N11

$$H_0: p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

$$H_1: p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

a) $n=1$

$$l = \frac{L_1}{L_0} = \frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{e-1} \{(0,1)\} \geq c$$

$$1 - x \geq \ln c(e-1)$$

$$x \leq 1 - \ln c(e-1) = A.$$

$G_{кр}: x \leq A;\ P(x \leq A / H_0) = \alpha$

$$\int_0^A p_0(x)dx = \alpha \Rightarrow A = \alpha$$

$G_{кр}: x \leq \alpha$

$\alpha_1 = \alpha.$

$$W = P(x \leq A / H_1) = \int_0^\alpha p_1(x)dx =$$

$$= \int_0^\alpha \frac{e^{1-x}}{e-1}dx = -\frac{e^{1-x}}{e-1}\Big|_0^\alpha = (1-e^{-\alpha})\frac{e}{e-1}$$

$$\alpha_2 = 1 - (1 - e^{-\alpha})\frac{e}{e-1}$$

b) $n=2.$

$$l = \frac{L_1}{L_0} = \frac{p_1(x_1)p_1(x_2)}{p_0(x_1)\cdot p_0(x_2)} = \frac{e^{1-x_1}\cdot e^{1-x_2}}{(e-1)^2} \geq c$$

$$e^{-x_2-x_1} \geq \frac{c(e-1)^2}{e^2}$$

$$-x_2 - x_1 \geq \ln\left(\frac{c(e-1)^2}{e^2}\right)$$

$G_{кр}: x_1 + x_2 \leq A.$

$P(x_1 + x_2 \leq A / H_0) = \alpha$

$$\iint_{x_1+x_2 \leq A} 1\, dx_1\, dx_2 = \frac{1}{2}A^2 = \alpha \Rightarrow A = \sqrt{2\alpha}$$

$G_{кр}: x_1 + x_2 \leq \sqrt{2\alpha},\quad \alpha_1 = \alpha$

$$W = P(x_1 + x_2 \leq A / H_1) = \iint_{x_1+x_2 \leq A} \frac{e^2}{(e-1)^2} e^{-x_1-x_2} dx_1\, dx_2 =$$

$$= \frac{e^2}{(e-1)^2}\int_0^A dx_1 \int_0^{A-x_1} e^{-x_2-x_1} dx_2 = \frac{e^2}{(e-1)^2}(1 - e^{-A} - e^{-A}A)$$

$$\alpha_2 = 1 - W = 1 - \frac{e^2}{(e-1)^2}(1 - e^{-A} - e^{-A}A)$$

c) $$l = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq c$$

$$\ln l = \sum_{i=1}^{n} \underbrace{\ln\left(\frac{p_1}{p_0}\right)}_{\eta_i} \geq \ln c$$

$$\eta_i = \ln\left(\frac{p_1}{p_0}\right) = \ln\left(\frac{e^{1-x_i}}{e-1}\right) = \ln\frac{e}{e-1} - x_i$$

$$\frac{\sum_{i=1}^{n} \eta_i - nM_{\eta_i}}{\sqrt{nD\eta_i}} \rightsquigarrow N(0,1).$$

$$P(\ln L \geq \ln C \mid H_0) = \alpha.$$

$$\ln L = n \ln\frac{e}{e-1} - \sum_{i=1}^{n} x_i \geq \ln C$$

$$G_{кр}: \sum_{i=1}^{n} x_i \leq A$$

$$P\left(\frac{\sum_{i=1}^{n} x_i - nM_x}{\sqrt{nDx}} \leq \frac{A - nM_x}{\sqrt{nDx}} \mid H_0\right) = \alpha.$$

для H_0: $M_x = \frac{1}{2}$

$D_x = \frac{1}{12}$.

$$\frac{A - \frac{n}{2}}{\sqrt{\frac{n}{12}}} = u_\alpha \qquad A = \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$G_{кр}: \sum_{i=1}^{n} x_i \leq \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} \qquad \alpha_1 = \alpha.$$

$$W = P\left(\sum_{i=1}^{n} x_i \leq A \mid H_1\right) = P\left(\frac{\sum_{i=1}^{n} x_i - nM_x}{\sqrt{nDx}} \leq \frac{A - nM_x}{\sqrt{nDx}} \mid H_1\right)$$

для H_1: $M_x = \int_0^1 \frac{e^{1-x}}{e-1} x\,dx = \frac{e-2}{e-1}$

$$M_{x^2} = \int_0^1 \frac{e^{1-x}}{e-1} x^2\,dx = \frac{2e-5}{e-1}$$

$$D_x = \frac{2e-5}{e-1} - \left(\frac{e-2}{e-1}\right)^2 = \frac{e^2-3e+1}{(e-1)^2}$$

$$W = \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx \to 1, \ n \to \infty, \text{ так как}$$

$$\beta = \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n\frac{e^2-3e+1}{(e-1)^2}}} \sim \frac{\sqrt{n}\,\overset{0,042}{\left(\frac{1}{2} - \frac{e-2}{e-1}\right)} \text{[illegible]}}{\sqrt{\frac{e^2-3e+1}{(e-1)^2}}} \xrightarrow[n \to +\infty]{} +\infty. \qquad u_\alpha < 0$$

2) Состоятельный.

d) $G_{кр}: x_{min} < C \qquad P(x_{min} < C \mid H_0) = \alpha$

$H_0: \xi \sim R(0,1)$.

$H_1: x_{min} \sim 1 - (1 - F(x))^n$.

$$1 - (1 - F(C))^n = \alpha$$

$$(1 - F(C))^n = 1 - \alpha$$

$$F(C) = 1 - \sqrt[n]{1-\alpha} = C$$

$$G_{кр}: x_{min} < 1 - \sqrt[n]{1-\alpha} \qquad \alpha_1 = \alpha$$

$$W = P(x_{min} < C \mid H_1)$$

$$F_1(x) = \int_{-\infty}^{x} p_1(t)\,dt = \int_0^x \frac{e^{1-t}}{e-1}\,dt = \frac{e}{e-1}(1 - e^{-x}) \qquad x \in (0,1)$$

$$W = 1 - (1 - F(C))^n = 1 - \left(1 + \frac{e^{1-x}}{e-1} - \frac{e}{e-1}\right)^n\Big|_{x=C} =$$

$$= 1 - \left(1 + \frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n$$

$$\alpha_2 = \left(1 + \frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n.$$

$$e^{\sqrt[n]{1-\alpha}} = \exp\left(\exp\left(\frac{1}{n}\ln(1-\alpha)\right)\right) = \exp\left(1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)\right) = e\left(1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)\right)$$

$$\alpha_2 = \left(1 + \frac{e}{e-1}\left(1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)\right) - \frac{e}{e-1}\right)^n = \left(1 + \frac{e}{e-1}\frac{1}{n}\ln(1-\alpha) - \underbrace{\frac{e}{e-1}o\left(\frac{1}{n}\right)}_{\xrightarrow[n\to\infty]{} 0}\right)^n$$

$$\alpha_2 \to e^{\frac{e}{e-1}\ln(1-\alpha)} = (1-\alpha)^{\frac{e}{e-1}} \neq 0, \ n \to \infty \Rightarrow \text{крит. несост.}$$