№6

$$P(x)=\begin{cases}\frac{\theta-1}{x^{\theta}} & x\geq 1\\ 0 & x<1\end{cases}$$

$$L(x,\theta)=\frac{(\theta-1)^n}{\prod_{i=1}^{n}x^{\theta}} \quad \ln L = n\ln(\theta-1)-\theta\sum_{i=1}^{n}\ln x_i$$

$$(\ln L)'_{\theta}=\frac{n}{\theta-1}-\sum_{i=1}^{n}x_i=0$$

$$(\ln L)''_{\theta\theta}=-\frac{n}{(\theta-1)^2}<0 \Rightarrow \theta=1+\frac{1}{\ln x_i} \Rightarrow \theta=1+\frac{1}{\ln x}\text{ - т. max.}$$

Дов. инт.

$\left(\int\frac{\theta-1}{x^{\theta}}dx\right)'_{\theta}=x^{1-\theta}\ln x \Rightarrow$ модель сильно регулярная.

$$\int\frac{\partial}{\partial\theta}\left(\frac{\theta-1}{x^{\theta}}\right)dx=x^{1-\theta}\ln x.$$

$$\int_1^{\tilde{x}}\frac{\theta-1}{x^{\theta}}dx=-\frac{1}{x^{\theta-1}}+1=\frac{1}{2} \quad \tilde{x}=\theta^{\frac{1}{\theta-1}}$$

$$g(\tilde{\theta})=2^{\frac{1}{\theta-1}}$$

$$\sqrt{n}\,\frac{g(\tilde{\theta})-g(\theta)}{\sigma(\theta)}\rightsquigarrow N(0,1).$$

$$\theta\xrightarrow{P}\sigma(\tilde{\theta})$$

$$\sigma(\tilde{\theta})=\sqrt{D^{T}g(\tilde{\theta})\,i^{-1}(\tilde{\theta})\,Dg(\theta)} \quad I(\theta)=M[((\ln p)'_{\theta})^2]=$$

$$=M\left[\left(\frac{1}{\theta-1}-\ln x\right)\right]=\int_1^{+\infty}\left(\frac{1}{\theta-1}-\ln x\right)^2p(x,\theta)dx=\int_1^{+\infty}\left(\frac{1}{\theta-1}-\ln x\right)^2\frac{\theta-1}{x^2}dx=$$

$=\frac{1}{(\theta-1)^2}$ непр. на $\theta>1$

$$Dg(\tilde{\theta})=\frac{-\ln 2\cdot 2^{\frac{1}{\theta-1}}}{\theta-1} \qquad \sigma(\tilde{\theta})=\frac{-\ln 2\cdot 2^{\frac{1}{\theta-1}}}{\theta-1}$$

$$\sqrt{n}\,\frac{g(\tilde{\theta})-g(\theta)}{\sigma(\tilde{\theta})}\rightsquigarrow N(0,1)$$

$$\frac{1.96\sigma(\theta)}{\sqrt{n}}+g(\tilde{\theta})<g(\theta)<\frac{1.96\sigma(\tilde{\theta})}{\sqrt{n}}+g(\tilde{\theta})$$

$$\sqrt{n}\,\frac{\tilde{\theta}-\theta}{\sigma(\theta)}\rightsquigarrow N(0,1)$$

$$\sigma(\theta)\xrightarrow{P}\sigma(\tilde{\theta})$$

$$\sigma(\tilde{\theta})=\cancel{\theta X}\,\theta-1 \Rightarrow \sqrt{n}\,\frac{\tilde{\theta}-\theta}{\theta-1}\rightsquigarrow N(0,1)$$

$$-\frac{1.96(\theta-1)}{\sqrt{n}}+1+\frac{1}{\ln x_i}<\theta<\frac{1.96(\theta-1)}{\sqrt{n}}+1+\frac{1}{\ln x_i}$$