

Пример 1  
Решить задачу линейного программирования

$$Z = x_1 + x_2 + 2x_3 \rightarrow \max$$

$$\begin{cases} x_1 + x_2 + x_3 = 8 \\ x_1 - x_2 \geq 4 \\ x_1 + 2x_2 \geq 6 \\ x_i \geq 0, i = 1, 2, 3 \end{cases}$$

$$\begin{cases} x_1 + x_2 + x_3 = 8 \\ x_1 - x_2 - x_4 = 4 \\ x_1 + 2x_2 - x_5 = 6 \\ x_i \geq 0, i = 1, 5 \end{cases} \quad \left( \begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 8 \\ 1 & -1 & 0 & -1 & 0 & 4 \\ 1 & 2 & 0 & 0 & -1 & 6 \end{array} \right) \sim$$
$$\left( \begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 8 \\ -1 & 1 & 0 & 1 & 0 & -4 \\ -1 & -2 & 0 & 0 & 1 & -6 \end{array} \right)$$

$$Z = x_1 + x_2 + 2(8 - x_1 - x_2) = 16 - x_1 - x_2 \rightarrow \max$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_3$	8	1	1	1	0	0	
$x_4$	-4	-1	1	0	1	0	
$x_5$	-6	-1	-2	0	0	1	$\leftarrow$
$Z$	16	1	1	0	0	0	
CO		1	1/2				

$$X' = (0, 0, 8, -4, -6) - \text{нелбгомаи}$$

$$Z(X') = 16$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_3$	5	1/2	0	1	0	1/2	
$x_4$	-7	-3/2	0	0	1	1/2	$\leftarrow$
$x_2$	3	1/2	1	0	0	-1/2	
$Z$	13	1/2	0	0	0	1/2	
CO		1/3				-	

$$X^2 = (0, 3, 5, -7, 0) - \text{нелбгомаи}$$

$$Z(X^2) = 13$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_3$	8/3	0	0	1			
$x_1$	14/3	1	0	0	-2/3	-1/3	
$x_2$	2/3	0	1	0			
$Z$	32/3	0	0	0	1/3	2/3	

$$5 \rightarrow 5 - \frac{1}{2} \cdot (-7) \cdot \left(-\frac{2}{3}\right) = \frac{8}{3}$$

$$3 \rightarrow 3 - \frac{1}{2} \cdot (-7) \cdot \left(-\frac{2}{3}\right) = \frac{2}{3}$$

$$13 \rightarrow 13 - \frac{1}{2} \cdot (-7) \cdot \left(-\frac{2}{3}\right) = \frac{32}{3}$$

$$0 \rightarrow 0 - \frac{1}{2} \cdot 1 \cdot \left(-\frac{2}{3}\right) = 1/3$$

$$1/2 \rightarrow \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} \cdot \left(-\frac{2}{3}\right) = \frac{4}{6} = \frac{2}{3}$$

$$X^3 = (14/3, 2/3, 8/3, 0, 0)$$

оптимальное решение

$$Z(X^3) = 32/3$$

$$Z_{\max} = Z(14/3, 2/3, 8/3) = 32/3$$

Пример 2

Решить задачу линейного программирования и сделать геометрическую интерпретацию поиска оптимального решения

$$Z = 3x_1 + 4x_2 \rightarrow \min$$

$$\begin{cases} x_1 + 3x_2 \geq 12 \\ 5x_1 + 4x_2 \geq 31 \\ 2x_1 + 3x_2 \geq 18 \\ x_1, x_2 \geq 0 \end{cases} \quad \begin{cases} x_1 + 3x_2 - x_3 = 12 \\ 5x_1 + 4x_2 - x_4 = 31 \\ 2x_1 + 3x_2 - x_5 = 18 \\ x_i \geq 0, i = 1, 5 \end{cases} \quad \left( \begin{array}{ccccc|c} 1 & 3 & -1 & 0 & 0 & 12 \\ 5 & 4 & 0 & -1 & 0 & 31 \\ 2 & 3 & 0 & 0 & -1 & 18 \end{array} \right) \sim$$
$$\sim \left( \begin{array}{ccccc|c} -1 & -3 & 1 & 0 & 0 & -12 \\ -5 & -4 & 0 & 1 & 0 & -31 \\ -2 & -3 & 0 & 0 & 1 & -18 \end{array} \right)$$

$$Z_1 = -Z = -3x_1 - 4x_2 \rightarrow \max$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_3$	-12	-1	-3	1	0	0	
$x_4$	-31	-5	-4	0	1	0	$\leftarrow$
$x_5$	-18	-2	-3	0	0	1	
$Z_1$	0	3	4	0	0	0	
CO		3/5	1				

$$X^1 = (0; 0; -12; -31; -18) - \text{нелбгомаи}$$

$$Z_1(X^1) = 0$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_3$	-29/5	0	-11/5	1	-1/5	0	$\leftarrow$
$x_1$	31/5	1	4/5	0	-1/5	0	
$x_5$	-28/5	0	-7/5	0	-2/5	1	
$Z_1$	-93/5	0	8/5	0	3/5	0	
CO			8/11		3		

$$-12 \rightarrow -12 - \frac{-1 \cdot (-31)}{-5} = \frac{-60 + 31}{5} = \frac{-29}{5}$$

$$-18 \rightarrow -18 - \frac{-2 \cdot (-31)}{-5} = \frac{-90 + 62}{5} = \frac{-28}{5}$$

$$0 \rightarrow 0 - \frac{-31 \cdot 3}{-5} = -\frac{93}{5}$$

$$-3 \rightarrow -3 - \frac{-1 \cdot (-4)}{-5} = \frac{-15 + 4}{5} = -\frac{11}{5}$$

$$-3 \rightarrow -3 - \frac{-4 \cdot (-2)}{-5} = \frac{-15 + 8}{5} = -\frac{7}{5}$$

$$4 \rightarrow 4 - \frac{-4 \cdot 3}{-5} = \frac{20 - 12}{5} = \frac{8}{5}$$

$$X^2 = (31/5, 0, -29/5, 0, -28/5) - \text{нелбгомаи}$$

$$Z_1(X^2) = -93/5$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_2$	29/11	0	1	-5/11	1/11	0	
$x_1$	45/11	1	0	4/11	-3/11	0	
$x_5$	-21/11	0	0	-7/11	-3/11	1	$\leftarrow$
$Z_1$	-251/11	0	0	8/11	5/11	0	
				8/7	5/3		

$$\frac{31}{5} \rightarrow \frac{31}{5} - \left(-\frac{29}{5}\right) \cdot \frac{4}{5} \cdot \left(-\frac{5}{11}\right) = \frac{341 - 116}{55} = \frac{225}{55} = \frac{45}{11}$$

$$-\frac{28}{5} \rightarrow -\frac{28}{5} - \left(-\frac{29}{5}\right) \cdot \left(-\frac{7}{5}\right) \cdot \left(-\frac{5}{11}\right) = \frac{-308 + 203}{55} = \frac{-105}{55} = -\frac{21}{11}$$

$$-\frac{93}{5} \rightarrow -\frac{93}{5} - \left(-\frac{29}{5}\right) \cdot \frac{8}{5} \cdot \left(-\frac{5}{11}\right) = \frac{-1023 - 232}{55} = \frac{-1255}{55} = -\frac{251}{11}$$

$$-\frac{1}{5} \rightarrow -\frac{1}{5} - \left(-\frac{1}{5}\right) \cdot \frac{4}{5} \cdot \left(-\frac{5}{11}\right) = \frac{-11 - 4}{55} = -\frac{15}{55} = -\frac{3}{11}$$

$$-\frac{2}{5} \rightarrow -\frac{2}{5} - \left(-\frac{1}{5}\right) \cdot \left(-\frac{7}{5}\right) \cdot \left(-\frac{5}{11}\right) = \frac{-22 + 7}{55} = -\frac{15}{55} = -\frac{3}{11}$$

$$3/5 \rightarrow \frac{3}{5} - \left(-\frac{1}{5}\right) \cdot \frac{8}{5} \cdot \left(-\frac{5}{11}\right) = \frac{33 - 8}{55} = \frac{25}{55} = \frac{5}{11}$$

$$X^3 = (45/11, 29/11, 0, 0, -21/11), Z_1(X^3) = -251/11$$

б.н.	1	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
$x_2$	4	0	1	0			
$x_1$	3	1	0	0			
$x_3$	3	0	0	1	3/4	-11/4	
$Z_1$	-25	0	0	0	1/2	8/7	

$$\frac{29}{11} \rightarrow \frac{29}{11} - \left(-\frac{5}{11}\right) \cdot \left(-\frac{21}{11}\right) \cdot \left(-\frac{11}{4}\right) = \frac{29 + 15}{11} = 4$$

$$\frac{45}{11} \rightarrow \frac{45}{11} - \frac{4}{11} \cdot \left(-\frac{21}{11}\right) \cdot \left(-\frac{11}{4}\right) = \frac{45 - 12}{11} = 3$$

$$-\frac{251}{11} \rightarrow -\frac{251}{11} - \left(-\frac{21}{11}\right) \cdot \frac{8}{11} \cdot \left(-\frac{11}{4}\right) = \frac{-251 - 24}{11} = -\frac{275}{11} = -25$$

$$\frac{5}{11} \rightarrow \frac{5}{11} - \left(-\frac{3}{11}\right) \cdot \frac{8}{11} \cdot \left(-\frac{11}{4}\right) = \frac{35 - 24}{22} = \frac{11}{22} = \frac{1}{2}$$

$$X^4 = (3; 4; 3; 0; 0) - \text{опт. рещи}$$

$$Z_1(3; 4; 3) = -25$$

$$Z_{\max} = Z(3; 4; 3) = 25$$

$$(1) x_1 + 3x_2 = 12$$

$x_1$	0	12
$x_2$	4	0

$$(2) 5x_1 + 4x_2 = 31$$

$x_1$	0	6,2
$x_2$	7,8	0

$$(3) 2x_1 + 3x_2 = 18$$

$x_1$	0	9
$x_2$	6	0

$$Z = 3x_1 + 4x_2$$

$$\text{grad } Z = (3, 4)$$

