Пример 2 из п.1.4.

$$\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 3 \\ 2x_1 - x_2 + x_4 = 2 \\ 3x_1 - x_3 - x_4 = -1 \end{cases}$$

$$\left(\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 3 \\ 2 & -1 & 0 & 1 & 2 \\ -3 & 0 & +1 & +1 & +1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 0 & 1 & -2 & 1 \\ 1 & -1/2 & 0 & 1/2 & 1 \\ 0 & -3/2 & 1 & 5/2 & 4 \end{array}\right) \sim$$

$\min\left(\frac{3}{2}, \frac{2}{2}\right) = 1$ $\qquad \min\left(\frac{1}{1}, \frac{4}{1}\right) = 1$

$$\left(\begin{array}{cccc|c} 0 & 0 & 1 & -2 & 1 \\ 1 & -1/2 & 0 & 1/2 & 1 \\ 0 & -3/2 & 0 & 9/2 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -2/3 & 1 & 0 & 7/3 \\ 1 & -1/3 & 0 & 0 & 2/3 \\ 0 & -1/3 & 0 & 1 & 2/3 \end{array}\right)$$

$\min\left(\frac{1}{1/2}, \frac{3}{9/2}\right) = \frac{6}{9}$

$$0 - \frac{2 \cdot 3/2}{9/2} = -2/3$$

$$-\frac{1}{2} + \frac{1/2 \cdot 3/2}{3/2} = \frac{-3+1}{6} = -\frac{1}{3}$$

$$1 + \frac{2 \cdot 3}{9/2} = 1 + \frac{4}{3} = 7/3$$

$$1 - \frac{3 \cdot \frac{1}{2}}{9/2} = 2/3$$

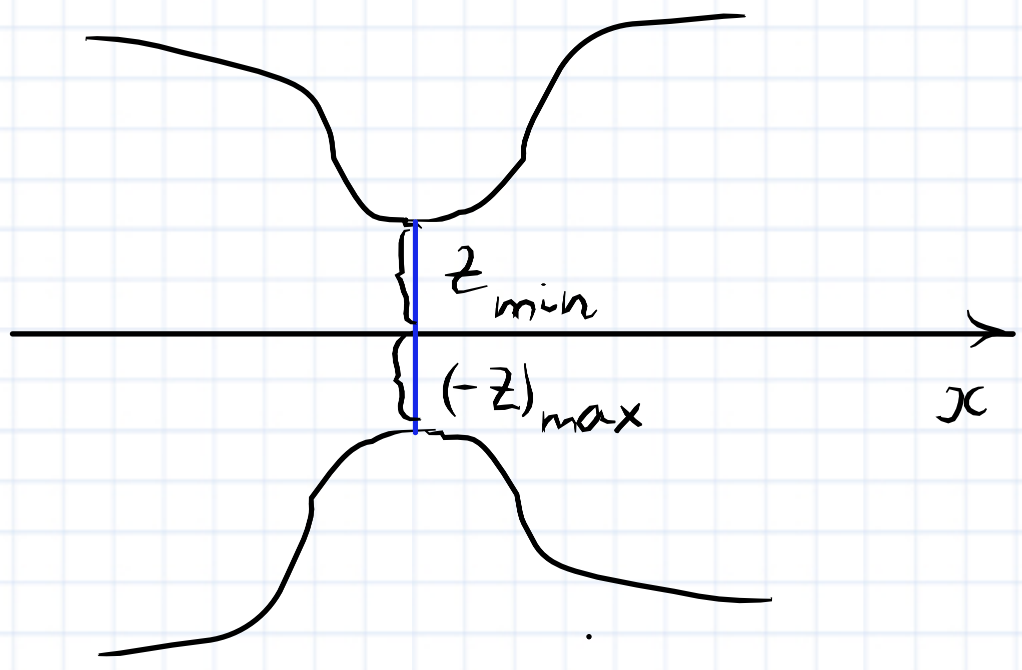
$X^1 = (2/3, 0, 7/3, 2/3)$ — ед. оп. реш.

$C_4^3 = \frac{4!}{3!\,1!} = 4$

Пример 1 из п.1.5.

$Z = 3x_1 - 2x_2 - 3x_3 \to \min$

$$\begin{cases} 2x_1 - x_2 + x_3 \geq 2 \\ 3x_1 + 2x_2 - 4x_3 \leq 6 \\ x_1 + x_2 + 5x_3 = 4 \\ x_1, x_2 \geq 0 \end{cases}$$



$x_3 = x_3' - x_3'', \quad x_3', x_3'' \geq 0$

$Z_1 = -Z = -3x_1 + 2x_2 + 3x_3' - 3x_3'' \to \max$

$$\begin{cases} 2x_1 - x_2 + x_3' - x_3'' - x_4 = 2 \\ 3x_1 + 2x_2 - 4x_3' + 4x_3'' + x_5 = 6 \\ x_1 + x_2 + 5x_3' - 5x_3'' = 4 \\ x_1, x_2, x_3', x_3'', x_4, x_5 \geq 0 \end{cases}$$

Пример 2 из п.1.5.

$Z = 2x_1 - x_2 - 3x_3 + 6x_4 - x_5 \to \max$

$$\begin{cases} x_1 + 3x_2 + x_3 + 4x_4 - x_5 = 12 \\ x_1 + 2x_2 + 3x_4 - x_5 = 6 \\ 3x_1 + 3x_2 + 16x_4 - 2x_5 = 26 \\ x_i \geq 0,\ i = 1, 2, \dots, 5 \end{cases} \qquad \left(\begin{array}{ccccc|c} 1 & 3 & 1 & 4 & -1 & 12 \\ 1 & 2 & 0 & 3 & -1 & 6 \\ 3 & 3 & 0 & 16 & -2 & 26 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccccc|c} 0 & 1 & 1 & 1 & 0 & 6 \\ 1 & 2 & 0 & 3 & -1 & 6 \\ 0 & -3 & 0 & 7 & 1 & 8 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left(\begin{array}{ccccc|c} 0 & 1 & 1 & 1 & 0 & 6 \\ 1 & -1 & 0 & 10 & 0 & 14 \\ 0 & -3 & 0 & 7 & 1 & 8 \end{array}\right) \end{array} \begin{cases} x_1 = 14 + x_2 - 10x_4 \geq 0 \\ x_3 = 6 - x_2 - x_4 \geq 0 \\ x_5 = 8 + 3x_2 - 7x_4 \geq 0 \end{cases}$$

общ. реш.

$$Z = 2(14 + x_2 - 10x_4) - x_2 - 3(6 - x_2 - x_4) + $$
$$+ 6x_4 - (8 + 3x_2 - 7x_4) = 2 + x_2 - 4x_4$$

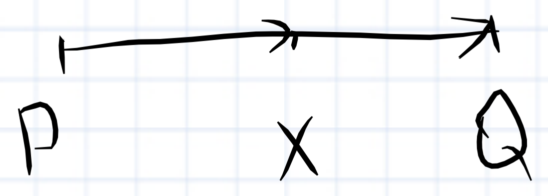
Симметр. форма:

$Z = 2 + x_2 - 4x_4 \to \max$

$$\begin{cases} -x_2 + 10x_4 \leq 14 \\ x_2 + x_4 \leq 6 \\ -3x_2 + 7x_4 \leq 8 \\ x_2, x_4 \geq 0 \end{cases}$$

Доказательство Теоремы 5

$P(p_1, \dots, p_n) \quad Q(q_1, \dots, q_n) \quad X(x_1, \dots, x_n) \in [PQ]$



$\overline{PQ} = (q_1 - p_1, \dots, q_n - p_n)$

$\overline{PX} = (x_1 - p_1, \dots, x_n - p_n)$

$\overline{PX} = \lambda \overline{PQ}, \quad 0 \leq \lambda \leq 1$

$(x_1 - p_1, \dots, x_n - p_n) = (\lambda(q_1 - p_1), \dots, \lambda(q_n - p_n))$

$x_i - p_i = \lambda(q_i - p_i) \quad i = \overline{1,n}$

$x_i = p_i + \lambda q_i - \lambda p_i = (1-\lambda)p_i + \lambda q_i, \quad i = \overline{1,n}$