Logic

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1 Propositional logic

Convention: In this course we write T for true and F for false.

Definition. 1.1. The language of propositional logic consists of following symbols: *propositional variables* denoted (mostly) by $p, q, \ldots$ or $p_1, p_2, \ldots, q_1, q_2, \ldots$ and the *connectives* $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$.

Definition. 1.2. A *propositional formula* is a string of symbols obatained in the following way

1. Any variable is a formula

2. If ϕ and ψ are formulas then so are $(\phi \wedge \psi), (\phi \vee \psi), (\neg \phi), (\phi \rightarrow \psi), (\phi \leftrightarrow \psi)$

3. Any formula is obtained in this way.

Definition. 1.3. A *truth function* of n variables is a function

$$f : \{T, F\}^n \rightarrow \{T, F\} \quad .$$

Exercize: How many functions are there for n variables?

Definition. 1.4. Suppose ϕ is a formula with variables $p_1, \ldots, p_n$ then we obtain a truth function $F_\phi : \{T, F\}^n \rightarrow \{T, F\}$ whose value at $(x_1, \ldots, x_n) \quad x_i \in \{T, F\}$ is the truth value of ϕ when p_i has value x_i. THe function F_ϕ is the *truth function of ϕ*.