{x^2+z^2}$

$f(\bar{x}_0) \Longleftrightarrow A_n(\alpha_0) = \bar{a}_n(\psi)$

$e^{iR\Theta_0} \Longleftrightarrow e^{i\varkappa_n r}$; $e^{i\frac{\pi}{4}\operatorname{sgn}\Theta_0''} = e^{-i\frac{\pi}{4}} = \sqrt{-i} = 1/\sqrt{i}$

$|\det \Theta_0''|^{-1/2} \Longleftrightarrow \cancel{\frac{1}{\sqrt{\varkappa_n \sin^2\psi}}} = \cancel{\frac{1}{\sqrt{\varkappa_n}(-\sin\psi)}}$

$$\Rightarrow -\sqrt{\varkappa_n}\sin\psi \geq 0$$

Для $\bar{u}_n$ получаем

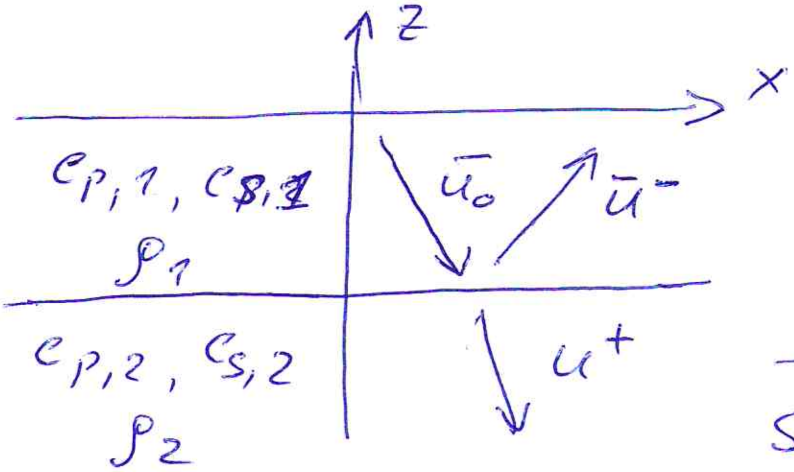
$$\bar{u}_n = \frac{\varkappa_n |\sin\varphi|}{\sqrt{2\pi i}} \bar{a}_n(\varphi) e^{i\varkappa_n r} / \sqrt{\varkappa_n r} \left[1 + O\left(\frac{1}{\varkappa_n r}\right)\right]$$

$$\varkappa_n r \to \infty, \quad n = 1, 2.$$

$$\bar{a}_n(\varphi) = \bar{A}_n(\alpha_n),$$

$$\alpha_n = -\varkappa_n \cos\varphi, \quad \pi < \varphi < 2\pi$$

2. Отражение $\bar{u}_0$ от границы $z=-h$



$$\sigma_n = -i\sqrt{\varkappa_n^2-\alpha^2},\quad |\alpha|<\varkappa_n$$

$$\varkappa_1=\sqrt{\omega/c_{p,1}}$$

$$\varkappa_2=\sqrt{\omega/c_{s,1}}$$

$$s_n = -i\sqrt{k_n^2-\alpha^2},\quad |\alpha|<k_n$$

$$k_1=\sqrt{\omega/c_{p,2}},$$

$$k_2=\sqrt{\omega/c_{s,2}}$$

В Фурье символах

$$\bar{u}_0=\sum_{n=1}^{2}\bar{A}_n(\alpha)e^{\sigma_n z}$$

(см. стр. 2-3)

Для $\bar{u}^-$ и $\bar{u}^+$ исходим из представл. Гельмгольца:

$$\bar{u}=\nabla\varphi+\mathrm{rot}\,\bar{\psi}$$

В плоском сл. $\bar{u}=(u,w)$, $\varphi=\varphi(x,z)$

$\bar{\psi}=(0,\psi,0)$, $\psi=\psi(x,z)$

тогда

$$\begin{cases} u=\dfrac{\partial\varphi}{\partial x}-\dfrac{\partial\psi}{\partial z} \\[2mm] w=\dfrac{\partial\varphi}{\partial z}+\dfrac{\partial\psi}{\partial x}\end{cases};\qquad \begin{cases}\Delta\varphi^-+\varkappa_1^2\varphi^-=0\\ \Delta\psi^-+\varkappa_2^2\psi^-=0\end{cases},\ \underline{z\geq -h}$$

$$\begin{cases}\Delta\varphi^++k_1^2\varphi^+=0\\ \Delta\psi^++k_2^2\psi^+=0\end{cases},\ z\leq -h$$

Соответственно

$$\begin{cases} U = -i\alpha\Phi - \Psi'_z \\ W = \Phi'_z - i\alpha\Psi \end{cases} \quad \left| \quad \begin{cases} \Phi^- = t_1 e^{-\sigma_1(z+h)} \\ \Psi^- = t_2 e^{-\sigma_2(z+h)}, \ \underline{z \geq -h} \end{cases} \right.$$

$$\begin{cases} \Phi^+ = t_3 e^{s_1(z+h)} \\ \Psi^+ = t_4 e^{s_2(z+h)}, \ z \leq -h \end{cases}$$

$$\begin{cases} U^- = t_1(-i\alpha)e^{-\sigma_1(z+h)} - t_2(-\sigma_2)e^{-\sigma_2(z+h)} \\ W^- = t_1(-\sigma_1)e^{-\sigma_1(z+h)} + t_2(-i\alpha)e^{-\sigma_2(z+h)}, \ \underline{z \geq} \end{cases}$$

$$\begin{cases} U^+ = t_3(-i\alpha)e^{s_1(z+h)} - t_4 s_2 e^{s_2(z+h)} \\ W^+ = t_3 s_1 e^{s_1(z+h)} + t_4(-i\alpha)e^{s_2(z+h)}, \ \underline{z \leq -} \end{cases}$$

Условие непрерывности $\bar{u}$ на границе $z=-h$ выглядит так:

$$[(\bar{u}_0 + \bar{u}^-) - \bar{u}^+]\big|_{z=-h} = 0 \text{ или}$$

$$(\bar{u}^- - \bar{u}^+)\big|_{z=-h} = -\bar{u}_0\big|_{z=-h} \Longrightarrow$$

$$\begin{cases} U^- - U^+ = -U_0 \\ W^- - W^+ = -W_0 \end{cases} \text{ при } z=-h$$

через ~~Φ, Ψ~~ $t_1, \dots, t_4$ получаем:

$$\left[\begin{aligned} &-i\alpha t_1 + \sigma_2 t_2 + i\alpha t_3 + s_2 t_4 = -U_0 \\ &-\sigma_1 t_1 - i\alpha t_2 - s_1 t_3 + i\alpha t_4 = -W_0 \end{aligned}\right.$$

Есть ещё два усл. равенства напряжений

$$(\bar\tau^- - \bar\tau^+)\big|_{z=-h} = -\bar\tau_0\big|_{z=-h}$$

$$\bar\tau = (\tau_{xz}, \sigma_z), \quad \left\{\begin{aligned} &\tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ &\sigma_z = \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial w}{\partial z} \end{aligned}\right.$$

$$\cancel{T^-\big|_{z=-h} = \mu_1\left[i\alpha\sigma_1 t_1 - \sigma_2^2 t_2\right]}$$

~~В пр. Ф.:~~

$$\cancel{\frac{\partial u}{\partial x} \to -i\alpha U}$$

В Фурье символах при $z=-h$:

$$\left.\begin{aligned} &\left[\begin{aligned} &(U^-)'_z = i\alpha\sigma_1 t_1 - \sigma_2^2 t_2 \quad + \\ &-i\alpha W^- = i\alpha\sigma_1 t_1 - \alpha^2 t_2 \end{aligned}\right. \\ &\left[\begin{aligned} &-i\alpha U^- = -\alpha^2 t_1 - i\alpha\sigma_2 t_2 \\ &(W^-)'_z = \sigma_1^2 t_1 + i\alpha\sigma_2 t_2 \end{aligned}\right. \end{aligned}\right\} \begin{aligned} &\mu_1 \\ &\text{для } \bar\tau^- \\ &\lambda_1 \quad + \\ &(\lambda_1 + 2\mu_1) \end{aligned}$$

Пусть $F[\bar\tau] = \bar T = (T_x, S_z)$, тогда

$$\begin{cases} T_x^- = \mu_1\left[2i\alpha\sigma_1 t_1 - (\sigma_2^2+\alpha^2)t_2\right] \\ S_z^- = \underbrace{(-\lambda_1\alpha^2 + (\lambda_1+2\mu_1)\sigma_1^2)}_{=\mu_1(\alpha^2+\sigma_2^2)} t_1 - \cancel{2i\alpha\sigma_2(\lambda_1+\mu_1)t_2} \quad \underline{+2\mu_1 i\alpha\sigma_2 t_2} \end{cases}$$

- для τ^+:

$$\begin{cases} (u^+)'_z = -i\alpha s_1 t_3 - s_2^2 t_4 & \mu_2 \\ -i\alpha W^+ = -i\alpha s_1 t_3 - \alpha^2 t_4 & \mu_2 \end{cases} +$$

$$\begin{cases} -i\alpha u^+ = -\alpha^2 t_3 + i\alpha s_2 t_4 & \lambda_2 \\ (W^+)'_z = s_1^2 t_3 - i\alpha s_2 t_4 & (\lambda_2+2\mu_2) \end{cases} +$$

$$\begin{cases} T_x^+ = \mu_2\left[-2i\alpha s_1 t_3 - (s_2^2+\alpha^2)t_4\right] \\ S_z^+ = \underbrace{(-\lambda_2\alpha^2 + (\lambda_2+2\mu_2)s_1^2)}_{=\mu_2(\alpha^2+s_2^2)} t_3 \cancel{+2i\alpha s_2(\lambda_2+\mu_2)t_4} \quad \underline{-2\mu_2 i\alpha s_2 t_4} \end{cases}$$

- для τ_0:

$$\bar{u}_0(-h) = \bar{A}_1 e^{-\sigma_1 h} + \bar{A}_2 e^{-\sigma_2 h}, \quad \bar{A}_n = \begin{pmatrix} -i\alpha P_n \\ R_n \end{pmatrix}$$

$$\begin{cases} (u_0^{\#})'_z = \underbrace{(-i\alpha)\sigma_1}P_1 e^{-\sigma_1 h} + \underbrace{(-i\alpha)\sigma_2}P_2 e^{-\sigma_2 h} \\ -i\alpha W_0 = -i\alpha R_1 \underbrace{e^{-\sigma_1 h}}_{=e_1} - i\alpha R_2 \underbrace{e^{-\sigma_2 h}}_{=e_2}, \quad \underline{e_n \stackrel{Df}{=} e^{-\sigma_n h}} \end{cases}$$

$$\left[\begin{aligned} -i\alpha U_0 &= -\alpha^2 P_1 e_1 - \alpha^2 P_2 e_2 \\ W_0' &= \sigma_1 R_1 e_1 + \sigma_2 R_2 e_2 \end{aligned}\right. \quad \left|\begin{matrix} \lambda_1 \\ + \\ (\lambda_1 + 2\mu_1) \end{matrix}\right.$$

$$\begin{cases} T_{x,0} = \mu_1 \left[-i\alpha(\sigma_1 P_1 + R_1)e_1 - i\alpha(\sigma_2 P_2 + R_2)e_2\right] \\ S_{z,0} = \left(-\lambda_1 \alpha^2 P_1 + (\lambda_1 + 2\mu_1)\sigma_1 R_1\right)e_1 + \\ \qquad + \left(-\lambda_1 \alpha^2 P_2 + (\lambda_1 + 2\mu_1)\sigma_2 R_2\right)e_2 \end{cases}$$

Условия равенства напряжений принимают вид:

$$\left\|\begin{aligned} & 2\mu_1 i\alpha\sigma_1 t_1 - \mu_1(\sigma_2^2 + \alpha^2)t_2 + \\ & + 2\mu_2 i\alpha s_1 t_3 + \mu_2(s_2^2 + \alpha^2)t_4 = \\ & = +\mu_1 i\alpha(\sigma_1 P_1 + R_1)e_1 + \mu_1 i\alpha(\sigma_2 P_2 + R_2)e_2 \end{aligned}\right. \quad \left|\begin{matrix} T_x^- \\ -T_x^+ \\ -T_{x,0} \end{matrix}\right.$$

$$\left\|\begin{aligned} & \mu_1(\alpha^2 + \sigma_2^2)t_1 \overset{+2\mu_1 i\alpha\sigma_2}{\cancel{-2(\lambda_1 + \mu_1)i\alpha\sigma_2}}\, t_2 + \\ & - \mu_2(\alpha^2 + s_2^2)t_3 + \overset{+2\mu_2 i\alpha s_2}{\cancel{2(\lambda_2 + \mu_2)i\alpha s_2}}\, t_4 = \\ & = \left[\lambda_1 \alpha^2 P_1 - (\lambda_1 + 2\mu_1)\sigma_1 R_1\right]e_1 + \\ & + \left[\lambda_1 \alpha^2 P_2 - (\lambda_1 + 2\mu_1)\sigma_2 R_2\right]e_2 \end{aligned}\right. \quad \left|\begin{matrix} S_z^- \\ -S_z^+ \\ -S_{z,0} \end{matrix}\right.$$

Сделаем правую часть для произв. падающего поля. Вобщем сл.

$$\bar{u}_0 = K \cdot \bar{Q} = \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} e^{\sigma_1 z} + \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} e^{\sigma_2 z}$$

$$\bar{u}_0(-h) = \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} e_1 + \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} e_2$$

$$\boxed{e_n = e^{-\sigma_n h}}$$

$$T_{x,0} = \mu_1 \left[(\sigma_1 u_1 - i\alpha w_1) e_1 + (\sigma_2 u_2 - i\alpha w_2) e_2 \right]$$

при $z = -h$

$$S_{z,0} = \left[\lambda_1 (-i\alpha) u_1 + (\lambda_1 + 2\mu_1) \sigma_1 w_1 \right] e_1 + \left[\lambda_1 (-i\alpha) u_2 + (\lambda_1 + 2\mu_1) \sigma_2 w_2 \right] e_2$$

из $\bar{f} = \begin{pmatrix} -\bar{u}_0 \\ -\bar{T}_0 \end{pmatrix} = \bar{f}_1 e_1 + \bar{f}_2 e_2$ получаем

$$\bar{f}_1 = \{ -u_1, -w_1, -\mu_1(\sigma_1 u_1 - i\alpha w_1), \lambda_1 i\alpha u_1 - \ell m_1 \sigma_1 w_1 \}^T$$

$$\bar{f}_2 = \{ -u_2, -w_2, -\mu_1(\sigma_2 u_2 - i\alpha w_2), \lambda_1 i\alpha u_2 - \ell m_1 \sigma_2 w_2 \}^T$$

здесь $\underline{\ell m_1 = \lambda_1 + 2\mu_1}$

В матричном виде

$$\| \; A \cdot \bar{t} = \bar{f} \; , \quad \bar{f} = \bar{f}_1 e_1 + \bar{f}_2 e_2 \; \text{(см. 11.1)}$$

$$A = \begin{bmatrix} -i\alpha & \sigma_2 & i\alpha & s_2 \\ -\sigma_1 & -i\alpha & -s_1 & i\alpha \\ 2\mu_1 i\alpha\sigma_1 & -\mu_1(\alpha^2+\sigma_2^2) & 2\mu_2 i\alpha s_1 & \mu_2(\alpha^2+s_2^2) \\ \mu_1(\alpha^2+\sigma_2^2) & \cancel{-2(\lambda_1+\mu_1)i\alpha\sigma_2} \; 2\mu_1 i\alpha\sigma_2 & -\mu_2(\alpha^2+s_2^2) & \cancel{2(\lambda_2+\mu_2)i\alpha s_2} \; 2\mu_2 i\alpha s_2 \end{bmatrix}$$

$$\bar{t} = (\; t_1 \quad t_2 \quad t_3 \quad t_3 \;)$$

Соответственно структуре $\bar{f}$ решение этой системы

$$\| \; \bar{t} = \bar{t}_1 e_1 + \bar{t}_2 e_2 \; , \text{ где } A\bar{t}_n = \bar{f}_n \, , \; n = 1,2$$

т.е. каждая компонента $t^{(j)}$ расщепляется на два слагаемых:

$$\| \; t^{(j)} = t_1^{(j)} e_1 + t_2^{(j)} e_2 \; , \text{ где } t_n^{(j)}:$$

$$\bar{t}_n = (t_n^{(1)}, t_n^{(2)}, t_n^{(3)}, t_n^{(4)}) , \; n = 1,2$$

3. Асимптотика отражённого поля $\bar{u}^-$

Вернёмся к виду $\bar{u}(\alpha, z)$:

$$\left\| \; \bar{u}^- = \bar{V}_1 \cdot t_1 e^{-\sigma_1 (z+h)} + \bar{V}_2 t_2 e^{-\sigma_2 (z+h)} \right.$$

$$\bar{V}_1 = \begin{pmatrix} -i\alpha \\ -\sigma_1 \end{pmatrix}, \quad \bar{V}_2 = \begin{pmatrix} -\sigma_2 \\ -i\alpha \end{pmatrix}$$

Df $\sigma_n = -i\hat{\sigma}_n$, $\hat{\sigma}_n = \sqrt{\varkappa_n^2 - \alpha^2}$

и учтём, что

$$\left| \begin{array}{l} t_1 = t_1^{(1)} e^{-\sigma_1 h} + t_2^{(1)} e^{-\sigma_2 h} \\ t_2 = t_1^{(2)} e^{-\sigma_1 h} + t_2^{(2)} e^{-\sigma_2 h} \end{array} \right. \Longrightarrow$$

$$\bar{u}^- = \bar{V}_1 \left[t_1^{(1)} e^{i\hat{\sigma}_1 (z+2h)} + t_2^{(1)} e^{i(\hat{\sigma}_2 h + \hat{\sigma}_1 (z+h))} \right] +$$

$$+ \bar{V}_2 \left[t_1^{(2)} e^{i(\hat{\sigma}_1 h + \hat{\sigma}_2 (z+h))} + t_2^{(2)} e^{i\hat{\sigma}_2 (z+2h)} \right]$$

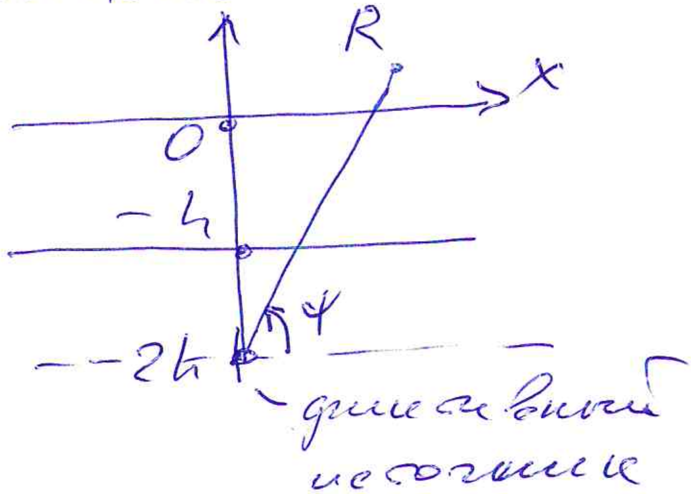
С учётом $e^{-i\alpha x}$ получим четыре вида фазовых ф-ций

$$\begin{cases} \theta_1 = \hat{\sigma}_1 (z+2h) - \alpha x \\ \theta_2 = \hat{\sigma}_2 h + \hat{\sigma}_1 (z+h) - \alpha x \\ \theta_3 = \hat{\sigma}_1 h + \hat{\sigma}_2 (z+h) - \alpha x \\ \theta_4 = \hat{\sigma}_2 (z+2h) - \alpha x \end{cases}$$

В полярных координатах

$$\begin{cases} x = R\cos\varphi \\ z+2h = R\sin\varphi \end{cases}$$

$$R = \sqrt{x^2+(z+2h)^2}$$



$$\begin{cases} \theta_1 = \hat{\sigma}_1 \sin\varphi - \alpha\cos\varphi \\ \theta_2 = (-\hat{\sigma}_1 h + \hat{\sigma}_2 h)/R + \hat{\sigma}_1 \sin\varphi - \alpha\cos\varphi \\ \theta_3 = (\hat{\sigma}_1 h - \hat{\sigma}_2 h)/R + \hat{\sigma}_2 \sin\varphi - \alpha\cos\varphi \\ \theta_4 = \hat{\sigma}_2 \sin\varphi - \alpha\cos\varphi \end{cases}$$

Стац. точки находятся из условия

$$\left.\theta'_m\right|_\alpha = 0 \quad \left| \quad (\hat{\sigma}_n)'_\alpha = -\frac{\alpha}{\hat{\sigma}_n} \quad \right| \quad \underline{\underline{b = h/R}}$$

$$\begin{cases} \theta'_1 = -\dfrac{\alpha}{\hat{\sigma}_1}\sin\varphi - \cos\varphi = 0 \\ \theta'_2 = \left(\dfrac{\alpha}{\hat{\sigma}_1} - \dfrac{\alpha}{\hat{\sigma}_2}\right) b - \dfrac{\alpha}{\hat{\sigma}_1}\sin\varphi - \cos\varphi = 0 \\ \theta'_3 = \alpha b\left(-\dfrac{1}{\hat{\sigma}_1} + \dfrac{1}{\hat{\sigma}_2}\right) - \dfrac{\alpha}{\hat{\sigma}_2}\sin\varphi - \cos\varphi = 0 \\ \theta'_4 = -\dfrac{\alpha}{\hat{\sigma}_2}\sin\varphi - \cos\varphi = 0 \end{cases}$$

Для θ_1 и θ_4 стац. точка находится в явном виде:

$$\alpha_0 = \pm \varkappa_n |\cos\psi|, \quad n = \begin{cases} 1 & \text{для } \theta_1 \\ 2 & \text{для } \theta_4 \end{cases}$$

Отбираем нужный корень.

1) $\underline{n = 1,\ \theta_1' = 0}$, $\alpha_0 = \pm \varkappa_1 |\cos\psi|$, $0 < \psi < \pi$

$$\hat{\sigma}_1(\alpha_0) = \sqrt{\varkappa_1^2 - \varkappa_1^2 \cos^2\psi} = \varkappa_1 \sin\psi > 0$$

$$\theta_1'(\alpha_0) = \frac{\mp \cancel{\varkappa_1} |\cos\psi|}{\cancel{\varkappa_1 \sin\psi}} \cdot \cancel{\sin\psi} - \cos\psi = 0$$

$\Rightarrow$ нижний знак при $0 < \psi < \frac{\pi}{2}$
верхний –"– при $\frac{\pi}{2} < \psi < \pi$

в целом $\overset{*}{\alpha}_0 = \begin{cases} -\varkappa_1 |\cos\psi| = -\varkappa_1 \cos\psi, & \text{I сл} \\ \varkappa_1 |\cos\psi| = -\varkappa_1 \cos\psi; & \text{II сл} \end{cases}$

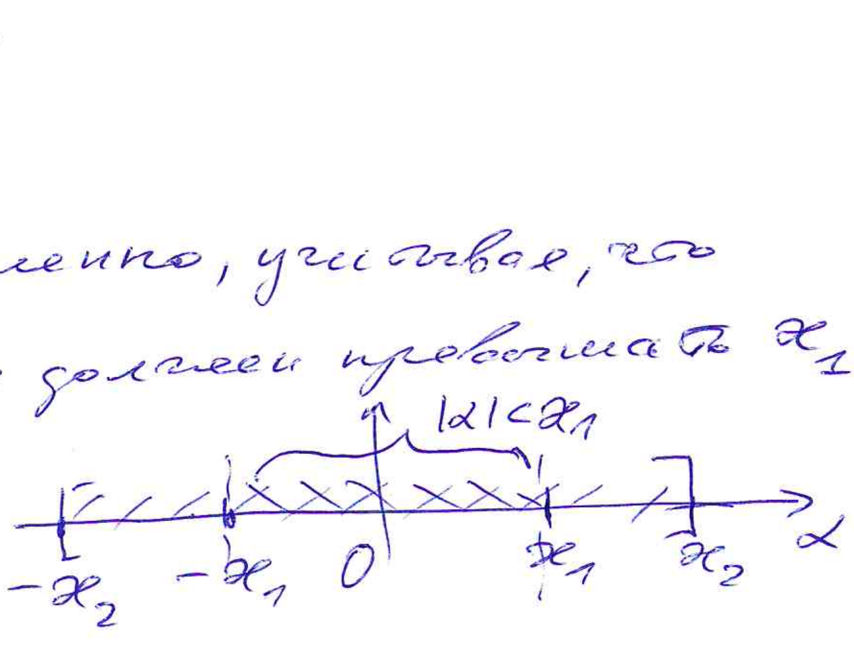
$$\boxed{\alpha_0 = -\varkappa_1 \cos\psi}$$

2) Аналогично для θ_4'

$$\boxed{\alpha_0 = -\varkappa_2 \cos\psi}$$

3) $\underline{\theta_2', \theta_3'}$ − численно, учитывая, что для $\hat{\sigma}_1$, $|\alpha_0|$ не должен превышать $\varkappa_1$.

$\underline{|\alpha| < \varkappa_2}$ − зона поиска



При $\alpha > \varkappa_1$, $\hat{b}_1 = \sqrt{\varkappa_1^2 - \alpha^2} = i\sqrt{\alpha^2 - \varkappa_1^2}$,

т.е. $b_1 = -i\hat{b}_1 = \sqrt{\alpha^2 - \varkappa_1^2} > 0 \Rightarrow$ убывающая из фаз. ф-ции.

$$\theta_2 = \begin{cases} (-\hat{b}_1 + \hat{b}_2)b + \hat{b}_1 \sin\psi - \alpha\cos\psi, & \underline{|\alpha| < \varkappa_1} \\ \hat{b}_2 b - \alpha\cos\psi & \underline{\varkappa_1 < |\alpha| < \varkappa_2} \end{cases}$$

$$\theta_3 = \begin{cases} (\hat{b}_1 - \hat{b}_2)b + \hat{b}_2 \sin\psi - \alpha\cos\psi, & |\alpha| < \varkappa_1 \\ -\hat{b}_2 b + \hat{b}_2 \sin\psi - \alpha\cos\psi, & \varkappa_1 < |\alpha| < \varkappa_2 \end{cases}$$