1. Вывести $(\exists x)(A \wedge B) \ \vdash \ (\exists x)A \wedge (\exists x)B$

$$\dfrac{\dfrac{\dfrac{\dfrac{A \wedge B \ \vdash \ A \wedge B}{A \wedge B \ \vdash \ A}\,(\text{уд} \wedge \text{пр})}{A \wedge B \ \vdash \ (\exists x)A}\,(\text{вв} \exists \text{пр})}{(\exists x)(A \wedge B) \ \vdash \ (\exists x)A}\,(\text{вв} \exists \text{лев}) \qquad \dfrac{\dfrac{\dfrac{A \wedge B \ \vdash \ A \wedge B}{A \wedge B \ \vdash \ B}\,(\text{уд} \wedge \text{лев})}{A \wedge B \ \vdash \ (\exists x)B}\,(\text{вв} \exists \text{пр})}{(\exists x)(A \wedge B) \ \vdash \ (\exists x)B}\,(\text{вв} \exists \text{лев})}{(\exists x)(A \wedge B) \ \vdash \ (\exists x)A \wedge (\exists x)B}\,(\text{вв} \wedge)$$

2. Вывести $\ \vdash \ s \approx s$, где s - терм.

   Выводится, используя правило дополнительной конкретизации правилу дополнительной конкретизации

$$\dfrac{\vdash \ x \approx x}{\vdash \ (x \approx x)^x_s}$$

1. Вывести $(\exists x)(\exists y)\psi \vdash (\exists y)(\exists x)\psi$

$$\cfrac{\cfrac{\cfrac{\cfrac{\psi \vdash \psi}{\psi \vdash (\exists x)\psi}\ (\text{вв } \exists \text{ прав})}{\psi \vdash (\exists y)(\exists x)\psi}\ (\text{вв } \exists \text{ прав})}{(\exists y)\psi \vdash (\exists y)(\exists x)\psi}\ (\text{вв } \exists \text{ лев})}{(\exists x)(\exists y)\psi \vdash (\exists y)(\exists x)\psi}\ (\text{вв } \exists \text{ лев})$$

2. Вывести $(\exists x)\varphi \to (\forall x)\psi \vdash (\forall x)(\varphi \to \psi)$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\varphi \vdash \varphi}{\varphi \vdash (\exists x)\varphi}}{(\exists x)\varphi \to (\forall x)\psi, \varphi \vdash (\exists x)\varphi} \quad \cfrac{(\exists x)\varphi \to (\forall x)\psi \vdash (\exists x)\varphi \to (\forall x)\psi}{(\exists x)\varphi \to (\forall x)\psi, \varphi \vdash (\exists x)\varphi \to (\forall x)\psi}}{(\exists x)\varphi \to (\forall x)\psi, \varphi \vdash (\forall x)\psi}\ ()}{(\exists x)\varphi \to (\forall x)\psi, \varphi \vdash (\forall x)\neg\varphi \vee (\forall x)\psi} \quad \cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\varphi \vdash \varphi}{\cdots}}{\varphi, (\forall x)\neg\varphi, \neg\psi \vdash \varphi} \quad \cfrac{\cfrac{\cfrac{\neg\varphi \vdash \neg\varphi}{(\forall x)\neg\varphi \vdash \neg\varphi}}{\cdots}}{\varphi, (\forall x)\neg\varphi, \neg\psi \vdash \neg\varphi}}{\varphi, (\forall x)\neg\varphi \vdash \psi}}{\cdots}}{(\exists x)\varphi \to (\forall x)\psi, \varphi, (\forall x)\neg\varphi \vdash \psi} \quad \cfrac{\cfrac{\psi \vdash \psi}{(\forall x)\psi \vdash \psi}\ (\text{вв } \forall \text{ лев}}{(\exists x)\varphi \to (\forall x)\psi, \varphi, (\forall x)\psi \vdash \psi}\ (\text{ут, ут, пер, пер}}{(\exists x)\varphi \to (\forall x)\psi, \varphi \vdash \psi}\ (\text{уд } \vee)}{(\exists x)\varphi \to (\forall x)\psi \vdash \varphi \to \psi}\ (\text{вв } \to)}{(\exists x)\varphi \to (\forall x)\psi \vdash (\forall x)(\varphi \to \psi)}\ (\text{вв } \forall \text{ пр})$$

3. Вывести $(\forall y)\psi \vdash (\forall x)(\psi)^x_y$