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SOM-R

Materials and Methods for Experiments 1-5

For these experiments, we recruited 1,000 participants from the Amazon.com Mechanical Turk online labor market. Our sample size was determined based on previous exploratory studies in which we found a sample size of 1,000 participants was sufficient to fit various discount functions to experimental data. We collected these 1,000 participants and stopped collecting further data. After collection was complete, we excluded data from participants whose IP address had provided more responses than the number of questions we had asked. This left us with data from 940 participants. We then also excluded any individual observations for which a participant had not responded to a specific question. As one participant had not responded to any questions, this left us with a final data set of 939 participants.

Within the experiments, participants were randomly assigned to one of the five task variants described in the main text. They made hypothetical choices, allowing us to use a large range of monetary rewards. We generated random questions that spanned dollar amounts from $0.01 to $100,000.00 and time amounts from 0 weeks to 6 weeks. The specific questions shown to each participant were randomly generated in order to make the four terms of Equation 1 in the main text orthogonal across participants.

We estimated the parameters of the ITCH model using standard maximum likelihood estimation techniques for Generalized Linear Models. To compare models, we performed a cross-validation analysis that repeatedly split the data into two randomly generated parts: the first part consisted of 75% of the data and was used to estimate the parameters of our model; the second part consisted of 25% of the data and was used to assess the model's fit to "held out" data (Kohavi, 1995). We assessed model fit using the Mean Absolute Deviation (MAD) between a

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model's prediction (which is a probability between 0 and 1) and the actual choice indicator (which is 0 or 1 depending on whether the LL option was chosen) because this metric is interpretable on the scale of probabilities and robust to outliers. (When model fit is assessed using held-out log likelihood and Root Mean Squared Error, the results are qualitatively similar).

We compared the ITCH model with seven other models of intertemporal choice: a baseline model that makes choices by flipping a biased coin, exponential discounting, hyperbolic discounting, generalized hyperbolic discounting, quasi-hyperbolic discounting, and a fixed cost model of delay (Benhabib, Bisin & Schotter, 2010). We also estimated the "DRIFT" model of Read et al. (2013).

Because the discounting models contain nonlinearities and impose theoretical limits on allowable model parameters, we estimated them via maximum likelihood using a constrained optimization suite in R that implements the L-BFGS-B algorithm (Zhu, Byrd, Lu, & Nocedal, 1997).

The functional forms of each model are listed below. In each of these models, the notation, L(z), represents the inverse logistic function of z:

$$L(z) = (1+e^{-z})^{-1}$$

The notation I(x) represents the indicator variable that is 1 if x is true and 0 otherwise. The variable a represents the logistic scaling parameter.

- Baseline model: $P(LL) = \pi$ (empirical mean)
- Exponential model: $P(LL) = L\left(a\left(x_2\delta^{t2} - x_1\delta^{t_1}\right)\right)$
- Hyperbolic model: $P(LL) = L(\mathrm{a}(x_2(1+\alpha t_2)^{-1} - x_1(1+\alpha t_1)^{-1}))$
- Quasi-hyperbolic model: $P(LL) = L(a(x_2\beta^{I(t_2>0)}\delta^{t_2} - x_1\beta^{I(t_1>0)}\delta^{t_1}))$
- Tradeoff model:

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$$P(LL) = L\left(a\left(\left(\log(1+\gamma_x x_2)/\gamma_x - \log(1+\gamma_x x_1\right)/\gamma_x - k(\log(1+\gamma_t t_2) - \log(1+\gamma_t t_1)/\gamma_t)\right)\right)$$

- DRIFT model: $P(LL) = L\left(\beta_0 + \beta_1(x_2 - x_1) + \beta_2 \frac{x_2 - x_1}{x_1} + \beta_3\left(\frac{x_2}{x_1}^{\frac{1}{t_2 - t_1}} - 1\right) + \beta_4(t_2 - t_1)\right)$

- ITCH model: $P(LL) = L\left(\beta_I + \beta_{xA}(x_2 - x_1) + \beta_{xR}\frac{x_2 - x_1}{x^*} + \beta_{tA}(t_2 - t_1) + \beta_{tR}\frac{t_2 - t_1}{t^*}\right)$

We find that the exponential, hyperbolic, quasi-hyperbolic and generalized hyperbolic models can be numerically unstable. To resolve this numeric instability, we compute the probability of an entire sequence of choices by bounding the per-choice probabilities estimated using the functional forms above at a value that is not 0 or 1. In our estimation, the exponential, hyperbolic, quasi-hyperbolic and generalized hyperbolic models perform better when bounded. Table S1 shows the results when bounded at 0.01 and 0.99. These bounded probabilities are computed as:

$$P(LL_b) = (1-\varepsilon)P(LL) + \frac{\varepsilon}{2}$$

For the non-heuristic models, we have also estimated variants in which utilities do not have constant variance logistic noise (as implied by the inverse logistic function), but instead have scale-invariant, homothetic noise. With homothetic noise, the probability of errors depends on the *ratio* of the utilities of the two goods, rather than on their absolute difference. This homothetic noise assumption gives the following functional forms:

- *Homothetic exponential model:* $P(LL) = L(a(\log(x_2\delta^{t_2}) - \log(x_1\delta^{t_1})))$
- *Homothetic hyperbolic model:* $P(LL) = L(a(\log(x_2(1+\alpha t_2)^{-1}) - \log(x_1(1+\alpha t_1)^{-1})))$
- *Homothetic quasi-hyperbolic model:* $P(LL) = L(a(\log(x_2\beta^{I(t_2>0)}\delta^{t_2}) - \log(x_1\beta^{I(t_1>0)}\delta^{t_1})))$

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The results comparing the heuristic models against these homothetic models are shown in Table S2.

All data and analysis code for this work is available at https://osf.io/9uxve/.

References

Benhabib, J., Bisin, A., & Schotter, A. (2010). Present-bias, quasi-hyperbolic discounting, and fixed costs. *Games and Economic Behavior 69(2)*, 205-223.

Mason, W., & Suri, S. (2012). Conducting behavioral research on Amazon's Mechanical Turk. *Behavior Research Methods 44(1)*, 1-23.

Zhu, C., Byrd, R.H., Lu, P., & Nocedal, J. (1997). Algorithm 778: L-BFGS-B: Fortran subroutines for large-scale bound-constrained optimization. *ACM Transactions on Mathematical Software (TOMS) 23(4)*, 550-560.

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SOM-R Tables

Experiment	Baseline	Exponential	Hyperbolic	Quasi-Hyperbolic	Tradeoff	DRIFT	ITCH
Pooled	0.4674 (0.0002)	0.4924 (0.0001)	0.4923 (0.0001)	0.4922 (0.0001)	0.4582 (0.0002)	0.4082 (0.0003)	0.3997 (0.0004)
1: Absolute $, Delay Framing	0.4106 (0.0007)	0.4593 (0.0012)	0.4564 (0.0016)	0.4581 (0.0016)	0.4068 (0.0008)	0.3271 (0.0009)	0.3213 (0.0011)
2: Relative $, Delay Framing	0.4682 (0.0003)	0.4766 (0.0023)	0.4779 (0.0022)	0.4794 (0.0022)	0.4599 (0.0004)	0.4006 (0.0007)	0.3899 (0.0008)
3: Standard MEL	0.4118 (0.0008)	0.4712 (0.0026)	0.4705 (0.0029)	0.4703 (0.0028)	0.4048 (0.0007)	0.3232 (0.0010)	0.3159 (0.0010)
4: Absolute $, Speedup Framing	0.4943 (0.0002)	0.4990 (0.0001)	0.4987 (0.0001)	0.4988 (0.0001)	0.4842 (0.0003)	0.4634 (0.0005)	0.4527 (0.0006)
5: Relative $, Speedup Framing	0.4941 (0.0002)	0.4847 (0.0003)	0.4846 (0.0003)	0.4847 (0.0003)	0.4862 (0.0003)	0.4529 (0.0005)	0.4410 (0.0005)

Table S1. Mean absolute deviations for all models under cross-validation using data from Experiments 1-5. Values in parentheses indicate variability under cross-validation. Per-choice probabilities are bounded at 0.01 and 0.99.

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	Exponential	Hyperbolic	Quasi-Hyperbolic
Pooled	0.4164 (0.0003)	0.4135 (0.0003)	0.4109 (0.0003)
1: Absolute $, Delay Framing	0.3525 (0.0009)	0.3597 (0.0009)	0.3479 (0.0009)
2: Relative $, Delay Framing	0.4026 (0.0007)	0.4013 (0.0008)	0.3989 (0.0007)
3: Standard MEL	0.3478 (0.0010)	0.3566 (0.0009)	0.3424 (0.0010)
4: Absolute $, Speedup Framing	0.4662 (0.0004)	0.4587 (0.0005)	0.4576 (0.0004)
5: Relative $, Speedup Framing	0.4534 (0.0005)	0.4473 (0.0005)	0.4487 (0.0006)

Table S2. Mean absolute deviations for classical models after assuming homothetic noise under cross-validation using data from Experiments 1-5. Values in parentheses indicate variability under cross-validation

Experiment	β_{I}	β_{xA}	β_{xR}	β_{tA}	β_{tR}
1: Absolute $, Delay Framing	-1.11 (0.08)	0.40 (0.07)	1.01 (0.07)	-0.22 (0.08)	-0.16 (0.08)
2: Relative $, Delay Framing	-0.59 (0.07)	0.20 (0.06)	0.84 (0.06)	-0.25 (0.07)	-0.17 (0.07)
3: Standard MEL	-1.13 (0.09)	0.44 (0.08)	1.05 (0.08)	-0.29 (0.08)	-0.19 (0.08)
4: Absolute $, Speedup Framing	-0.23 (0.06)	0.20 (0.06)	0.46 (0.06)	-0.18 (0.06)	-0.30 (0.06)
5: Relative $, Speedup Framing	-0.23 (0.06)	0.23 (0.06)	0.58 (0.06)	-0.18 (0.06)	-0.23 (0.06)
Pooled	-0.59 (0.03)	0.27 (0.03)	0.73 (0.03)	-0.21 (0.03)	-0.21 (0.03)

Table S3. Decision weights for the ITCH model separated by experiment. Values in parentheses indicate standard errors. Input variables are z-scored.