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UAS Metode Numerik

1.a. $\frac{dy}{dx} = \frac{2x}{y} - xy \quad y(0) = 1$

exact = $y(x) = \sqrt{2 - e^{-x^2}} \Rightarrow y(1) = \sqrt{2 - e^{-1^2}} =$ ~~0,7945~~
$= 1,27754$

i. Euler

$y_{n+1} = y_n + hf(x_n, y_n)$

$y_1 = y_0 + hf(x_n, y_n)$

$y_1 = 1 + 0,5\left(\frac{2.\cancel{0}}{1} - 0.1\right)$

$= 1 + 0 = 1$

ii Heun

$y_{n+1} = y_n + h/2 (k_1 + k_2)$

$y_1 = y_0 + h/2 (k_1 + k_2)$

$= 1 + 0,25 (0,5)$

$= 1,125$

- $k_1 = f(x_n, y_n) = 0$
- $k_2 = f(x_n + h, y_0 + k_1 h)$

$= \frac{2(0,5)}{1} - 0,5(1) = 0,5$

iii Akurasi

Euler $= \frac{1,27754 - 1}{1,27754} \times 100\%$

$= 0,2172 = 21,72\%$

Heun $= \frac{1,27754 - 1,125}{1,27754} \times 100\%$

$= 0,1194 = 11,94\%$

~~Euler~~

Peningkatan akurasi

$21,72 - 11,94 =$

$9,78\%$

Heun meningkat

$9,78\%$

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b. $$\frac{d^2y}{dt^2} - (1-y^2)\frac{dy}{dt} + y = 0$$

$$y(0) = y'(0) = 1$$

$$y'' - (1-y^2)\, y' + y = 0$$

$u = y$, $v = y'$

$u' = v$ $\quad u(0) = 1$

$v' = -u + (1-u^2)\, v$; $v(0) = 1$

$\cancel{v' = -u + (}$

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2. a. $f(3,4)$ polinomial kuadrat

a. i.

x	1	2	3
y	0	5	7

ii. $b_0 = 0$

$$b_1 = \frac{5-0}{2-1} = 5$$

$$b_2 = \frac{\frac{7-5}{3-2} - \frac{5-0}{2-1}}{3-1} = \frac{2-5}{2} = -1,5$$

$$f(x) = b_0 + b_1(x - x_0) + b_2(x-x_0)(x-x_1)$$

$$= 0 + 5(x-1) - 1,5(x-1)(x-2)$$

$$= \cancel{1,5x^2 + 0,5x - 2} \quad -1,5x^2 + 9,5x - 8$$

iii $1,5(3,4)^2 + 9,5(3,4) - 8 = \cancel{17,04}\ 6,96$

b. i.

x	1	2	3
y	0	5	7

ii

$$\frac{(x-1)}{(1-2)}\frac{(x-3)}{(\cancel{1}-2)}0 + \frac{(x-1)}{(2-1)}\frac{(x-3)}{(2-3)}5 + \frac{(x-1)}{(3-1)}\frac{(x-2)}{(3-2)}7$$

$$\cancel{f(x) = \frac{3x^2 + 8x + 9}{2}} \quad \cancel{= 1,5x^2 - 6x + 4,5}$$

$$f(x) = \frac{-3x^2}{2} + \frac{19x}{2} - 8$$

$\cancel{= 1,49}$

iii $= f(3,4) = \cancel{1,44}\ 6,96$

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Kedua metode menghasilkan nilai yg sama

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3. Hitung

$$\int_0^{\frac{\pi}{2}} (8 + 4\cos x)\, dx$$

a. eksak = $4 \int_0^{\frac{\pi}{2}} \cos x \, dx + 8x \int_0^{\frac{\pi}{2}} 1 \, dx$

$= 4 \sin x \Big]_0^{\pi/2} + 8 \quad x \int_0^{\pi/2} 1 \, dx$

$= 4 + 8 \quad x \int_0^{\pi/2} 1 \, dx$

$= 4 + 8x \Big]_0^{\pi/2}$

$= 4 + 4\pi$

b.

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