

1. Вывести  $(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \vdash (\forall x)P(x)$

$$\begin{array}{c}
\frac{(P(x))_t^x, t \approx x \vdash (P(x))_x^x}{P(t), x \approx t \vdash P(x)} \text{ (зам. экв.)} \\
\frac{P(t), x \approx t \vdash P(x)}{P(g(y)), x \approx g(y) \vdash P(x)} \text{ (своб. конкр.)} \\
\frac{P(g(y)), x \approx g(y) \vdash P(x)}{(P(g(x)))_y^x, x \approx g(y) \vdash P(x)} \\
\frac{(P(g(x)))_y^x, x \approx g(y) \vdash P(x)}{(\forall x)P(g(x)), x \approx g(y) \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{(\forall x)P(g(x)), x \approx g(y) \vdash P(x)}{(\forall x)P(g(x)), (\exists y)x \approx g(y) \vdash P(x)} \text{ (}\exists \text{ лев)} \\
\frac{(\forall x)P(g(x)), (\exists y)x \approx g(y) \vdash P(x)}{(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \vdash P(x)}{(\forall x)P(g(x)), (\forall x)(\exists y)x \approx g(y) \vdash (\forall x)P(x)} \text{ (}\forall \text{ прав)}
\end{array}$$

2. Вывести  $(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\forall x)(\exists y)R(x, y) \vdash (\exists x)P(x)$

$$\begin{array}{c}
\frac{(R(x, y))_x^y \vdash R(x, x)}{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash R(x, x)} \text{ (yт, пер)} \quad \frac{(R(x, y) \rightarrow P(y))_x^y \vdash R(x, x) \rightarrow P(x)}{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z \vdash R(x, x) \rightarrow P(x))} \text{ (экв. зам.)} \\
\frac{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash R(x, x)) \quad ((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash R(x, x) \rightarrow P(x))}{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash R(x, x) \rightarrow P(x))} \text{ (yт)} \\
\frac{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash R(x, x) \rightarrow P(x))}{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash P(x))} \text{ (yд. } \rightarrow \text{)} \\
\frac{((R(x, y) \wedge R(y, z) \rightarrow P(y))_x^z, (R(x, y))_x^y \vdash P(x))}{((\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)))_x^y, (R(x, y))_x^y \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{((\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)))_x^y, (R(x, y))_x^y \vdash P(x)}{(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (R(x, y))_x^y \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (R(x, y))_x^y \vdash P(x)}{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (R(x, y))_x^y \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (R(x, y))_x^y \vdash P(x)}{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\exists y)R(x, y) \vdash P(x)} \text{ (}\exists \text{ лев)} \\
\frac{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\exists y)R(x, y) \vdash P(x)}{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\forall x)(\exists y)R(x, y) \vdash P(x)} \text{ (}\forall \text{ лев)} \\
\frac{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\forall x)(\exists y)R(x, y) \vdash P(x)}{(\forall x)(\forall y)(\forall z)(R(x, y) \wedge R(y, z) \rightarrow P(y)), (\forall x)(\exists y)R(x, y) \vdash (\exists x)P(x)} \text{ (}\exists \text{ прав)}
\end{array}$$