SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

F21SA
Statistical Modelling and Analysis

Semester 1 - 2021/2022

Duration: 24 hours

Question	Marks
1	9
2	6
3	7
4	4
5	8
6	6
Total Marks	40

Attempt ALL SIX questions.
To receive full credit you must show your work and explain your answers.

Excerpts from Cambridge Statistical Tables are provided on pages 7–14

1. Three companies, A, B and C, produce industrial pipes. The city council regularly orders pipes needed for ongoing construction projects. Over the years, they have collected the following information:

- They receive 10% of the pipes from A and each pipe from A has a 2% chance of being faulty.
- They receive 30% of the pipes from B and each pipe from B has a 0.5% chance of being faulty.
- They receive 60% of the pipes from C and each pipe from C has a 1% chance of being faulty.

Use this information to calculate:

(a) The probability that a randomly selected pipe is faulty. [2 marks]

(b) The conditional probability that a randomly selected pipe was supplied by A given that it is not faulty. [3 marks]

(c) Which supplier would you use for an order of 100 pipes, given that the pipes from A cost $\$80$ per pipe, the pipes from B cost $\$150$ per pipe and the pipes from C cost $\$120$ per pipe? Here we assume that all the pipes in the order have to come from one supplier and that receiving a faulty pipe causes losses of $\$100000$ if there is even one faulty pipe in the order (but the losses are not increased if there are more faulty pipes than one). [4 marks]

[Total 9 marks]

[PLEASE TURN OVER]

2. A certain subspecies of abalones can have between 3 and 7 rings. The probability distribution of the number of rings is given by

x	3	4	5	6	7
$P(X = x)$	θ	θ	2θ	$\frac{1}{2} - \theta$	$\frac{1}{2} - 3\theta$

for some parameter $\theta \in \mathbb{R}$. A sample of 40 specimens was collected and the measured numbers of rings were:

rings	3	4	5	6	7
frequency	4	3	9	15	9

(a) Find the method of moments estimator for θ. [2 marks]

(b) Find the maximum likelihood estimator for θ. [4 marks]

[Total 6 marks]

[PLEASE TURN OVER]

3. A group of biologists collected 15 specimens of a certain plant and measured their height in cm. They obtained

$$\sum_{i=1}^{15} x_i = 424.22 \qquad \text{and} \qquad \sum_{i=1}^{15} x_i^2 = 12086.41$$

The biologists believe that the height has distribution $N(28, 5)$.

(a) Find a 95% confidence interval for the population variance and comment on whether this belief is justified. [3 marks]

(b) The experiment was subsequently reviewed by a different group of biologists who discovered that the two specimens with the largest heights of 33.97 cm and 34.04 cm were incorrectly classified and in fact belong to a different species. Hence their heights should be removed from the sample. With the modified data, how does the confidence interval for the population variance change? Is the belief that the population variance equals 5 still justified? [4 marks]

[Total 7 marks]

4. The number of Heavy Goods Vehicles (HGVs) passing through a busy bridge has been recorded for 15 consecutive days.

1908	1993	2110	2145	2009	2043	2037	1889
1922	2066	2025	1979	2039	2085	1912	

Based on this data, the government wants to test the hypothesis that the mean daily number of HGVs passing through that bridge is 2000. Design an appropriate hypothesis test and comment on whether the data supports the hypothesis. [4 marks]

[Total 4 marks]

[PLEASE TURN OVER]

5. An ice cream factory is trying to determine whether increasing the amount of milk fat solids in their products increases the number of sales. They have collected the following data:

Milk fat solids in % (x)	10	11	12	13	14	15	16	17	18	19
Litres of ice cream sold (y)	56	64	62	57	51	49	46	41	35	36

For these data

$$\sum x_i = 145,\ \sum x_i^2 = 2185,\ \sum y_i = 497,\ \sum y_i^2 = 25645,\ \sum x_i y_i = 6945$$

(a) Calculate S_{xx}, S_{yy}, S_{xy} and the fitted linear regression equation of increase in y on x. [3 marks]

(b) Calculate the Pearson's correlation coefficient between y and x and comment on the result. [2 marks]

(c) Find a 99% confidence interval for the estimate of the intercept ($\hat{\alpha}$). [3 marks]

[Total 8 marks]

[PLEASE TURN OVER]

6. Let $\boldsymbol{x} = (x_1, \dots, x_n)$ denote a realised sample from a Poisson distribution with unknown parameter $\theta \in (0, \infty)$. The likelihood of a single observation $x_k \in \mathbb{N}$ is

$$L(\theta; x_k) = \frac{\theta^{x_k}}{x_k!} \exp(-\theta)\,.$$

Assume that observations are i.i.d. given the value of θ.

(a) Let α, $\beta > 0$. Assuming a $\mathrm{Gamma}(\alpha, \beta)$ prior for θ, derive the posterior distribution for θ given the observed data $\boldsymbol{x}$. [3 marks]

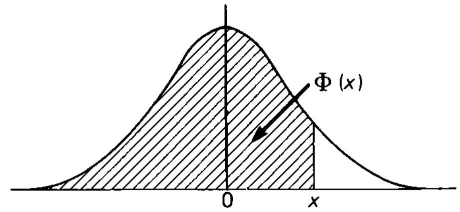
(b) Consider the posterior mean as an estimator of the parameter θ. What can you say about the bias of this estimator? Can the posterior variance also be used as an estimator of θ? [3 marks]

[Total 6 marks]

[END OF PAPER]

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

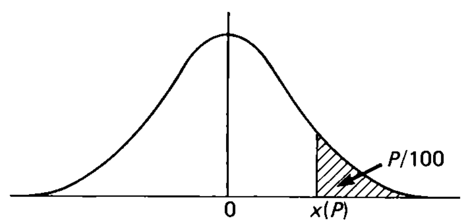
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

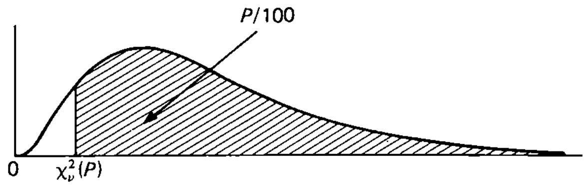
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	99·95	99·9	99·5	99	97·5	95	90	80	70	60
$\nu = 1$	$0{\cdot}0^{6}3927$	$0{\cdot}0^{5}1571$	$0{\cdot}0^{4}3927$	$0{\cdot}0^{3}1571$	$0{\cdot}0^{3}9821$	0·003932	0·01579	0·06418	0·1485	0·2750
2	0·001000	0·002001	0·01003	0·02010	0·05064	0·1026	0·2107	0·4463	0·7133	1·022
3	0·01528	0·02430	0·07172	0·1148	0·2158	0·3518	0·5844	1·005	1·424	1·869
4	0·06392	0·09080	0·2070	0·2971	0·4844	0·7107	1·064	1·649	2·195	2·753
5	0·1581	0·2102	0·4117	0·5543	0·8312	1·145	1·610	2·343	3·000	3·655
6	0·2994	0·3811	0·6757	0·8721	1·237	1·635	2·204	3·070	3·828	4·570
7	0·4849	0·5985	0·9893	1·239	1·690	2·167	2·833	3·822	4·671	5·493
8	0·7104	0·8571	1·344	1·646	2·180	2·733	3·490	4·594	5·527	6·423
9	0·9717	1·152	1·735	2·088	2·700	3·325	4·168	5·380	6·393	7·357
10	1·265	1·479	2·156	2·558	3·247	3·940	4·865	6·179	7·267	8·295
11	1·587	1·834	2·603	3·053	3·816	4·575	5·578	6·989	8·148	9·237
12	1·934	2·214	3·074	3·571	4·404	5·226	6·304	7·807	9·034	10·18
13	2·305	2·617	3·565	4·107	5·009	5·892	7·042	8·634	9·926	11·13
14	2·697	3·041	4·075	4·660	5·629	6·571	7·790	9·467	10·82	12·08
15	3·108	3·483	4·601	5·229	6·262	7·261	8·547	10·31	11·72	13·03
16	3·536	3·942	5·142	5·812	6·908	7·962	9·312	11·15	12·62	13·98
17	3·980	4·416	5·697	6·408	7·564	8·672	10·09	12·00	13·53	14·94
18	4·439	4·905	6·265	7·015	8·231	9·390	10·86	12·86	14·44	15·89
19	4·912	5·407	6·844	7·633	8·907	10·12	11·65	13·72	15·35	16·85
20	5·398	5·921	7·434	8·260	9·591	10·85	12·44	14·58	16·27	17·81
21	5·896	6·447	8·034	8·897	10·28	11·59	13·24	15·44	17·18	18·77
22	6·404	6·983	8·643	9·542	10·98	12·34	14·04	16·31	18·10	19·73
23	6·924	7·529	9·260	10·20	11·69	13·09	14·85	17·19	19·02	20·69
24	7·453	8·085	9·886	10·86	12·40	13·85	15·66	18·06	19·94	21·65
25	7·991	8·649	10·52	11·52	13·12	14·61	16·47	18·94	20·87	22·62
26	8·538	9·222	11·16	12·20	13·84	15·38	17·29	19·82	21·79	23·58
27	9·093	9·803	11·81	12·88	14·57	16·15	18·11	20·70	22·72	24·54
28	9·656	10·39	12·46	13·56	15·31	16·93	18·94	21·59	23·65	25·51
29	10·23	10·99	13·12	14·26	16·05	17·71	19·77	22·48	24·58	26·48
30	10·80	11·59	13·79	14·95	16·79	18·49	20·60	23·36	25·51	27·44
32	11·98	12·81	15·13	16·36	18·29	20·07	22·27	25·15	27·37	29·38
34	13·18	14·06	16·50	17·79	19·81	21·66	23·95	26·94	29·24	31·31
36	14·40	15·32	17·89	19·23	21·34	23·27	25·64	28·73	31·12	33·25
38	15·64	16·61	19·29	20·69	22·88	24·88	27·34	30·54	32·99	35·19
40	16·91	17·92	20·71	22·16	24·43	26·51	29·05	32·34	34·87	37·13
50	23·46	24·67	27·99	29·71	32·36	34·76	37·69	41·45	44·31	46·86
60	30·34	31·74	35·53	37·48	40·48	43·19	46·46	50·64	53·81	56·62
70	37·47	39·04	43·28	45·44	48·76	51·74	55·33	59·90	63·35	66·40
80	44·79	46·52	51·17	53·54	57·15	60·39	64·28	69·21	72·92	76·19
90	52·28	54·16	59·20	61·75	65·65	69·13	73·29	78·56	82·51	85·99
100	59·90	61·92	67·33	70·06	74·22	77·93	82·36	87·95	92·13	95·81

TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

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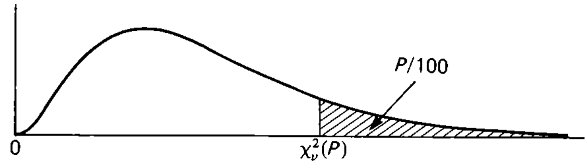
If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

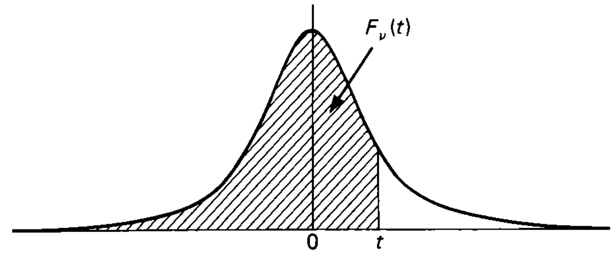
TABLE 9. THE t-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{-\infty}^{t} \frac{ds}{(1 + s^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

$F_\nu(t)$ is the probability that a random variable, distributed as t with ν degrees of freedom, will be less than or equal to t. When $t < 0$ use $F_\nu(t) = 1 - F_\nu(-t)$, the t distribution being symmetric about zero.

The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance (see Table 4). When ν is large interpolation in ν should be harmonic.



Omitted entries to the right of tabulated values are 1 (to four decimal places).

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$t =$ 0·0	0·5000	$t =$ 4·0	0·9220	$t =$ 0·0	0·5000	$t =$ 4·0	0·9714	$t =$ 0·0	0·5000	$t =$ 4·0	0·9860
·1	·5317	4·2	·9256	·1	·5353	·1	·9727	·1	·5367	·1	·9869
·2	·5628	4·4	·9289	·2	·5700	·2	·9739	·2	·5729	·2	·9877
·3	·5928	4·6	·9319	·3	·6038	·3	·9750	·3	·6081	·3	·9884
·4	·6211	4·8	·9346	·4	·6361	·4	·9760	·4	·6420	·4	·9891
0·5	0·6476	5·0	0·9372	0·5	0·6667	4·5	0·9770	0·5	0·6743	4·5	0·9898
·6	·6720	5·5	·9428	·6	·6953	·6	·9779	·6	·7046	·6	·9903
·7	·6944	6·0	·9474	·7	·7218	·7	·9788	·7	·7328	·7	·9909
·8	·7148	6·5	·9514	·8	·7462	·8	·9796	·8	·7589	·8	·9914
·9	·7333	7·0	·9548	·9	·7684	·9	·9804	·9	·7828	·9	·9919
1·0	0·7500	7·5	0·9578	1·0	0·7887	5·0	0·9811	1·0	0·8045	5·0	0·9923
·1	·7651	8·0	·9604	·1	·8070	·1	·9818	·1	·8242	·1	·9927
·2	·7789	8·5	·9627	·2	·8235	·2	·9825	·2	·8419	·2	·9931
·3	·7913	9·0	·9648	·3	·8384	·3	·9831	·3	·8578	·3	·9934
·4	·8026	9·5	·9666	·4	·8518	·4	·9837	·4	·8720	·4	·9938
1·5	0·8128	10·0	0·9683	1·5	0·8638	5·5	0·9842	1·5	0·8847	5·5	0·9941
·6	·8222	10·5	·9698	·6	·8746	·6	·9848	·6	·8960	·6	·9944
·7	·8307	11·0	·9711	·7	·8844	·7	·9853	·7	·9062	·7	·9946
·8	·8386	11·5	·9724	·8	·8932	·8	·9858	·8	·9152	·8	·9949
·9	·8458	12·0	·9735	·9	·9011	·9	·9862	·9	·9232	·9	·9951
2·0	0·8524	12·5	0·9746	2·0	0·9082	6·0	0·9867	2·0	0·9303	6·0	0·9954
·1	·8585	13·0	·9756	·1	·9147	·1	·9871	·1	·9367	·1	·9956
·2	·8642	13·5	·9765	·2	·9206	·2	·9875	·2	·9424	·2	·9958
·3	·8695	14·0	·9773	·3	·9259	·3	·9879	·3	·9475	·3	·9960
·4	·8743	14·5	·9781	·4	·9308	·4	·9882	·4	·9521	·4	·9961
2·5	0·8789	15	0·9788	2·5	0·9352	6·5	0·9886	2·5	0·9561	6·5	0·9963
·6	·8831	16	·9801	·6	·9392	·6	·9889	·6	·9598	·6	·9965
·7	·8871	17	·9813	·7	·9429	·7	·9892	·7	·9631	·7	·9966
·8	·8908	18	·9823	·8	·9463	·8	·9895	·8	·9661	·8	·9967
·9	·8943	19	·9833	·9	·9494	·9	·9898	·9	·9687	·9	·9969
3·0	0·8976	20	0·9841	3·0	0·9523	7·0	0·9901	3·0	0·9712	7·0	0·9970
·1	·9007	21	·9849	·1	·9549	·1	·9904	·1	·9734	·1	·9971
·2	·9036	22	·9855	·2	·9573	·2	·9906	·2	·9753	·2	·9972
·3	·9063	23	·9862	·3	·9596	·3	·9909	·3	·9771	·3	·9973
·4	·9089	24	·9867	·4	·9617	·4	·9911	·4	·9788	·4	·9974
3·5	0·9114	25	0·9873	3·5	0·9636	7·5	0·9913	3·5	0·9803	7·5	0·9975
·6	·9138	30	·9894	·6	·9654	·6	·9916	·6	·9816	·6	·9976
·7	·9160	35	·9909	·7	·9670	·7	·9918	·7	·9829	·7	·9977
·8	·9181	40	·9920	·8	·9686	·8	·9920	·8	·9840	·8	·9978
·9	·9201	45	·9929	·9	·9701	·9	·9922	·9	·9850	·9	·9979
4·0	0·9220	50	0·9936	4·0	0·9714	8·0	0·9924	4·0	0·9860	8·0	0·9980

TABLE 9. THE *t*-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$t =$ 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5374	·5379	·5382	·5384	·5386	·5387	·5388	·5389	·5390	·5391	·5391
·2	·5744	·5753	·5760	·5764	·5768	·5770	·5773	·5774	·5776	·5777	·5778
·3	·6104	·6119	·6129	·6136	·6141	·6145	·6148	·6151	·6153	·6155	·6157
·4	·6452	·6472	·6485	·6495	·6502	·6508	·6512	·6516	·6519	·6522	·6524
0·5	0·6783	0·6809	0·6826	0·6838	0·6847	0·6855	0·6861	0·6865	0·6869	0·6873	0·6876
·6	·7096	·7127	·7148	·7163	·7174	·7183	·7191	·7197	·7202	·7206	·7210
·7	·7387	·7424	·7449	·7467	·7481	·7492	·7501	·7508	·7514	·7519	·7523
·8	·7657	·7700	·7729	·7750	·7766	·7778	·7788	·7797	·7804	·7810	·7815
·9	·7905	·7953	·7986	·8010	·8028	·8042	·8054	·8063	·8071	·8078	·8083
1·0	0·8130	0·8184	0·8220	0·8247	0·8267	0·8283	0·8296	0·8306	0·8315	0·8322	0·8329
·1	·8335	·8393	·8433	·8461	·8483	·8501	·8514	·8526	·8535	·8544	·8551
·2	·8518	·8581	·8623	·8654	·8678	·8696	·8711	·8723	·8734	·8742	·8750
·3	·8683	·8748	·8793	·8826	·8851	·8870	·8886	·8899	·8910	·8919	·8927
·4	·8829	·8898	·8945	·8979	·9005	·9025	·9041	·9055	·9066	·9075	·9084
1·5	0·8960	0·9030	0·9079	0·9114	0·9140	0·9161	0·9177	0·9191	0·9203	0·9212	0·9221
·6	·9076	·9148	·9196	·9232	·9259	·9280	·9297	·9310	·9322	·9332	·9340
·7	·9178	·9251	·9300	·9335	·9362	·9383	·9400	·9414	·9426	·9435	·9444
·8	·9269	·9341	·9390	·9426	·9452	·9473	·9490	·9503	·9515	·9525	·9533
·9	·9349	·9421	·9469	·9504	·9530	·9551	·9567	·9580	·9591	·9601	·9609
2·0	0·9419	0·9490	0·9538	0·9572	0·9597	0·9617	0·9633	0·9646	0·9657	0·9666	0·9674
·1	·9482	·9551	·9598	·9631	·9655	·9674	·9690	·9702	·9712	·9721	·9728
·2	·9537	·9605	·9649	·9681	·9705	·9723	·9738	·9750	·9759	·9768	·9774
·3	·9585	·9651	·9694	·9725	·9748	·9765	·9779	·9790	·9799	·9807	·9813
·4	·9628	·9692	·9734	·9763	·9784	·9801	·9813	·9824	·9832	·9840	·9846
2·5	0·9666	0·9728	0·9767	0·9795	0·9815	0·9831	0·9843	0·9852	0·9860	0·9867	0·9873
·6	·9700	·9759	·9797	·9823	·9842	·9856	·9868	·9877	·9884	·9890	·9895
·7	·9730	·9786	·9822	·9847	·9865	·9878	·9888	·9897	·9903	·9909	·9914
·8	·9756	·9810	·9844	·9867	·9884	·9896	·9906	·9914	·9920	·9925	·9929
·9	·9779	·9831	·9863	·9885	·9901	·9912	·9921	·9928	·9933	·9938	·9942
3·0	0·9800	0·9850	0·9880	0·9900	0·9915	0·9925	0·9933	0·9940	0·9945	0·9949	0·9952
·1	·9819	·9866	·9894	·9913	·9927	·9936	·9944	·9949	·9954	·9958	·9961
·2	·9835	·9880	·9907	·9925	·9937	·9946	·9953	·9958	·9962	·9965	·9968
·3	·9850	·9893	·9918	·9934	·9946	·9954	·9960	·9965	·9968	·9971	·9974
·4	·9864	·9904	·9928	·9943	·9953	·9961	·9966	·9970	·9974	·9976	·9978
3·5	0·9876	0·9914	0·9936	0·9950	0·9960	0·9966	0·9971	0·9975	0·9978	0·9980	0·9982
·6	·9886	·9922	·9943	·9956	·9965	·9971	·9976	·9979	·9982	·9984	·9986
·7	·9896	·9930	·9950	·9962	·9970	·9975	·9979	·9982	·9985	·9987	·9988
·8	·9904	·9937	·9955	·9966	·9974	·9979	·9983	·9985	·9987	·9989	·9990
·9	·9912	·9943	·9960	·9971	·9977	·9982	·9985	·9988	·9989	·9991	·9992
4·0	0·9919	0·9948	0·9964	0·9974	0·9980	0·9984	0·9987	0·9990	0·9991	0·9992	0·9993
·1	·9926	·9953	·9968	·9977	·9983	·9987	·9989	·9991	·9993	·9994	·9995
·2	·9932	·9958	·9972	·9980	·9985	·9988	·9991	·9993	·9994	·9995	·9996
·3	·9937	·9961	·9975	·9982	·9987	·9990	·9992	·9994	·9995	·9996	·9996
·4	·9942	·9965	·9977	·9984	·9989	·9991	·9993	·9995	·9996	·9996	·9997
4·5	0·9946	0·9968	0·9979	0·9986	0·9990	0·9993	0·9994	0·9995	0·9996	0·9997	0·9998
·6	·9950	·9971	·9982	·9988	·9991	·9994	·9995	·9996	·9997	·9998	·9998
·7	·9953	·9973	·9983	·9989	·9992	·9994	·9996	·9997	·9997	·9998	·9998
·8	·9957	·9976	·9985	·9990	·9993	·9995	·9996	·9997	·9998	·9998	·9999
·9	·9960	·9978	·9986	·9991	·9994	·9996	·9997	·9998	·9998	·9999	·9999
5·0	0·9963	0·9979	0·9988	0·9992	0·9995	0·9996	0·9997	0·9998	0·9998	0·9999	0·9999

TABLE 9. THE t-DISTRIBUTION FUNCTION

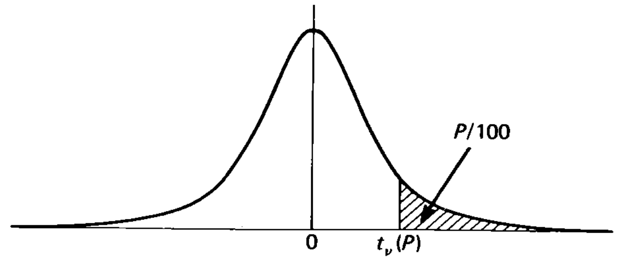
$\nu =$	15	16	17	18	19	20	24	30	40	60	∞
$t =$ 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5392	·5392	·5392	·5393	·5393	·5393	·5394	·5395	·5396	·5397	·5398
·2	·5779	·5780	·5781	·5781	·5782	·5782	·5784	·5786	·5788	·5789	·5793
·3	·6159	·6160	·6161	·6162	·6163	·6164	·6166	·6169	·6171	·6174	·6179
·4	·6526	·6528	·6529	·6531	·6532	·6533	·6537	·6540	·6544	·6547	·6554
0·5	0·6878	0·6881	0·6883	0·6884	0·6886	0·6887	0·6892	0·6896	0·6901	0·6905	0·6915
·6	·7213	·7215	·7218	·7220	·7222	·7224	·7229	·7235	·7241	·7246	·7257
·7	·7527	·7530	·7533	·7536	·7538	·7540	·7547	·7553	·7560	·7567	·7580
·8	·7819	·7823	·7826	·7829	·7832	·7834	·7842	·7850	·7858	·7866	·7881
·9	·8088	·8093	·8097	·8100	·8103	·8106	·8115	·8124	·8132	·8141	·8159
1·0	0·8334	0·8339	0·8343	0·8347	0·8351	0·8354	0·8364	0·8373	0·8383	0·8393	0·8413
·1	·8557	·8562	·8567	·8571	·8575	·8578	·8589	·8600	·8610	·8621	·8643
·2	·8756	·8762	·8767	·8772	·8776	·8779	·8791	·8802	·8814	·8826	·8849
·3	·8934	·8940	·8945	·8950	·8954	·8958	·8970	·8982	·8995	·9007	·9032
·4	·9091	·9097	·9103	·9107	·9112	·9116	·9128	·9141	·9154	·9167	·9192
1·5	0·9228	0·9235	0·9240	0·9245	0·9250	0·9254	0·9267	0·9280	0·9293	0·9306	0·9332
·6	·9348	·9354	·9360	·9365	·9370	·9374	·9387	·9400	·9413	·9426	·9452
·7	·9451	·9458	·9463	·9468	·9473	·9477	·9490	·9503	·9516	·9528	·9554
·8	·9540	·9546	·9552	·9557	·9561	·9565	·9578	·9590	·9603	·9616	·9641
·9	·9616	·9622	·9627	·9632	·9636	·9640	·9652	·9665	·9677	·9689	·9713
2·0	0·9680	0·9686	0·9691	0·9696	0·9700	0·9704	0·9715	0·9727	0·9738	0·9750	0·9772
·1	·9735	·9740	·9745	·9750	·9753	·9757	·9768	·9779	·9790	·9800	·9821
·2	·9781	·9786	·9790	·9794	·9798	·9801	·9812	·9822	·9832	·9842	·9861
·3	·9819	·9824	·9828	·9832	·9835	·9838	·9848	·9857	·9866	·9875	·9893
·4	·9851	·9855	·9859	·9863	·9866	·9869	·9877	·9886	·9894	·9902	·9918
2·5	0·9877	0·9882	0·9885	0·9888	0·9891	0·9894	0·9902	0·9909	0·9917	0·9924	0·9938
·6	·9900	·9903	·9907	·9910	·9912	·9914	·9921	·9928	·9935	·9941	·9953
·7	·9918	·9921	·9924	·9927	·9929	·9931	·9937	·9944	·9949	·9955	·9965
·8	·9933	·9936	·9938	·9941	·9943	·9945	·9950	·9956	·9961	·9966	·9974
·9	·9945	·9948	·9950	·9952	·9954	·9956	·9961	·9965	·9970	·9974	·9981
3·0	0·9955	0·9958	0·9960	0·9962	0·9963	0·9965	0·9969	0·9973	0·9977	0·9980	0·9987
·1	·9963	·9966	·9967	·9969	·9971	·9972	·9976	·9979	·9982	·9985	·9990
·2	·9970	·9972	·9974	·9975	·9976	·9978	·9981	·9984	·9987	·9989	·9993
·3	·9976	·9977	·9979	·9980	·9981	·9982	·9985	·9988	·9990	·9992	·9995
·4	·9980	·9982	·9983	·9984	·9985	·9986	·9988	·9990	·9992	·9994	·9997
3·5	0·9984	0·9985	0·9986	0·9987	0·9988	0·9989	0·9991	0·9993	0·9994	0·9996	0·9998
·6	·9987	·9988	·9989	·9990	·9990	·9991	·9993	·9994	·9996	·9997	·9998
·7	·9989	·9990	·9991	·9992	·9992	·9993	·9994	·9996	·9997	·9998	·9999
·8	·9991	·9992	·9993	·9993	·9994	·9994	·9996	·9997	·9998	·9998	·9999
·9	·9993	·9994	·9994	·9995	·9995	·9996	·9997	·9997	·9998	·9999	
4·0	0·9994	0·9995	0·9995	0·9996	0·9996	0·9996	0·9997	0·9998	0·9999	0·9999	
·1	·9995	·9996	·9996	·9997	·9997	·9997	·9998	·9999	·9999	·9999	
·2	·9996	·9997	·9997	·9997	·9998	·9998	·9998	·9999	·9999		
·3	·9997	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999		
·4	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999			
4·5	0·9998	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999				

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1 + t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P (%)	40	30	25	20	15	10	5%	2·5	1%	0·5	0·1	0·05
ν = 1	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291