# Tracking of Two Connected Inverted Pendulum on Carts by Using A Fast Terminal Sliding Mode Control with Fixed-time Convergence

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Ehsan Aslmostafa
École Centrale de Nantes


Mohammad Javad Mirzaei
École Centrale de Nantes


Mohammad Ali Badamchizadeh
University of Tabriz


Sehraneh Ghaemi
University of Tabriz

# Tracking of Two Connected Inverted Pendulum on Carts by Using A Fast Terminal Sliding Mode Control with Fixed-time Convergence

Ehsan Aslmostafa
*Faculty of Electrical and Computer Engineering*
University of Tabriz
Tabriz, Iran
e_aslemostafa95@ms.tabrizu.ac.ir

Mohammad Javad Mirzaei
*Faculty of Electrical and Computer Engineering*
University of Tabriz
Tabriz, Iran
mjmirzaei95@ms.tabrizu.ac.ir

Mostafa Asadollahi
*Faculty of Electrical and Computer Engineering*
University of Tabriz
Tabriz, Iran
m.asadollahi@tabrizu.ac.ir

Mohammad Ali Badamchizadeh
*Faculty of Electrical and Computer Engineering*
University of Tabriz
Tabriz, Iran
mbadamchi@tabrizu.ac.ir

Sehraneh Ghaemi
*Faculty of Electrical and Computer Engineering*
University of Tabriz
Tabriz, Iran
ghaemi@tabrizu.ac.ir

***Abstract*—To study the underactuated systems, two connected inverted pendulum on carts (TCIPC) which is also a suitable benchmark to model various problems of control theory is utilized. In this work, a fast-terminal sliding mode controller (FTSMC) investigates the tracking problem of TCIPC. The designed controller makes the state trajectories of the system track the desired trajectories. Fixed-time sliding manifolds are appropriately proposed for the purpose to attain a better convergence in the reaching phase. In this case, the convergence time is independent of initial conditions, in contrast to the recent conventional controllers. Finally, in the simulation result, the robustness and the effectiveness of the proposed controller is shown.**

***Index Terms*—Fast-terminal Sliding Mode Control, Underactuated systems, Two connected inverted pendulum on carts (TCIPC), Fixed-time convergence, Tracking problem**

## I. Introduction

Underactuated systems are known as systems with having fewer control inputs than the possible degrees of freedom (DOF). They exist in extensive groups of practical systems like cart-pendulum system [1]. Since underactuated systems need less number of actuators, their implementation cost and intricacy are low [2]. Two inverted pendulum mounted on movable carts is a common example of such systems which has been studied since the 1950s [3]. Dynamics of the TCIPC is analogous to the dynamics of the various control system. Therefore, it can be an appropriate benchmark to investigate diverse control methods like backstepping technique and variable structure control (VSC) [4], [5]. The main goal in previous literature was to control the TCIPC in order to stabilize the equilibrium point. Many control paradigms address the TCIPC system. Some examples such as, practical implementations using linear control techniques in [6], the linear optimal control in [7] trajectory tracking in [8], and the pole placement method in [9].

Sliding mode control (SMC) is robust control approach that has constructively been employed to control various nonlinear systems such as underactuated systems [10], robotic manipulators [11], and chaotic systems [12]. method has been effectively applied to the control of linear and nonlinear systems such as robotic manipulators [11], underactuated systems [10], and chaotic systems [12]. The main features of an SMC can be listed as

- Insensitive and robust to the bounded perturbations
- Fast response and better transient performance
- Simple implementation in comparison with some other control approaches

In a conventional sliding mode control (CSMC), the robust tracking problem is ensured after the system states get to the sliding surface(s), which means that, the robustness is not guaranteed in the reaching phase. By employing a linear sliding surface(s), system asymptotic stability in the sliding phase can be attained and the system states will converge asymptotically to the system equilibria [13]. Compared with the mentioned SMC, the terminal sliding mode control (TSMC) is much stronger in achieving finite-time convergence and fast response [14]. Since TSMC increases the convergence rate in the neighborhood of the origin, this control approach is suitable for designing a higher accuracy controller [15]. In contrast to CSMC, the TSMC is designed through a set of recursive equations which are converged in finite time. Much attention is paid to the utilization of it in various control problems [16]. Nonetheless, whenever the system states are not close to the origin, TSMC may not have similar convergence behavior as CSMC. To have both robustness and a fast convergence rate, the FTSMC method has been proposed [17]. However, it is still required that the FTSMC structure have additional consideration on the reaching phase and the robust performance. It is noteworthy that the attained convergence in SMC can be any of the two asymptotic or finite-time convergence [18], relying on the chosen sliding surface(s). Moreover, it mainly has dependency on the initial values of the system states. By increasing the control gain of the present control, the convergence rate is also increased. Although the convergence rate has been increased, but it is still dependent on the initial values' parameters. Many controllers are suffering from possible limitations over their bandwidth. This shortcoming motivated the control engineers to develop such controllers in which the convergence time is free from system initial values. This emerged the so-called fixed-time convergence theory [19]. Till now, we have not found any application of the aforementioned technique to the TCIPC system, it motivates us to offer the current work.

In this paper, to consider the robust tracking problem of nonlinear TCIPC systems, FTSMC is combined with fixed-time sliding manifolds. According to Lyapunov stability theory, the existence of the proposed FTSMC is ensured around the fixed-time sliding surfaces in finite-time intervals.

The organization of this work is divided as below. In Section 2, the problem preliminaries and descriptions are given. In Section 3, an FTSMC is proposed to accomplish the tracking problem for the TCIPC, besides stability of the tracking problem and the fixed-time manifolds are analyzed via Lyapunov theorem. In Section 4, the simulation results are shown, and in Section 5, the research findings are concluded.

## II. Preliminaries and Problem description

We consider the TCIPC system (see Fig. 1), the position of the pivot is assumed as a function of time and can have different value across the length $l$ of the pendulums. The input applied at the pivot point of each pendulum is equal to the torque $U_1$ and $U_2$.

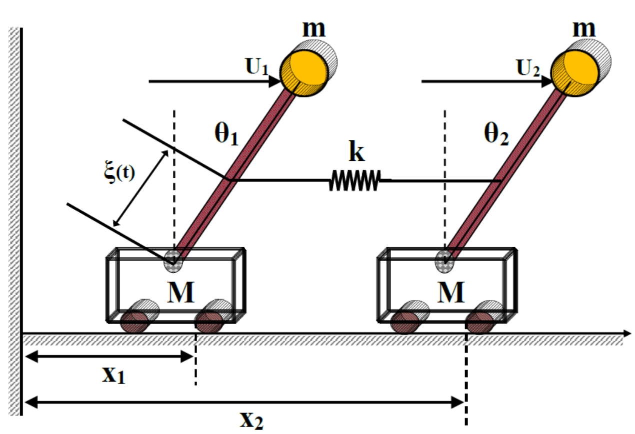


Fig. 1. Two connected inverted pendulums on carts.

The dynamic of the TCIPC can be described as follow

$$\begin{cases} \ddot{\theta}_1(t) = \frac{g}{cl}\theta_1 + \frac{1}{cml^2}U_1 - \frac{m}{M}\dot{\theta}_1^2 sin(\theta_1) \\ \qquad +(\frac{k(\xi(t)-cl)}{cml^2})(-\xi(t)\theta_1 + \xi(t)\theta_2 - \tilde{x}_1 + \tilde{x}_2) \\ \ddot{\theta}_2(t) = \frac{g}{cl}\theta_2 + \frac{1}{cml^2}U_2 - \frac{m}{M}\dot{\theta}_2^2 sin(\theta_2) + \\ \qquad +(\frac{k(\xi(t)-cl)}{cml^2})(-\xi(t)\theta_2 + \xi(t)\theta_1 - \tilde{x}_2 + \tilde{x}_1) \end{cases} \tag{1}$$

where $\theta_1$, $\theta_2$ are the angles of inverted pendulums and $\dot{\theta}_1$ ,$\dot{\theta}_2$ are the angular velocities of the inverted pendulums with respect to vertical axes, the parameters $U_1$ and $U_2$ are the control inputs applied to the two inverted pendulums to attain the tracking problem, $c = 1 - \frac{M}{m+M}$, $k$ is the connected spring and $g$ is the gravity constant. The parameter $\xi(t)$ is an intermediate variable, which is working as a function of time, and $\xi(t) \in [0, cl]$ and has the value equals to $1 - \frac{M}{M+m}$.

By considering $[x_{11}, x_{21}]^T = [\theta_1, \theta_2]^T$ and $[x_{12}, x_{22}]^T = [\dot{\theta}_1, \dot{\theta}_2]^T$ the state space dynamical model of the considered system is obtained as follows:

$$\begin{cases} \dot{x}_{11}(t) = x_{12}(t) \\ \dot{x}_{12}(t) = \Xi_1(x_{11}, x_{12}, x_{21}, x_{22}) + \mathcal{K}_1 U_1(t) \\ \dot{x}_{21}(t) = x_{22}(t) \\ \dot{x}_{22}(t) = \Xi_2(x_{11}, x_{12}, x_{21}, x_{22}) + \mathcal{K}_2 U_2(t) \\ y_1 = x_{11}, \;\; y_2 = x_{21} \end{cases} \tag{2}$$

where

$$\Xi_1(x_{11}, x_{12}, x_{21}, x_{22}) = \frac{g}{cl}x_{11}(t) - \frac{m}{M}x_{12}^2(t) sin(x_{11}(t))$$
$$+ \frac{k(\xi(t) - cl)}{cml^2}(-\xi(t)x_{11}(t) + \xi(t)x_{21}(t) - \tilde{x}_1(t) + \tilde{x}_2(t)),$$
$$\Xi_2(x_{11}, x_{12}, x_{21}, x_{22}) = \frac{g}{cl}x_{11}(t) - \frac{m}{M}x_{12}^2(t) sin(x_{11}(t))$$
$$+ \frac{k(\xi(t) - cl)}{cml^2}(-\xi(t)x_{11}(t) + \xi(t)x_{21}(t) - \tilde{x}_1(t) + \tilde{x}_2(t)),$$
$$\mathcal{K}_1 = \mathcal{K}_2 = \frac{1}{cml^2}, \;\; \xi(t) = sin(\omega t)$$
$$\tilde{x}_1 = sin(\omega_1 t), \;\; \tilde{x}_2 = sin(\omega_2 t) + \mathcal{F}.$$

The main aim of the proposed finite-time method is to control each pendulum with mass $m$ freely such that every pendulum can track the desired reference signal in finite time, while the carts and the connected spring are moving. By considering the tracking errors as $e_1(t) = y_1(t) - y_{r1}(t)$ and $e_2(t) = y_2(t) - y_{r2}(t)$, the system model (2) can be obtained in the error-state space structure through the subsequent dynamical equations:

$$\begin{cases} \dot{e}_{11}(t) = e_{12}(t) \\ \dot{e}_{12}(t) = \Xi_1 - \frac{d^2 y_1(t)}{dt^2} + \mathcal{K}_1 U_1(t) \\ \dot{e}_{21}(t) = e_{22}(t) \\ \dot{e}_{22}(t) = \Xi_2 - \frac{d^2 y_2(t)}{dt^2} + \mathcal{K}_2 U_2(t) \end{cases} \tag{3}$$

Therefore, the tracking problem for error system (3) should be considered. To have control over error-state system (3) the input signals are taken as $U_1(t)$ and $U_2(t)$ in the function of defined tracking error states. Accordingly, if the proposed finite-time controllers stabilize the error-states system (3), and the defined errors reach to the origin in given finite time, then the two connected inverted pendulum on carts have been tracked the desired trajectories through the designed finite time controllers. In the following, some lemmas which are used in the design procedure have been presented.

**Lemma 1.** *The following system is considered*

$$\dot{\varpi} = f(\varpi, t), \qquad \varpi(0,t) = 0, \qquad \varpi \in U_0 \subset R^n. \tag{4}$$

*Let the $\upsilon(\varpi)$ as a positive definite function in an adjacency of a system origin, in which constant numbers $\mathcal{C}, \rho \in \mathbb{R}$ and $\mathcal{C} > 0, \rho \in (0,1)$ such that $\dot{\varpi}(t) \leq -\mathcal{C}\varpi(t)$, then the system can be finite-time stable, and the following inequality shows the upper bound of the settling time [20].*

$$T \leq \frac{V^{1-\rho}(0)}{\mathcal{C}(1-\rho)} \tag{5}$$

**Lemma 2.** *Let $\mathcal{A}_i \in \mathbb{R}$ for $i = 1, \cdots, n$ and $0 < \kappa < 16$, then the following inequality is satisfied [15]:*

$$\sum_{i=1}^{n} |\mathcal{A}_i|^{\kappa} \geq \Big(\sum_{i=1}^{n} |\mathcal{A}_i|\Big)^{\kappa} \tag{6}$$

The principal aim of the presented article mainly is the combination of the terminal SMC approach and fixed-time integral sliding manifolds to accomplish the finite-time convergence of error states to the origin of the system. The privilege of the proposed method is attainment of finite-time tracking problem of TCIPC that can be accomplished in the sense of fixed-time convergence for sliding phase.

## III. Control Design

This section addresses the design procedure of the proposed FTSMC with fixed-time convergence for the finite-time tracking of the TCIPC system. In general, an SMC technique is designed in two stages. First, the sliding manifolds with suitable dynamical properties are designed, then the control inputs are developed to guarantee that the system states arrive at the sliding manifolds and remain on them.

### A. Fixed-time sliding manifold

According to the defined error-state dynamical system (3), the fixed-time integral sliding manifolds are chosen as the following equations.

$$\begin{aligned} S_1(t) &= e_{12}(t) + \int_0^t \Big(C_{11} sig^{\alpha_{11}}(e_{11}) + C_{12} sig^{\alpha_{12}}(e_{12}) \\ &\qquad + B_{11} sig^{\beta_{11}}(e_{11}) + B_{12} sig^{\beta_{12}}(e_{12})\Big)\Big) d\varphi \\ S_2(t) &= e_{22}(t) + \int_0^t \Big(C_{21} sig^{\alpha_{21}}(e_{21}) + C_{22} sig^{\alpha_{22}}(e_{22}) \\ &\qquad + B_{21} sig^{\beta_{21}}(e_{21}) + B_{22} sig^{\beta_{22}}(e_{22}))\Big) d\varphi \end{aligned} \tag{7}$$

where $sig^{\alpha}(\mathcal{G}) = |\mathcal{G}|^{\alpha} sgn(\mathcal{G})$ and $sgn(\mathcal{G})$ denotes the sign function. The parameters $B_{ij}$ and $C_{ij}$ for $i,j = 1,2$ are positive constants. The above surfaces are fixed-time convergent, provided that the following polynomials are satisfied, such that $S_1^2 + B_{12}S_1 + B_{11}$, $S_1^2 + C_{12}S_1 + C_{11}$, $S_2^2 + B_{22}S_2 + B_{21}$, and $S_2^2 + C_{22}S_1 + C_{21}$ are Hurwitz [21]. The parameters $\alpha_{ij} = \frac{\tilde{\alpha}_{ij}\alpha}{2\alpha - \tilde{\alpha}_{ij}}$ and $\beta_{ij} = \frac{\tilde{\beta}_{ij}\beta}{2\beta - \tilde{\beta}_{ij}}$, where $\tilde{\alpha}_{ij} \in (0,1)$ and $\tilde{\beta}_{ij} \in (1, 1+\epsilon)$ in which $i,j = 1,2$, $\epsilon > 0$ is small enough, and the constant parameters $\alpha = \beta = 1$.

The conditions $S_1(t) = 0$ and $S_2(t) = 0$ is satisfied whenever the system states get to the sliding manifolds. If the error-state system remain on the sliding manifolds, i.e., if $S_1(t) = 0$, then $\dot{S}_1(t)$, and $S_2(t) = 0$, then $\dot{S}_2(t) = 0$ . Hence, the dynamics of the suggested FTSMC in the presence of fixed-time convergence can be regarded as:

$$\begin{aligned} \dot{e}_{12}(t) &= -\Big(C_{11} sig^{\alpha_{11}}(e_{11}) + C_{12} sig^{\alpha_{12}}(e_{12}) \\ &\qquad + B_{11} sig^{\beta_{11}}(e_{11}) + B_{12} sig^{\beta_{12}}(e_{12})\Big) \\ \dot{e}_{22}(t) &= -\Big(C_{21} sig^{\alpha_{21}}(e_{21}) + C_{22} sig^{\alpha_{22}}(e_{22}) \\ &\qquad + B_{21} sig^{\beta_{21}}(e_{21}) + B_{22} sig^{\beta_{22}}(e_{22})\Big) \end{aligned} \tag{8}$$

### B. Fast terminal sliding mode controller

The goal of the designed scheme is to make the state trajectories of the TCIPC (2) track the desired signals so that reach the fixed-time sliding manifolds (7) in a given finite time and stay on it forever. Here, to address the tracking problem of TCIPC, a sliding control rule is defined for each inverted pendulum, and the finite-time stability for each one of them is guaranteed. The fast terminal SMC protocols for the nonlinear defined error-state system (3) with the fixed-time integral sliding surfaces (7) are designed as follows

$$\begin{aligned} U_1 &= -\mathcal{K}_1^{-1}(\Xi_1 - \frac{d^2 y_1(t)}{dt^2} + S_1 + \zeta sig^{\kappa}(S_1) \\ &\quad + C_{11} sig^{\alpha_{11}}(e_{11}) + C_{12} sig^{\alpha_{12}}(e_{12}) \\ &\quad + B_{11} sig^{\beta_{11}}(e_{11}) + B_{12} sig^{\beta_{12}}(e_{12})) \\ U_2 &= -\mathcal{K}_2^{-1}(\Xi_2 - \frac{d^2 y_2(t)}{dt^2} + S_2 + \zeta sig^{\kappa}(S_2) \\ &\quad + C_{11} sig^{\alpha_{11}}(e_{11}) + C_{12} sig^{\alpha_{12}}(e_{12}) \\ &\quad + B_{11} sig^{\beta_{11}}(e_{11}) + B_{12} sig^{\beta_{12}}(e_{12})) \end{aligned} \tag{9}$$

According to the designed finite-time controller, it is guaranteed that the nonlinear defined tracking error dynamics (3) states reach to the proposed integral sliding manifolds (7) in finite convergence time and remain on them correspondingly.

**Theorem 1.** *Consider the error dynamics* (3)*, which have been defined for tracking problem of TCIPC. If the control inputs are selected as* (9)*, then the system states will be achieved $S_1 = 0$ and $S_2 = 0$ in the sense of Lemma 1, and also will stay on the defined fixed-time sliding manifolds.*

*Proof.* The Lyapunov function candidate is chosen as follows.

$$V(t) = 0.5\Big(S_1^2(t) + S_2^2(t)\Big) \tag{10}$$

By taking the time derivative of V(t), it yields

$$\dot{V}(t) = S_1\dot{S}_1 + S_2\dot{S}_2 \tag{11}$$

Taking the derivative of ISM manifolds (7) and inserting the obtained error-state system (3) yield

$$\begin{aligned}\dot{V}(t) = S_1\Big(&\Xi_1 + \mathcal{K}_1U_1 + C_{11}sig^{\alpha_{11}}(e_{11}) + C_{12}sig^{\alpha_{12}}(e_{12})\\ &+ B_{11}sig^{\beta_{11}}(e_{11}) + B_{12}sig^{\beta_{12}}(e_{12})\Big)\\ + S_2\Big(&\Xi_2 + \mathcal{K}_2U_2 + C_{21}sig^{\alpha_{21}}(e_{21}) + C_{22}sig^{\alpha_{22}}(e_{22})\\ &+ B_{21}sig^{\beta_{21}}(e_{21}) + B_{22}sig^{\beta_{22}}(e_{22})\Big)\end{aligned} \tag{12}$$

According to the error-state system dynamics, using proposed control laws (9) yields

$$\begin{aligned}\dot{V}(t) &= -S_1\Big(S_1 + \zeta sig^{\kappa}(S_1)\Big) - S_2\Big(S_2 + \zeta sig^{\kappa}(S_2)\Big)\\ &= -S_1^2 - S_2^2 - \zeta|S_1|^{\kappa+1} - \zeta|S_2|^{\kappa+1}\end{aligned} \tag{13}$$

Taking Lemma 2 into account gives

$$\begin{aligned}\dot{V}(t) &= -S_1^2 - S_2^2 + \zeta\Big(|S_1|^2\Big)^{\frac{\kappa+1}{2}} - \zeta\Big(|S_2|^2\Big)^{\frac{\kappa+1}{2}}\\ &\leq -S_1^2 - S_2^2 - \zeta\Big(|S_1|^2 + |S_2|^2\Big)^{\frac{\kappa+1}{2}}\\ &\leq -2\Big(V(t) + \zeta\big(V(t)\big)^{\frac{\kappa+1}{2}}\Big)\end{aligned} \tag{14}$$

According to the Lemma 1, the tracking problem for the defined error-state system (3) is accomplished in the sense of finite-time stability with the proposed controls (9). This shows that the sliding surfaces are converging to zero in finite time. This verifies the proof. □

## IV. Analysis and Numerical Simulation

In the present section, numerical simulations have been performed to demonstrate the effectiveness of the proposed finite-time tracking problem. The TCIPC design parameters as selected as $g = 9.8$, $M = 10$, $m = 10$, $l = 1$, $c = 0.5$, $\omega = 5$, $\omega_1 = 2$, $\omega_2 = 2$, $\mathcal{F} = 1$, and $k = 1$. Initial conditions for considered underactuated system (2) are chosen as $[x_{11}, x_{12}, x_{21}, x_{22}]^T = [0.3, -0.3, -0.5, 0.5]^T$.

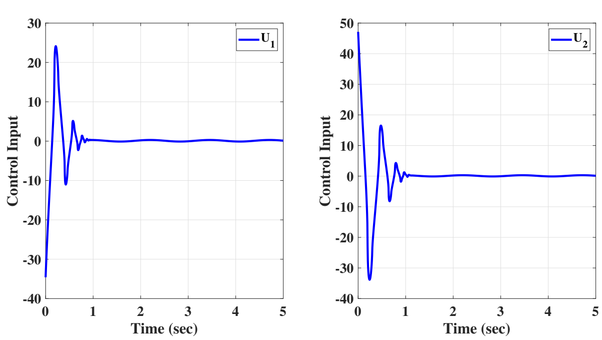


Fig. 2. Control inputs for two connected inverted pendulums on carts.

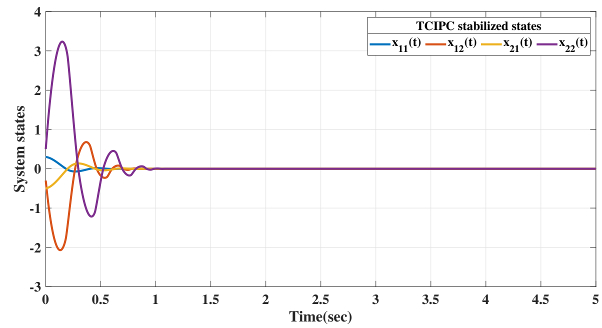


Fig. 3. Time history of the system states by using the proposed controller.

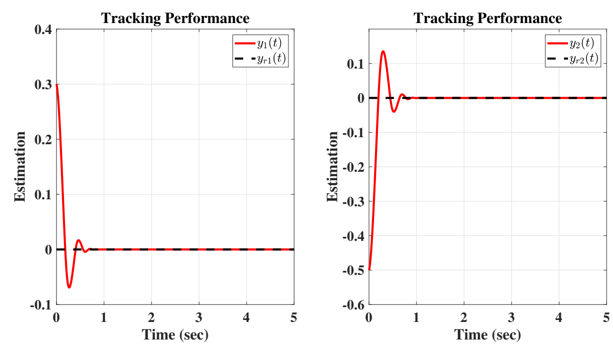


Fig. 4. Time history of the tracking errors by using the proposed FTSMC.

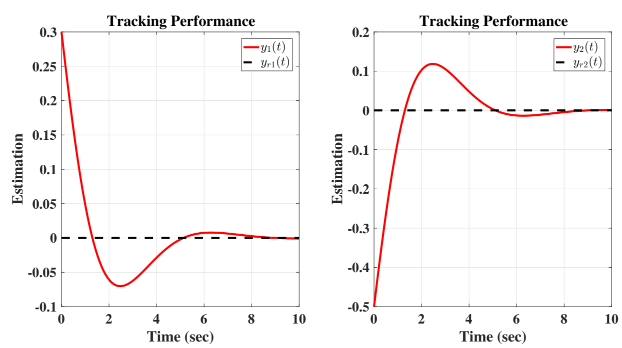


Fig. 5. Time history of the tracking errors by using a CSMC.

The designed fast-terminal sliding mode controller parameters are selected as $B_{11} = B_{21} = 32$, $B_{12} = B_{22} = 5$, $C_{11} = C_{21} = 32$, $C_{12} = C_{22} = 5$, $\alpha_{11} = \alpha_{21} = 7/13$, $\alpha_{21} = \alpha_{22} = 0.7$, $\beta_{11} = \beta_{21} = 21/20$, $\beta_{21} = \beta_{22} = 21/19$ $\zeta = 5$ and $\kappa = 0.9$. If the desired signal set as zero the tracking problem can be regarded as a stabilization problem, in this case, the reference signals have been selected as $y_{r1} = y_{r2} = 0$ in order to investigate output stabilization of the proposed controller. In the simulation procedure, the desired signals to be tracked are set as zero. The trajectory of the control input is displayed in Fig. 2. In Fig. 3, the time behaviors of the system states has been shown. The error states convergent to 0 in a finite time with the proposed controller is shown in Fig. 4, which means that the error-state system (3) can achieve finite-time tracking problem with accurate performance and much faster tracking realization compare with the conventional sliding mode controller depicted in Fig. 5.

## V. Conclusion

In this article tracking problem for two connected inverted pendulum on carts was investigated by an FTSMC with fixed-time sliding manifolds. The stability of the proposed fixed-time manifolds is proved in the sense of Lyapunov theory. The designed control can achieve the tracking problem for TCIPC in finite time. Numerical simulations demonstrated the robustness and validity of theoretical investigations completely.

## References


[1] A. Zhang, J. She, J. Qiu, C. Yang, and F. Alsaadi, "Design of motion trajectory and tracking control for underactuated cart-pendulum system," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 8, pp. 2458–2470, 2019.

[2] I. Fantoni, R. Lozano, and R. Lozano, *Non-linear control for underactuated mechanical systems*. Springer Science & Business Media, 2002.

[3] W. Zhong and H. Rock, "Energy and passivity based control of the double inverted pendulum on a cart," in *Proceedings of the 2001 IEEE International Conference on Control Applications (CCA'01)(Cat. No. 01CH37204)*. IEEE, 2001, pp. 896–901.

[4] S. Rudra and R. K. Barai, "Robust adaptive backstepping control of inverted pendulum on cart system," *International journal of control and automation*, vol. 5, no. 1, pp. 13–26, 2012.

[5] E. Doskocz, Y. Shtessel, and C. Katsinis, "Mimo sliding mode control of a robotic" pick and place" system modeled as an inverted pendulum on a moving cart," in *Proceedings of Thirtieth Southeastern Symposium on System Theory*. IEEE, 1998, pp. 379–383.

[6] H. N. Iordanou and B. W. Surgenor, "Experimental evaluation of the robustness of discrete sliding mode control versus linear quadratic control," *IEEE Transactions on control systems technology*, vol. 5, no. 2, pp. 254–260, 1997.

[7] B. Ata and R. Coban, "Linear quadratic optimal control of an inverted pendulum on a cart using artificial bee colony algorithm: an experimental study," *Cukurova University Journal of the Faculty of Engineering and Architecture*, vol. 32, no. 2, pp. 109–123, 2017.

[8] M. Ramirez-Neria, Z. Gao, H. Sira-Ramirez, R. Garrido-Moctezuma, and A. Luviano-Juarez, "Trajectory tracking for an inverted pendulum on a cart: an active disturbance rejection control approach," in *2018 Annual American Control Conference (ACC)*. IEEE, 2018, pp. 4881–4886.

[9] I. Siradjuddin, Z. Amalia, B. Setiawan, R. P. Wicaksono, and E. Yudaningtyas, "Stabilising a cart inverted pendulum system using pole placement control method," in *2017 15th International Conference on Quality in Research (QiR): International Symposium on Electrical and Computer Engineering*. IEEE, 2017, pp. 197–203.

[10] M. Zhang, Y. Zhang, and X. Cheng, "An enhanced coupling pd with sliding mode control method for underactuated double-pendulum overhead crane systems," *International Journal of Control, Automation and Systems*, vol. 17, no. 6, pp. 1579–1588, 2019.

[11] Q. V. Doan, T. D. Le, and A. T. Vo, "Synchronization full-order terminal sliding mode control for an uncertain 3-dof planar parallel robotic manipulator," *Applied Sciences*, vol. 9, no. 9, p. 1756, 2019.

[12] M. Asadollahi, A. R. Ghiasi, and M. A. Badamchizadeh, "Adaptive synchronization of chaotic systems with hysteresis quantizer input," *ISA transactions*, vol. 98, pp. 137–148, 2020.

[13] S. Ahmed, H. Wang, and Y. Tian, "Adaptive fractional high-order terminal sliding mode control for nonlinear robotic manipulator under alternating loads," *Asian Journal of Control*, 2020.

[14] M. Zhihong and X. H. Yu, "Terminal sliding mode control of mimo linear systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, no. 11, pp. 1065–1070, 1997.

[15] A. Modiri and S. Mobayen, "Adaptive terminal sliding mode control scheme for synchronization of fractional-order uncertain chaotic systems," *ISA Transactions*, 2020.

[16] H. Rabiee, M. Ataei, and M. Ekramian, "Continuous nonsingular terminal sliding mode control based on adaptive sliding mode disturbance observer for uncertain nonlinear systems," *Automatica*, vol. 109, p. 108515, 2019.

[17] M. Labbadi and M. Cherkaoui, "Robust adaptive backstepping fast terminal sliding mode controller for uncertain quadrotor uav," *Aerospace Science and Technology*, vol. 93, p. 105306, 2019.

[18] X. Yu and M. Zhihong, "Fast terminal sliding-mode control design for nonlinear dynamical systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 49, no. 2, pp. 261–264, 2002.

[19] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time stabilization: Implicit lyapunov function approach," *Automatica*, vol. 51, pp. 332–340, 2015.

[20] X. Liu, S. He, P. Lu, H. Liu, and C. Du, "Finite-time consensus for multi-agent systems with nonlinear dynamics under euler digraph via pinning control," *International Journal of Systems Science*, pp. 1–11, 2021.

[21] J. P. Mishra, X. Yu, and M. Jalili, "Arbitrary-order continuous finite-time sliding mode controller for fixed-time convergence," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 65, no. 12, pp. 1988–1992, 2018.