Exuberant and Uninformed: How Financial Markets (Mis-)Allocate Capital during Booms

Ilja Kantorovitch*
Universitat Pompeu Fabra & Barcelona GSE

Job Market Paper

October 2, 2020

Click here for the latest version

Abstract

I develop a neoclassical growth model in which investment is intermediated by a stock market. Boundedly-rational traders save by purchasing shares of firms. Crucially, traders acquire information to guide their trading decision. The acquisition of information determines the financial market's ability to allocate more capital to productive firms and pins down total factor productivity (TFP). I study the different responses of information acquisition to two aggregate shocks. Sentiment shocks, defined as correlated errors in the information traders receive, *crowd-out* information acquisition. In contrast, productivity shocks *crowd-in* information. The decentralized equilibrium is constrained-inefficient as (i) bounded-rationality in the form of overconfidence distorts prices, and (ii) traders do not acquire information to increase TFP. Traders acquire less information than is socially optimal when firms are relatively heterogeneous and financial markets are noisy. The optimal policy in response to shocks is to "lean against sentiment." In particular, asset purchases improve allocative efficiency when used in response to negative sentiment shocks, but decrease efficiency when used against negative productivity shocks.

Keywords: Financial Markets; Information Production; Misallocation; Macroeconomics; Booms

JEL Codes: D80, E32, E44, G14

*Email Address: ilja.kantorovitch@upf.edu. I want to thank Vladimir Asriyan, Alberto Martin and Victoria Vanasco for their guidance and advice on this project. I acknowledge support from the German National Academic Foundation throughout my studies. I also thank Edouard Schaal, Isaac Baley, Filippo Ippolito and the participants of the CREi Macro Lunch and the 1st NuCamp Virtual PhD Workshop for their insightful comments and feedback.

1 Introduction

Financial markets play a central role in allocating capital to its most productive uses, yet they do not fulfill this role equally well at all times. In the last 20 years, financial markets have gone through booms and busts that have been difficult to justify on fundamental grounds alone (Martin and Ventura, 2018).[1] Against this backdrop, there are growing concerns that such booms led to a deterioration in capital allocation.[2] The argument goes as follows: during booms, agents acquire less information, so that markets become less informative and, ultimately, the allocation of resources worsens. It is impossible to fully evaluate this narrative without a macroeconomic theory of information acquisition and its macroeconomic effects. The goal of this paper is to provide such a model.

I develop a tractable model in which agents have dispersed information about the productivity of firms and investment is intermediated by a stock market. With a few exceptions,[3] financial markets' role as aggregators of dispersed information has received little attention in macroeconomics.[4] The primary reason is that most standard models of informative financial markets use non-optimizing agents, such as noise traders, a feature that is not straightforward to reconcile with general equilibrium. As a result, the macroeconomic impact of information acquisition in financial markets is not well understood and some open questions remain. For example, do financial markets amplify or dampen shocks in such an environment? Can financial markets themselves be a source of volatility? Which factors determine the allocative efficiency of financial markets? Does this allocative efficiency change over the cycle? I use my model to answer these questions and shed light on whether and how macro-prudential policy can improve the allocative efficiency of financial markets.

The model is populated by traders who work when young and invest in shares of firms with idiosyncratic production technology, which use these resources to fund investment. To inform their trading decision, traders exert effort to receive a noisy signal about firm productivity, where the precision of the signal increases with effort. The resulting trades make asset prices informative about firm productivity, which aids the allocation of capital according to firm productivity.

[1]For example, the dot-com bubbles in the US and the housing bubbles in the US and Southern Europe.

[2]Gopinath et al. (2017) document a rise in capital misallocation during the housing boom has been documented for Southern Europe and García-Santana et al. (2020) study it for Spain. Doerr (2018) provides similar evidence for the US. Gorton and Ordoñez (2020) relate credit booms in general to changes to total factor productivity. They find that "bad" booms can lead to a decrease in TFP.

[3]Some exceptions are David, Hopenhayn, and Venkateswaran (2016), Peress (2014), and Straub and Ulbricht (2018).

[4]The idea of markets as aggregators of dispersed information dates back to Hayek (1945): "The economic problem of society is (...) rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality."

I first characterize the equilibrium and show that the allocation of resources depends on information acquisition. In particular, TFP rises with information acquisition. I then show how information acquisition changes in response to two macroeconomic shocks, on sentiment and productivity. Sentiment shocks, defined as correlated noise shocks, are meant to capture a range of phenomena that drive asset prices away from their fundamental value, such as herding, network effects, social learning, extrapolative expectations, or bubbles (see Kindleberger and Aliber, 2011; Shiller, 2015, 2017). Sentiment and productivity shocks lead to similar co-movements in output, investment, wages, and asset prices. However, they have different effects on information acquisition and, therefore, on the allocative efficiency of capital between firms.

The first main result is that sentiment shocks, whether they make agents optimistic or pessimistic about firm productivity, tend to crowd-out information acquisition. There are two channels through which sentiment shocks affect the incentive to acquire information. The first and for larger shocks dominant channel is that sentiment shocks make trading less information-sensitive as a larger number of shares become increasingly mispriced. Intuitively, an imprecise yet unbiased private signal is sufficient to identify grossly mispriced firms. Moreover, firms that remain fairly priced and for which, therefore, precise information is valuable must appear in comparison unproductive when the economy is hit by a positive sentiment shocks. Therefore, fairly priced firms attract less capital, which discourages information acquisition where it is otherwise most valuable. The second channel works through aggregate investment as positive sentiment shocks increase investment for each traded firm, which increases the stakes for each trade and, in turn, incentivizes information acquisition. All positive sentiment shocks crowd-out information acquisition for reasonable calibrations, whereas negative sentiment initially crowd-in information acquisition and, once the negative shock is large enough, crowd-out information.

The second main result is that positive (negative) productivity shocks *crowd-in* (-out) information acquisition. This result points to financial markets as amplifiers of fundamental shocks, similar to the net worth channel (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). As aggregate earnings and investment increase in response to a positive productivity shock, traders choose to acquire more precise information because the stakes increase for each trade. Therefore, the allocative efficiency of financial markets improves over and beyond the initial shock.

The third main result is that the social planner's optimal policy is to "lean against sentiment" as sentiment-driven fluctuations in information acquisition and investment are inefficient. Taxes or subsidies on transactions or dividends achieve this objective by correcting asset prices for distorted beliefs. As noted earlier, sentiment shocks crowd-out information and thereby worsen the allocative efficiency of financial markets. Therefore, the social planner subsidizes information acquisition in response to negative and positive sentiment shocks to maintain financial markets' allocative efficiency. In reality, policymakers are hesitant to "lean against the wind" in fear of choking off productive investment, as they cannot distinguish sentiment from productivity in real-time. The model suggests that past fluctuations in price-earnings ratios can be used as an indicator of future sentiment shocks, if aggregate shocks are persistent.

In particular, asset purchases can have both efficiency improving and destroying effects depending on which shocks drive the economy. By reducing the supply of assets, purchase programs change the identity of the marginal buyer, which biases prices upward. Therefore, asset purchases can cancel out the downward bias from negative sentiment shocks. This does not only revive investment but also recovers the incentives to acquire information as asset prices become undistorted. However, asset purchases can also be a source of distortion and discourage information acquisition when used in response to a negative productivity shock. In this case, asset purchases increase investment but worsen allocative efficiency through a decrease in information acquisition over and beyond the initial shock. Therefore, my model supports arguments for and against quantitative easing, as the effect of purchase programs crucially depends on the source of economic fluctuations.

From a normative perspective, I show that information acquisition is always at an inefficient level in the decentralized equilibrium due to a mismatch of motives for acquiring information between the social planner and traders. The social planner seeks to improve allocative efficiency through information production, whereas traders take prices as given and ignore this effect. Instead, traders are motivated by a speculative motive, as they aim to buy shares that are affected by a negative sentiment shock. Such speculative profits are not socially valuable, as they are transfers between risk-neutral traders. Therefore, the social planner's and the traders' information choice never coincide and whether information acquisition is too low or too high in the decentralized equilibrium depends on parameters. In particular, information acquisition is too low when firms are relatively heterogeneous, and financial markets are efficient aggregators of information.

This paper's methodological contribution is to provide a tractable general equilibrium macroeconomic model with utility-maximizing agents, in which information acquisition drives the allocative efficiency of financial markets and, therefore, TFP. A small behavioral deviation is sufficient to avoid introducing non-maximizing agents such as noise traders. Concretely, overconfident traders receive signals on firm productivity that contain both idiosyncratic and correlated noise but think that their signal contains only idiosyncratic noise. This misperception motivates traders to acquire costly information as they believe to have an informational edge over the market. This assumption is empirically motivated, as is discussed further below, and is essential to avoid the Grossman-Stiglitz paradox.[5] Moreover, using a single trader type is especially tractable, whereas standard models of informative financial markets are notoriously intractable. The proposed framework is flexible enough to study cash-in-the-market pricing, leverage, and rational bubbles, among other topics. However, this paper limits itself to studying the two-way feedback between the aggregate state of the economy and information acquisition in financial markets.

Providing empirical evidence for the core mechanism at play is challenging, as this model relates three unobservable variables to each other, namely sentiment/productivity, information acquisition, and TFP. Nonetheless, Dávila and Parlatore (2020) suggest that financial market efficiency, as captured by price informativeness,[6] has indeed been increasingly volatile in the US since the late 1990s, as can be seen in Figure 16 in the Appendix. Although price informativeness has been steadily increasing since the 1990s, it experienced a rapid increase during the dot-com boom, but dropped again below trend as the housing bubble was inflating. This drop ended with the Great Recession and another spike in price informativeness in 2009. These movements in price informativeness roughly coincide with high growth in total factor productivity (TFP) from 1995 to 2005 and decrease from 2006 to 2008 as can be seen in Figure 17. Together, these stylized facts suggest a connection between economic fluctuations, financial market efficiency as measured through price informativeness, and TFP growth.

1.1 Literature Review

This paper is part of the literature on the link between information production and the business cycle (Ambrocio, 2019; Asriyan, Laeven, and Martin, 2019; Chousakos, Gorton, and Ordonez, 2020; Farboodi and Kondor, 2019; Veldkamp, 2005).[7] The overarching theme is that incentives to produce information vary over the cycle, which in turn shapes the business cycle. Similar to Asriyan, Laeven, and Martin (2019), non-fundamental booms crowd-out information from financial markets, whereas in this paper also non-fundamental busts crowd-out information. Also Chousakos, Gorton, and Ordonez (2020) find that information production is low during both depressions and booms.

This paper builds on the literature on informative financial markets (Albagli, Hellwig,

[5] The Grossman-Stiglitz paradox states that no equilibrium exists in models of financial markets with costly information acquisition when there is no noise to keep prices from being perfectly revealing.

[6] Price informativeness relates to the precision of information embodied in asset prices.

[7] See also Angeletos, Lorenzoni, and Pavan (2010), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), Gorton and Ordonez (2014), Ordonez (2013), Straub and Ulbricht (2018), and Van Nieuwerburgh and Veldkamp (2006) for related work.

and Tsyvinski, 2011a; Grossman and Stiglitz, 1980; Kyle, 1985; Vives, 2010). In this literature, limits-to-arbitrage keep arbitrageurs from eliminating all mispricing and, therefore, asset prices are noisy functions of fundamentals. The closest paper in this literature is Albagli, Hellwig, and Tsyvinski (2011a). Similarly, traders in this model are also risk-neutral, subject to exogenous position limits and asset prices are distorted. Whereas their paper includes rational and noise traders, traders in this model are boundedly-rational. A number of papers study informative financial markets in a macroeconomic context (Albagli, Hellwig, and Tsyvinski, 2017; Asriyan, 2020; David, Hopenhayn, and Venkateswaran, 2016; Peress, 2014; Straub and Ulbricht, 2018). Different to these papers, I study the effects of aggregate shocks in connection to information acquisition and the allocation of capital.

This paper contributes to the literature on the feedback between financial markets and the real economy, as surveyed in Bond, Edmans, and Goldstein (2012). This literature mostly studies the role of secondary markets as sources of information, for example to inform managers (Dow and Gorton, 1997; Holmström and Tirole, 1993). Similar to Dow, Goldstein, and Guembel (2017), I study the two-way feedback between the financial and real economy when information is chosen endogenously. Different to most papers in the literature, financial markets directly allocate capital in this paper instead of only providing information.

There is ample empirical evidence that asset prices indeed provide valuable information. See Morck, Yeung, and Yu (2013) for a survey on the literature that uses "non-synchronicity" as a measure of price-informativeness.[8] Morck, Yeung, and Yu (2000) find that more developed countries have stock markets that are more informative. Focusing instead on the cross-section of firms, Durnev, Morck, and Yeung (2004) find that non-synchronicity is positively related to corporate investment's efficiency. More recently, Bai, Philippon, and Savov (2016) and Farboodi, Matray, et al. (2020) show that prices have become better predictors of corporate earnings in the US since the 1960s. The latter emphasize that this has been mainly the case for large growth firms. Dávila and Parlatore (2020) provide a micro-founded measure of price informativeness. They find that prices have become more informative since the 1990s, but price informativeness has also become increasingly volatile since the late 1990s. Finally, Bennett, Stulz, and Wang (2020) provide evidence that price informativeness increases firm productivity.

In this model, traders suffer from correlation neglect. This bias has been studied in the literature and documented repeatedly in experimental settings (Brandts, Giritligil, and Weber, 2015; Chandrasekhar, Larreguy, and Xandri, 2012; Enke and Zimmermann, 2019; Eyster et al., 2018; Grimm and Mengel, 2018). When receiving information from multiple sources,

[8] Non-synchronicity has been suggested by Roll (1988) as a measure of firm-specific information in asset prices. The main idea is that the more volatility in asset prices stem from firm-specific factors, the more informative are prices about firms.

neglecting correlated noise in the signals can lead to an overly precise posterior. Therefore, correlation neglect leads to overconfidence, which plays a central role in the literature on behavioral biases, especially in relation to financial markets (Daniel and Hirshleifer, 2015; Glaser and Weber, 2010).

2 Model

2.1 Households and Traders

The model is populated by overlapping generations of households indexed by $i \in [0,1]$. As is common in the New-Keynesian literature, I assume that each household i consists of a unit mass of traders indexed by $ij \in [0,1] \times [0,1]$ (see for example Blanchard and Galí, 2010). Households pool resources, borrow on behalf of traders and distribute consumption equally, whereas traders individually maximize the utility for the household given by

$$U_{it} = C_{it,t} + \delta \mathbb{E}\left\{C_{it,t+1}\right\} - \int_0^1 IA(\beta_{ijt})dj, \tag{1}$$

where $C_{it,t}$ is youth consumption, $C_{it,t+1}$ is old age consumption, $\delta \in (0,1)$ is the discount factor and $\int_0^1 IA(\beta_{ijt})dj$ are information acquisition costs which are introduced in more detail in a later section.

When young, traders each inelastically supply one unit of labor and receive a wage W_t and buy shares of intermediate good firms in a competitive financial market. To avoid unbounded demands by risk-neutral traders, demand for each stock is limited to the interval $[\kappa_L, \kappa_H]$ where $\kappa_L \leq 0$ and $\kappa_H > 1$.[9] Traders also choose the precision β_{ijt} of a noisy signal of firm productivity to inform their trading decision subject to a utility cost $IA\left(\beta_{ijt}\right)$. Finally, the household lends and borrows through risk-less bonds with return R_{t+1}.

2.2 Technologies

2.2.1 Final Good Sector

There are many identical final good firms owned by households. The production function for the final good, which also serves as the numéraire, is Cobb-Douglas over labor and a

[9]See Albagli, Hellwig, and Tsyvinski (e.g., 2011a) and Dow, Goldstein, and Guembel (2017) for similar approaches and Appendix C for a further elaboration.

CES-aggregate of intermediate goods. Aggregate output is

$$Y_t = L^{1-\alpha} \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}}, \tag{2}$$

where $\theta \in (0, \infty)$ is the elasticity of substitution between varieties and α is the share of intermediate goods. Y_{jt} is an intermediate good produced by firm j. The final good can be consumed or invested in firm-capital.

2.2.2 Intermediate Good Sector

For each generation there is a unit mass of intermediate good firms $j \in [0, 1]$ with production function

$$Y_{jt} = A_{jt-1}^{\frac{\theta}{\theta-1}} K_{jt}, \tag{3}$$

where K_{jt} is firm-capital and $\ln(A_{jt-1}) \overset{iid}{\sim} \mathcal{N}(a_{t-1}, \sigma_a^2)$ is firm productivity. Capital takes time to build, such that investment takes place in t but production in $t+1$, and depreciates fully after production. Each firm sells a unit mass of claims to total firm-revenue to households and finances capital investment with the proceeds,[10]

$$P_{jt} = K_{jt}. \tag{4}$$

2.2.3 Information

Trader ij is only active in the market for shares of firm j, for which she is an expert as she receives a signal

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt} + \varepsilon_{jt}}{\sqrt{\beta_{ijt}}}, \tag{5}$$

where $a_{jt} \overset{iid}{\sim} \mathcal{N}(a_t, \sigma_a^2)$ is firm productivity, $\eta_{ijt} \overset{iid}{\sim} \mathcal{N}(0, 1)$ is idiosyncratic noise, $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}(\varepsilon_t, \sigma_\varepsilon^2)$ is correlated noise which is interpreted as *sentiment*, and β_{ijt} is a information precision parameter chosen by trader ij.[11] Both idiosyncratic and correlated noise are *iid* over time and across markets; idiosyncratic noise is also *iid* between traders. A high realization of η_{ijt} means that trader ij is optimistic about firm j relative to other traders in the same market. Similarly, a high realization of ε_{jt} means that traders in market j are collectively too optimistic.

[10] See Appendix D for a micro-foundation and further discussion.
[11] See section 7.1 for the effect of uncertainty about aggregate shocks.

Assumption 1 (***Overconfidence***)**.** *From the perspective of trader ij the information structure is*

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt}}{\sqrt{\beta_{ijt}}}$$
$$s_{-ijt} = a_{jt} + \frac{\eta_{-ijt} + \varepsilon_{jt}}{\sqrt{\beta_{-ijt}}}.$$

Following Assumption 1, traders believe that sentiment ε_{jt} drives the beliefs of all traders except their own. As a result, traders are overconfident and willing to acquire costly information to exploit mispricing induced through sentiment shocks ε_{jt}.[12] Finally, trader ij chooses the precision of her private signal β_{ijt} subject to a convex cost function $IA(\beta_{ijt})$ with standard properties $IA(0) = 0$, $IA'(0) = 0$, $IA''(\cdot) > 0$.

2.2.4 Aggregate Shocks

Two classes of shocks drive the economy. *Aggregate productivity shocks* move the mean of the distribution of firm-specific productivity shocks, $a_{jt} \sim \mathcal{N}(a_t, \sigma_a^2)$, and *aggregate sentiment shocks* drive the mean of firm-specific sentiment shocks, $\varepsilon_{jt} \sim \mathcal{N}(\varepsilon_t, \sigma_\varepsilon^2)$, similar to Angeletos, Lorenzoni, and Pavan (2010). The sentiment shock ε_t is meant to capture a range of phenomena that lead to non-fundamental price movements in financial markets, e.g., herding, informational cascades, social learning, bubbles, liquidity trading, etc. (see Kindleberger and Aliber, 2011; Shiller, 2015, 2017).

For simplicity, traders perfectly observe aggregate shocks $\{a_t, \varepsilon_t\}$ before their information acquisition decision, but firm-specific shocks $\{a_{jt}, \varepsilon_{jt}\}$ need to be learned. The law of motions for the aggregate shocks are irrelevant for this setup, as the dynamic model is a repetition of static problems. It follows that the information set of trader ij consists of the private signal s_{ijt}, share prices $\{P_{jt}\}$ for all markets $j \in [0, 1]$ and the mean and variances of firm-specific shocks $\{a_t, \varepsilon_t\}$, i.e., $\mathcal{I}_{ij} = \{s_{ijt}, \{P_{jt}\}, a_t, \varepsilon_t\}$. In other words, traders have rational beliefs about aggregates, but disagree about the productivity of intermediate good firms based on public information in form of prices and private signals.

[12] This assumption is necessary to avoid the Grossman-Stiglitz paradox Grossman and Stiglitz (1980). It states that informationally efficient markets are impossible in the absence of noise when information is costly. In that case, markets would already reveal all information and, therefore, destroy the incentive to acquire costly information in the first place.

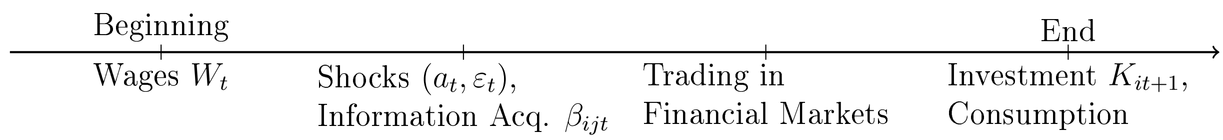

Figure 1: Intraperiod timing.

2.3 Timing

The timing is laid out in Figure 1. At the beginning of each period, young traders work in the final good sector and receive wage W_t. Then, traders choose the precision of their signal and the financial market opens. At the end of the period, investment takes place and consumption takes place.

2.4 Notation

Traders think that their private signal does not contain correlated noise ε_{jt} as in Assumption 1. Therefore, expectations that condition on the private signal are distorted and, for example, trader ij's expectations about revenue are by $\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}, P_{jt}\right\}$.

The determinants of functions are usually omitted to save on notation. For example, the firm $j's$ revenue is denoted by Π_{jt+1} instead of $\Pi\left(A_{jt}, K_{jt+1}, Y_{t+1}\right)$. A_{jt} is indexed by t instead of $t+1$ as traders can learn about firm productivity in period t.

2.5 Household's and Trader's Problem

Household i takes the interest rate R_{t+1} as given and decides how much to borrow or lend. Furthermore, households are also prone to the behavioral bias of Assumption 1, but do not observe the private signals of traders in the household. The household's problem is then

$$\max_{B_{it+1}} \quad C_{it,t} + \delta\tilde{\mathbb{E}}_t\left\{C_{it,t+1}\right\} - \int_0^1 IA\left(\beta_{ijt}\right)dj \tag{P1}$$

$$s.t. \quad C_{it,t} = W_t - \int_0^1 x_{ijt}P_{jt}dj - B_{it+1} \tag{6}$$

$$C_{it,t+1} = \int_0^1 x_{ijt}\Pi_{jt+1}dj + R_{t+1}B_{it+1} \tag{7}$$

$$C_{it,t}, C_{it,t+1} \geq 0. \tag{8}$$

Households optimally choose how to much lend or borrow subject to the budget constraints during youth and old age. The first constraint (6) states that today's consumption is equal to wages W_t minus the costs of buying stocks $\int_0^1 x_{ijt} P_{jt} dj$ and saving through the bond market B_{it+1}. Constraint (7) states that old age consumption is equal to revenue $\int_0^1 x_{ijt} \Pi_{jt+1} dj$ plus income from lending on the bond market $R_{t+1} B_{it+1}$. Although household i is overly optimistic of the return of its portfolio due to overconfidence, each household correctly values the portfolio of all other households. Therefore, limiting borrowing by the natural borrowing constraint as in (8) rules out default.

Household i's optimal saving decision is given by

$$B_{it+1} \begin{cases} = -\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}} & \text{if } R_{t+1} < \frac{1}{\delta} \\ \in \left[-\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}}, W_t - \int_0^1 x_{ijt} P_{jt} dj \right] - & \text{if } R_{t+1} = \frac{1}{\delta} \\ = W_t - \int_0^1 x_{ijt} P_{jt} dj & \text{if } R_{t+1} > \frac{1}{\delta} \end{cases}. \tag{9}$$

If the interest rate R_{t+1} is below $\frac{1}{\delta}$, it is optimal to borrow as much as possible. If the interest is equal to $\frac{1}{\delta}$, household i is indifferent between borrowing and saving. Finally, if the interest rate is above $\frac{1}{\delta}$, it is optimal to save as much as possible. Plugging (6) and (7) into (P1) and using the solution for the saving decision (9) yields the problem of trader ij

$$\max_{x_{ijt}, \beta_{ijt}} \quad \max\{1, \delta R_{t+1}\} \tilde{\mathbb{E}}_t \left\{ x_{ijt} \left(\frac{1}{R_{t+1}} \Pi_{jt+1} - P_{jt} \right) \right\} - IA(\beta_{ijt}) \tag{P2}$$

$$s.t. \quad x_{ijt} \in [\kappa_L, \kappa_H] \tag{10}$$

$$\beta_{ijt} \geq 0, \tag{11}$$

where terms that do not depend on the decision by trader ij were dropped. Trader ij chooses demand for share j x_{ijt} subject to the position limits (10) and decides on the information precision β_{ijt} subject to a non-negativity constraint. Traders can use the household's pooled resources and borrow through the household for trading. The first term $\max\{1, \delta R_{t+1}\}$ reflects that then $R_{t+1} > \frac{1}{\delta}$ the value of an additional unit of wealth is above one.

3 Equilibrium Characterization

3.1 Input Markets

Wages and intermediate good prices are determined competitively,

$$W_t = \frac{\partial Y_t}{\partial L} = (1-\alpha) Y_t \tag{12}$$

$$\rho_{jt} = \frac{\partial Y_t}{\partial Y_{jt}} = \alpha Y_t^{\alpha_Y} Y_{jt}^{-\frac{1}{\theta}}, \tag{13}$$

where $\alpha_Y = \frac{\alpha\theta-\theta+1}{\alpha\theta}$. Wages are equal to a share $(1-\alpha)$ of output. The price for intermediate good j is downward sloping in the quantity produced of the same good. Finally, the revenue of intermediate good firm j is given by

$$\Pi_{jt+1} = \rho_{jt+1} Y_{jt+1}. \tag{14}$$

3.2 Trader's Decisions

Trading Trader ij sells $-\kappa_L$ shares if the price P_{jt} is above her expectations of revenue Π_{jt+1} using the interest rate on bonds R_{t+1} as the benchmark rate, is indifferent between buying or selling when they coincide and buys κ_H shares when her expectations exceed the price,

$$x\left(s_{ijt}, P_{jt}\right) = \begin{cases} \kappa_L & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} < P_{jt} \\ \in \left[\kappa_L, \kappa_H\right] & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} = P_{jt} \\ \kappa_H & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} > P_{jt} \end{cases}. \tag{15}$$

Information Acquisition As laid out in (15), the trading decision is driven by the realization of the private signal s_{ijt} relative to the price P_{jt}. Consequently, trader ij chooses information precision β_{ijt} to improve her ability to identify profitable trading opportunities. A central object in this context is the subjective probability of buying conditional on realizations of productivity a_{jt}, sentiment ε_{jt}, trader ij's information choice β_{ijt} and the symmetric choice of all other traders in the market β_{jt}. Taking expectations with respect to the realizations of the idiosyncratic noise η_{ijt} yields the probability of buying

$$\mathcal{P}\left\{x_{ijt} = \kappa_H | a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\right\} = \int_{-\infty}^{\infty} \phi\left(\eta_{ijt}\right) 1_{\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt}, P_{jt}\} > P_{jt}} d\eta_{ijt} \tag{16}$$

where $\phi(\cdot)$ is the standard-normal pdf.[13]

[13] A more detailed derivation can be found in Appendix A.

The first order condition for the information acquisition decision is obtained after plugging (15) into (P2). Evaluating the expectations with respect to the realizations of the idiosyncratic noise η_{ijt} and taking the symmetric information acquisition decisions of all other traders as given ($\beta_{-ijt} = \beta_{jt}$) leads to first-order condition

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right) = \max\left\{1, \delta R_{t+1}\right\} \tilde{\mathbb{E}}_t \left\{ \left(\kappa_H - \kappa_L\right) \underbrace{\frac{\partial \mathcal{P}\left\{x_{ijt} = \kappa_H | a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\right\}}{\partial \beta_{ijt}}}_{\text{Change in the probability of buying}} \underbrace{\left(\frac{1}{R_{t+1}} \Pi_{jt+1} - P_{jt}\right)}_{\text{Profit}} \right\} = IA'\left(\beta_{ijt}\right). \tag{17}$$

The marginal benefit of acquiring information consists of two parts. The first is the probability of buying in state $(a_{jt}, \varepsilon_{jt})$ given information choices $(\beta_{ijt}, \beta_{jt})$. The second component is the profit given by the difference between the net-present value of firm-revenue minus the price of the stock.

3.3 Financial Market

Market-Clearing At the symmetric equilibrium ($\forall j : \beta_{ijt} = \beta_{jt}$), traders buy κ_H shares whenever their private signal is above some threshold, $\hat{s}\left(P_{jt}\right)$, are indifferent between buying and selling when their private signal coincides with the threshold and sell otherwise. After normalizing the supply of shares in each market j to one, the market-clearing condition becomes

$$\kappa_H \left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right) - \kappa_L \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right) = 1, \tag{18}$$

which can be used to solve for the threshold signal

$$\hat{s}\left(P_{jt}\right) = a_{jt} + \frac{\varepsilon_{jt} + \Phi^{-1}\left(\frac{\kappa_H - 1}{\kappa_H + \kappa_L}\right)}{\sqrt{\beta_{jt}}}. \tag{19}$$

Price Signal Traders learn from prices, which is equivalent to observing a noisy signal of the form

$$z_{jt} = \hat{s}\left(P_{jt}\right) - \frac{\Phi^{-1}\left(\frac{\kappa_H - 1}{\kappa_H + \kappa_L}\right)}{\sqrt{\beta_{jt}}} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}. \tag{20}$$

When the price P_{jt} is high, traders realize that this can be due to two reasons: either firm j is productive (high a_{jt}) or other traders are very optimistic (high ε_{jt}). Therefore, prices are a noisy signal of firm productivity. The combination of dispersed information and position limits for asset demand ensure that the signal is normally distributed as $z_{jt} \sim \mathcal{N}(a_{jt}, \sigma_\varepsilon^2/\beta_{ijt})$

for all values of κ_L and κ_H. I call z_{jt} the *price signal* and expectations condition rather on z_{jt} instead of P_{jt}.

A crucial object in my analysis is the precision of the price signal $\beta_{jt}\sigma_\varepsilon^{-2}$, also referred to as price informativeness in the literature. If $\beta_{jt}\sigma_\varepsilon^{-2}$ is high, financial markets efficiently aggregate information and asset prices are informative about firm productivity. This naturally leads to productive firms receiving on average more capital which improves the allocative efficiency of financial markets. I focus on the endogenous component β_{jt}.

As is evident now, the values of κ_H and κ_L do not matter for the price signal z_{jt}. They only pin down the identity of the marginal trader, which has a predictable effect on the price. For instance, the marginal trader is relatively optimistic for $\kappa_H - \kappa_L > 2$, which means that price is set by a trader who received a private signal with positive idiosyncratic noise ($\eta_{ijt} > 0$). As a result, the price would be upward biased.[14] Choosing $\kappa_H = 2$ and $\kappa_L = 0$ ensures that the choice of position limits does not introduce a bias in share prices as the marginal trader has on average unbiased beliefs ($\eta_{ijt} = 0$).

The following proposition shows that the described equilibrium is unique. Moreover, the price P_{jt} is equal to the valuation of the *marginal* trader who is just indifferent between buying or not and observed the private signal $s_{ijt} = z_{jt}$. Any trader who is more optimistic than the marginal trader ($s_{ijt} > z_{jt}$) buys two shares, whereas more pessimistic traders buy nothing.

Proposition 1. *Observing P_{jt} is equivalent to observing the signal (20) whenever K_{jt+1} is non-decreasing in z_{jt}. In the unique equilibrium, in which demand $x\left(s_{ijt}, P_{jt}\right)$ is non-increasing in P_{jt}, the price is equal to the valuation of the trader with the private signal $s_{ijt} = z_{jt}$,*

$$P\left(z_{jt}\right) = \frac{1}{R_{t+1}} \tilde{\mathbb{E}} \left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{21}$$

3.4 Bond and Capital Market

The net supply of bonds is equal to zero, $\int_0^1 B_{it+1} di = 0$. Moreover, as all households are ex-ante identical, positions in bond markets are zero for all households, $\forall i : B_{it+1} = 0$. There is no excess demand or supply for bonds whenever the return on bonds R_{t+1} is equal to the return that traders expect to earn on the stock market. This is the case whenever

$$R_{t+1} = \frac{\int_0^1 \tilde{\mathbb{E}} \left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\} dj}{\int_0^1 P_{jt} dj}, \tag{22}$$

[14]This mechanism plays an important role in Fostel and Geanakoplos (2012) and Simsek (2013) and is treated more in-depth in Appendix C.

which is derived by integrating (18) on both sides.

The aggregate value of the stock market is equal to the aggregate capital stock as all revenue from financial markets is invested by firms as follows from aggregating (4),

$$\int_0^1 P_{jt} dj = K_{t+1}. \tag{23}$$

3.5 Equilibrium Definition

I focus on symmetric competitive equilibria, in which all traders choose the same information precision for all markets ($\forall ij : \beta_{ijt} = \beta_t$) and expect all traders to choose the same.

Definition 1. *A symmetric, competitive equilibrium consists of prices* $\{W_t, \rho_{jt+1}, P_{jt}, P_t, R_{t+1}\}$ *and allocations* $\{B_{it+1}, x_{ijt}, \beta_{ijt}, K_{jt+1}\}$ *such that:*

1. *Given prices* $\{W_t, \rho_{jt+1}, P_{jt}, R_{t+1}\}$ *and allocations* $\{x_{ijt}, \beta_{ijt}\}$*,* B_{it+1} *solves the household's problem P1.*

2. *Given prices* $\{P_{jt}, R_{t+1}\}$ *and allocations* $\{B_{it+1}, \beta_{jt}, K_{jt+1}\}$*,* $\{x_{ijt}, \beta_{ijt}\}$ *solve the trader's problem P2.*

3. *Prices are such that markets for labor, intermediate goods, shares, bonds and capital clear, i.e., (12), (13), (18), (22) and (23) hold.*

4 Properties of the Equilibrium

In the following, I work out the properties of the equilibrium abstracting from the information acquisition decision until the next section. I focus on how the the allocation of capital can be expressed in terms of beliefs of the marginal trader and how these beliefs respond to shocks both idiosyncratic and aggregate. Next, I demonstrate how the allocation of capital through the stock market leads to total factor productivity, which depends on the information choice. Finally, I show that the market allocation is distorted and derive the constrained-efficient allocation.

As shown in (21), the beliefs of the marginal trader determine share prices. Therefore, they play a central role for the allocation of capital both in the cross-section and aggregate. The marginal trader's expectations are a weighted sum of the realization of both idiosyncratic and aggregate productivity and sentiment shocks,

$$\ln \tilde{\mathbb{E}} \{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\} = \omega_p (\beta_{jt}) a_t + \omega_a (\beta_{jt}) a_{jt} + \omega_\varepsilon (\beta_{jt}) (\varepsilon_{jt} - \varepsilon_t) + \omega_{s\varepsilon} (\beta_{jt}) \varepsilon_t + \frac{1}{2} \mathbb{V}_{jt}. \tag{24}$$

The weights $\{\omega_p(\beta_{jt}), \omega_a(\beta_{jt}), \omega_\varepsilon(\beta_{jt}), \omega_{s\varepsilon}(\beta_{jt})\}$ depend on information acquisition β_{jt}. $\mathbb{V}_{jt}$ is related to the uncertainty of the marginal trader and does not depend on shocks.[15]

The first two terms capture the effect of aggregate and idiosyncratic productivity shocks. If traders do not acquire information ($\beta_{jt} = 0$), traders only rely on their prior a_t ($\omega_p(0) = 1$ and $\omega_a(0) = 0$). As traders acquire more information, they shift weight from their prior to the realization of firm productivity ($\lim_{\beta_{jt}\to\infty} \omega_a(\beta_{jt}) = 1$). This leads to a higher sensitivity of the allocation of capital to firm-specific productivity shocks and improves the allocative efficiency of financial markets.

In contrast to the weights on productivity shocks, the weights on sentiment shocks are hump-shaped in β_{jt}. If traders do not acquire information, they do not have a signal to learn from and, therefore, their expectations cannot be moved by noise ($\omega_\varepsilon(0) = \omega_{s\varepsilon}(0) = 0$). For perfect information, traders receive signals that do not contain noise in the first place ($\lim_{\beta_{jt}\to\infty} \omega_\varepsilon(\beta_{jt}) = \omega_{s\varepsilon}(\beta_{jt}) = 0$). If β_{jt} goes to either extreme, both idiosyncratic and aggregate sentiment shocks do not affect the beliefs of traders.

The aggregate sentiment shock ε_t moves the beliefs of traders although ε_t is common knowledge. This effect stems from the behavioral bias in Assumption 1. Traders correct the price signal z_{jt} for the aggregate sentiment shock, but mistakenly believe that their private signal s_{ijt} is unaffected by sentiment and, therefore, do not correct their private signal in a similar way.

4.1 Capital Allocation and TFP

The results so far can be combined to derive the allocation of capital and total factor productivity in equilibrium as captured in the following Proposition.

Proposition 2 (Market Allocation). *Under the market allocation:*

(i) Firm-capital is given by

$$K_{jt+1} = \frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^\theta}{\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^\theta dj} K_{t+1}. \tag{25}$$

(ii) The aggregate production function is

$$Y_t = A\left(a_{t-1}, \beta_{t-1}\right) K_t^\alpha \tag{26}$$

[15] See Appendix A for derivations.

with aggregate total factor productivity

$$\ln A\left(a_{t-1},\beta_{t-1}\right)=\underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right)}_{exogenous}+\underbrace{\kappa^a\left(\beta_{t-1}\right)\sigma_a^2-\kappa^\varepsilon\left(\beta_{t-1}\right)\sigma_\varepsilon^2}_{allocative\ efficiency}. \tag{27}$$

(iii) $A\left(a_{t-1},\beta_{t-1}\right)$ *can be locally decreasing in* β_{t-1} *if* σ_ε^2 *is large enough and is monotonically increasing otherwise.*

The first part of the Proposition highlights that capital is allocated to firms with higher realizations of the price signal z_{jt} whether it is driven by sentiment or productivity. Moreover, changes to aggregate investment K_{t+1} lead a proportional change in firm-capital for all firms. Consequently, total factor productivity (TFP) has both an exogenous and endogenous component. The exogenous component is related to the realization of the aggregate productivity shock a_t, which mechanically increases productivity of all firms. The endogenous component captures instead the allocational efficiency of financial markets, which is determined by aggregate information acquisition β_t.

However, the market does not allocate capital efficiently given the available information. As traders are overconfident, expectations in (25) condition also on the private signal s_{ijt}, although it is uninformative after observing z_{jt}. In other words, P_{jt} behaves as if the precision of the market signal z_{jt} was $\beta_{jt}+\beta_{jt}\sigma_\varepsilon^{-2}$, although its true precision is $\beta_{jt}\sigma_\varepsilon^{-2}$. Therefore, the price *overreacts* to the price signal z_{jt}.[16]

This distortion can be so severe that an increase in information acquisition β_t leads to a decrease in TFP as seen in Figure 2. As traders acquire more precise information, they also wrongly put more weight on their private signal. The overall effect on TFP depends on the balance between the beneficial effect of an increase in price informativeness $\beta_t\sigma_\varepsilon^{-2}$ and an increased weight on the private signal.

This price distortion leads to ex-ante misallocation of capital, i.e., output can be increased by reallocating capital between firms given the same publicly available information $\{z_{jt}\}$. A social planner would use the available information efficiently, leading to the *constrained-*

[16] This distortion has been studied intensively in Albagli, Hellwig, and Tsyvinski (2011a, 2015) and is called the "information aggregation wedge". Its general equilibrium implications are studied in Albagli, Hellwig, and Tsyvinski (2017). In contrast to this paper, their model features a combination of rational and noise traders. Therefore, the information aggregation wedge does not require a behavioral price-setting traders. Furthermore, it arises in any informative financial market model in which traders learn from both a heterogeneous private signal and the price. It does not arise in models in which the information set of informed agents is homogeneous (Grossman and Stiglitz, 1980) or in models in which traders do not observe the price before submitting market orders (Kyle, 1985). In the former case informed agents cannot learn anything from the price and in the latter it is not possible to learn from the price before trading. Both of these models restrict the analysis to linear models, whereas non-linearity arises naturally in macroeconomic models and therefore a different model is used here.

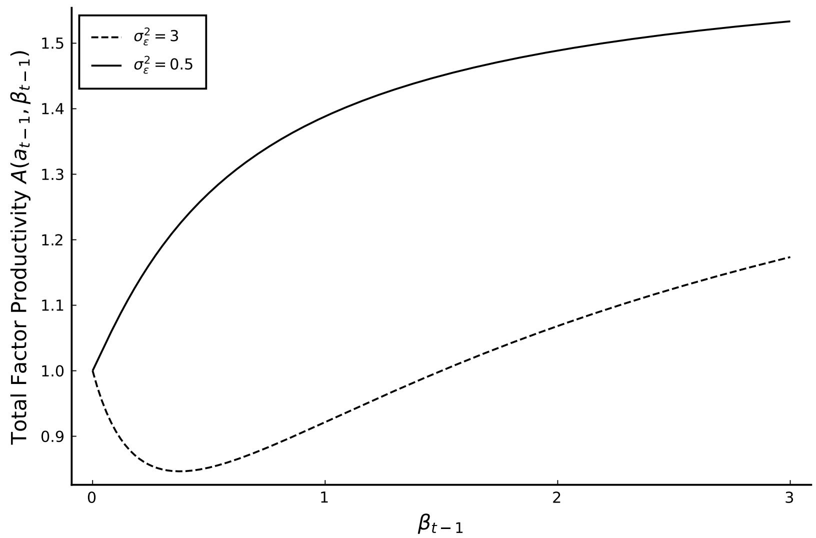

Figure 2: Total factor productivity as defined in (27). If the variance of sentiment shocks σ_ε^2 is sufficiently large, financial markets may worsen allocative efficiency relative to the case in which capital is equally distributed between firms ($\beta_t = 0$).

efficient allocation summarized in the following Proposition.

Proposition 3 (**Constrained-Efficient Allocation**). *Under the constrained-efficient allocation*

(i) Firm-capital is

$$K_{jt+1}^{eff} = \frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta} dj} K_{t+1}. \tag{28}$$

(ii) TFP $A^{eff}\left(a_{t-1}, \beta_{t-1}\right)$ is monotonically increasing in aggregate information acquisition β_{t-1}.

(iii) $A^{eff}\left(a_{t-1}, \beta_{t-1}\right) \geq A\left(a_{t-1}, \beta_{t-1}\right)$, with strict inequality for interior values of β_{t-1}.

The *constrained-efficient* allocation assigns the correct precision $\beta_t \sigma_\varepsilon^{-2}$ to the price signal z_{jt}. Relative to the market allocation, the constrained-efficient allocation redistributes capital from firms that were previously too large to firms that were too small as seen in Figure 3. Moreover, TFP is monotonically increasing in aggregate information acquisition β_{t-1} under the constrained-efficient allocation, because the interaction with the behavioral bias is removed.

The following Corollary provides conditions under which the market and constrained-efficient allocation coincide.

Corollary 1. *(i) symmetric information acquisition in market j β_{jt} goes to zero or infinity.*

(ii) the variance of firm-specific productivity shocks σ_a^2 goes to zero or infinity.

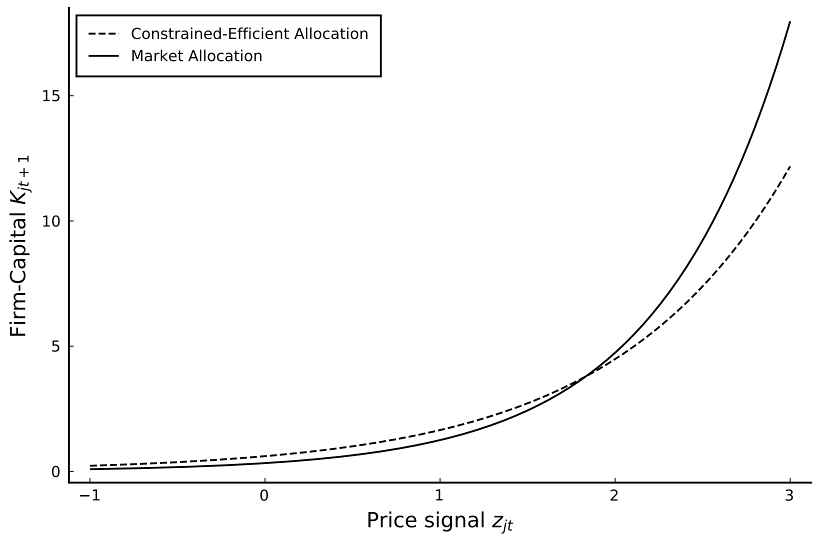

Figure 3: Market allocation of capital K_{jt} as in 25 and the efficient allocation K_{jt}^{eff} as in 28.

(iii) the variance of firm-specific sentiment shocks σ_ε^2 goes to zero.

As Corollary 1 shows, the behavioral bias disappears both when households have perfect information or no information at all ($\beta_{jt} \in \{0, \infty\}$), as in both cases traders put zero weight on their private signal. There is also no distortion if the prior is arbitrarily noisy ($\sigma_a^2 \to \infty$), as in that case both the market and the efficient allocation put full weight on the price signal z_{jt}. If the prior is arbitrarily precise ($\sigma_a^2 \to 0$), the weight is zero for both. Finally, if the variance of sentiment shocks goes to zero, financial markets perfectly aggregate information as the price signal z_{jt} converges to firm productivity a_{jt}.

4.2 Aggregate Investment

Aggregate investment in this economy is in one of two regions. In the first region, traders consume during youth and investment is pinned down by $R_{t+1} = \frac{1}{\delta}$. In the second region, the interest rate is so high ($R_{t+1} > \frac{1}{\delta}$) that traders exhaust their wages for investment. Finally, $R_{t+1} < \frac{1}{\delta}$ cannot arise in equilibrium as investment would collapse to zero and the interest rate R_{t+1} would go to infinity. Taken together, aggregate investment is equal to

$$K_{t+1} = \min\left\{ \left(\alpha\delta A_t^{\alpha_Y} \left(\int_0^1 \tilde{\mathbb{E}}\left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^\theta dj \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\alpha}}, W_t \right\}. \tag{29}$$

Aggregate shocks and information acquisition determine investment in the elastic region. Aggregate productivity and sentiment shocks increase investment as traders expect all firms to be more productive. An increase in aggregate information acquisition β_t has ambivalent effects, as it may increase or decrease TFP A_t and the average expectations of firm productivity $\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta} dj$ may be hump-shaped in β_t.

5 Main Results

As laid out in the prior section, the model has several sources of non-monotonicity. TFP may be locally decreasing in aggregate information acquisition β_t, but also aggregate investment K_{t+1} may be non-monotonic in β_t. These effects are not due to a friction that can easily be removed, but rather arise through imperfect aggregation of information in a market with dispersed information.

Economic intuition tells us that better information usually leads to better economic outcomes. Indeed, the model allows for this intuition to hold by restricting the parameter space. As Corollary 1 shows, the distortion vanishes as the variance of firm-specific sentiment shocks σ_ε^2 goes to zero. Therefore, there must be some threshold $M > 0$, such that whenever σ_ε^2 is below that threshold, TFP and aggregate investment K_{t+1} are increasing in β_t for a neutral stance of sentiment ($\varepsilon_t = 0$). For the following analysis, I assume that this is the case as captured in the following Assumption.

Assumption 2. *The variance of firm-specific sentiment shocks σ_ε^2 is low enough such that*

(i) $\frac{\partial A(a_t,\beta_t)}{\partial \beta_t} > 0$.

(ii) for $\varepsilon_t = 0$: $\frac{\partial K_{t+1}(\beta_t)}{\partial \beta_t} \geq 0$.

5.1 Aggregate Shocks and Information Acquisition

Recent experiences during stock and credit booms have raised concerns about increasing capital misallocation during these episodes (Doerr, 2018; Gopinath et al., 2017; Gorton and Ordoñez, 2020). This model can be used as a laboratory to think about the effects of productivity and sentiment shocks that may drive such episodes and their effect on the incentive to acquire information and thereby allocative efficiency. The following Proposition starts with the effect of aggregate sentiment shocks.

Proposition 4. *There exists a threshold $\bar{\varepsilon}$ such that:*

(i) Information acquisition is increasing in the sentiment shock if $\varepsilon_t < \bar{\varepsilon}$

(ii) Information acquisition is decreasing in the sentiment shock if $\varepsilon_t > \bar{\varepsilon}$

where the threshold $\bar{\varepsilon}$ is negative for $\theta > \frac{1}{1-\alpha}$ and positive for $\theta < \frac{1}{1-\alpha}$.

Proposition 4 shows that the effect of relatively small sentiment shocks ($\varepsilon_t \approx 0$) on information acquisition is ambiguous and depends on the parameters of the model. However, sentiment shocks always crowd-out information acquisition once they are sufficiently large. Moreover, note that aggregate sentiment shocks do not affect price informativeness directly but only through information acquisition.

At first, it may seem surprising that *aggregate* sentiment shocks crowd-out information acquisition, especially as in this model, *firm-specific* sentiment shocks incentivize information acquisition in the first place. This is because knowledge about an *aggregate* sentiment shock changes the incentive to acquire *firm-specific* information. In particular, there are two direct channels through which sentiment shocks affect the incentive to acquire information:

1. Sentiment shocks make valuations more extreme and, therefore, trading less *information-sensitive.* A relatively imprecise yet unbiased signal is sufficient to identify grossly mispriced firms and trade accordingly. Moreover, sentiment shocks make subtle mispricing rarer, for which precise information is helpful as shown in Figure 4. This effects *crowds-out* information acquisition for positive and negative sentiment shocks equally. Moreover, firms with such subtle mispricing must appear relatively unproductive in an otherwise exuberant market and consequently attract less capital as in Figure 5. This *relative size* effect *crowds-out* information acquisition for positive sentiment shocks as learning about smaller firms is unattractive.

2. Aggregate sentiment shocks increase aggregate investment K_{t+1}, which leads to an increase in the *absolute size* of all firms and a rise in the incentive to acquire information.

To further build intuition for this result, I use (25) in (17) to rewrite the marginal benefit of information acquisition evaluated at the symmetric equilibrium,

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \tilde{\mathbb{E}}\left\{ \underbrace{\frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}}\Bigg|_{\beta_{ijt}=\beta_{jt}}}_{Information\text{-}Sensitivity} \underbrace{\left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}}}_{Relative\ Size} \underbrace{K_{t+1}^{\alpha}}_{Absolute\ Size} \left(A_{jt} - \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}\right) \right\}. \tag{30}$$

The *information-sensitivity* channel materializes through the interaction of the change in the buying probability with the distribution of firm-specific sentiment shocks ε_{jt}. In the symmetric equilibrium ($\beta_{ijt} = \beta_{jt}$), traders expect to buy whenever they are more optimistic than the marginal trader, i.e., $s_{ijt} \geq z_{jt} \iff \eta_{ijt} \geq \varepsilon_{jt}$. The resulting probability of buying is $\Phi(-\varepsilon_{jt})$ where $\Phi(\cdot)$ is the standard-normal cdf. Consequently, the derivative of the buying

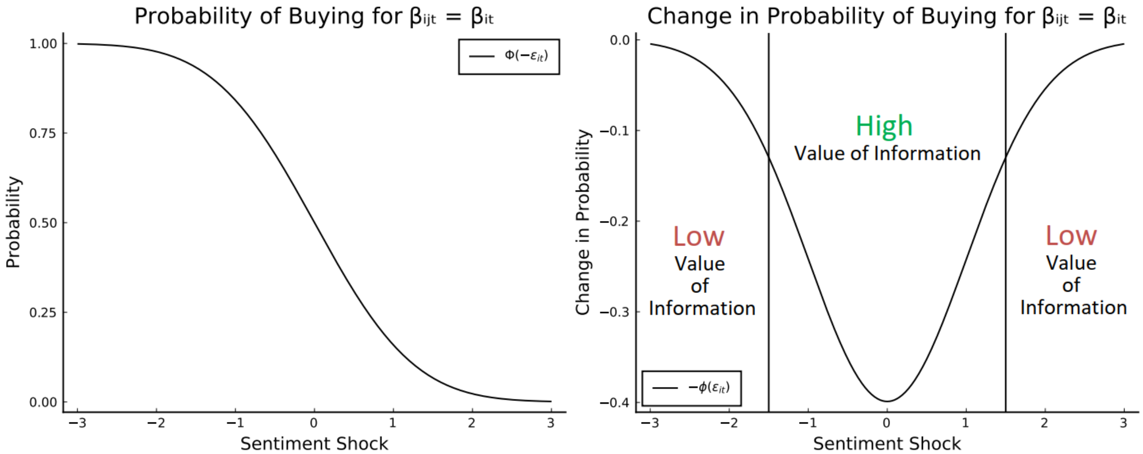

Figure 4: Left panel: probability of buying depending on the realization of the firm-specific sentiment shock ε_{jt}. Right panel: The derivative of the probability of buying. The trading decision is most information-sensitive, i.e., varies most with the realization of the sentiment shock ε_{jt} around zero.

probability with respect to the realization of the firm-specific sentiment shock ε_{jt} is $-\phi(\varepsilon_{jt})$ where $\phi(\cdot)$ is the standard-normal pdf. As shown in Figure 4, the trading decision is most elastic for relatively small realizations of the firm-specific sentiment shock ε_{jt}. However, aggregate sentiment shocks push the distribution of ε_{jt} to the more inelastic regions toward the extremes.

Formally, this effect can be captured by multiplying the change in the buying probability with the distribution of sentiment shocks

$$\phi\left(\varepsilon_{jt}\right) f\left(\varepsilon_{jt}\right) \propto \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\} \tilde{f}(\varepsilon_{jt}), \tag{31}$$

where $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ and $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of ε_{jt} as if its distribution was $\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. The *information-sensitivity* channel is captured by the term $\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}$, which is symmetrically decreasing around zero. Somewhat surprisingly, the decline in *information-sensitivity* does not depend on the actual pass-through of sentiment shocks to expectations. The reason can be found in the trading decision, which neither depends on the actual mispricing caused by sentiment shocks but only on the realization of the firm-specific sentiment shock ε_{jt}. Therefore, aggregate sentiment shocks can discourage information acquisition, even if they do not significantly affect actual prizes.

The additional effect of a decline in information-sensitivity on the *relative size* of firms, for which information remains valuable, is captured by taking expectations of the relative

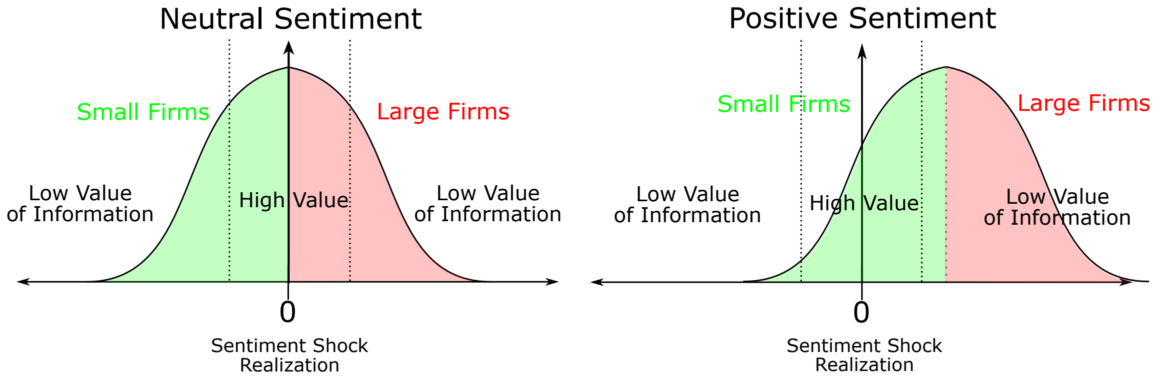

Figure 5: Firms that are fairly priced and for which information is valuable are in the center of the firm-size distribution under neutral sentiment ($\varepsilon_t = 0$). In contrast, for positive sentiment shocks, the same firms are in the left part of the firm-size distribution as they appear to be unproductive relative to other firms.

firm-size $\left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}}$ with the density $\tilde{f}\left(\varepsilon_{jt}\right)$,

$$\int_0^1 \tilde{f}(\varepsilon_{jt}) \left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}} dj \propto \exp\left\{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t\right\}, \tag{32}$$

where $\omega_{s\varepsilon} = \frac{\sqrt{\beta_t}}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$. For a positive sentiment shock, information acquisition becomes effectively directed towards smaller firms, which weakens the incentive to acquire information, which is illustrated in Figure 5 and formally captured by $\exp\left\{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t\right\}$.

The *relative size* effect is increasing in the elasticity of substitution and in the pass-through of aggregate sentiment shocks $\omega_{s\varepsilon}$, which is non-monotonic in information acquisition β_t. If intermediate goods are close substitutes, firms that are perceived as unproductive attract very little capital. Moreover, if aggregate sentiment shocks have a large effect on expectations, these priced firms will be even smaller and information acquisition less attractive.

The *absolute size* effect is captured by changes in aggregate investment. Restricting our attention to shocks for which $K_{t+1} < W_t$ leads to

$$K_{t+1}^{\alpha} \propto \exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{33}$$

As long as traders do not fully invest their wages, the *absolute size* effect can be captured by $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Intuitively, the effect on investment is stronger when α and therefore the returns to scale increase. A further increase in the sentiment shock is ineffectual for the *absolute size* channel once traders fully invest their wages, but incentivizes nonetheless more information acquisition through an increase in the value of resources as captured by $\max\left\{1, R_{t+1}\delta\right\}$ in (P2).

Putting all three effects together allows to write the marginal benefit of information acquisition for a given symmetric information acquisition choice ($\beta_{ijt} = \beta_{jt}$) as

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{ \underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information-Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}} \right\}. \tag{34}$$

For the empirically plausible calibration $\theta - 1 > \frac{\alpha}{1-\alpha}$, positive sentiment shocks always *crowd-out* information, as the increase in aggregate investment is dominated by a larger decrease in size of fairly priced firms. Conversely, negative sentiment shocks initially *crowd-in* information, as fairly priced firms turn out to be relatively large although aggregate investment goes down. Finally, the *information-sensitivity* channel always dominates for large shocks.

Productivity shocks have quite different effects on the incentive to acquire information. Whereas sentiment shocks affect trading in multiple ways, productivity shocks leave the buying decision unaffected. The reason is that traders believe that sentiment shocks affect only other traders, whereas productivity shocks affect all traders. The only channel through which productivity shocks change the incentive to acquire information is through an increase in aggregate investment (*absolute size* channel) and dividends for all firms. This result is captured in the following Proposition.

Proposition 5. *Positive (negative) productivity shocks crowd-in (out) information.*

The model provides a rationale for the different impact of "good" and "bad" booms as in Gorton and Ordoñez (2020). Whereas productivity-driven "good" booms increase information acquisition and improve allocative efficiency, sentiment-driven "bad" booms *crowd-out* information and increase capital misallocation. The results of both Propositions 4 and 5 are pictured in Figure 6.

5.2 Real Feedback

Financial markets do not only react to aggregate shocks, but also shape the economy's response to aggregate shocks. In the following, aggregate shocks hit an economy that is in steady state. Whether shocks amplify or dampen the effect of shocks on output is determined relative to an economy for which the information choice is fixed at the endogenous steady state level β^*.

In the economy with fixed information precision β^* the only effect of aggregate shocks is the direct effect on TFP and investment. Positive shocks of both types increase investment,

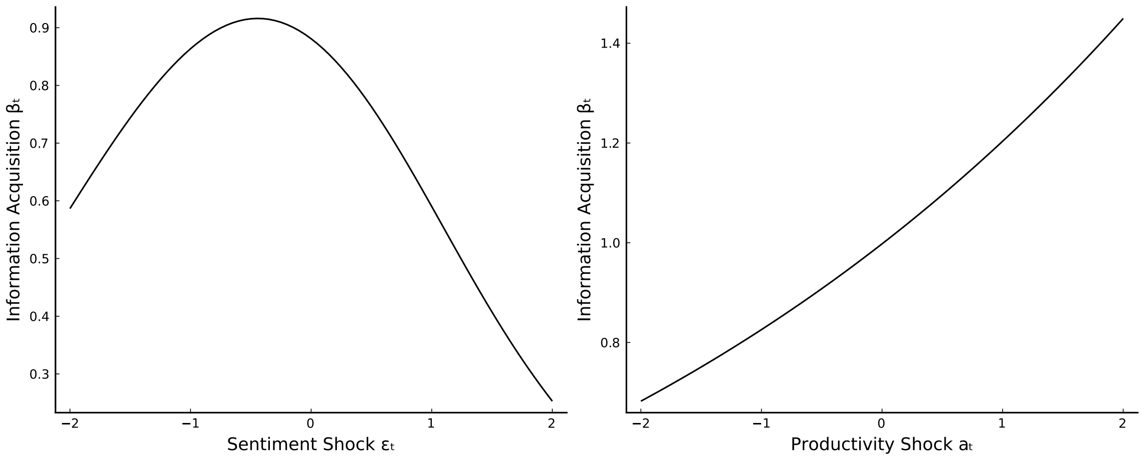

Figure 6: Information acquisition is non-monotonic in the sentiment shock, with the peak $\bar{\varepsilon}$ being negative for $\theta > \frac{1}{1-\alpha}$. Information acquisition is monotonically increasing in the productivity shock.

whereas only productivity shocks also have a direct effect on TFP. The opposite is true for negative shocks, which depress investment and TFP in the case of productivity shocks. Whereas the direct effect of aggregate shocks are straightforward, the indirect effects are more subtle.

There are two indirect effects of sentiment shocks, which lead to a non-monotonic response of the economy. First, sentiment shocks affect the allocative efficiency of financial markets through their effect on information acquisition, which also decreases investment. The cost of misallocation through a decrease in information acquisition depends on the elasticity of substitution between intermediate goods. If the elasticity of substitution is large, misallocation between firms is costly. Moreover, a high elasticity of substitution also lead to a stronger decrease in information acquisition for a positive sentiment shock. In contrast, the costs of misallocating capital are low if the elasticity of substitution is small.

The second effect concerns the pass-through of sentiment shocks. Since traders are unaffected by sentiment if they acquire either no or perfect information ($\beta_t \in \{0, \infty\}$), the effect of a given sentiment shock on beliefs must be maximized for an interior value of information precision. Therefore, a change in information acquisition by traders may increase or decrease the effect of a given sentiment shock on their beliefs, which depends on whether steady state information precision β^* is above or below the threshold $\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$. If β^* is above (below) the threshold, then the effect of aggregate sentiment shocks is locally decreasing (increasing) in information acquisition. For example, a positive sentiment shock crowds-out information acquisition, which leads to an amplification of the shock if the resulting precision choice β^* is still above the threshold $\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$.

The results for the case with $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$ are captured in the following Proposition and are visualized in Figure 7.

Proposition 6. *(i) For $\theta > \frac{1}{1-\alpha}$ and $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$, information acquisition dampens positive sentiment shocks.*

(ii) Large positive sentiment shocks eventually lead to a decrease in aggregate investment if $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$.

The second result of Proposition 6 captures that the costs of misallocation must be eventually so large that they outweigh the investment-stimulating effect of sentiment shocks. Moreover, the direct effect of sentiment shocks vanishes as sentiment shocks grows large, as long as information acquisition declines fast enough. This result is captured in the following Corollary.

Corollary 2. *If information acquisition declines fast enough as sentiment shocks grow large, then aggregate investment approaches its level without information acquisition $\beta_t = 0$ and sentiment shock $\varepsilon_t = 0$. Formally,*

$$\lim_{\varepsilon_t \to \pm\infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0 \Rightarrow \lim_{\varepsilon_t \to \pm\infty} K(\beta_t(\varepsilon_t), \varepsilon_t) = K(0,0).$$

These results may strike at first as counter-intuitive as sufficiently large positive sentiment shocks possibly decrease prices and output, because the decrease information acquisition can outweigh the expansionary effect of sentiment shocks and eventually drive the pass-through of sentiment shocks to zero. Moreover, this section studies only *anticipated* sentiment shocks. If the same shock was unknown prior to the information acquisition decision, positive sentiment shocks would unambiguously increase investment as in the economy with exogenous information precision.

Similar forces are active for negative shocks with the exception that negative sentiment shocks initially crowd-in information acquisition if the elasticity of substitution is large enough ($\theta > \frac{1}{1-\alpha}$). If strong enough, this indirect effect can even lead to negative sentiment shocks being initially expansionary. In contrast, if the elasticity of substitution is relatively small ($\theta < \frac{1}{1-\alpha}$), then negative sentiment shocks always crowd-out information acquisition and are, therefore, initially amplified.

Similar to the previous section, the indirect effect of productivity shocks leads generally to amplification. As follows from Proposition 5, positive productivity shocks crowd-in information acquisition, which leads to an improvement in the allocation of capital and incentivizes additional investment. Therefore, compared to the economy with fixed information precision, the reaction of both output and investment to a productivity shock are larger if information

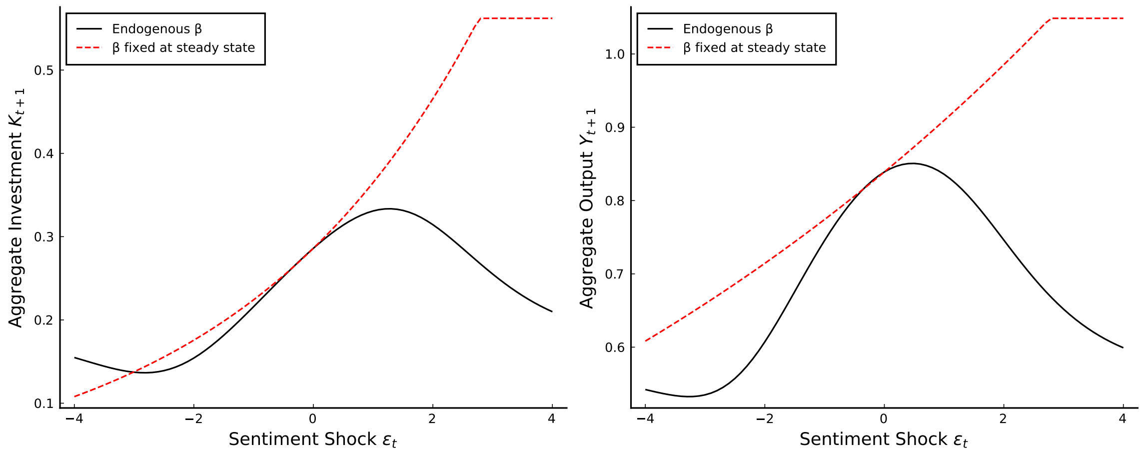

Figure 7: Whether information acquisition dampens or amplifies sentiment shocks depends on the size of the shock and parameters. As information acquisition affects both allocative efficiency and the pass-through of sentiment shocks, large sentiment shocks eventually drive information so low that investment and output decrease.

precision is allowed to adjust, as can be seen in Figure 8. This result is captured in the following Proposition.

Proposition 7. *Productivity shocks are amplified by the endogenous choice of information precision.*

5.2.1 Numerical Illustration

This section provides a numerical illustration of booms driven by productivity and sentiment shocks, focusing on the region of parameters and shocks for which sentiment shocks are expansionary and dampened by information acquisition. To capture the notion of booms, aggregate shocks build up over time according to an auto-regressive process

$$y_t = \begin{cases} \rho y_{t-1} + \zeta & t \in [0, B] \\ 0 & \text{otherwise.} \end{cases},$$

where y_t is the aggregate shock, ζ is a constant innovation and $\rho \in (0, 1)$ is the persistence. After the boom is over, the aggregate shock returns to a neutral stance and remains there.

The expansionary effect of sentiment shocks is dampened as can be seen in Figure 9. Optimistic expectations lead to an increase in investment, but traders decide to cut back on information acquisition, which decreases the allocative efficiency of financial markets. In total, output still increases because the sentiment shock leads to an offsetting increase in investment. In this case, the endogenous response of traders dampens the effect of a positive

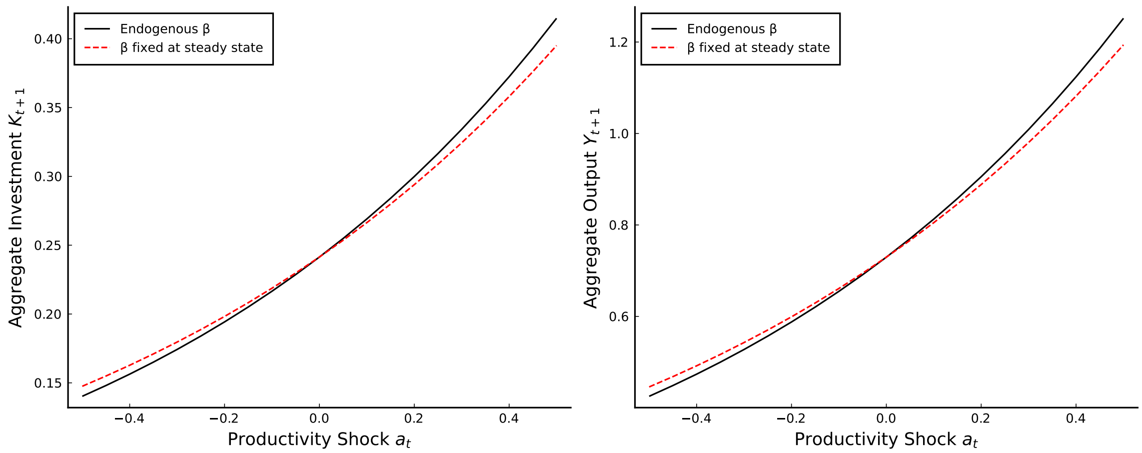

Figure 8: Productivity shocks crowd-in information acquisition, which leads to an additional increase in TFP and Aggregate Investment K_{t+1}. As a result, the effect of productivity shocks is amplified.

sentiment shock.

In contrast, productivity-driven booms are generally amplified by an increase in information acquisition as seen in Figure 10. This mirrors the result from Proposition 8. Expectations of higher productivity tomorrow cause an increase in investment today, which triggers more information acquisition. As a result, the endogenous response of traders amplifies the effect of productivity shocks. Times of high productivity are also times in which financial markets allocate capital efficiently.

6 Is there a Role for Policy?

After studying the positive properties of the model, I turn now to the normative implications. There are two sources of inefficiency in this model. First, traders' overconfidence distorts the allocation of capital between firms as described in section 4.1 and lets aggregate sentiment shocks drive investment. Second, information acquisition is not socially optimal, because traders take share prices as given and do not take into account the positive effect of their information acquisition on TFP. Instead, traders acquire information to maximize their trading profits which amount to transfers between risk-neutral agents. Whether information is above or below its socially optimal level depends on the social value of information which might be above or below its value for speculative purposes.

This section proceeds in the following steps. First, I solve the social planner's problem under perfect information about aggregate states. Next, I consider the optimal intervention if the social planner can only steer the information choice, but not aggregate investment.

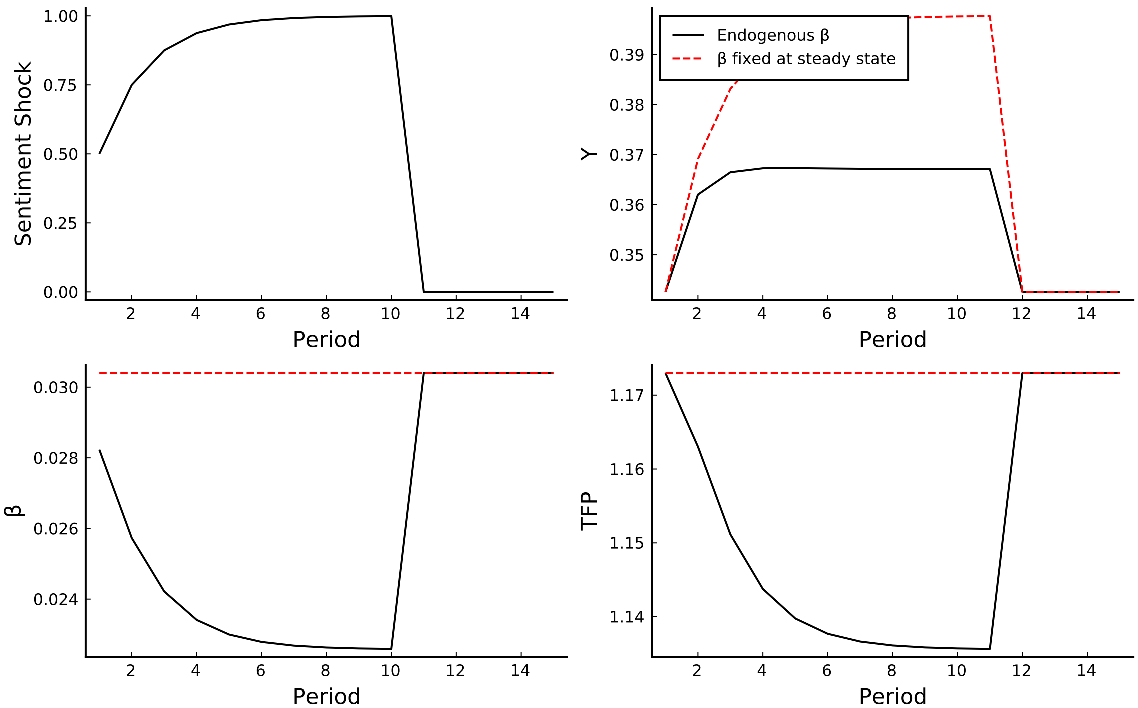

Figure 9: Sentiment-driven booms are dampened by information acquisition.

Afterwards I consider implementations of the social planner's allocation and also cover the effects of asset purchases through the social planner.

6.1 Social Planner's Problem

I abstract from well-known inter-generational trade-offs by using a two-period model for the welfare analysis. Traders are born with an endowment and choose in which firms to invest. Production takes place in the second period and the final good sector combines intermediate goods into the final good without labor,

$$Y_1 = \left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}}. \tag{35}$$

The setup is otherwise identical to the main model.

The social planner chooses consumption, information acquisition and investment in the aggregate and cross-section to maximize social welfare, defined as the sum of utilities of all traders,

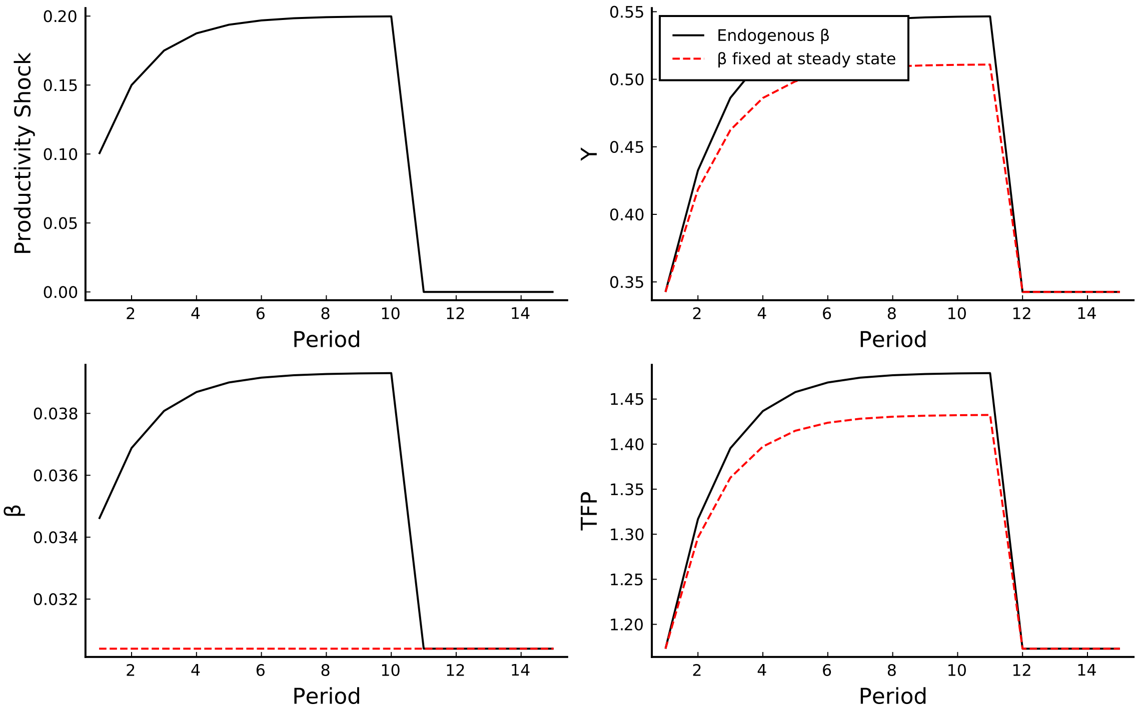

Figure 10: Productivity-driven booms are amplified by the information acquisition.

$$\max_{C_0, C_1, \beta_{j0}, K_{j1}} \quad C_0 + \delta \mathbb{E}_0 \{C_1\} - \int_0^1 IA(\beta_{j0})\, dj \tag{SP1}$$

$$s.t. \quad K_1 = W_0 - C_0 \tag{36}$$

$$C_1 \leq Y_1(\{K_{j1}\}, \{\beta_{j0}\}) \tag{37}$$

$$C_0 \leq W_0 \tag{38}$$

$$C_0, C_1, \beta_{j0}, K_{j1} \geq 0. \tag{39}$$

Constraint (36) states that aggregate capital in period 1 is equal to endowments W_0 minus youth consumption C_0. (37) and (38) are resource constraints. (39) are non-negativity constraints on consumption, information acquisition and capital. The solution to the social planner's problem is given in the following Proposition.

Proposition 8. *The social planner's allocation under perfect information about aggregate shocks* $\{a_0, \varepsilon_0\}$ *is given by* $\{C_0^{SP}, K_{j1}^{SP}, K_1^{SP}, \beta_0^{SP}\}$, *where*

$$K_{j1}^{SP} = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}^{\theta}}{\int_0^1 \mathbb{E}\{A_{j0}|z_{j0}\}^{\theta}\, dj} K_1^{SP} \tag{40}$$

leading to aggregate output

$$Y_1^{SP} = A_0^{SP} \left(K_1^{SP}\right)^{\alpha} \quad \textit{with } A_0^{SP} = \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} di\right)^{\frac{\alpha}{\theta-1}}. \tag{41}$$

The interest rate is

$$R_1^{SP} = \alpha A_0^{SP} \left(K_1^{SP}\right)^{\alpha-1}, \tag{42}$$

which leads to aggregate investment

$$K_1^{SP} = \min\left\{W_0, \left(\alpha\delta A_0^{SP}\right)^{\frac{1}{1-\alpha}}\right\}. \tag{43}$$

The symmetric information acquisition choice is

$$\textit{for all } \beta_{j0} = \beta_0^{SP} : \delta \frac{\partial A_0^{SP}}{\partial \beta_0^{SP}} \left(K_1^{SP}\right)^{\alpha} = \frac{\partial I A_0}{\partial \beta_0^{SP}}. \tag{44}$$

The social planner fixes the two aforementioned inefficiencies. First, the social planner distributes capital optimally by attributing the correct precision to the price signal z_{jt} as in (28) and (40). As a result, ex-ante marginal products of capital are equalized between firms. This reallocation of capital leads to an increase in TFP as in Proposition 3 compared to the competitive allocation. Second, the social planner chooses information acquisition β_0^{SP} to increase TFP instead of speculative profits. Given that the social planner optimally distributes capital between firms as in (40), an increase in β_0^{SP} has always beneficial effects on A_0^{SP}.

6.2 Information Choice

An important difference in the information acquisition decision between the social planner and the market allocation is driven by a mismatch of objectives. Formally, comparing the expressions for the marginal benefit of increasing information precision under the social planner and the market allocation leads to

$$\frac{MB^{SP}\left(\beta_0\right)}{\widetilde{MB}\left(\beta_{ij0}, \beta_{j0}\right)\Big|_{\beta_{ij0}=\beta_{j0}=\beta_0}} = \frac{\delta \frac{\partial A_0}{\partial \beta_0} K_1^{\alpha}}{\max\left\{1, \delta R_1\right\} 2\tilde{\mathbb{E}}_0\left\{\frac{\partial \mathcal{P}\{x_{ij0}=2\}}{\partial \beta_{ij0}}\left(\frac{1}{R_1}\Pi_{j1} - P_{j0}\right)\right\}}, \tag{45}$$

where $MB^{SP}(\cdot)$ is the marginal benefit of increasing β_0 from the perspective of the social planner and $\widetilde{MB}(\cdot)$ is the trader's marginal benefit of increasing β_{ij0}.

The expression above shows that the social planner and traders are driven by different motives in their information acquisition decision. The social planner seeks to improve allocative efficiency through information production, whereas traders take prices as given and ignore this effect. Instead, traders are motivated by a trading motive, as they aim to buy shares that are affected by a negative sentiment shock. Such trading profits are not socially valuable, as they are transfers between risk-neutral traders. Indeed, these two motives can even be in conflict with each other, as an increase in allocative efficiency eventually also leads to a decrease in trading profits. Therefore, the social planner's and the traders' information choice never coincide as captured in the following Proposition.

Proposition 9. *Information acquisition in the decentralized equilibrium is always inefficiently high or low.*

6.2.1 Intervention only through the Information Choice

In the following analysis, the social planner cannot intervene directly in financial markets in response to shocks but only dictates traders how much information to acquire. In this case, the first-order condition for the social planner's choice of β_0 is given by

$$\frac{\partial A_0}{\partial \beta_0} K_1^{\alpha} + \left(\delta \alpha A_0 K_1^{\alpha-1} - 1\right) \frac{\partial K_1}{\partial \beta_0} = \frac{\partial IA\left(\beta_0\right)}{\partial \beta_0}, \tag{46}$$

where aggregate investment is determined as in (29). In comparison to (44), the social planner targets A_0 but also seeks to steer aggregate investment K_1 through her choice of β_0. Generally, an increase in β_0 improves the allocation of capital and incentivizes more investment. Moreover, as explained in section 5.2, the pass-through of sentiment shocks depends on β_0 as well as more precise information reduces the impact of sentiment shocks when β_0 is above $\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$.

The social planner generally increases information acquisition in response to both negative and positive sentiment shocks for two reasons. First, a positive sentiment shock increases investment, which incentivizes more information acquisition. Second, the social planner seeks to stabilize inefficient fluctuations in investment that are caused by sentiment shocks. As a result, this effect is asymmetric for positive and negative shocks, as positive shocks increase investment, whereas negative shocks lower investment. If information acquisition is sufficiently high in the steady state ($\beta^* > \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$), an increase in β_0 can lower the pass-through of sentiment shocks and their effect on investment.

In contrast, the social planner's choice in response to productivity shocks is pro-cyclical. An increase in exogenous productivity a_0 incentivizes information acquisition in two ways. To illustrate the first, note that TFP can be decomposed two parts, $A_0 = A_0\left(a_0\right) A_0\left(\beta_0\right)$, where the first is exogenously driven by a_0 and the second is related to allocative efficiency through β_0. An increase in a_0 therefore amplifies the improvement in the allocative efficiency through

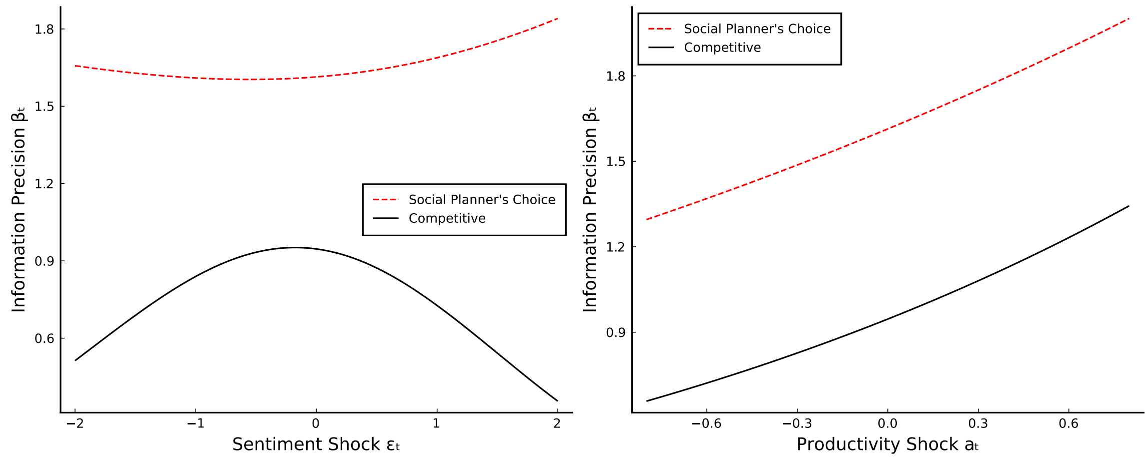

Figure 11: Information acquisition under the social planner and in the competitive economy depending on the realization of aggregate shocks. Parameters are $\sigma_a^2 = 1$, $\sigma_\varepsilon^2 = 0.2$, $\theta = 3$, $\alpha = 0.33$ and the cost function is $IA\left(\beta_{ijt}\right) = a\beta_{ijt}^b$ with $a = 0.01$ and $b = 2$.

an increase in β_0. Second, positive productivity shocks lead to an increase in investment which incentivizes more information acquisition.

The social planner's choice is shown in comparison to the competitive equilibrium in Figure 11. For the chosen parameters, the social planner chooses generally more precise information. Sentiment shocks widen the difference between the social planner's choice and the competitive outcome, whereas productivity shocks leave the gap largely unchanged.

Whether the social planner chooses higher or lower information acquisition than the competitive equilibrium depends on the variances of sentiment and productivity shocks. As can be seen in Figure 12, the social planner's choice of β_0 is initially increasing in the variance of sentiment shocks σ_ε^2 as the social planner counters a decrease in TFP. Once σ_ε^2 exceeds a certain level, the social planner chooses not to acquire information ($\beta_0 = 0$). This discontinuity is due to the price distortion explained in section 4.1 and the resulting non-monotonicity of TFP as in Figure 2. The distortion becomes so severe that distributing capital equally between firms and saving on information acquisition costs is preferable to misallocating capital through the financial market.

In contrast, the social planner's choice is monotonically increasing in the variance of productivity shocks σ_a^2 as more firm-heterogeneity increases the benefits of allocating capital efficiently. When σ_a^2 is low relative to the variance of sentiment shocks σ_ε^2, the social planner's choice for information precision may be below the market outcome. In other words, the speculative motive may incentivize traders to acquire excessively precise information.

If capital is distributed efficiently between firms according to 40, then there is no discontinuity for the social planner's choice of β_0 as the variance of sentiment shocks σ_ε^2 grows

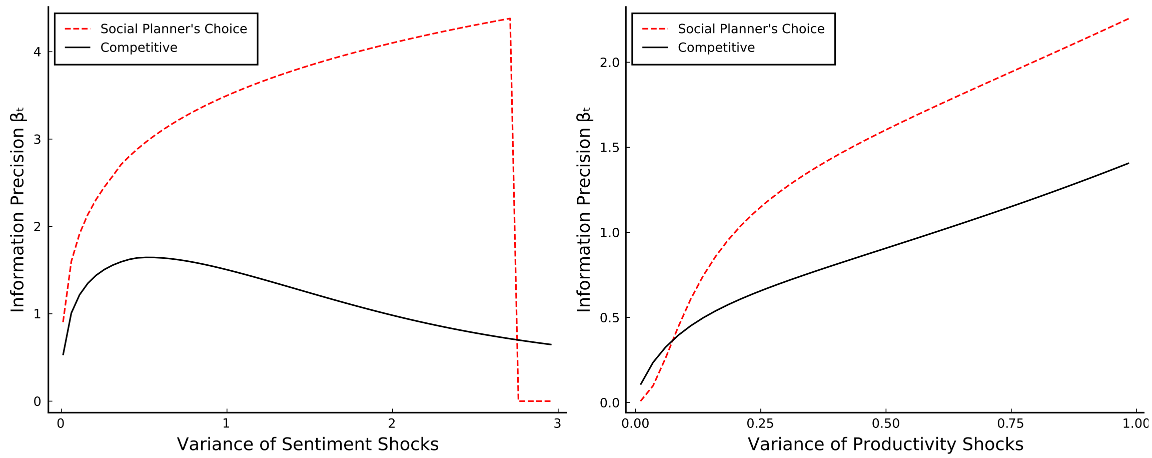

Figure 12: Information acquisition under the social planner and in the competitive economy depending on variances of sentiment and productivity shocks. Parameters are $\sigma_a^2 = 1$ in the left plot and $\sigma_\varepsilon^2 = 0.2$ in the right plot, $\theta = 3$, $\alpha = 0.33$ and the cost function is $IA(\beta_{ijt}) = a\beta_{ijt}^b$ with $a = 0.01$ and $b = 2$.

large as in Figure 13. Now, the planner's choice is continuous and clearly hump-shaped in σ_ε^2 as noisier markets eventually decrease the marginal benefit of increasing β_0. As before, the social planner's choice is lower than the competitive outcome whenever σ_a^2 is low relative to σ_ε^2.

6.3 Implementation

In this section I investigate how the social planner can implement the centralized allocation through the use of taxes and subsidies. Net proceeds and costs of taxes and subsidies are distributed lump-sum between old traders.

The social planner can apply a tax/subsidy on dividend income to achieve the constrained-efficient allocation of capital. Under this state-dependent tax/subsidy, traders receive

$$\Pi_{j1}^{DE} = \tau^{Bias}(z_{j0})\,\Pi_{j1}, \quad \text{where } \tau^{Bias}(z_{j0}) = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}}{\tilde{\mathbb{E}}\{A_{j0}|x_{ij0} = z_{j0}, z_{j0}\}}. \tag{47}$$

As seen in Figure 3, $\tau^{Bias}(z_{j0})$ is a subsidy on dividends whenever $K_{j1}^{eff} < K_{j1}$. If the social planner has information about aggregate shocks, the tax/subsidy corrects also for aggregate sentiment shocks through the marginal trader's expectations $\tilde{\mathbb{E}}\{A_{j0}|x_{ij0} = z_{j0}, z_{j0}\}$. A tax (subsidy) can lower (increase) investment in response to a positive (negative) sentiment shock.

Moreover, a tax/subsidy $\tau^{Info}(\beta_{ij0})$ on information acquisition is sufficient to induce the

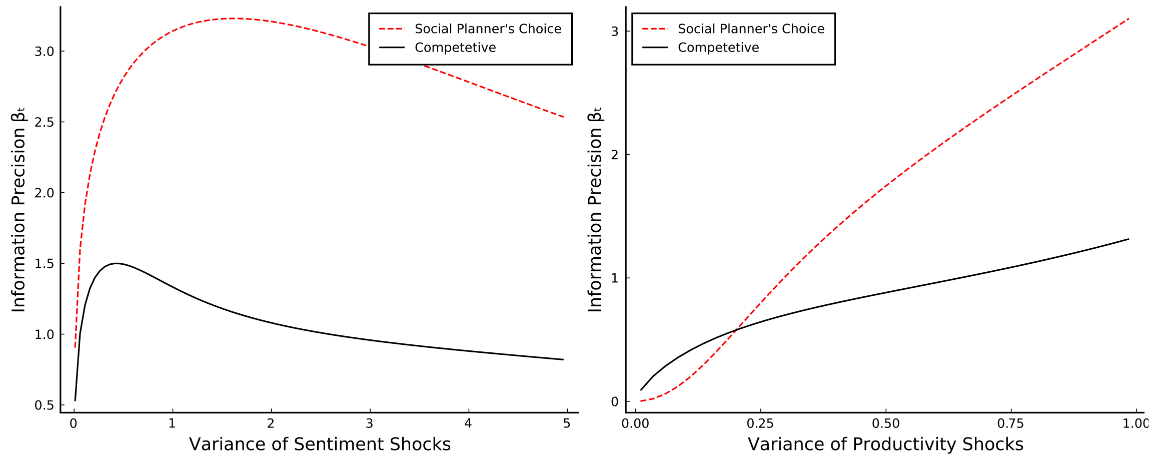

Figure 13: Information acquisition under the social planner and in the competitive economy depending on variances of sentiment and productivity shocks. Parameters are $\sigma_a^2 = 1$ in the left plot and $\sigma_\varepsilon^2 = 1$ in the right plot, $\theta = 3$, $\alpha = 0.33$ and the cost function is $IA(\beta_{ijt}) = a\beta_{ijt}^b$ with $a = 0.01$ and $b = 2$.

socially optimal level,

$$\frac{\partial IA^{DE}(\beta_{ij0})}{\partial \beta_{ij0}} = \tau^{Info}(\beta_{ij0}) \frac{\partial IA(\beta_{ij0})}{\partial \beta_{ij0}}, \quad \tau^{Info}(\beta_{ij0}) = \frac{\widetilde{MB}(\beta_{ij0}, \beta_{j0})\Big|_{\beta_{ij0}=\beta_{j0}}}{\delta \frac{\partial A_0}{\partial \beta_0} K_1^\alpha}. \tag{48}$$

Applying the after-tax marginal cost leads directly to the first order condition as in (44). The results are summarized in the following Proposition.

Proposition 10. *The social planner allocation* $\left\{K_1^{SP}, K_{j1}^{SP}, \beta_0^{SP}\right\}$ *can be implemented through taxes/subsidies (47) and (48).*

6.4 Alternative Instruments

In the following, I investigate instruments that are more common in recent policy discussions, such as transaction taxes and asset purchases.

6.4.1 Transaction Taxes

Since Tobin (1972), financial transaction taxes have been discussed with the objective of reducing volatility by making short-term speculation less profitable. This analysis is inapplicable here as assets are short-lived and only traded once. Nonetheless, a transaction tax can be used to drive a wedge between how much traders pay for shares and how much is invested in capital. The following Proposition shows how such a transaction tax can be used to stabilize investment against sentiment shocks and reallocate capital across firms.

Corollary 3. *(i) Aggregate investment can be stabilized with respect to sentiment shocks through a transaction tax,*

$$K_{j1}^{DE} = \tau^{Trans}(\varepsilon_0) P_{j0}, \quad \tau^{Trans}(\varepsilon_0) = \exp\{-\omega_{s\varepsilon}\varepsilon_0\}, \quad \omega_{s\varepsilon} = \frac{\sqrt{\beta_0}}{\sigma_a^{-2} + \beta_0 (1 + \sigma_\varepsilon^{-2})}. \tag{49}$$

(ii) The dividend tax/subsidy (47) can be substituted by a state-dependent transaction tax,

$$K_{j1}^{DE} = \tau^{Trans}(P_{j0}) P_{j0}, \quad \tau^{Trans}(P_{j0}) = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}}{\tilde{\mathbb{E}}\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\}}. \tag{50}$$

6.4.2 Shaping Incentives for Information Acquisition

Traders are taking a gamble when they decide to invest in a given asset. The social planner can incentivize (discourage) information acquisition by increasing (decreasing) the stakes for each trade. This idea can be implemented through a redistribution of dividends between over- and underperforming firms as shown in the following Corollary.

Corollary 4. *A state-dependent tax/subsidy $\tau(a_{jt}, z_{jt})$ on dividends with the properties,*
(i) No price distortions: $\tilde{\mathbb{E}}\{\tau(a_{j0}, z_{j0}) \Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\} = \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\}$
(ii) Monotonicity of beliefs: $\frac{\partial \tilde{\mathbb{E}}\{\tau(a_{j0}, z_{j0})\Pi_{j1}|s_{ij0}, z_{j0}\}}{\partial s_{ij0}} > 0$
(iii) Monotonicity of prices: $\frac{\partial \tilde{\mathbb{E}}\{\tau(a_{j0}, z_{j0})\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\}}{\partial z_{j0}} > 0$
increases (decreases) $\widetilde{MB}(\beta_{ij0}, \beta_{j0})$ *when*

$$\tau(a_{j0}, z_{j0}) \geq (\leq) 1 \iff \Pi_{j1} \geq \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\} \text{ and } \frac{\partial \mathcal{P}\{x_{ij0} = 2|a_{j0}, \varepsilon_{j0}, \beta_{ij0}, \beta_{j0}\}}{\partial \beta_{ij0}} \geq 0$$

$$\tau(a_{j0}, z_{j0}) \leq (\geq) 1 \iff \Pi_{j1} \leq \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\} \text{ and } \frac{\partial \mathcal{P}\{x_{ij0} = 2|a_{j0}, \varepsilon_{j0}, \beta_{ij0}, \beta_{j0}\}}{\partial \beta_{ij0}} \leq 0$$

and there exists for all z_{j0} some a_{j0} for which the inequalities are strict.

Intuitively, the social planner can make assets more or less risky by taxing/subsidizing dividends depending on realized productivity and market expectations. For example, subsidizing dividend payments of over-performing firms and taxing under-performing firms makes any investment riskier and information acquisition more attractive. To avoid distorting prices, subsidies and taxes must offset each other in expectations.

As an illustration, the following combination of a tax $\tau(a_{j0}, z_{j0})$ and a lump-sum transfer

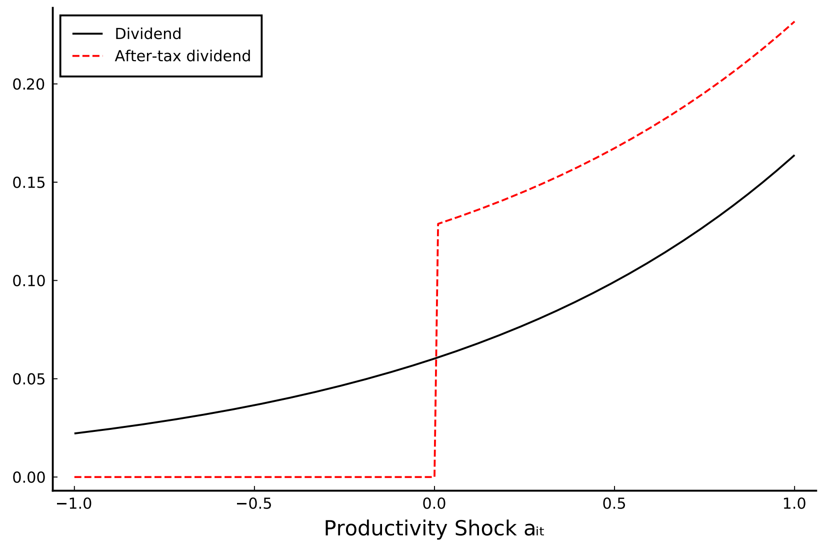

Figure 14: The tax schedule incentivizes information acquisition.

$T(a_{j0}, z_{j0})$ encourage information acquisition,

$$\tau(a_{j0}, z_{j0}) = \begin{cases} 0 & a_{j0} < z_{j0} \\ 1 & a_{j0} \geq z_{j0} \end{cases}$$

$$T(a_{j0}, z_{j0}) = \begin{cases} 0 & a_{j0} < z_{j0} \\ \tilde{\mathbb{E}}\left\{\Pi_{j1} | a_{j0} < z_{j0}, s_{ij0} = z_{j0}, z_{j0}\right\} & a_{j0} \geq z_{j0} \end{cases}$$

where I assumed $a_0 = -\frac{\sigma_a^2}{2}$ as a normalization. In words, the tax is confiscatory if the realization of the productivity shock a_{j0} is below the price signal z_{j0}, i.e., the firm disappoints market expectations. The expected tax revenue from the perspective of the marginal trader are transferred to buyers if the realization of a_{j0} is above z_{j0}, i.e., the firm exceeds market expectations. A tax schedule that incentivizes information acquisition, therefore, increases both the potential down- and upside of any trade. The before- and after-tax dividend schedule is shown in Figure 14 for the case with $z_{j0} = 0$.

Information acquisition can be discouraged by flattening the dividend function instead. A straightforward and common implementation is through a progressive dividend tax in combination with the deduction of losses from realized gains, effectively offsetting part of the incurred losses by reducing the tax owed. In the model, the social planner can completely crowd-out information acquisition by buying up all shares and selling shares that are a stake in aggregate output. As there is no aggregate uncertainty, such shares would pay a deterministic dividend and traders do not acquire information.

6.4.3 Asset Purchases

In the last decade, central banks have repeatedly used asset purchases to stabilize financial markets, spur growth, and inflation (see Gagnon, Sack, et al., 2018, for a brief overview). These interventions were accompanied by concerns that asset purchases might harm market efficiency and lead to an increase in capital misallocation (e.g., Fernandez, Bortz, and Zeolla, 2018). Although this model is too stylized to give a full assessment of asset purchases, it can be used to shed light on the effect of asset purchases on information acquisition in financial markets.

In this model, asset purchases have real effects by exploiting that information is dispersed between traders and can discourage information acquisition. The mechanism works as follows. Asset purchases lead to an upward shift in the identity of the marginal trader, who becomes more optimistic, and consequently, asset prices increase. Additionally, asset purchases have an effect on information acquisition if they are announced. Then, the resulting reduction in the asset supply discourages information acquisition, similar to a positive sentiment shock. Therefore, this model can provide a rationale for the concerns about asset purchases and a possible decline in market efficiency.

However, the equivalence of asset purchases and sentiment shocks shows that asset purchases can be used to offset negative sentiment shocks. When a sufficiently large negative sentiment shock hits the economy, traders anticipate that prices will be depressed, which discourages information acquisition as trading becomes less information-sensitive. The central bank can offset the downward bias on asset prices through the negative sentiment shock by purchasing assets. This counter-measure can lead to unbiased prices, which restores the incentive to acquire information for traders. This logic is captured in the following Proposition and is visualized in Figure 15.

Proposition 11. *Let the social planner acquire* $d^{SP} \in (-1, 1)$ *units of assets, such that* $1 - d^{SP}$ *shares are left for traders. Then,*

(i) asset purchases ($d^{SP} > 0$*) undo negative sentiment shocks both in terms of investment and information acquisition.*

(ii) asset sales ($d^{SP} < 0$*) undo positive sentiment shocks both in terms of investment and information acquisition.*

In other words, asset purchases and sales can *increase* market efficiency by countering sentiment shocks. The point about asset sales is relevant for central banks in deciding when to start shrinking the size of their balance sheets. Central banks can avoid the adverse effects of asset sales by waiting until sentiment has reached a more neutral level. A reduction in asset holdings can then even increase market efficiency.

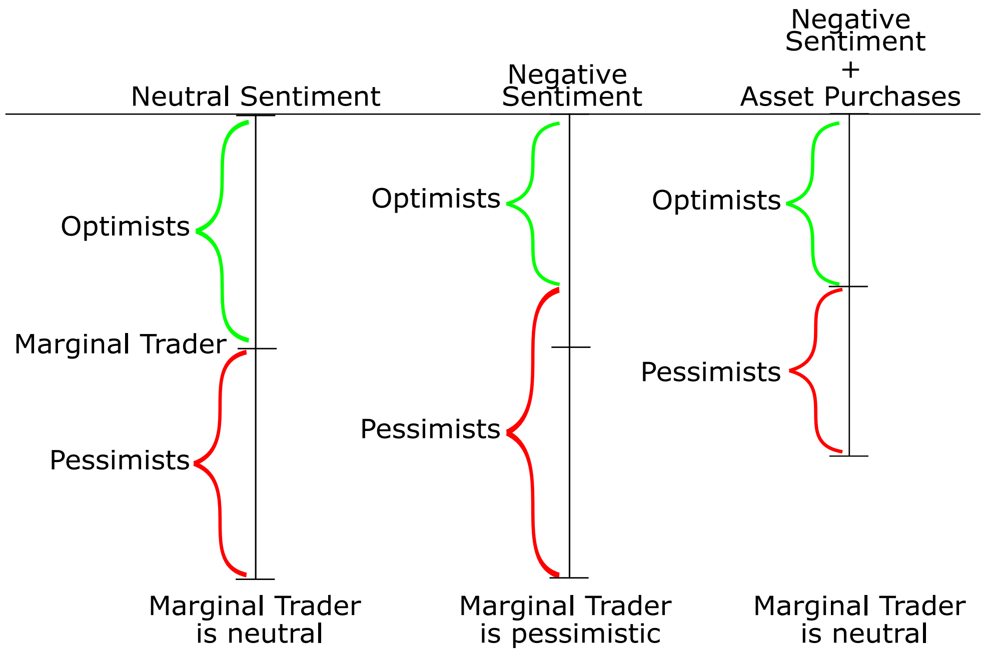

Figure 15: Asset purchases counter negative sentiment shocks.

7 Discussion

7.1 Uncertainty

The analysis so far assumed that traders observed aggregate states perfectly before they decided on information precision. This assumption is not crucial for the results, which also hold when traders have only imperfect information about aggregate states when they take their information acquisition decision.

The simplest setting to think about the effects of uncertainty is to reveal aggregate shocks after the information acquisition decision but before trading. Furthermore, assume that aggregate productivity and sentiment shocks are auto-correlated.[17] Then, the law of motions for aggregate shocks are given by

$$a_t = \rho_a a_{t-1} + \xi_t^a \tag{51}$$

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \xi_t^\varepsilon \tag{52}$$

where $\rho_a \in (0,1)$ and $\rho_\varepsilon \in (0,1)$ capture the persistence of aggregate shocks and $\xi_t^a \sim \mathcal{N}\left(0, \sigma_{\xi a}^2\right)$ and $\xi_t^\varepsilon \sim \mathcal{N}\left(0, \sigma_{\xi\varepsilon}^2\right)$ are the corresponding innovations. Traders are able to

[17] An alternative would be not to reveal aggregate shocks before trading takes place. In this setting traders learn from private and public signals also about aggregate states. Similarly, the social planner can use publicly available information to guide her interventions. The insights are broadly the same as in the case when aggregate shocks are revealed after the information acquisition decision.

learn about past aggregate states by observing past aggregate investment K_t and output Y_t. Whereas K_t is moved by both productivity and sentiment, output Y_t reacts only to productivity after controlling for K_t^α. For example, if investment was high but output disappointing, investment must have been driven by a positive sentiment shock. The prior for traders about aggregate states is then given by

$$a_t|a_{t-1} \sim \mathcal{N}\left(\rho_a a_{t-1}, \sigma_{\xi a}^2\right) \tag{53}$$

$$\varepsilon_t|\varepsilon_{t-1} \sim \mathcal{N}\left(\rho_\varepsilon \varepsilon_{t-1}, \sigma_{\xi\varepsilon}^2\right). \tag{54}$$

In this setting, past sentiment shocks generate expectations about future sentiment shocks. The analysis of Proposition 4 still applies, as traders evaluate the value of information for different realizations of the sentiment shock ε_t.

The social planner analysis is not substantially changed under aggregate uncertainty if the social planner has to decide on using policy instruments before the aggregate shocks are revealed. Indeed, uncertainty is irrelevant for interventions that offset the effect of sentiment shocks on information acquisition, as only *anticipated* sentiment shocks can affect information acquisition. Therefore, the optimal tax or subsidy does not depend on the realization of the sentiment shock, but only on the traders' prior.

The social planner has to weigh the costs and benefits of taxes or subsidies on investment in different states, as aggregate investment depends on the realization of the sentiment shock. Information acquisition has an additional role in this setting as it can play an insulating role against sentiment shocks. If traders have sufficiently precise signals, then the effect of sentiment shocks is diminished. This is especially attractive as the social planner is uncertain about the concrete realization of sentiment shocks.

7.2 Information Structure

The model presented here assumes that traders wrongly believe that their private signal is free of correlated noise, i.e., traders are overconfident about the informativeness of their private signal. Whereas it is empirically reasonable to assume that behavioral biases play a role in financial markets, the main reason for this assumption is tractability and introducing noise in asset prices to motivate information acquisition.[18] The standard assumption in the literature is to split the population of traders in to rational arbitrageurs and non-optimizing noise traders. I do not follows this approach as it is difficult to reconcile with general equilibrium.

In the following I walk through different assumptions for the information structure and

[18] See Grossman and Stiglitz (1980) for why noise in asset markets is necessary to incentivize information acquisition.

their relationship to information aggregation and acquisition.

7.2.1 Exogenous Public Signals

The simplest case is one in which traders do not have private signals but instead observe public signals of the form $z_{jt} = a_{it+1} + \varepsilon_{jt}/\sqrt{\beta_{jt}}$. This setting mirrors the allocation in Proposition 3. However, it has nothing to say about the origin of the signal. How does it come about, and what determines its precision?

7.2.2 Heterogeneous Private Signals

To say something how information is aggregated, endow traders with heterogeneous private signals,

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt} + \varepsilon_{jt}}{\sqrt{\beta_{jt}}}.$$

Rationality Observing the asset price is isomorphic to observing $z_{jt} = \int s_{ijt} dj$. Therefore, the information set of rational traders upon observing the price is $\{z_{jt}\}$, since the private signal becomes uninformative after observing z_{jt}. Yet, setting up this equilibrium required that traders use their private signal for their buying decision. Since traders are always indifferent between buying or not, the indifference can be broken in favor of buying whenever $s_{ijt} \geq z_{jt}$.[19] The resulting allocations are the same as with an exogenous public signal.

The main drawback of this approach is that it rules out costly information acquisition. If prices are already fully revealing, why pay for a non-informative private signal? However, if no one pays for information, prices cannot be informative, and paying for a private signal becomes profitable again. Therefore, an equilibrium with costly information acquisition and without noise does not exist, which is called the Grossman-Stiglitz paradox (Grossman and Stiglitz, 1980).

Overconfidence To overcome this problem, assume that traders think that their private signal does not contain correlated noise. Therefore, they do not discard it after observing the price signal z_{jt}. The posterior of trader ij is

$$a_{jt}|s_{ijt}, z_{jt} \sim \mathcal{N}\left(\frac{\beta_{ijt}s_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}z_{jt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}\right).$$

It follows that traders have posteriors that are too precise, as they think that their private signal remains informative after observing z_{jt}. This misperception motivates traders to invest in their private signal with the anticipation of trading profits. Finally, this bias leads to an

[19]This class of equilibria is referred to as "fully revealing rational expectations equilibrium" and they are studied in Grossman (1976).

overreaction of prices to the price signal z_{jt} as described in section 4.1. This price distortion appears also in rational models in which traders learn from prices and have heterogeneous private signals.[20] The main focus of this paper, however, is not on the price distortion, but rather time-varying price-informativeness and allocational efficiency of financial markets.

8 Conclusion

I develop a novel framework that allows me to study aggregate shocks and information acquisition in financial markets embedded in a neoclassical growth model. In such a model, total factor productivity has an endogenous component that depends on the decentralized information acquisition by traders. When asset prices are more informative, more capital is allocated to the most productive firms and total factor productivity increases. I prove that whereas the endogenous response of traders amplifies productivity shocks, sentiment shocks can have ambiguous effects on information acquisition. Sentiment driven booms (exuberance) crowd-out information acquisition, as trading becomes more one-sided and less information-sensitive. The decrease in information worsens the allocative efficiency of financial markets. Negative sentiment shocks, however, initially crowd-in information, but eventually also crowd-out and worsen deep depressions.

The optimal policy is to lean against sentiment and therefore subsidize/tax dividend income or transactions when asset markets are depressed/exuberant. Asset purchasing programs stand out as a policy instrument as they can address both investment and information acquisition. Reducing the supply of assets changes the identity of the marginal trader and biases prices upward. This effect can cancel out the downward bias on prices through a negative sentiment shock, which also reinstates incentives to acquire precise information. These interventions, however, require that the social planner can differentiate between productivity and sentiment shocks. Asset purchases can also be a source of distortion when used in response to a negative productivity shock, which discourages information acquisition and worsens financial markets' allocative efficiency. Therefore, the social planner's ability to intervene successfully depends crucially on her knowledge of the aggregate state of the economy.

Finally, information acquisition is always at an inefficient level in the decentralized from a normative perspective. Traders acquire information to increase their trading profits, which are a wash from the social planner's perspective. In contrast, the social planner acquires information to improve the allocative efficiency, which traders ignore as they take prices as given. This mismatch of motives leads to a situation in which traders and the social

[20] For a more detailed discussion, see Albagli, Hellwig, and Tsyvinski (2011a, 2015).

planner's information choice never coincide. Generally, information acquisition is too low in the decentralized equilibrium when firms are heterogeneous and financial markets aggregate information efficiently.

References

Albagli, Elias, Christian Hellwig, and Aleh Tsyvinski (2011a). *A Theory of Asset Prices based on Heterogeneous Information*. NBER Working Paper No. 17548. National Bureau of Economic Research.

— (2011b). *Information Aggregation, Investment, and Managerial Incentives*. Tech. rep. National Bureau of Economic Research.

— (2015). *A Theory of Asset Prices Based on Heterogeneous Information*. Working Paper.

— (2017). *Imperfect Financial Markets and Shareholder Incentives in Partial and General Equilibrium*. NBER Working Paper No. 23419. National Bureau of Economic Research.

Ambrocio, Gene (2019). "Rational exuberance booms". *Review of Economic Dynamics* 35, pp. 263–282.

Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan (2010). *Beauty Contests and Irrational Exuberance: A Neoclassical Approach*. NBER Working Paper No. 15883. National Bureau of Economic Research.

Asriyan, Vladimir (2020). *Balance Sheet Channel with Information-Trading Frictions in Secondary Markets*. Working Paper. CREi.

Asriyan, Vladimir, Luc Laeven, and Alberto Martin (2019). "Collateral Booms and Information Depletion". Barcelona GSE Working Paper nº 1064.

Bai, Jennie, Thomas Philippon, and Alexi Savov (2016). "Have financial markets become more informative?" *Journal of Financial Economics* 122, pp. 625–654.

Basu, Susanto, John G Fernald, and Miles S Kimball (2006). "Are Technology Improvements Contractionary?" *American Economic Review* 96.5, pp. 1418–1448.

Bennett, Benjamin, René Stulz, and Zexi Wang (2020). "Does the stock market make firms more productive?" *Journal of Financial Economics* 136, pp. 281–306.

Bernanke, Ben and Mark Gertler (1989). "Agency Costs, Net Worth, and Business Fluctuations". *American Economic Review* 79.1, pp. 14–31.

Blanchard, Olivier and Jordi Galí (2010). "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment". *American Economic Journal: Macroeconomics* 2.2, pp. 1–30.

Bond, Philip, Alex Edmans, and Itay Goldstein (2012). "The Real Effects of Financial Markets". *Annual Review of Financial Economics* 4, pp. 339–360.

Brandts, Jordi, Ayça Ebru Giritligil, and Roberto A. Weber (2015). "An experimental study of persuasion bias and social influence in networks". *European Economic Review*.

Chandrasekhar, Arun G, Horacio Larreguy, and Juan Pablo Xandri (2012). *Testing Models of Social Learning on Networks: Evidence From a Framed Field Experiment*. Tech. rep.

Chousakos, Kyriakos, Gary Gorton, and Guillermo Ordonez (2020). *The Macroprudential Role of Stock Markets*. Tech. rep. UPenn mimeo.

Daniel, Kent and David Hirshleifer (Sept. 2015). “Overconfident investors, predictable returns, and excessive trading”. *Journal of Economic Perspectives*. Vol. 29. 4. American Economic Association, pp. 61–88.

David, Joel M, Hugo A Hopenhayn, and Venky Venkateswaran (2016). “Information, Misallocation, and Aggregate Productivity”. *Quarterly Journal of Economics* 131.2, pp. 943–1005.

Dávila, Eduardo and Cecilia Parlatore (2020). *Identifying Price Informativeness*. Working Paper No. 25210. National Bureau of Economic Research.

Doerr, Sebastian (2018). “Collateral, Reallocation, and Aggregate Productivity: Evidence from the U.S. Housing Boom”. *SSRN Electronic Journal*, pp. 1–66.

Dow, James, Itay Goldstein, and Alexander Guembel (2017). “Incentives for Information Production in Markets where Prices Affect Real Investment”. *Journal of the European Economic Association* 15.4, pp. 877–909.

Dow, James and Gary Gorton (1997). “Stock Market Eefficiency and Economic Efficiency: is there a Connection?” *Journal of Finance* 52.3, pp. 1087–1129.

Durnev, Art, Randall Morck, and Bernard Yeung (Feb. 2004). “Value-Enhancing Capital Budgeting and Firm-specific Stock Return Variation”. 59.1, pp. 65–105.

Enke, Benjamin and Florian Zimmermann (2019). “Correlation Neglect in Belief Formation”. *Review of Economic Studies* 86.1, pp. 313–332.

Eyster, Erik, Georg Weizsäcker, H U Berlin, Klaus M Schmidt, and Matthew Rabin (2018). *An Experiment On Social Mislearning*. Tech. rep.

Fajgelbaum, Pablo D, Edouard Schaal, and Mathieu Taschereau-Dumouchel (2017). “Uncertainty traps”. *Quarterly Journal of Economics* 132.4, pp. 1641–1692.

Farboodi, Maryam and Péter Kondor (2019). “Rational Sentiments and Economic Cycles”.

Farboodi, Maryam, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran (2020). *Where Has All the Data Gone?* Tech. rep. National Bureau of Economic Research.

Fernandez, Rodrigo, Pablo Bortz, and Nicolas Zeolla (2018). *The Politics of Quantitative Easing*. Tech. rep. The Centre for Research on Multinational Corporations.

Fostel, Ana and John Geanakoplos (2012). “Tranching, CDS, and asset prices: How financial innovation can cause bubbles and crashes”. *American Economic Journal: Macroeconomics* 4.1, pp. 190–225.

Gagnon, Joseph E, Brian Sack, et al. (2018). *QE: A User's Guide*. Tech. rep. Peterson Institute for International Economics.

García-Santana, Manuel, Enrique Moral-Benito, Josep Pijoan-Mas, and Roberto Ramos (2020). "Growing Like Spain: 1995–2007". *International Economic Review* 61.1, pp. 383–416.

Glaser, Markus and Martin Weber (2010). "Overconfidence". *Behavioral finance: Investors, corporations, and markets*, pp. 241–258.

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez (2017). "Capital Allocation and Productivity in South Europe". *Quarterly Journal of Economics* 132.4, pp. 1915–1967.

Gorton, Gary and Guillermo Ordonez (2014). "Collateral Crises". *American Economic Review* 104.2, pp. 343–378.

Gorton, Gary and Guillermo Ordoñez (2020). "Good Booms, Bad Booms". *Journal of the European Economic Association* 18.2, pp. 618–665.

Grimm, Veronika and Friederike Mengel (Oct. 2018). "Experiments on Belief Formation in Networks". *Journal of the European Economic Association*.

Grossman, Sanford (1976). "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information Author". *Journal of Finance* 31.2.

Grossman, Sanford and Joseph Stiglitz (1980). "On the impossibility of informationally efficient markets". *American Economic Review* 70.3, pp. 393–408.

Hayek, Friedrich August (1945). "The use of knowledge in society". *American Economic Review* 35.4, pp. 519–530.

Holmström, Bengt and Jean Tirole (1993). "Market liquidity and performance monitoring". *Journal of Political Economy* 101.4, pp. 678–709.

Jordà, Òscar, Moritz Schularick, and Alan M Taylor (2010). *Financial Crises, Credit Booms, and External Imbalances: 140 Years of Lessons*. NBER Working Paper No. 16567. National Bureau of Economic Research.

Keys, Benjamin J, Tanmoy Mukherjee, Amit Seru, and Vikrant Vig (2010). "Did securitization lead to lax screening? Evidence from subprime loans". *Quarterly Journal of Economics* 125.1, pp. 307–362.

Kindleberger, Charles P and Robert Z Aliber (2011). *Manias, panics and crashes: a history of financial crises*. Palgrave Macmillan.

Kiyotaki, Nobuhiro and John Moore (1997). "Credit Cycles". *Journal of Political Economy* 105.2, pp. 211–248.

Krishnamurthy, Arvind and Tyler Muir (2017). *How Credit Cycles across a Financial Crisis*. Tech. rep. 23850. National Bureau of Economic Research.

Kyle, Albert S (1985). "Continuous Auctions and Insider Trading". *Econometrica*, pp. 1315–1335.

Martin, Alberto and Jaume Ventura (2018). “The Macroeconomics of Rational Bubbles: A User’s Guide”. *Annual Review of Economics* 10, pp. 505–539.

Morck, Randall, Bernard Yeung, and Wayne Yu (2000). “The information content of stock markets: Why do emerging markets have synchronous stock price movements?” *Journal of Financial Economics* 58.1-2, pp. 215–260.

— (2013). “R 2 and the Economy ”. *Annual Review of Financial Economics* 5.1, pp. 143–166.

Ordonez, Guillermo (2013). “The Asymmetric Effects of Financial Frictions”. *Journal of Political Economy* 121.5, pp. 844–895.

Peress, Joel (2014). “Learning from Stock Prices and Economic Growth”. *Review of Financial Studies* 27.10, pp. 2998–3059.

Roll, Richard (July 1988). “R2”. *The Journal of Finance* 43.3, pp. 541–566.

Shiller, Robert J (2015). *Irrational exuberance*. Princeton university press.

— (2017). “Narrative economics”. *American Economic Review* 107.4, pp. 967–1004.

Simsek, Alp (2013). “Belief disagreements and collateral constraints”. *Econometrica* 81.1, pp. 1–53.

Straub, Ludwig and Robert Ulbricht (2018). “Endogenous Uncertainty and Credit Crunches”.

Tobin, J. (1972). *The New Economics One Decade Older: The Eliot Janeway Lectureson Historical Economics in Honour of Joseph Schumpeter*. Princeton University Press.

Van Nieuwerburgh, Stijn and Laura Veldkamp (2006). “Learning asymmetries in real business cycles”. *Journal of monetary Economics* 53.4, pp. 753–772.

Veldkamp, Laura (2005). “Slow Boom, Sudden Crash”. *Journal of Economic theory* 124.2, pp. 230–257.

Vives, Xavier (2010). *Information and learning in markets: the impact of market microstructure*. Princeton University Press.

A Trading

I assume that every family i consists of many traders indexed by $ij \in [0,1]$. The information set of each trader consists of $\{s_{ijt}, \{z_{jt}\}, a_t, \varepsilon_t\}$, i.e., traders observe their private signal, all public signals and the aggregate states. This setting allows that traders have rational expectations about aggregates, but still disagree about firm-specific variables, which motivates trade. I impose that $\kappa_L = 0$ and $\kappa_H = 2$.

The beliefs of traders about firm productivity A_{jt} are relevant for their trading decision, where

$$\tilde{\mathbb{E}}\{A_{jt}|s_{ijt}, z_{jt}\} = \exp\left\{\omega_{p,ijt}a_t + \omega_{s,ijt}s_{ijt} + \omega_{z,ijt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{ijt}\right\}$$
$$\tilde{\mathbb{E}}\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\} = \exp\left\{\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{jt}\right\}$$

where $a_{jt} \sim \mathcal{N}(a_t, \sigma_a^2)$, $\varepsilon_{jt} \sim \mathcal{N}(\varepsilon_t, \sigma_\varepsilon^2)$ and ω-terms are the corresponding Bayesian weights,

$$\begin{aligned}
\omega_{z,ijt} &= \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}} & \omega_{z,jt} &= \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \\
\omega_{s,ijt} &= \frac{\beta_{ijt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}} & \omega_{s,jt} &= \frac{\beta_{jt}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \\
\omega_{p,ijt} &= \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}} & \omega_{p,jt} &= \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}},
\end{aligned}$$

and $\{\mathbb{V}_{jt}, \mathbb{V}_{ijt}\}$ stand for posterior uncertainty

$$\mathbb{V}_{ijt} = \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \mathbb{V}_{jt} = \frac{1}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}}.$$

Alternatively, the beliefs of the marginal trader who observed $s_{ijt} = z_{jt}$ can be expressed as a function of shocks,

$$\ln\tilde{\mathbb{E}}\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\} = \omega_{p,jt}a_t + \omega_{\varepsilon,it}\varepsilon_t + \omega_{a,jt}a_{jt} + \frac{\omega_{a,jt}}{\sqrt{\beta_{jt}}}(\varepsilon_{jt} - \varepsilon_t) + \frac{1}{2}\mathbb{V}_{jt},$$

where $\omega_{a,jt} = \omega_{z,jt} + \omega_{s,jt}$ and $\omega_{\varepsilon,jt} = \omega_{s,jt}/\sqrt{\beta_{jt}}$.

Trader ij buys shares of firm j whenever

$$\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt}, z_{jt}\} \geq \tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt} = z_{jt}, z_{jt}\}$$

which is equivalent to

$$\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}, z_{jt}\right\} \geq \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}.$$

The above inequality leads to

$$\begin{aligned}\omega_{p,ijt}a_t+\omega_{s,ijt}s_{ijt}+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{\mathbb{V}_{ijt}}{2} &\geq \omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{\mathbb{V}_{jt}}{2}\\ \iff \eta_{ijt} \geq \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}&\left(\omega_{p,ijt}a_t+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{\mathbb{V}_{ijt}}{2}\right)+\sqrt{\beta_{ijt}}a_{jt}\\ &-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{\mathbb{V}_{jt}}{2}\right)\end{aligned}$$

Since η_{ijt} is standard-normally distributed, the perceived probability of buying can be written in closed form

$$\begin{aligned}\mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}=\Phi\Bigg(&-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{ijt}\right)+\sqrt{\beta_{ijt}}a_{jt}\\ &-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\Bigg),\end{aligned}$$

where $\Phi(\cdot)$ is the standard-normal cdf and the weights on the realizations of shocks depend on the information precision choice of trader ij and of all other traders $-ij$. For a symmetric information choice ($\beta_{ijt}=\beta_{jt}$), the buying probability can be simplified to

$$\mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}\big|_{\beta_{ijt}=\beta_{jt}}=\Phi\left(-\varepsilon_{jt}\right).$$

Traders think that they are more likely to buy shares when the realization of the sentiment shock is relatively low and shares are therefore cheap relative to their fundamental value.

Finally, traders choose their information precision taking the symmetric choice of all other traders as given. The derivative of the probability of buying with respect to β_{ijt} is

$$\begin{aligned}\frac{\partial\mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}}{\partial\beta_{ijt}}=\phi\Bigg(&-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{ijt}\right)+\sqrt{\beta_{ijt}}a_{jt}\\ &-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_t}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\Bigg)\\ &*\Bigg(-\frac{1}{2\beta_{ijt}^{3/2}}\left(\sigma_a^{-2}a_t+\beta_{jt}\sigma_\varepsilon^{-2}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\right)+\frac{a_{jt}}{2\sqrt{\beta_{ijt}}}\\ &-\left(\frac{1}{\sqrt{\beta_{ijt}}}-\frac{1}{2\beta_{ijt}^{3/2}}\left(\mathbb{V}_{ijt}\right)^{-1}\right)\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\Bigg)\end{aligned} \tag{55}$$

where $\phi(\cdot)$ is the standard normal pdf. For a symmetric information choice ($\beta_{ijt}=\beta_{jt}$) this expression can be simplified to

$$\left.\frac{\partial \mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}}{\partial \beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} = \phi\left(\varepsilon_{jt}\right)\left[\frac{1}{2\sqrt{\beta}_{it}}\left(a_{jt}+z_{jt}\right)-\frac{1}{\sqrt{\beta_{jt}}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right].$$

B Figures

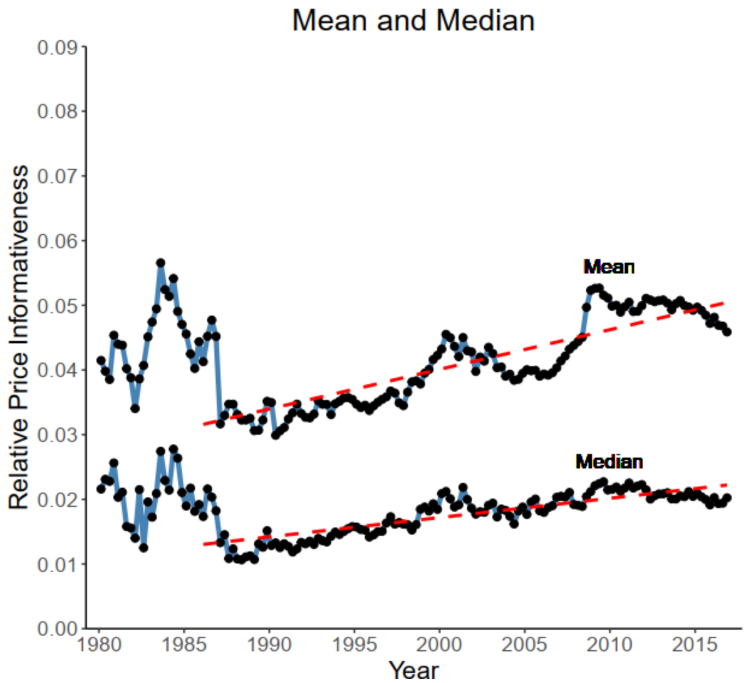

Figure 16: Dávila and Parlatore (2020) derive an identification strategy to measure price informativeness from a general, linear asset pricing equation and stochastic process for shocks. Relative prices informativeness is defined here as the Kalman gain that on observer without a private signal would put on the price signal, which corresponds in this model to $\frac{\beta_{jt}\sigma_{\varepsilon}^{-2}}{\sigma_a^{-2}+\beta_{jt}\sigma_{\varepsilon}^{-2}}$. Their estimate suggests that stock prices in the US have become steadily more informative since the late 1980s with an increase in the series's volatility since the late 1990s. Asset prices became rapidly more informative between 1997 and 2001, followed by a decrease until 2007. These fluctuations roughly coincide with an increase during the dot-com boom and decrease during the housing boom. Price informativeness spiked again in 2008-2009 and has been declining after that. The observations before 1987 follow a different pattern due to a change in the data set's composition.

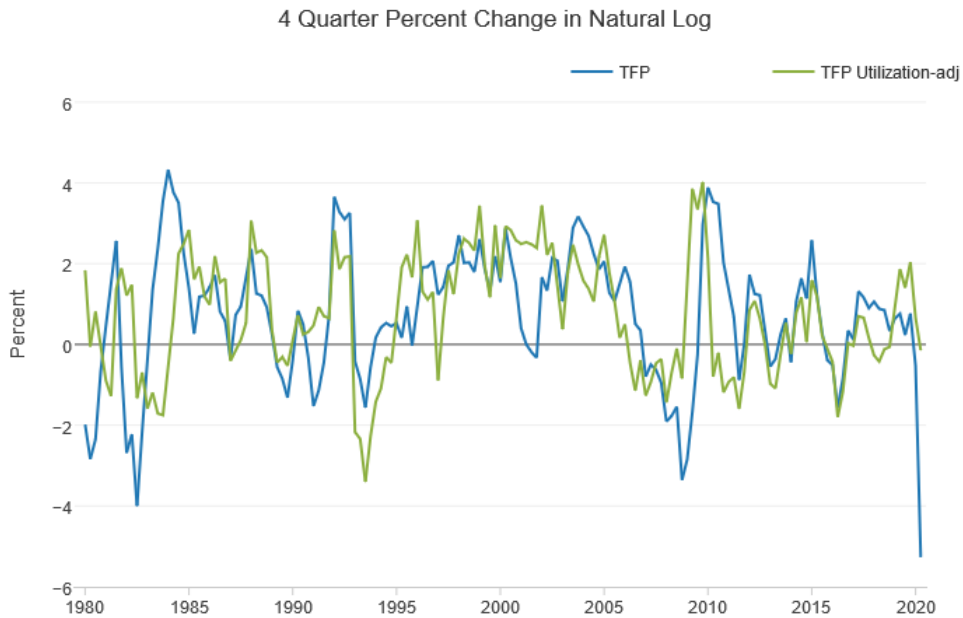

Figure 17: Change in quarterly TFP with (green) and without (blue) utilization-adjustment for both labor and capital following Basu, Fernald, and Kimball (2006). A period of high TFP growth between 1995 and 2005 was followed by a sharp slowdown and even negative growth between 2006 and 2008. Source: Federal Reserve of San Francisco.

C Position Limits

C.1 Exogenous Position Limits

In the main text I have assumed that traders can buy up to two units of each stock. Assume now that traders' position limits are given by $x_{ijt} \in [0, \kappa_H]$. Consider first some special cases.

Let $\kappa_H \in [0, 1)$. In that case the traders are collectively not able to clear the market. The result is that the stock price collapses to zero, all traders acquire κ_H units of firm j's stock, and the price is uninformative because it does not vary with firm productivity. Similarly, if $\kappa_H = 1$, traders are able to clear the market, but the same outcome arises.

In contrast, if there are no upper limits to how much traders can buy ($\kappa_H = \infty$), the most optimistic trader alone can clear the whole market. Expectations about dividends and the interest rate R_{t+1} go to infinity, but prices are finite. Information becomes useless for traders because the probability of buying in any given market is zero.

To avoid these edge cases, I focus position limits for which the market clearing condition gives an interior solution for the threshold, i.e., $\kappa_H \in (1, \infty)$. The market clearing condition leads to the threshold $\hat{s}\left(P_{jt}\right)$,

$$\kappa_H\left(1-\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right)-a_{jt}\right)-\varepsilon_{jt}\right)\right)=1.$$

$$\iff \hat{s}\left(P_{jt}\right)=a_{jt}+\frac{\varepsilon_{jt}+\Phi^{-1}\left(1-\frac{1}{\kappa_H}\right)}{\sqrt{\beta_{jt}}}$$

The resulting expectations of dividends can be written by multiplying the price under $\kappa_H=2$ with a factor related to κ_H,

$$\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}=z_{jt}+\Phi^{-1}\left(1-\frac{\kappa_H}{2}\right)/\sqrt{\beta_{jt}},z_{jt}\right\}=\underbrace{\exp\left\{\Phi^{-1}\left(1-\frac{1}{\kappa_H}\right)/\sqrt{\beta_{jt}}\right\}^{\theta}}_{\text{bias through choice of position limits}}\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}=z_{jt},z_{jt}\right\}.$$

Consequently, the interest rate is also distorted,

$$R_{t+1}=\exp\left\{\Phi^{-1}\left(1-\frac{\kappa_H}{2}\right)/\sqrt{\beta_{jt}}\right\}^{\theta}\frac{\int_0^1\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}=z_{jt},z_{jt}\right\}dj}{K_{t+1}}.$$

For $\kappa_H=2$ the marginal trader is neither optimistic nor pessimistic and, therefore, the bias due to the choice of position limits is equal to zero.

Holding K_{t+1} constant leads to an unchanged allocation of capital,

$$\begin{aligned}K_{jt+1}&=\frac{\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}=z_{jt}+\Phi^{-1}\left(1-\frac{1}{\kappa_H}\right)/\sqrt{\beta_{jt}},z_{jt}\right\}}{R_{t+1}}\\&=\frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}}{\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj}K_{t+1}.\end{aligned}$$

A different interest rate will affect aggregate investment through 29. If buyers are relatively optimistic ($\kappa_H>2$), then the interest and aggregate investment increase. Setting $\kappa_H=2$ is for the model without the information choice inconsequential and only avoids introducing a multiplicative factor for expectations of dividends.

For the information acquisition decision, the choice of position limits has similar effects as aggregate sentiment shocks or reductions in asset supply. The main idea is the same: when the aggregate sentiment shock is positive, trader expect to buy in fewer states of the world, making information less valuable. The same effect is present when setting the position limit $\kappa_H>2$; however, it is counteracted by traders acquiring more units if they trade, which is absent in the case of sentiment shocks. Depending on which effect dominates, the maximum information choice is achieved for $\kappa_H<2$ or $\kappa_H>2$.

Position limits affect the analysis for sentiment shocks and revert the logic outlined in the main text. For example, assume that $\kappa_H = 1+\eta$, where $\eta > 0$ is a small number. Then almost all traders need to buy shares to clear the market. It follows that trading is information-insensitive because all traders expect to buy κ_H units of nearly all firms irrespective of the private signal. Different from the intuition before, a positive sentiment shock makes traders think that the trading decision becomes more information-sensitive. Recall that the trading decision is most information-sensitive if the ex-ante probability of buying is $\frac{1}{2}$. As the increase in the sentiment shock pushes the ex-ante probability of trading towards $\frac{1}{2}$ from below, a sentiment shock can make the trading decision more information-sensitive.

The choice of $\kappa_H = 2$ in the main text guarantees that the marginal trader is, on average, neither optimistic nor pessimistic absent sentiment shocks. Moreover, considering aspects outside of the model, excess or lack of demand can lead to entry or exit of traders because prices are predictably under-/overpriced. It can also lead to additional entry or exit of firms for the same reason. Both forces tend to undo the effects of too much/little demand. Finally, denoting position limits in units of shares is mainly an analytical simplification when including risk-neutral traders in the financial markets.

C.2 Short-Selling

Short-selling was ruled out in the main text for analytical convenience, but does not change the results of the model. Assume that traders can take also negative positions, such that $x_{ijt} \in [-2, 2]$. The market clearing condition becomes

$$\underbrace{2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right)}_{\text{buying}} - \underbrace{2\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)}_{\text{selling}} = 1,$$

and the threshold is

$$\hat{s}\left(P_{jt}\right) = a_{jt} + \frac{\varepsilon_{jt} + \Phi^{-1}\left(\frac{1}{4}\right)}{\sqrt{\beta_{jt}}}.$$

In contrast to before, more traders need to be buying to clear the market, because previously inactive traders now short stocks and thereby increase their supply. Therefore, short-selling leads to a lower price because the marginal trader will be more pessimistic than before. The bias can be avoided by letting traders take asymmetric position, e.g., $x_{ijt} \in [-2, 4]$, in which case the marginal trader still is identified by the signal $\hat{s}\left(P_{jt}\right) = a_{jt} + \varepsilon_{jt}/\sqrt{\beta_{jt}}$.

C.3 Endogenous Position Limits

Finally, let traders choose position limits $x_{ijt} \in [\kappa_L, \kappa_H]$ subject to a cost $c^L\left(\kappa_L\right)$ and $c^H\left(\kappa_H\right)$ before trading takes place. One interpretation funds and credit lines have to be allocated between markets, which can be costly. It may be, however, valuable if traders expect that some markets are under- or overpriced. For example, if market j is hit by a positive sentiment shock, traders may want to extend their ability to short-sell in this market while reducing their ability to buy. Generally, this possibility will tend to imperfectly counteract the effects of sentiment shocks.

The effect on the information acquisition decision is more subtle. Consider as a partial equilibrium example that trader ij received private information that shares of firm j will be underpriced. In anticipation of a depressed market, trader ij extends her ability to buy, but completely forgoes short-selling. Intuitively, the opportunity cost of buying when prices are too high is captured by $(\kappa_H - \kappa_L) * Loss$. Therefore, the value of information is increasing in $\kappa_H - \kappa_L$. Whether the adjustment of position limits increases information acquisition depends, therefore, on whether $\kappa_H - \kappa_L$ is increased as a result.

More formally, the expected trading profits can be written as

$$\widetilde{EU}\left(\beta_{ijt}, \beta_{jt}\right) = \tilde{\mathbb{E}}\left\{\left(\kappa_H \mathcal{P}\left\{\text{Buy}\right\} + \kappa_L \mathcal{P}\left\{\text{Sell}\right\}\right) * \text{Profits}\right\}.$$

Because there are no trading costs, it must be that $\mathcal{P}\left\{\text{Sell}\right\} = 1 - \mathcal{P}\left\{\text{Buy}\right\}$:

$$\widetilde{EU}\left(\beta_{ijt}, \beta_{jt}\right) = \tilde{\mathbb{E}}\left\{\left[\left(\kappa_H - \kappa_L\right) \mathcal{P}\left\{\text{Buy}\right\}\right] * \text{Profits}\right\} - \kappa_L \tilde{\mathbb{E}}\left\{\text{Profits}\right\}.$$

Taking the derivative with respect to β_{ijt} yields,

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right) = \tilde{\mathbb{E}}\left\{\left[\left(\kappa_H - d_L\right) \frac{\partial \mathcal{P}\left\{\text{Buy}\right\}}{\partial \beta_{ijt}}\right] * \text{Profits}\right\}.$$

The marginal benefit to acquire information is proportional to $\kappa_H - \kappa_L$. Therefore, if traders decide to expand $\kappa_H - \kappa_L$ in response to a shock, it will tend to increase information acquisition.

D Intermediate Good Firms

D.1 Micro-Foundation

Intermediate good firms sell their whole revenue stream to traders to focus the analysis on information frictions. This assumption can be micro-founded by assuming that there are at least two entrepreneurs without private wealth for each variety j. Entrepreneurs need to turn to financial markets for funding their project, but the market is competitive in the sense that at most one entrepreneur for each variety j can sell her shares to traders. A mechanism chooses the entrepreneur who promises the highest rate of return on her shares. If there is a tie, the successful entrepreneur is chosen at random among the entrepreneurs that offer the highest return.

Formally, the entrepreneurs problem is

$$\max_{K_{jkt+1}, D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right)} C_{jkt} + \delta \mathbb{E}\left\{C_{jkt+1}\right\} \tag{56}$$

$$s.t. \quad K_{jkt+1} + C_{jkt} \leq P_{jkt}. \tag{57}$$

$$C_{jkt+1} \leq \Pi_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) - D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) \tag{58}$$

$$C_{jkt}, C_{jkt+1}, K_{jkt+1}, D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) \geq 0 \tag{59}$$

where

$$P_{jkt} = \begin{cases} 0 & \text{if } \exists k' \neq k : R_{jkt+1} < R_{jk't+1} \\ \begin{cases} 0 & \text{w.p. } 1 - \frac{1}{|k''|} \\ \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) | s_{ijt} = z_{jt}, z_{jt}\right\} & \text{w.p. } \frac{1}{|k''|} \end{cases} & \text{if } \exists k'' \neq k : R_{jkt+1} = R_{jk''t+1} \text{ and } \forall k' \neq k : R_{jkt+1} \geq R_{jk't+1} \\ \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) | s_{ijt} = z_{jt}, z_{jt}\right\} & \text{if } \forall k' \neq k : R_{jkt+1} > R_{jk't+1} \end{cases} . \tag{60}$$

The entrepreneur maximizes her utility over consumption today and tomorrow with the same utility function as households.[21] When young, entrepreneurs can either consume or invest in their firm. When old, entrepreneurs pay out a dividend D_{jkt+1} and consume what remains of revenue Π_{jkt+1}.

The entrepreneur is only able to sell her shares at a positive price if she offers the highest return in market j. If the entrepreneur promises a lower rate of return R_{jkt+1} than some other entrepreneur k', she will not be able to sell her shares and raise nothing. If she promises the highest rate of return in the economy, but other entrepreneur promise the same rate of return, she will be able to sell her shares with probability $1/|k''|$ where $|k''|$ is the number of entrepreneurs which promise the highest return. If only she promises the highest return, she will be able to sell her shares with probability one. Finally, the rate of return is given by

$$R_{jkt+1} = \frac{\mathbb{E}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right)\right\}}{K_{jkt+1}}. \tag{61}$$

There is perfect competition between entrepreneurs because productivity A_{jt} is attached to the variety j instead of the entrepreneur jk and all entrepreneurs sell at the same price

[21] Entrepreneurs can only raise funds by selling claims to revenue and cannot borrow or lend otherwise. This setting guarantees that asset prices are an invertible function of z_{jt}, a noisy signal of firm productivity, without which the equilibrium in the financial market does not exist. See Albagli, Hellwig, and Tsyvinski (2011b, 2017) for a discussion of this issue.

ρ_{jt+1} . Therefore, the only equilibrium is one in which at least two entrepreneurs choose

$$D_{jkt+1}\left(A_{jt},K_{jkt+1}\right)=\Pi_{jkt+1}\left(A_{jt},K_{jkt+1}\right) \tag{62}$$

$$K_{jkt+1}=P_{jkt} \tag{63}$$

It is easy to verify that this is the only equilibrium. Any entrepreneur k who chooses (62) and (63) can only deviate by either investing less or paying a lower dividend. It follows there exists another entrepreneur who promises a higher return on investment and following (60) entrepreneur k is unable to sell her shares. Similarly, any entrepreneur who does not choose (62) and (63) does not have a profitable deviation. Choosing to invest less or promising a lower dividend leads to no change, as the rate of return is only further depressed. Investing more or promising a higher dividend is similarly inconsequential as long as the entrepreneur does not choose (62) and (63). If she chooses to deviate to (62) and (63), she still earns zero profits but gets to produce with positive probability. Therefore, (62) and (63) are an equilibrium.

To show that at least two entrepreneurs choosing (62) and (63) is the only equilibrium, it is necessary to show that profitable deviations exist for all other choices of investment and dividends. First, consider that only one entrepreneur k chooses (62) and (63) and all others either invest strictly less or pay a lower dividend in some states. Then entrepreneur k can raise her profits by either investing less or promising a lower dividend while still promising the highest rate of return. Second, assume that all entrepreneurs choose an investment and dividend policy that leads to positive profits for the entrepreneur in at least some states. In this case, there is a profitable deviation for any entrepreneur k, as she invest more or pay a larger dividend to promise the highest rate of return while keeping positive profits. Therefore, the only equilibrium is given by at least two entrepreneurs choosing (62) and (63).

D.2 Entrepreneurs with Market Power: Equity

Alternatively, assume there is only one entrepreneur per variety. Furthermore, entrepreneurs are patient and restricted to selling equity contracts as captured in the following Assumption.

Assumption 3 (***Equity Contracts***)**.** *Entrepreneurs can only sell claims to a fraction* $\lambda_{jt}\left(P_{jt},P_t\right)\in[0,1]$ *of firm-revenue.*

The share of revenue that is sold to the market is allowed to depend on price P_{jt} and on the aggregate value of stock market, P_t. The entrepreneur's maximization problem is

$$\max_{\lambda_{jt}, K_{jt+1}} \quad \mathbb{E}\left\{C_{jt+1}|P_{jt}\right\} \tag{64}$$

$$C_{jt+1} \leq \Pi\left(A_{jt}, K_{jt+1}\right) - D\left(A_{jt}, K_{jt+1}\right) \tag{65}$$

$$D\left(A_{jt}, K_{jt+1}\right) = \lambda_{jt}\left(P_{jt}, P_t\right)\Pi\left(A_{jt}, K_{jt+1}\right) \tag{66}$$

$$0 \leq K_{jt+1} \leq P_{jt}, \tag{67}$$

The entrepreneur maximizes her consumption, which consists of firm-revenue $\Pi\left(A_{jt}, K_{jt+1}\right)$ after paying dividends $D\left(A_{jt}, K_{jt+1}\right)$ subject to constraints (65), (66) and (67). The first constraint states that consumption can be at most revenue minus dividends. The second constraint follows from Assumption 3. The last constraint imposes non-negativity on investment and states that entrepreneurs cannot borrow additional funds from other sources. Plugging in the constraints into the objective yields the simplified problem

$$\max_{\lambda_{jt}(P_{jt}, P_t), K_{jt+1}} \quad \mathbb{E}\left\{\Pi_{jt+1} - \lambda_{jt}\left(P_{jt}, P_t\right)\Pi_{jt+1}|z_{jt}\right\}$$
$$0 \leq K_{jt+1} \leq P_{jt}$$

The asset price P_{jt} can be expressed as

$$P_{jt} = \alpha\frac{\lambda_{jt}\left(P_{jt}, P_t\right)}{R_{t+1}}Y_t^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}K_{jt+1}^{\frac{\theta-1}{\theta}}.$$

It is optimal for the entrepreneur to invest everything she raises, which allows to write firm-capital as

$$P_{jt} = K_{jt+1} = \left(\alpha\frac{\lambda_{jt}\left(P_{jt}, P_t\right)}{R_{t+1}}Y_t^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}\right)^{\theta}.$$

Plugging this back into the entrepreneur's problem leads to a simplified problem

$$\max_{\lambda_{jt}(P_{jt}, P_t)} \left(1 - \lambda_{jt}\left(P_{jt}, P_t\right)\right)\lambda_{jt}\left(P_{jt}, P_t\right)^{\theta-1}\left(1 - \alpha^{\theta-1}\right)\mathbb{E}\left\{A_{jt}|z_{jt}\right\}\left(\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}\frac{Y_t^{\alpha_Y}}{R_{t+1}}\right)^{\theta-1}.$$

The first order condition to the simplified problem is

$$
\begin{aligned}
\frac{\partial}{\partial \lambda_{jt}} \left(1-\lambda_{jt}\left(P_{jt},P_t\right)\right)\lambda_{jt}\left(P_{jt},P_t\right)^{\theta-1} &= 0\\
\Rightarrow (\theta-1)\,\lambda_{jt}\left(P_{jt},P_t\right)^{\theta-2}-\theta\lambda_{jt}\left(P_{jt},P_t\right)^{\theta-1} &= 0\\
\Rightarrow \forall j,t: \quad \lambda_{jt} &= \frac{\theta-1}{\theta}
\end{aligned}
$$

Therefore, all entrepreneurs irrespective of firm-specific and aggregate asset prices sell a constant fraction $\lambda_{jt}=\frac{\theta-1}{\theta}$ of revenue to the financial market. The resulting dividend per share is

$$D_{jt+1}=\alpha\frac{\theta-1}{\theta}Y_{t+1}^{\alpha_Y}A_{jt}K_{jt+1}^{\frac{\theta-1}{\theta}}.$$

Assigning market power to entrepreneurs, therefore, effectively leads to a mark-up on the price of the intermediate good as traders only receive a fraction $\frac{\theta-1}{\theta}$ of firm-revenue for completely funding firm investment. The effect is to depress investment, which can be undone through an ad-valorem subsidy of $\tau=\frac{\theta}{\theta-1}$ in the market for intermediate goods.

D.3 Entrepreneurs with Market Power: Credit Markets

The main focus of this paper is to study booms that are caused by productivity or sentiment. There is an extensive literature that studies such booms in credit markets. The model can be extended to cover debt securities that are centrally traded instead of stock markets. Assume that the entrepreneur's technology is given by

$$Y_{jt}=\begin{cases} A_t^{\frac{\theta-1}{\theta}}K_{jt} & \text{w.p. } \pi_{jt}\\ 0 & \text{w.p. } 1-\pi_{jt}\end{cases}.$$

Different to before, entrepreneurs run projects that are either successful and give a certain return on capital or fail and produce nothing. Success or failure is determined by the realization of a normally distributed variable,

$$P\left(Y_{jt}>0\right)=P\left(a_{jt}>\bar{a}\right)=\Phi\left(\frac{a_t-\bar{a}}{\sqrt{\sigma_a^2}}\right)=\pi_t.$$

The entrepreneur's project succeeds whenever $a_{jt}\sim\mathcal{N}\left(a_t,\sigma_a^2\right)$ has a realization above the threshold $\bar{a}$. Households have dispersed information about the firm-specific shock $s_{ijt}=a_{jt}+\frac{\eta_{ijt}+\varepsilon_{jt}}{\sqrt{\beta_{ijt}}}$ where η_{ijt} is idiosyncratic noise and ε_{jt} is correlated noise. Same as before, traders suffer from correlation neglect and perceive only their own signal to be $s_{ijt}=a_{jt}+\eta_{ijt}/\sqrt{\beta_{ijt}}$.

The households problem is the same as in the main model.

To finance their projects, entrepreneurs issue a unit mass of debt securities with the payoff

$$X_{jt} = \begin{cases} \lambda_{jt} & \text{if } Y_{jt} > 0 \\ 0 & \text{otherwise} \end{cases}.$$

The security pays an amount λ_{jt} when the project succeeds and zero otherwise.[22] The entrepreneur maximizes the revenue that she can keep in case of success after repaying debt obligations

$$\begin{aligned} &\max_{\lambda_{jt}, K_{jt+1}} \quad \rho_{jt+1} Y_{jt+1}\left(a_{jt}, K_{jt+1}\right) - X_{jt}\left(a_{jt}, \lambda_{jt}\right) \\ &0 \leq K_{jt+1} \leq P_{jt}. \end{aligned}$$

The entrepreneur invests all raised funds, $K_{jt+1} = P_{jt}$. Using $P\left(a_{jt} > \bar{a} | s_{ijt} = z_{jt}, z_{jt}\right) = P\left(\tilde{a}_{jt+1} > \bar{a}\right)$ where $\tilde{a}_{it+1} \sim \mathcal{N}\left(\tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}, \mathbb{V}\right)$ are the posterior beliefs of the marginal trader, the price of debt and firm-capital can then be written as

$$K_{jt+1} = P_{jt} = \frac{\lambda_{jt}}{R_{t+1}} \Phi\left(\frac{\tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}}{\sqrt{\mathbb{V}}}\right), \tag{68}$$

where $\mathbb{V} = \left(\sigma_a^{-2} + \beta_{jt}\left(1 + \sigma_\varepsilon^{-2}\right)\right)^{-1}$is the posterior uncertainty. The solution to the entrepreneur's problem is

$$\lambda_{jt} = \left(\frac{\theta - 1}{\theta} \alpha Y_{t+1}^{\alpha_Y} A_{t+1}\right)^{\theta} \frac{\Phi\left(\frac{\tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}}{\sqrt{\mathbb{V}}}\right)^{\theta - 1}}{\left(R_{t+1}\right)^{\theta - 1}}, \tag{69}$$

which depends on the market valuation of debt or equivalently the interest rate that entrepreneur j faces. Using (68) and (69) in the expression for firm-revenue allows to express the entrepreneur's decision as a fraction of output,

$$\frac{\lambda_{jt}}{\rho_{jt+1} Y_{jt+1}\left(a_{jt}, K_{jt+1}\right)} = \frac{\theta - 1}{\theta}. \tag{70}$$

This result recovers the optimal equity contract from section D.2.

In contrast to the model with equity, there is an additional channel through which shocks affect information acquisition. The binary payoff function introduces changes in the vari-

[22] Quantity and payoffs can be interchanged by denoting the mass of securities by λ_{jt} and the payoff in the good state is normalized to one. Instead, the quantity is normalized to one and the payoff is allowed to vary.

ance of outcomes for firms driven by productivity and sentiment shocks. The variance of outcomes is captured by $\pi_{jt}(1-\pi_{jt})$, whereas riskiness normally would only be captured by the probability of failure, $1-\pi_{jt}$. Intuitively, a project is entirely safe whenever the probability of success, π_{jt}, is equal to one. In this case, learning about the firm-specific shock, a_{jt}, is inconsequential. The same reasoning applies if the project is sure to fail ($\pi_{jt}=0$). Therefore, the effect of changes to a_t is ambiguous. Positive shocks to a_t trigger additional information acquisition only when π_{jt} was low before, but they crowd-out information when debt becomes safe as a consequence. Therefore, the aggregate shocks affect the (perceived) riskiness of debt.[23]

Although this model abstracts from banks and credit intermediation, it replicates the main stylized facts of credit booms before financial crises. First, credit booms are episodes of sharp increases in lending and economic activity (Jordà, Schularick, and Taylor, 2010). This is the case in the model presented here, as the volume of credit increases in response to a positive aggregate shock. As a result, investment and economic activity increase. Second, credit becomes riskier as lending standards are relaxed, and riskier firms get access to credit (Keys et al., 2010). In response to a sentiment shock, all firms are considered to be safer than they actually are. Because there is more scope for a change in beliefs for relatively risky firms, the sentiment shock leads to a disproportionate increase in funding for risky firms (low π_{jt}). Third, credit spreads decrease in the boom phase before a financial crisis (Krishnamurthy and Muir, 2017). In the case for sentiment driven booms, all firms are perceived to be safer than they actually are and, therefore, spreads are low.

E Proofs

Proof of Proposition 1. The proof follows the same steps as the proof for Proposition 1 in Albagli, Hellwig, and Tsyvinski (2017) as the financial market in this model is isomorphic to their version. It is repeated here for completeness. The only difference is that K_{jt+1} depends on the price signal z_{jt}, whereas k in Albagli, Hellwig, and Tsyvinski (2017) is determined before trading takes place. Therefore, it is necessary to assume that $K_{jt+1}(z_{jt})$ is non-decreasing in z_{jt} as the price might otherwise be not invertible, which is confirmed ex-post.

There must be a threshold $\hat{s}(P_{jt})$ such that all households with $s_{ijt} \geq \hat{s}(P_{jt})$ find it profitable to buy two units of share j and otherwise abstain from trading. It follows that the price must be equal to the valuation of the trader who is just indifferent between buying or

[23] An alternative interpretation is that productivity shocks affect productivity conditional on success, A_{t+1}. In this case, productivity shocks would have no effect on the riskiness of debt and behave similar to a productivity shock in the model with equity markets.

not,

$$P_{jt} = \tilde{\mathbb{E}} \left\{ D\left(A_{jt}, K_{jt+1}\right) | s_{ijt} = \hat{s}\left(P_{jt}\right), P_{jt} \right\}.$$

This monotone demand schedule leads to total demand $2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right)$. Equalizing total demand with a normalized supply of one leads to the market clearing condition

$$2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right) = 1$$

with the unique solution $\hat{s}\left(P_{jt}\right) = z_{jt} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}$. If P_{jt} is pinned down by z_{jt}, then P_{jt} is invertible, given that K_{jt+1} is non-decreasing in z_{jt}. It follows then that observing P_{jt} is equivalent to observing $z_{jt} \sim \mathcal{N}\left(a_{jt}, \beta_{jt}^{-1}\sigma_{\varepsilon}\right)$. Traders treat the signal z_{jt} and their private signal $s_{ijt} \sim \mathcal{N}\left(a_{jt}, \beta_{ijt}^{-1}\right)$ as mutually independent. Using this result, the price can be restated as

$$P\left(z_{jt}, K_{jt+1}\right) = \tilde{\mathbb{E}} \left\{ D\left(A_{jt}, K_{jt+1}\right) | s_{ijt} = z_{jt}, z_{jt} \right\}$$

where posterior expectations of trader ij are given by

$$a_{jt} | s_{ijt}, z_{jt} \sim \mathcal{N}\left(\frac{\sigma_a^{-2} a_t + \beta_{ijt} s_{ijt} + \beta_{jt}\sigma_{\varepsilon}^{-2} z_{jt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_{\varepsilon}^{-2}}, \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_{\varepsilon}^{-2}}\right).$$

It remains to show uniqueness of the above solution. Start with the assumption that demand $d\left(s_{ijt}, P_{jt}\right)$ is non- increasing in P_{jt}. It follows that $\hat{s}\left(P_{jt}\right)$ is non-decreasing in P_{jt}. There are two cases to differentiate. First, if $\hat{s}\left(P_{jt}\right)$ is strictly increasing in P_{jt}, then the price is indeed uniquely pinned-down by z_{jt} and invertible; it can be expressed like above. Secondly, assume that the threshold is flat over some interval, such that $\hat{s}\left(P_{jt}\right) = \hat{s}$ over some interval $P_{jt} \in (P', P'')$ for $P' \neq P''$. Furthermore, choose $\epsilon > 0$ small enough such that $\hat{s}\left(P_{jt}\right)$ is increasing to the left and right of the interval, i.e., over $P_{jt} \in (P' - \epsilon, P')$ and $P_{jt} \in (P'', P'' + \epsilon)$. In these regions, $\hat{s}\left(P_{jt}\right)$ is monotonically increasing in P_{jt}, which is uniquely pinned down by z_{jt} and invertible; observing the price is equivalent to observing the signal z_{jt}. In this case the price can be expressed as before for $z_{jt} \in \left(\hat{s}\left(P' - \varepsilon\right), \hat{s}\right)$ and $z_{jt} \in \left(\hat{s}, \hat{s}\left(P'' + \epsilon\right)\right)$. This leads to a contradiction to the assumption that $P' \neq P''$, because $P\left(z_{jt}, K_{jt+1}\right)$ is continuous and monotonically increasing in z_{jt}. Therefore, $\hat{s}\left(P_{jt}\right)$ cannot be flat and the above solution is indeed unique. □

Proof of Proposition 2. (i) Using (4) in (21) leads to the expression for firm-capital

$$K_{jt+1} = \left(\frac{\alpha Y_{t+1}^{\alpha_Y}}{R_{t+1}} \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}\right)^{\theta}. \tag{71}$$

Plugging R_{t+1} from (22) into (21) using (71) leads to

$$K_{jt+1} = \frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj} K_{t+1}.$$

Finally, plugging this expression for firm capital into the aggregate production function leads to

$$\begin{aligned}
Y_t &= \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \left(\int_0^1 A_{jt-1} K_{jt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \frac{\left(\int_0^1 A_{jt-1} \tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1} = z_{jt-1}, z_{jt-1}\right\}^{\theta-1} dj\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1} = z_{jt-1}, z_{jt-1}\right\}^{\theta} dj\right)^{\alpha}} K_t^{\alpha} \\
&= A_{t-1} L^{1-\alpha} K_t^{\alpha}
\end{aligned}$$

where total factor productivity is

$$\begin{aligned}
A_{t-1} &= \frac{\left(\int_0^1 A_{jt-1} \tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1} = z_{jt-1}, z_{jt-1}\right\}^{\theta-1} dj\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1} = z_{jt-1}, z_{jt-1}\right\}^{\theta} dj\right)^{\alpha}} \\
&= \exp\left\{\theta a_{t-1} + \left(\left(\theta-1\right)\omega_a + 1\right)^2 \frac{\sigma_a^2}{2} + \left(\theta-1\right)^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} + \left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_{t-1} + \frac{\left(\theta-1\right)}{2}\mathbb{V}\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&: \exp\left\{\theta a_{t-1} + \theta^2\omega_a^2 \frac{\sigma_a^2}{2} + \theta^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} + \theta\omega_{s\varepsilon}\varepsilon_{t-1} + \frac{\theta}{2}\mathbb{V}\right\}^{\alpha} \\
&= \exp\left(\frac{\alpha\theta}{\theta-1} a_{t-1} + \left(\frac{\alpha\theta}{\theta-1}\left(\left(\theta-1\right)\omega_a + 1\right)^2 - \alpha\theta^2\omega_a^2\right)\frac{\sigma_a^2}{2} + \left(\alpha\theta\left(\theta-1\right) - \alpha\theta^2\right)\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{\alpha\theta}{\theta-1} a_{t-1} + \alpha\theta\left(\left(\theta-1\right)\omega_a^2 + 2\omega_a + \frac{1}{\theta-1} - \theta\omega_a^2\right)\frac{\sigma_a^2}{2} - \alpha\theta\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{1}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right) + \omega_a\left(2-\omega_a\right)\frac{\sigma_a^2}{2} - \omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}\right)^{\alpha\theta}.
\end{aligned}$$

The weights $\{\omega_a, \omega_\varepsilon, \omega_{s\varepsilon}\}$ and $\mathbb{V}$ are derived in Appendix A. Finally, total factor productivity can be expressed as

$$\ln A_{t-1}\left(a_{t-1}, \beta_{t-1}\right) = \underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right)}_{\text{exogenous}} + \underbrace{\kappa^a\left(\beta_{t-1}\right)\sigma_a^2 - \kappa^\varepsilon\left(\beta_{t-1}\right)\sigma_\varepsilon^2}_{\text{allocative efficiency}}$$

where $\kappa^a(\beta_{t-1}) = \frac{\omega_a(2-\omega_a)}{2}$ and $\kappa^\varepsilon(\beta_{t-1}) = \frac{\omega_\varepsilon^2}{2}$.

(iii) $\omega_a(\beta_t)$ is monotonically increasing in β_t. In the case with $\beta_t = 0$ or $\beta_t \to \infty$, no noise enters the posterior of traders. Therefore, the Bayesian weight on realizations of correlated noise, $\omega_\varepsilon(\beta_t)$, must be hump-shaped. It follows then from (27) that if σ_ε^2 is large enough relative to σ_a^2, TFP is an inversely hump-shaped function of β_t. Reversely, if σ_ε^2 is small enough relative to σ_a^2, $A(a_{t-1}, \beta_{t-1})$ is monotonically increasing in β_{t-1}. □

Lemma 1. *Denote the Bayesian weights* $\omega_a^{eff} = \frac{\beta\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}}, \omega_\varepsilon^{eff} = \frac{\sqrt{\beta}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}}$ *and posterior uncertainty* $\mathbb{V}^{eff} = \frac{1}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}}$, *then*

$$\underbrace{\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2}_{=Var(\mathbb{E}\{a_{jt}|z_{jt}\})} + \underbrace{\mathbb{V}^{eff}}_{Var(a_{jt}|z_{jt})} = \underbrace{\sigma_a^2}_{Var(a_{jt})}.$$

Proof.

$$\begin{aligned}
\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \mathbb{V}^{eff} &= \frac{\beta_t^2\sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}\right)^2}\sigma_a^2 + \frac{\beta_t\sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}\right)^2}\sigma_\varepsilon^2 + \frac{1}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}} \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + 2\beta_t\sigma_\varepsilon^{-2} + \beta_t^2\sigma_\varepsilon^{-4}\sigma_a^2\right) \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-4} + 2\beta_t\sigma_\varepsilon^{-2}\sigma_a^{-2} + \beta_t^2\sigma_\varepsilon^{-4}\right)\sigma_a^2 \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + \beta_t\sigma_\varepsilon^{-2}\right)^2\sigma_a^2 \\
&= \sigma_a^2.
\end{aligned}$$

□

Proof of Proposition 3. (i) An efficient allocation of capital equalizes marginal products between firms. Demand for firm-capital follows from the following maximization problem

$$\max_{K_{jt}} \alpha Y_t^{\alpha_Y}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} K_{jt}^{\frac{\theta-1}{\theta}} - R_{t+1}K_{jt}$$

with the first-order condition

$$K_{jt} = \left(\frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}}{R_{t+1}}\alpha Y_t^{\alpha_Y}\right)^\theta.$$

Integrating over all firms on both sides yields

$$R_{t+1} = \left(\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^\theta dj\right)^{\frac{1}{\theta}} \alpha Y_t^{\alpha_Y} K_t^{-\frac{1}{\theta}}.$$

Plugging this expression back into the first order condition leads to the constrained-efficient allocation

$$K_{jt} = \frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta} dj} K_t.$$

(ii) Plugging (28) into (26) leads to

$$Y_t = A_{t-1}^{eff} K_t^{\alpha},$$

where the constrained-efficient level of total factor productivity is

$$A_{t-1}^{eff} = \frac{\left(\int_0^1 A_{jt}\mathbb{E}\left\{A_{jt}|z_{jt-1}\right\}^{\theta-1} di\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt-1}\right\}^{\theta} di\right)^{\alpha}}.$$

The analytical expression can be obtained by evaluating the conditional expectations and using the constrained-efficient Bayesian weights and posterior uncertainty,

$$\omega_p^{eff} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}, \quad \omega_a^{eff} = \frac{\beta_{t-1}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}, \quad \omega_\varepsilon^{eff} = \frac{\sqrt{\beta_{t-1}}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}, \quad \mathbb{V}^{eff} = \frac{1}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}$$

which leads to

$$
\begin{aligned}
A_{t-1}^{eff} &= \frac{\left(\int_0^1 A_{it-1}\mathbb{E}\left\{A_{it-1}|z_{it-1}\right\}^{\theta-1} di\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \mathbb{E}\left\{A_{it-1}|z_{it-1}\right\}^{\theta} di\right)^{\alpha}} \\
&= \exp\left\{\theta a_{t-1} + \left((\theta-1)\,\omega_a^{eff}+1\right)^2 \frac{\sigma_a^2}{2} + (\theta-1)^2\left(\omega_\varepsilon^{eff}\right)^2\frac{\sigma_\varepsilon^2}{2} + \frac{(\theta-1)}{2}\mathbb{V}^{eff}\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&: \exp\left\{\theta a_{t-1} + \theta^2\left(\omega_a^{eff}\right)^2\frac{\sigma_a^2}{2} + \theta^2\left(\omega_\varepsilon^{eff}\right)^2\frac{\sigma_\varepsilon^2}{2} + \frac{\theta}{2}\mathbb{V}^{eff}\right\}^{\alpha} \\
&= \exp\left(\frac{\alpha\theta}{\theta-1}a_{t-1} + \left(\frac{\alpha\theta}{\theta-1}\left((\theta-1)\,\omega_a^{eff}+1\right)^2 - \alpha\theta^2\left(\omega_a^{eff}\right)^2\right)\frac{\sigma_a^2}{2} + \left(\alpha\theta\,(\theta-1)-\alpha\theta^2\right)\left(\omega_\varepsilon^{eff}\right)^2\frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{\alpha\theta}{\theta-1}a_{t-1} + \alpha\theta\left((\theta-1)\left(\omega_a^{eff}\right)^2 + 2\omega_a^{eff} + \frac{1}{\theta-1} - \theta\left(\omega_a^{eff}\right)^2\right)\frac{\sigma_a^2}{2} - \alpha\theta\left(\omega_\varepsilon^{eff}\right)^2\frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{1}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right) + 2\omega_a^{eff}\frac{\sigma_a^2}{2} - \left(\left(\omega_a^{eff}\right)^2\frac{\sigma_a^2}{2} + \left(\omega_\varepsilon^{eff}\right)^2\frac{\sigma_\varepsilon^2}{2}\right)\right)^{\alpha\theta} \\
&\overset{\text{Lemma1}}{=} \exp\left(\frac{1}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right) + 2\left(\omega_a^{eff}\right)\frac{\sigma_a^2}{2} - \left(\frac{\sigma_a^2 - \frac{1}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}}{2}\right)\right)^{\alpha\theta} \\
&= \exp\left(\frac{1}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right) + 2\omega_a^{eff}\frac{\sigma_a^2}{2} - \omega_a^{eff}\frac{\sigma_a^2}{2}\right)^{\alpha\theta} \\
&= \exp\left(\frac{1}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right) + \omega_a^{eff}\frac{\sigma_a^2}{2}\right)^{\alpha\theta}.
\end{aligned}
$$

TFP under the efficient allocation of capital can be similarly decomposed in two expressions,

$$
\ln A_{t-1}^{eff} = \underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right)}_{\text{exogenous}} + \underbrace{\alpha\theta\omega_a^{eff}\frac{\sigma_a^2}{2}}_{\text{allocative efficiency}}.
$$

It follows that

$$
\frac{\partial\omega_a^{eff}}{\partial\beta_{t-1}} > 0 \Rightarrow \frac{\partial A_{t-1}^{eff}}{\partial\beta_{t-1}} > 0.
$$

(iii) As under both allocations capital is distributed equally between firms for $\beta_{t-1} = 0$, total factor productivity also coincides,

$$
A_{t-1}^{eff} = A_{t-1} = \exp\left(\frac{\alpha\theta}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right)\right).
$$

In the perfect information ($\beta_{t-1} = \infty$) case the efficient and market allocation also coincide,

$$A_{t-1}^{eff} = A_{t-1} = \exp\left(\frac{1}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right) + \frac{\sigma_a^2}{2}\right)^{\alpha\theta}.$$

For $\beta_t \in (0,\infty)$, TFP under the constrained-efficient capital allocation must be higher than under the market allocation, as is the allocation is explicitly derived to maximize firm-production given the available information information, which guarantees that ex-ante marginal products of capital are equalized given an aggregate level of investment.[24] $\square$

Proof of Corollary 1. The distortion vanishes of the expectations of the marginal trader $\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}$ and $\mathbb{E}\left\{A_{jt}|z_{jt}\right\}$ coincide, i.e., $K_{jt+1} = K_{jt+1}^{eff}$.

(i) When the private signal is infinitely noisy, both the expectations converge to the unconditional expectation,

$$\lim_{\beta\to 0}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \lim_{\beta\to 0}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = \mathbb{E}\left\{A_{jt}\right\}.$$

When the private signal is infinitely precise, both expectations converge to the actual realization of A_{jt},

$$\lim_{\beta\to \infty}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \lim_{\beta\to \infty}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = A_{jt}$$

(ii) When the variance of firm-specific productivity shocks goes to zero, i.e., the prior becomes arbitrarily precise, both expectations converge to $\exp\left\{a_t\right\}$

$$\lim_{\sigma_a^2\to 0}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \lim_{\beta\to 0}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = \exp\left\{a_t\right\}.$$

When the variance of firm-specific productivity shocks goes to infinity, i.e., the prior becomes arbitrarily noisy, both allocations coincide because they put full weight on the price signal z_{jt},

$$\lim_{\sigma_a^{-2}\to 0}\omega_z = \lim_{\sigma_a^{-2}\to 0}\omega_z^{eff} = 1$$

where

$$\omega_z = \frac{\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2} + \beta_t\left(1+\sigma_\varepsilon^{-2}\right)}, \quad \omega_z^{eff} = \frac{\beta_t\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_t\sigma_\varepsilon^{-2}}.$$

(iii) When the variance of firm-specific sentiment shocks goes to zero, financial markets perfectly aggregate dispersed information as the precision of the price signal goes to infinity. $\square$

[24] Direct proof is in the making.

Lemma 2 (Joining two Normal PDFs). *Let $f(\varepsilon_{jt})$ be the pdf of $\varepsilon_{jt} \sim \mathcal{N}(\varepsilon_t, \sigma_\varepsilon^2)$ and $\phi(\cdot)$ the standard-normal pdf. Then*

$$f(\varepsilon_{jt})\,\phi(\varepsilon_{jt}) = \exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}\sqrt{\frac{1}{2\pi(1+\sigma_\varepsilon^2)}}\tilde{f}(\varepsilon_{jt})$$

where $\tilde{f}(\varepsilon_{jt})$ is the transformed pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$.

Proof. Write out the pdfs explicitly,

$$\begin{aligned}
\phi(\varepsilon_{jt}) &= \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}\right\} \\
f(\varepsilon_{jt}) &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\exp\left\{-\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{2\sigma_\varepsilon^2}\right\} \\
f(\varepsilon_{jt})\,\phi(\varepsilon_{jt}) &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}-\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{2\sigma_\varepsilon^2}\right\}.
\end{aligned}$$

Rearranging the term inside the exponential function,

$$
\begin{aligned}
\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{\sigma_\varepsilon^2}+\varepsilon_{jt} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\varepsilon_t+\varepsilon_t^2}{\sigma_\varepsilon^2}+\varepsilon_{jt}\\
\text{join fractions} &= \frac{\left(1+\sigma_\varepsilon^2\right)\varepsilon_{jt}-2\varepsilon_t\varepsilon_{jt}+\varepsilon_t^2}{\sigma_\varepsilon^2}\\
\text{divide by } \left(1+\sigma_\varepsilon^2\right) &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\\
\text{add and substract} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\\
\text{exchange terms} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\\
\text{join paranthesis again} &= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{1-\frac{1}{1+\sigma_\varepsilon^2}}{\sigma_\varepsilon^2}\varepsilon_t^2\\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\frac{1+\sigma_\varepsilon^2-1}{1+\sigma_\varepsilon^2}}{\sigma_\varepsilon^2}\varepsilon_t^2\\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}
\end{aligned}
$$

This allows to write

$$
\begin{aligned}
f\left(\varepsilon_{jt}\right)\phi\left(\varepsilon_{jt}\right) &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}}{2}-\frac{\left(\varepsilon_{jt}-\varepsilon_t\right)^2}{2\sigma_\varepsilon^2}\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\}\exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\sqrt{\frac{\sigma_\varepsilon^2}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\frac{1}{\sqrt{2\pi\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\}\exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}).
\end{aligned}
$$

□

Lemma 3 (Auxiliary Results Market Allocation)**.** *Denote the Bayesian weights* $\omega_a = \frac{\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)}$, $\omega_\varepsilon = \frac{\omega_a}{\sqrt{\beta_{jt}}}$, $\omega_{z\varepsilon,it} = \frac{\sqrt{\beta_{jt}}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\sqrt{\beta_{jt}}\left(1+\sigma_\varepsilon^{-2}\right)}$, *and posterior uncertainty* $\mathbb{V} = \frac{1}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$, *then*

(i) $\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}+\mathbb{V}=\sigma_a^2$,

(ii) $\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}=\sigma_a^2-\mathbb{V}=\omega_a\sigma_a^2$,

(iii) $\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2}=\omega_{z\varepsilon}$.

Proof. (i)

$$
\begin{aligned}
\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}+\mathbb{V} &= \frac{\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}+\frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}+\frac{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-2}+2\beta\left(1+\sigma_\varepsilon^{-2}\right)+\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-4}+2\sigma_a^{-2}\beta\left(1+\sigma_\varepsilon^{-2}\right)+\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \frac{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \sigma_a^2
\end{aligned}
$$

(ii) The first equality follows from (i). Then

$$\begin{aligned}
\sigma_a^2 - \mathbb{V} &= \sigma_a^2 - \frac{1}{\sigma_a^{-2} + \beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\sigma_a^2\sigma_a^{-2} + \sigma_a^2\beta\left(1+\sigma_\varepsilon^{-2}\right) - 1}{\sigma_a^{-2} + \beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2} + \beta\left(1+\sigma_\varepsilon^{-2}\right)}\sigma_a^2 \\
&= \omega_a\sigma_a^2.
\end{aligned}$$

(iii) $\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2} = \frac{\omega_\varepsilon}{\sigma_\varepsilon^2\left(1+\sigma_\varepsilon^{-2}\right)} = \frac{\omega_\varepsilon\sigma_\varepsilon^{-2}}{\left(1+\sigma_\varepsilon^{-2}\right)} = \omega_{z\varepsilon}$. □

Lemma 4. *In the symmetric equilibrium with $\beta_{ijt} = \beta_{jt}$ for $K_{t+1} < W_t$,*

(i) Sentiment shocks ε_t affect the marginal benefit of information acquisition through three channels,

$$MB\left(\beta_{ijt}, \beta_{jt}\right)\big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{ \underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\text{Information-Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative}\text{Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}} \right\}.$$

(ii) Productivity shocks a_t increase the marginal benefit of information acquisition,

$$MB\left(\beta_{ijt}, \beta_{jt}\right)\big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\left(\frac{\alpha\theta-\theta+1}{\theta-1} + \frac{\alpha}{1-\alpha} + 1\right)a_t\right\}.$$

Proof of Lemma 4. *(i)* Assume $a_t = 0$ without loss of generality. The marginal benefit to increasing β_{ijt} is

$$\begin{aligned}
MB\left(\beta_{ijt}, \beta_{jt}\right)\big|_{\beta_{ijt}=\beta_{jt}} = &\int_{-\infty}^{\infty} g\left(a_{jt}\right)\int_{-\infty}^{\infty} f\left(\varepsilon_{jt}\right)\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} \alpha A_t^{\alpha_Y} \\
&* \frac{\left(A_{jt} - \mathbb{E}\left\{A_{jt}|s_{ijt} = z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^\theta di\right)^{\frac{\theta-1}{\theta}}} K_{t+1}^\alpha d\varepsilon_{jt} da_{jt},
\end{aligned} \tag{72}$$

where $g\left(a_{jt}\right)$ is the pdf of $a_{jt} \sim \mathcal{N}\left(0, \sigma_a^2\right)$ and $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$. The most immediate effect comes from changes to aggregate investment K_{t+1}^α. For $\delta R_{t+1} = 1$,

$$K_{t+1}^\alpha = \left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^\theta di\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}} \propto \exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{73}$$

The *Scale* channel is summarized by $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Next, the derivative of the probability of buying at $\beta_{ijt} = \beta_{jt}$ is

$$\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} = \phi\left(\varepsilon_{jt}\right)\left(\frac{\omega_{p,it}}{\sqrt{\beta_{jt}}}a_{jt} + \frac{\varepsilon_{jt}}{2\beta_{jt}} - \frac{\omega_{\varepsilon,it}\varepsilon_{jt} - \omega_{z\varepsilon,it}\varepsilon_t}{\sqrt{\beta_{jt}}} + \frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}\right),$$

where $\phi\left(\cdot\right)$ is the standard-normal pdf. Combine $f\left(\varepsilon_{jt}\right)$ with $\phi\left(\varepsilon_{jt}\right)$ using Lemma (2),

$$\phi\left(\varepsilon_{jt}\right)f\left(\varepsilon_{jt}\right) = \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}), \tag{74}$$

where $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of a fictional variable $\varepsilon_{jt} \sim \mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. The *Information-Sensitivity* channel is summarized by $\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}$. For the rest of the proof, substitute

$$\begin{aligned}\varepsilon_{jt} &= \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\\ d\varepsilon_{jt} &= \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}dx\end{aligned}$$

Substitute ε_{jt} out of $\left.\frac{\partial\mathcal{P}\{x_{ijt}=2\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}$,

$$\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} = \frac{\omega_{p,it}}{\sqrt{\beta_{jt}}}a_{jt} + \left(\frac{1}{2\beta_{jt}} - \frac{\omega_{\varepsilon,it}}{\sqrt{\beta_{jt}}}\right)\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{\varepsilon_t}{2\beta_{jt}\left(1+\sigma_\varepsilon^2\right)} + \frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}$$

Substitute ε_{jt} out of $\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}$,

$$\begin{aligned}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} &= \exp\left\{\omega_a a_{jt} + \omega_\varepsilon\varepsilon_{jt} - \omega_{z\varepsilon}\varepsilon_t + \frac{1}{2}\mathbb{V}\right\}\\ &= \exp\left\{\omega_a a_{jt} + \omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \cancel{\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2} - \omega_{z\varepsilon}\varepsilon_t} + \frac{1}{2}\mathbb{V}\right\}\\ &= \exp\left\{\omega_a a_{jt} + \omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{1}{2}\mathbb{V}\right\}\end{aligned}$$

Substitute ε_{jt} out of the firm-specific multiplier for firm-capital,

$$
\begin{aligned}
\frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}di\right)^{\frac{\theta-1}{\theta}}} &= \exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+(\theta-1)\,\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}-(\theta-1)\,\omega_\varepsilon\varepsilon_t-\frac{(\theta-1)\,\theta}{2}\left(\omega_a^2\sigma_a^2+\omega_\varepsilon^2\sigma_\varepsilon^2\right)\right\} \\
&\propto \exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x-(\theta-1)\,\omega_\varepsilon\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\varepsilon_t\right\} \\
&= \exp\left\{(\theta-1)\,\omega_a a_i+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\} \\
&= \exp\left\{(\theta-1)\,\omega_a a_i+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x\right\}\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\} \qquad (75)
\end{aligned}
$$

where I used Lemma 3 (iii) repeatedly. The *Size* channel is summarized through $\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\}$. It remains to show that there are no other terms in $MB\left(\beta_{ijt},\beta_{jt}\right)$ that depend on ε_t. It is sufficient to show that

$$
\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\tilde{f}\left(\varepsilon_{jt}\right)\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}di\right)^{\frac{\theta-1}{\theta}}}d\varepsilon_{jt}da_{jt}\stackrel{!}{=}0.
$$

Substituting ε_{jt} out leads to

$$
\begin{aligned}
&\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\cancel{\frac{1}{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}}\phi\left(x\right)\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}di\right)^{\frac{\theta-1}{\theta}}}\cancel{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}dx\,da_{jt} \\
&\propto\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\phi\left(x\right)\left(A_{jt}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}^{\theta}\right)dx\,da_{jt} \\
&=\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\phi\left(x\right)\left(\exp\left\{\left((\theta-1)\,\omega_a+1\right)a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\frac{(\theta-1)}{2}\mathbb{V}\right\}-\exp\left\{\theta\omega_a a_{jt}+\theta\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\frac{\theta}{2}\mathbb{V}\right\}\right)dx\,da_{jt} \\
&\propto\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\phi\left(x\right)\left(\exp\left\{\left((\theta-1)\,\omega_a+1\right)a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x-\frac{\mathbb{V}}{2}\right\}-\exp\left\{\theta\omega_a a_{jt}+\theta\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x\right\}\right)dx\,da_{jt} \\
&=\int_{-\infty}^{\infty}g\left(a_{jt}\right)\left(\exp\left\{\left((\theta-1)\,\omega_a+1\right)a_{jt}+\frac{(\theta-1)^2\,\omega_\varepsilon^2}{2}\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\frac{\mathbb{V}}{2}\right\}-\exp\left\{\theta\omega_a a_{jt}+\frac{\theta^2\omega_\varepsilon^2}{2}\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}\right)da_{jt} \\
&=\exp\left\{\frac{\left((\theta-1)\,\omega_a+1\right)^2}{2}\sigma_a^2+\frac{(\theta-1)^2\,\omega_\varepsilon^2}{2}\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\frac{\mathbb{V}}{2}\right\}-\exp\left\{\frac{\theta^2\omega_a^2}{2}\sigma_a^2+\frac{\theta^2\omega_\varepsilon^2}{2}\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}.
\end{aligned}
$$

It remains to show that

$$
\left((\theta-1)\,\omega_a+1\right)^2\sigma_a^2+(\theta-1)^2\,\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\mathbb{V}\stackrel{!}{=}\theta^2\omega_a^2\sigma_a^2+\theta^2\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}.
$$

Using Lemma 3 (i), the LHS is equal to

$$
\begin{aligned}
&\left(\left(\theta^2-2\theta+1\right)\omega_a^2+2\left(\theta-1\right)\omega_a\cancel{+1}\right)\sigma_a^2+\left(\theta^2-2\theta+1\right)\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\cancel{-\sigma_a^2}+\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\\
=&\left(\left(\theta^2-2\theta+2\right)\omega_a^2+2\left(\theta-1\right)\omega_a\right)\sigma_a^2+\left(\theta^2-2\theta+2\right)\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\\
=&2\left(\theta-1\right)\omega_a\sigma_a^2+\left(\theta^2+2\left(1-\theta\right)\right)\left(\omega_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)\\
=&2\left(\theta-1\right)\omega_a\sigma_a^2+\left(\theta^2+2\left(1-\theta\right)\right)\left(\omega_a\sigma_a^2\right)\\
=&\theta^2\omega_a\sigma_a^2.
\end{aligned}
$$

Using Lemma 3 (ii), the RHS is equal to

$$
\frac{\theta^2\omega_a^2}{2}\sigma_a^2+\frac{\theta^2\omega_\varepsilon^2}{2}\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}=\theta^2\omega_a\sigma_a^2.
$$

Combining both confirms the conjecture. The marginal benefit of information acquisition depends on ε_t only through the multiplicative effects in (73), (74) and (75), such that

$$
MB\left(\beta_{ijt},\beta_{jt}\right)\big|_{\beta_{ijt}=\beta_{jt}}\propto\exp\left\{\underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Scale}}\quad\underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information-Sensitivity}}\quad\underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Size}}\right\}.
$$

(ii) Follow the same strategy as in *(i)*. Start with the expressions for aggregate investment, K_{t+1}^α, and productivity $A_t^{\alpha_Y}$ in (72). For $\delta R_{t+1}=1$, they are equal to

$$
A^{\alpha_Y}K_{t+1}^\alpha=A^{\alpha_Y}\left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^\theta di\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}},
$$

where

$$
A_t^{\alpha_Y}\propto\exp\left\{\frac{\alpha\theta-\theta+1}{\theta-1}a_t\right\},\quad\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^\theta di\right)^{\frac{1}{\theta}}\propto\exp\left\{a_t\right\}.
$$

Putting both together yields

$$
A_t^{\alpha_Y}K_{t+1}^\alpha\propto\exp\left\{\left(\frac{\alpha\theta-\theta+1}{\theta-1}+\frac{\alpha}{1-\alpha}\right)a_t\right\}.
$$

Again, using substitution with

$$a_{jt} = \sqrt{\sigma_a^2} y + a_t,$$

It follows that

$$\begin{aligned} A_{jt} &= \exp\{a_t\} \exp\left\{\sqrt{\sigma_a^2} y\right\} \\ \mathbb{E}\{A_{jt}|s_{ijt} = z_{jt} z_{jt}\} &\propto \exp\{a_t\} \end{aligned}$$

which yields

$$\frac{\left(A_{jt} - \mathbb{E}\{A_{jt}|s_{ijt} = z_{jt} z_{jt}\}\right) \tilde{\mathbb{E}}\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\}^{\theta} di\right)^{\frac{\theta-1}{\theta}}} \propto \exp\{a_t\}$$

The change in the trading probability does *not* depend on a_t

$$\left.\frac{\partial \mathcal{P}\{d_{ij} = 2|a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\}}{\partial \beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} = \phi(\varepsilon_{jt}) \left(\frac{\omega_p}{\sqrt{\beta_{jt}}} \sqrt{\sigma_a^2} y + \frac{\varepsilon_{jt}}{2\beta_{jt}} - \frac{\omega_\varepsilon \varepsilon_{jt} - \omega_{z\varepsilon} \varepsilon_t}{\sqrt{\beta_{jt}}} + \frac{1}{2} \frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}\right).$$

Finally,

$$MB(\beta_{ijt}, \beta_{jt})|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\left(\frac{\alpha\theta - \theta + 1}{\theta - 1} + \frac{\alpha}{1-\alpha} + 1\right) a_t\right\}$$

$\square$

Proof of Proposition 4*.* *(i)* and *(ii)* follow from Lemma 4 *(i)*. The cutoff can be derived by taking the derivative with respect to ε_t,

$$\frac{\partial}{\partial \varepsilon_t}\left(-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)} - (\theta - 1)\omega_{s\varepsilon}\varepsilon_t + \frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right) \stackrel{!}{=} 0$$

$$\begin{aligned} \iff -\frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2} - (\theta-1)\omega_{s\varepsilon} + \frac{\alpha}{1-\alpha}\omega_{s\varepsilon} = 0 \\ \iff \frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2} = \left(\frac{1}{1-\alpha} - \theta\right)\omega_{s\varepsilon} \\ \iff \bar{\varepsilon} = \left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{1-\alpha} - \theta\right)\omega_{s\varepsilon} \end{aligned}$$

where $\omega_{s\varepsilon} = \frac{\sqrt{\beta_t}}{\sigma_a^{-2} + \beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$. For $\varepsilon_t < \bar{\varepsilon}$, information acquisition β_t is increasing in ε_t. For

$\varepsilon_t > \bar{\varepsilon}$, information acquisition β_t is decreasing in ε_t. $\square$

Proof of Proposition 5. Follows from Lemma 4 *(ii)*. $\square$

Proof of Proposition 6. *(i)* Using the result from Proposition 4 *(ii)* and the assumption that $\theta > \frac{1}{1-\alpha}$, it must be that positive sentiment shocks crowd-out information acquisition. Moreover, as $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$, it must be that the pass-through of aggregate sentiment shock $\omega_{s\varepsilon}$ is smaller for when the information choice is allowed to adjust compared to the fixed information level β^*. As a result, sentiment shocks are dampened by information acquisition in financial markets, as less precise information by itself leads to less investment and the sentiment shock is less powerful through a decrease in the pass-through.

(ii) $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$ guarantees that the pass-through of sentiment shocks goes faster to zero than the sentiment shock goes to infinity, i.e., the direct effect of sentiment shocks on investment disappears as shocks become arbitrarily large. Moreover, Lemma 4 *(i)* shows that through the information-sensitivity effect $\lim_{\varepsilon_t \to \infty} \beta_t(\varepsilon_t) = 0$. $\square$

Proof of Corollary 2. Follows directly from Proposition 6 *(ii)*. $\square$

Proof of Proposition 7. Follows from Proposition 5 and Assumption 2. $\square$

Proof of Proposition 8. The social planner's allocation is given by equalizing the marginal products of capital for each firm given the market signals $\{z_{jt}\}$. The maximization problem of the social planner for firm-capital allocation is therefore

$$\max_{K_{j1}} \mathbb{E}\left\{ \left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} |z_{jt} \right\} - R_1^{SP} K_{j1},$$

for some interest rate R_1^{SP}. The resulting first order condition for firm-capital is

$$K_{j1}^{SP} = \left(\left(R_1^{SP}\right)^{-1} \alpha Y_1^{\alpha_Y} \mathbb{E}\left\{A_{j0}|z_{j0}\right\} \right)^{\theta}.$$

Integrating on both sides yields

$$R_1^{SP} = \alpha Y_1^{\alpha_Y} \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj \right)^{\frac{1}{\theta}} \left(K_1^{SP}\right)^{-\frac{1}{\theta}}.$$

Substituting R_1^{SP} out of K_{j1}^{SP} yields (40). Following the same steps as in the proof for Proposition 3, leads to

$$Y_1^{SP} = A_0^{SP} K_1^{\alpha}, \quad \text{where } A_0^{SP} = \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj \right)^{\frac{\alpha}{\theta-1}}.$$

Substituting Y_1^{SP} out of the expression for R_1^{SP} then leads to (42). trader consumption follows as in 43 using R_1^{SP} instead of R_{t+1}. Finally, taking K_1^{SP} as given and plugging in Y_1^{SP} in (SP1), (44) follows after taking the derivative with respect to β_t. □

Proof of Proposition 9. Follows directly from (45) as $\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}=\beta_t} \neq MB^{SP}\left(\beta_t\right)$. □

Proof of Proposition 10. Show that the decentralized allocations coincide with the social planner's allocations. The proof follows the same steps as the derivation of the equilibrium in the main section. Households receive from firm j the dividend

$$\hat{\Pi}_{j1} = \tau^{Bias}\left(z_{j0}\right)\Pi_{j1} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}}\alpha Y_1^{\alpha_Y} A_{j0} K_{j1}^{\frac{\theta-1}{\theta}}$$

and expected dividends become

$$\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0}=z_{j0},z_{j0}\right\} = \alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\} K_{j1}^{\frac{\theta-1}{\theta}}.$$

The price is using $P_{j0}=K_{j1}$

$$P_{j0} = \frac{1}{R_1}\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0}=z_{j0},z_{j0}\right\} = \left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta}.$$

This allows to adjust the expression for expected dividends,

$$\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0}=z_{j0},z_{j0}\right\} = \left(\frac{1}{R_1}\right)^{\theta-1}\left(\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta},$$

which is then used in the expression for the interest rate R_1

$$\begin{aligned}
R_1 &= \frac{\int_0^1 \tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0}=z_{j0},z_{j0}\right\} dj}{\int_0^1 P_{j0}dj} \\
&= \left(\frac{1}{R_1}\right)^{\theta-1}\left(\alpha Y_1^{\alpha_Y}\right)^{\theta}\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj K_1^{-1} \\
R_1 &= \alpha Y_1^{\alpha_Y}\left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} K_1^{-\frac{1}{\theta}}.
\end{aligned}$$

Using this result again in the expression for the price yields

$$K_{j1}^{DE} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj} K_1 = K_{j1}^{SP}.$$

As in the main text, the entrepreneur chooses $\lambda_{jt} = \frac{\theta-1}{\theta}$. Plugging this into the expression for the interest rate R_1 and substituting $Y_1^{\alpha_Y}$ leads to

$$R_1 = \alpha \left(\int_0^1 \mathbb{E} \left\{ A_{j0} | z_{j0} \right\}^{\theta} dj \right)^{\frac{1}{\theta}} K_1^{\alpha-1} = R_1^{SP}.$$

This result also leads directly to $K_1^{DE} = K_1^{SP}$. Finally, the first order condition for information acquisition of trader ij is

$$MB\left(\beta_{ij0}, \beta_{j0}\right) = \frac{\partial IA^{DE}}{\partial \beta_{ij0}} = \tau^{Info}\left(\beta_{ij0}, \beta_{j0}\right) \frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}} = \frac{MB\left(\beta_{ij0}, \beta_{j0}\right)}{\left.\frac{\partial Y_1}{\partial \beta_0}\right|_{\beta_0=\beta_{ij0}}} \frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}}$$

$$\iff \left.\frac{\partial Y_1}{\partial \beta_0}\right|_{\beta_0=\beta_{ij0}} = \frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}}$$

which is the same first-order condition as in (44) and therefore $\beta_0^{DE} = \beta_0^{SP}$. $\square$

Proof of Corollary 3. *(i)* First, denote $\omega_{s\varepsilon} = \frac{\sqrt{\beta}}{\sigma_a^{-2} + \beta\left(1+\sigma_\varepsilon^{-2}\right)}$ as the weight on the correlated noise in the private signal. The transaction tax/subsidy $\tau^{Trans}(\varepsilon_0) = \exp\left\{-\omega_{s\varepsilon}\varepsilon_0\right\}$ leads to traders paying P_{j0} but only $\tau^{Trans} P_{j0}$ is collected by the entrepreneur. The transaction tax/subsidy is aimed to stabilize aggregate asset prices with respect to aggregate sentiment shocks. It is a tax when traders are exuberant and a subsidy when they are depressed. The proof follows the same steps as for Proposition 10 with the difference that $K_{j1} = \tau^{Trans} P_{j0}$ and therefore firm-capital is

$$K_{j1} = \left(\frac{1}{R_1} \alpha Y_1^{\alpha_Y} \tilde{\mathbb{E}} \left\{ A_{j0} | s_{ij0} = z_{j0}, z_{j0} \right\} \tau^{Trans}(\varepsilon_0) \right)^{\theta}$$

Following the same steps as before, the interest rate is

$$R_1 = \alpha Y_1^{\alpha_Y} \left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{j0} | s_{ij0} = z_{j0}, z_{j0} \right\}^{\theta} dj \right)^{\frac{1}{\theta}} \tau^{Trans}(\varepsilon_0) K_1^{-\frac{1}{\theta}}.$$

Since $\left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{j0} | s_{ij0} = z_{j0}, z_{j0} \right\}^{\theta} dj \right)^{\frac{1}{\theta}} \propto \exp\left\{\omega_{s\varepsilon}\varepsilon_0\right\}$, it follows that the transaction tax/subsidy $\tau^{Trans}(\varepsilon_0) = \exp\left\{-\omega_{s\varepsilon}\varepsilon_0\right\}$ keeps the interest rate R_1 from moving with the aggregate sentiment shock ε_t and stabilizes, therefore, aggregate investment with respect to sentiment shocks.

(ii) Similarly, allow now the transaction tax to vary with the share price,

$$\tau^{Trans}\left(P_{j0}\right)=\frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}}.$$

Same as before, the traders pays P_{j0} but only $\tau^{Trans}\left(P_{j0}\right)P_{j0}$ is collected by the entrepreneur. Firm-capital is then equal to

$$\begin{aligned}K_{j1}&=\left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}\tau^{Trans}\left(P_{j0}\right)\right)^{\theta}\\&=\left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta}\end{aligned}$$

It remains to show that also the interest rate R_{t+1} coincides with R_t^{SP}. The aggregate market values of the stock market and capital stock are given by

$$\begin{aligned}K_1&=\int_0^1\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}dj\left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\right)^{\theta}\\P_0=\int_0^1P_{j0}dj&=\int_0^1\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta-1}dj\left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\right)^{\theta}\\\Rightarrow P_t&=\frac{\int_0^1\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta-1}dj}{\int_0^1\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}dj}K_1\end{aligned}$$

This allows to write the interest rate and substitute P_t

$$\begin{aligned}R_1&=\alpha Y_1^{\alpha_Y}\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0},z_{j0}\right\}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta-1}dj\right)^{\frac{1}{\theta}}P_0^{-\frac{1}{\theta}}\\&=\alpha Y_1^{\alpha_Y}\left(\int_0^1\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}dj\right)^{\frac{1}{\theta}}K_1^{-\frac{1}{\theta}}.\end{aligned}$$

It follows the transaction tax $\tau^{Trans}\left(P_{j0}\right)$ corrects for the mispricing between firms and stabilizes aggregate investment with respect to the sentiment shock. □

Proof of Corollary 4. The marginal benefit of information acquisition after applying the tax/subsidy $\tau\left(a_{j0},z_{j0}\right)$ is

$$\widetilde{MB}\left(\beta_{ij0},\beta_{j0}\right)\propto\tilde{\mathbb{E}}\left\{2\frac{\partial\mathcal{P}\left\{x_{ij0}=2\right\}}{\partial\beta_{ij0}}\left(\tau\left(a_{j0},z_{j0}\right)\Pi_{j1}-\tilde{\mathbb{E}}\left\{\tau\left(a_{j0},z_{j0}\right)\Pi_{j1}|s_{ij0}=z_{j0},z_{j0}\right\}\right)\right\}.$$

Assume that a tax fulfills the following conditions

$$\tau\left(a_{j0}, z_{j0}\right) \geq (\leq)\, 1 \iff \Pi_{j1} \geq \tilde{\mathbb{E}}\left\{\Pi_{j1} | s_{ij0} = z_{j0}, z_{j0}\right\} \text{ and } \frac{\partial \mathcal{P}\left\{x_{ij0} = 2\right\}}{\partial \beta_{ij0}} \geq 0$$
$$\tau\left(a_{j0}, z_{j0}\right) \leq (\geq)\, 1 \iff \Pi_{j1} \leq \tilde{\mathbb{E}}\left\{\Pi_{j1} | s_{ij0} = z_{j0}, z_{j0}\right\} \text{ and } \frac{\partial \mathcal{P}\left\{x_{ij0} = 2\right\}}{\partial \beta_{ij0}} \leq 0$$

and for all z_{j0} there is at least some a_{j0} for which the inequalities are strict. In the first case, $\tau\left(a_{j0}, z_{j0}\right) \geq 1$ whenever the profits $\Pi_{j1} - \tilde{\mathbb{E}}\left\{\tau\left(a_{j0}, z_{j0}\right) \Pi_{j1} | s_{ij0} = z_{j0}, z_{j0}\right\}$ are positive and acquiring additional information leads to an increase of the probability of trading in that state. The same reasoning applies for $\tau\left(a_{j0}, z_{j0}\right) \leq 1$ and losses. This set of taxes increases $\widetilde{MB}(\beta_{ij0}, \beta_{j0})$. The reverse reasoning applies when $\tau\left(a_{j0}, z_{j0}\right) \geq 1$ for losses and $\tau\left(a_{j0}, z_{j0}\right) \leq 1$ for gains, which leads to a decrease in $\widetilde{MB}\left(\beta_{ij0}, \beta_{j0}\right)$.

□

Proof of Proposition 11. Let the social planner buy $d^{SP} \in (-1, 1)$ units of shares in all markets. The market clearing condition for market j becomes

$$2\left(1 - \Phi\left(\sqrt{\beta_{j0}}\left(\hat{s}\left(P_{j0}\right) - a_{j0}\right) - \varepsilon_{j0}\right)\right) = 1 - d^{SP},$$

Keeping position limits fixed, the exogenous demand d^{SP} changes the identity of the marginal trader. If the social planner purchases more assets, the marginal trader becomes more optimistic on average. The threshold signal becomes,

$$\hat{s}\left(P_{j0}, d^{SP}\right) = a_{j0} + \frac{\varepsilon_{j0} + \Phi^{-1}\left(\frac{1+d^{SP}}{2}\right)}{\sqrt{\beta_{j0}}}.$$

It follows immediately that asset purchases or sales with $d^{SP} = 2\Phi\left(-\varepsilon_0\right) - 1$ ensure that the marginal trader holds unbiased beliefs,

$$\hat{s}\left(P_{j0}, d^{SP}\right) = a_{j0} + \frac{\varepsilon_{j0} - \varepsilon_0}{\sqrt{\beta_{j0}}}.$$

As a result, the asset purchases/sells force the trader to correct also the private signal for the sentiment shock. It follows that prices are unbiased and aggregate investment is at the level in absence of the sentiment shock.

Traders expect to buy in equilibrium whenever $s_{ijt} > \hat{s}\left(P_{j0}, d^{SP}\right)$. Asset purchases/sells reverting the threshold for the private signal towards its level in absence of the aggregate sentiment shock $\varepsilon_0 = 0$, effectively undoing any change to the incentive to acquire information, because the trader thinks that she is unaffected by the sentiment shock and markets became unaffected by the sentiment shock.

$\square$