GMSK Demodulators Based on Laurent Decomposition

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***Abstract*—This paper explores the demodulation of Gaussian Minimum Shift Keying (GMSK) signals, widely used in GSM and satellite communications. We analyze the signal structure and its approximation using Laurent Decomposition, which represents the Continuous Phase Modulation (CPM) signal as a sum of Pulse Amplitude Modulation (PAM) components. This decomposition allows for significant reductions in receiver complexity by enabling the use of Viterbi algorithms with reduced states. We compare optimal, sub-optimal, and improved sub-optimal demodulation techniques, analyzing their Bit Error Rate (BER) performance against Signal-to-Noise Ratio (SNR). Finally, a comparison with QPSK modulation highlights the trade-offs between spectral efficiency and power amplifier linearity.**

***Index Terms*—GMSK, Laurent Decomposition, Viterbi Algorithm, CPM, GSM, Demodulation.**

I. Introduction

In telecommunications, a signal carrying information must pass through a transmission medium between a transmitter and a receiver. The signal is modulated to transform it into a form adapted to the channel, varying the amplitude, frequency, or phase of a carrier wave.

There are generally two types of modulation:

- **Analog Modulation:** Ensures transmission quality for analog information (voice, image) but often lacks efficiency or precision.
- **Digital Modulation:** Ensures maximum bit rate with an acceptable Bit Error Rate (BER).

Our study focuses on **GMSK** (Gaussian Minimum Shift Keying), a digital modulation from the FSK family. GMSK is famously associated with the GSM 2G mobile system ($BT = 0.3$), contributing to high power efficiency in mobile devices. It is also used in CDPD ($BT = 0.5$) and satellite communications (CCSDS standards).

This paper studies GMSK demodulators based on Laurent decomposition, specifically for BT values of 0.3 and 0.5.

II. Expression and Form of a GMSK Signal

GMSK is a Continuous Phase Modulation (CPM) with a modulation index $h = 0.5$. The GMSK signal is obtained by filtering the rectangular pulse of an MSK modulator with a Gaussian filter.

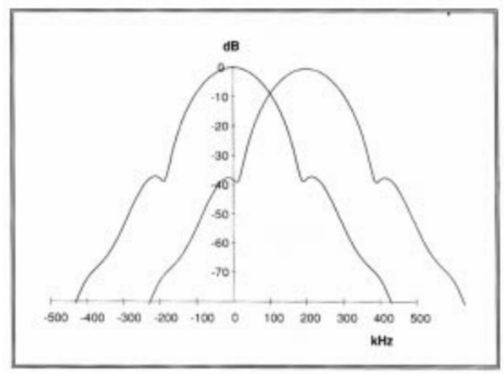

Fig. 1. Spectrum of a GMSK modulation for GSM with adjacent center frequencies.

The resulting signal is represented by:

$$s(t) = a_I(t)\cos\left(\frac{\pi t}{2T}\right)\cos(2\pi f_c t) - a_Q(t)\sin\left(\frac{\pi t}{2T}\right)\sin(2\pi f_c t) \quad (1)$$

By analogy, it can be written as:

$$s_c(t,\alpha) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \phi(t,\alpha)) \quad (2)$$

Where T is the symbol time, E is the symbol energy, and the phase $\phi(t,\alpha)$ is:

$$\phi(t,\alpha) = 2\pi h \sum_{k=-\infty}^{\infty} \alpha_k q(t-kT) \quad (3)$$

Here, $q(t)$ is the integral of the Gaussian-filtered pulse $g(t)$:

$$g(t) = \frac{1}{K} g_0(t - LT/2) \quad (4)$$

$$g_0(t) = \mathrm{erf}(\beta_0(t)) - \mathrm{erf}(\beta_m(t)) \quad (5)$$

Where $\mathrm{erf}(x)$ is the error function. B is the -3dB bandwidth. Lower BT values yield a compact spectrum but higher Inter-Symbol Interference (ISI).

III. Laurent Decomposition

In 1986, Pierre A. Laurent demonstrated that any CPM signal can be expressed as a finite sum of Pulse Amplitude

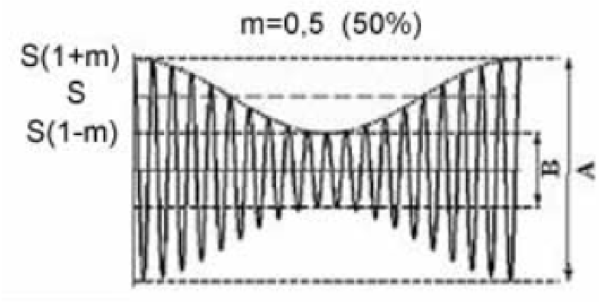

Fig. 2. Example of a signal with modulation index 0.5.

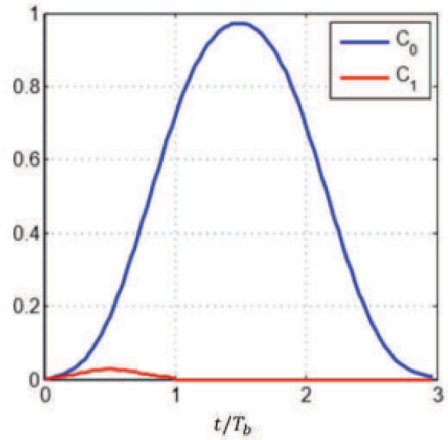

Fig. 3. Laurent Pulses for GMSK $BT = 0.5$. Note that C_0 contains the majority of the energy.

Modulation (PAM) signals. This significantly reduces the complexity of synchronous detection.

The signal is decomposed into $2^{(L-1)}$ PAM components:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M+1} a_{m,i} h_m(t - iT) \tag{6}$$

Where $a_{m,i}$ are the Laurent coefficients and $h_m(t)$ are the Laurent Pulses defined by:

$$h_m(t) = c(t) \prod_{l=1}^{L-1} c(t + lT + \gamma_{m,l} \cdot LT) \tag{7}$$

The main pulse $c(t)$ is roughly sinusoidal.

The decomposition relies on the representation of the CPM signal phase as a sum of tilted phase pulses. The primary pulse $C_0(t)$ essentially represents the main lobe of the Gaussian spectrum, while the secondary pulse $C_1(t)$ accounts for the primary side-lobe interference. The specific duration of these pulses is determined by the correlation length L. As L increases, the number of required pulses increases exponentially (2^{L-1}), but the energy concentration in the first pulse (C_0) remains dominant, justifying the truncation used in sub-optimal receivers.

A. *Case* $BT = 0.5, L = 2$

The GMSK signal can be expressed as the superposition of two pulses, C_0 and C_1.

$$\begin{cases} C_0(t) = \sin(\phi(t)) \sin(\phi(t+T)) & 0 \le t \le 3T \\ C_1(t) = \sin(\phi(t)) \sin(\phi(t+3T)) & LT \le t \end{cases} \tag{8}$$

The pulse C_0 is dominant. Consequently, a simplified receiver may consider only the first pulse, as it transmits approximately 95% of the energy.

B. *Case* $BT = 0.3, L = 3$

Similar to the previous case, h_0 transmits the majority of the information. We deduce that the modulated GMSK signal is effectively a superposition of several PAM signals, with energy concentrated in the first few components.

IV. Demodulation

Using Laurent's work, we model the transmitter output as a sum of PAM signals.

A. *Optimal Demodulation*

Optimal demodulation utilizes the Viterbi algorithm. The principle is Maximum Likelihood Sequence Estimation (MLSE). Assuming an AWGN channel ($r(t) = s(t) + b(t)$), the receiver filters the signal. The Viterbi algorithm then finds the most probable state sequence by calculating the minimum Euclidean distance in the trellis.

The Viterbi algorithm operates by maintaining a set of survivor paths through the trellis. At each time step k, the receiver calculates the branch metrics corresponding to the transition from state S_{k-1} to S_k. These metrics are accumulated to form path metrics. The algorithm then selects the path with the minimum accumulated Euclidean distance. For GMSK with $h = 0.5$, the trellis structure is binary, but the presence of memory in the Gaussian filter expands the state space to 2^{L-1} states.

Laurent decomposition simplifies the structure and calculation while maintaining BER performance.

B. *Sub-optimal and Improved Demodulation*

Sub-optimal: Since the first PAM pulse (C_0) contains $\sim 95\%$ of the energy, we can neglect the others. This reduces the number of states in the Viterbi algorithm, lowering complexity.

Improved Sub-optimal: When SNR is low (high interference), the basic sub-optimal model fails. By including the second pulse (C_1, containing 2-5% energy), we improve performance significantly while keeping complexity lower than the fully optimal solution.

As seen in Fig. 4, the optimal demodulator performs best, but the gap widens as SNR decreases.

V. Comparison with QPSK and Conclusion

Comparing GMSK with Quadrature Phase Shift Keying (QPSK):

- **Amplification:** QPSK has a variable envelope and requires linear power amplifiers. GMSK has a constant

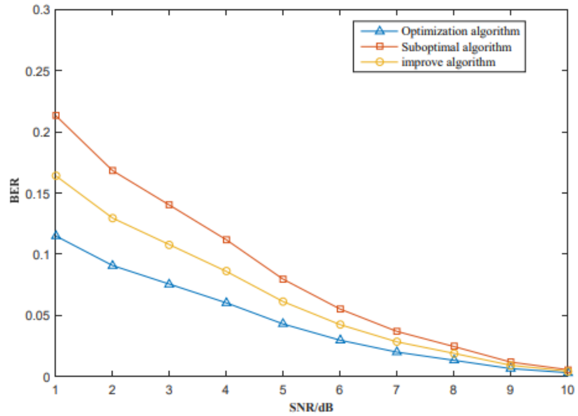

Fig. 4. BER vs SNR for $BT = 0.3$. Optimal demodulation yields the lowest error rate.

envelope, allowing the use of non-linear (Class C) amplifiers. This results in much lower power consumption, crucial for battery-operated mobile devices.

- **Spectral Efficiency:** GMSK is highly spectrally efficient.
- **Drawbacks:** GMSK suffers from higher Inter-Symbol Interference (ISI) and requires more complex equalization (e.g., Viterbi) compared to QPSK to achieve the same reliability.

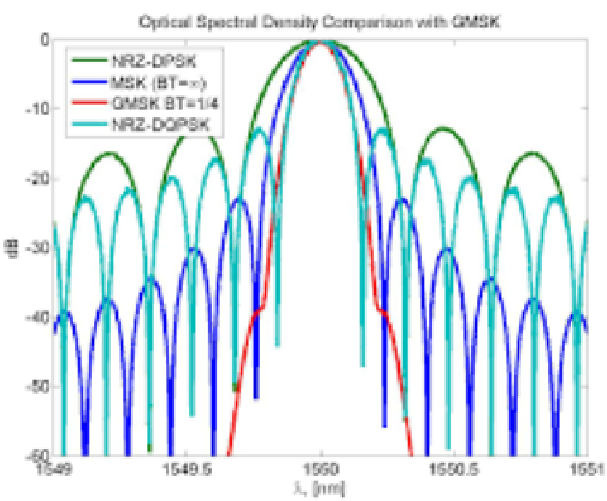

Fig. 5. Spectral density comparison of GMSK vs other modulations.

In conclusion, Laurent decomposition offers a powerful method to implement GMSK demodulators that balance computational complexity with performance, bridging the gap between theoretical optimality and practical hardware constraints.

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