Exuberant and Uninformed: How Financial Markets (Mis-)Allocate Capital during Booms

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Abstract

I develop a macroeconomic model with a central emphasis on the informational role of financial markets. Economic agents save by purchasing financial claims on firms. Crucially, agents produce information about firm productivity to guide their trading decisions. In the aggregate, this information determines the financial market's ability to allocate more capital to productive firms and, thus, pins down total factor productivity (TFP). Using this framework, I study how information varies in response to fundamental (productivity) and non-fundamental (sentiment) macroeconomic shocks. Both lead to similar co-movements in output, asset prices, and investment but affect traders' information production differently. Productivity booms crowd in information and, thus, amplify the initial shock by further increasing TFP. In contrast, sentiment shocks, defined as waves of optimism or pessimism, crowd out information production, which dampens sentiment booms through a decrease in TFP. I show that information production in the competitive equilibrium is generally constrained inefficient for two reasons. First, each agent produces information to extract rents from others (*rent-extracting behavior*). Second, atomistic agents fail to internalize that their information production helps improve capital allocation and TFP, which is partially revealed through prices (*information spillover*). As an application, I show that asset purchase programs can be an effective way to address the financial market inefficiencies. Finally, looking through the lens of the model, the US dot-com boom of the late 1990s appears to have been driven by productivity, whereas the US housing boom of the mid 2000s was driven by sentiment.

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JEL Codes: D80, E32, E44, G14

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1 Introduction

Financial markets play a central role in allocating capital to its most productive uses. Yet, they do not always fulfill this role well. The last three decades, for instance, have been characterized by successive booms and busts in financial markets.[1] These cycles have been difficult to justify on fundamental grounds alone (Martin and Ventura, 2018). Against this backdrop, there are growing concerns that such booms lead to the deterioration of capital allocation, ultimately reducing aggregate productivity.[2] The general narrative is as follows: during booms, the perception is that all investments perform well. As a result, agents are less prone to produce information about specific investments, and markets eventually become less informative, thereby worsening the allocation of resources in the economy.[3] Though suggestive, this narrative is loose and cannot be fully evaluated without a theory of information production and its macroeconomic effects. The goal of this paper is to provide such a theory.

In this paper, I develop a tractable macroeconomic framework in which financial markets play the key role of aggregating information that is dispersed among economic agents. The framework's central feature is that information is endogenous, in a sense that agents can decide to engage in costly information production. The framework's novelty is to study the two-way feedback between macroeconomic conditions and agents' incentives to produce information.

I model a dynamic economy populated by firms with heterogeneous productivity and households, which consist of many traders. Households decide on borrowing and saving. Traders decide which firms to invest in, but they have imperfect information about firm productivity. To make their investment decisions, traders combine their private information with a public signal provided by financial markets, which effectively aggregates all traders' information.

The model is based on two core assumptions. First, traders agree on realizations of aggregate shocks but disagree about the distribution of firm productivity. Whereas the former part is for simplicity, the latter is central for motivating trade. In particular, traders' private information features both idiosyncratic and correlated noise. The idiosyncratic noise captures trader-specific information and drives disagreement. In contrast, the correlated noise stands

[1] For example, the dot-com bubble in the US and the housing bubbles in the US and Southern Europe.

[2] For instance, Gopinath et al. (2017) and García-Santana et al. (2020) have found that the credit and asset price boom in Southern Europe, preceding the global financial crisis, had coincided with a rise in capital misallocation; Doerr (2018) provides such evidence for the US. Relatedly, Borio et al. (2015) show that credit booms tend to also coincide with misallocation of labor.

[3] There are several studies that point to a decline in information production and quality in explaining a worsening of investment efficiency, for example, Asea and Blomberg (1998), Keys et al. (2010), and Becker, Bos, and Roszbach (2020).

for a common "sentiment" across traders.[4] Second, to incentivize information production in equilibrium by avoiding the well-known Grossman-Stiglitz paradox (see Grossman and Stiglitz, 1980),[5] traders are assumed to be overconfident.[6] Formally, each trader believes the noise in her private information to be entirely idiosyncratic, allowing her to exploit mispricing due to sentiment. In a nutshell, each trader believes that she is not prone to sentiments even though she understands that everyone else is.

I find that information production crucially depends on the state of the economy. In particular, I study how information production reacts to two types of macroeconomic shocks: sentiment and productivity. Sentiment shocks, defined as waves of optimism or pessimism, formally drive the correlated noise in traders' private information. Sentiment and productivity shocks lead to similar co-movements in output, investment, and asset prices. However, they affect information production differently. Information is central in my model, as more precise information strengthens the correlation between the size of a firm and its productivity, thereby raising allocative efficiency. Consequently, an economy with higher information production allocates more capital to more productive firms and has higher aggregate productivity.

In particular, information production increases in productivity but is non-monotonic in sentiment. Productivity increases information production due to a scale effect. Since high productivity raises the optimal size of a firm, it also boosts the benefits of producing more precise information about it. From the viewpoint of an individual trader, producing more information is valuable if it significantly impacts the trader's investment decisions. However, if sentiment regarding a specific firm is too high or too low, producing more information is likely to not yield much. In particular, even without precise information, a trader knows not to invest in firms where sentiment is high (i.e., firms that are "overvalued") and to invest in firms where sentiment is low (i.e., firms that are "undervalued"). Thus, extreme sentiments discourage the production of information.

Finally, while productivity booms are endogenously amplified by information production's

[4]From an economic standpoint, this sentiment is meant to capture a range of phenomena that drive asset prices away from their fundamental value, such as herding, network effects, social learning, extrapolative expectations, or bubbles (see Kindleberger and Aliber, 2015; Shiller, 2015, 2017).

[5]The Grossman-Stiglitz paradox states that no equilibrium exists in models of financial markets with costly information production without noise that keeps prices from being perfectly revealing. If prices reveal all information, traders have no reason to produce costly information. However, prices cannot be informative if traders do not produce information. Therefore, no equilibrium exists. Many models of informative financial markets (Grossman and Stiglitz, 1980; Kyle, 1985; Albagli, Hellwig, and Tsyvinski, 2021) circumvent this problem by introducing so-called noise traders. These agents are non-optimizing, which makes them difficult to embed in a general equilibrium model.

[6]Some form of noise in asset prices is indispensable to motivate trade and information production in financial markets. I formalize and micro-found such noise by introducing correlated noise in traders' signals and assuming that traders are overconfident.

effect on capital allocation, sentiment booms may be dampened. Productivity booms *crowd in* information and improve allocative efficiency, thereby further increasing productivity. Sentiment booms, however, *crowd out* information and worsen allocative efficiency, thereby decreasing productivity. My finding is consistent with the empirical evidence that booms can fuel resource misallocation (e.g., Gopinath et al., 2017; Doerr, 2018), suggesting that such booms are driven by sentiment. It also captures a dichotomy of booms as in Gorton and Ordoñez (2020) but stresses source of booms is the essential factor.

On the normative front, information production is too high or too low in the competitive equilibrium due to the presence of two externalities. First, there can be too much information because traders produce information to gain at the expense of other traders (*rent-extracting behavior*). Second, there can also be too little information because traders do not reap the benefits of improved capital allocation through collective information production (*information spillover*). Which effect dominates depends on whether the allocation of capital is important for aggregate productivity. For example, if firms produce similar goods, allocating capital to the most productive firms becomes exceedingly important. Yet, this is exactly when the competitive equilibrium features little information production.

Moreover, my model sheds light on two current policy debates. First, it suggests that policymakers should tax investment during sentiment-driven booms, which can be identified by increasingly synchronous asset price movements. This policy prescription of "leaning against the wind" is often criticized on informational grounds:[7] namely, it requires the policymaker to be able to distinguish sentiment- from productivity-driven booms in real-time (e.g., Mishkin et al., 2011). My model suggests that, although they look similar in many respects, both types of booms can be distinguished through their effects on information production. In particular, less informative asset prices display more synchronous movements, which can identify sentiment booms. In contrast, productivity booms lead to more asynchronous asset price movements.

A second policy debate refers to the effects of large-scale asset purchases by central banks. There is the widespread perception that asset purchases can distort prices and worsen the allocation of resources.[8] My model yields a simple yet robust insight: whether this concern is justified depends on whether asset purchases reduce or aggravate the aggregate mispricing of assets. By reducing the asset supply in the hands of traders, asset purchases change the marginal trader's identity and thus raise equilibrium prices. If asset prices were ini-

[7]See Cecchetti et al. (2000).

[8]See da Silva and Rungcharoenkitkul (2017), DNB (2017), Fernandez, Bortz, and Zeolla (2018), Borio and Zabai (2018), Acharya et al. (2019), and Kurtzman and Zeke (2020). In particular, the Dutch central bank argues in their 2016 annual report (DNB, 2017): "*The large-scale purchase programmes and the flood of liquid assets has set the risk compass in financial markets spinning, with misallocations as a result.*"

tially depressed due to low productivity, the common perception laid out above is correct. By distorting prices upward, asset purchases discourage information production and thus worsen the allocation of capital. However, if asset prices were initially depressed due to negative sentiment, asset purchases *reduce* aggregate mispricing. Indeed, by undoing the effects of negative sentiment, asset purchases fuel information production, thereby improving the resource allocation.

Finally, the paper makes a methodological contribution by providing a tractable macroeconomic model of information production and aggregation, where financial market informativeness plays an important role for macroeconomic dynamics. With a few exceptions,[9] the role of financial markets as aggregators of dispersed information has received little attention in macroeconomics.[10] The primary reason is that most standard models of informative financial markets rely on non-optimizing agents, such as noise traders, which are not straightforward to reconcile with general-equilibrium analysis. Instead, my model relies on a small behavioral deviation – overconfidence – which means that traders do not adequately perceive the idiosyncratic and correlated components in their signals. This misperception motivates them to produce costly information as they believe in having an informational edge over the market. This simple assumption is grounded on empirical evidence,[11] and it avoids the Grossman-Stiglitz paradox.

1.1 Literature Review

A recent literature studies the link between information production and the business cycle (Veldkamp, 2005; Ambrocio, 2020; Farboodi and Kondor, 2020; Chousakos, Gorton, and Ordoñez, 2020; Gorton and Ordoñez, 2020; Asriyan, Laeven, and Martin, forthcoming).[12] In contrast to Gorton and Ordoñez (2020), my model shows that the source of fluctuations is important for the relationship between the cycle and information production.

This paper builds on the literature on informative financial markets (Grossman and Stiglitz, 1980; Kyle, 1985; Vives, 2008; Albagli, Hellwig, and Tsyvinski, 2021). In this literature, limits to arbitrage keep arbitrageurs from fully eliminating mispricing and, there-

[9] Some exceptions are Peress (2014), David, Hopenhayn, and Venkateswaran (2016), and Straub and Ulbricht (2018).

[10] The idea of markets as aggregators of dispersed information dates back to Hayek (1945): *"The economic problem of society is (...) rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know. Or, to put it briefly, it is a problem of the utilization of knowledge which is not given to anyone in its totality."*

[11] See for example Eyster et al. (2018), Grimm and Mengel (2020), and Enke and Zimmermann (2019).

[12] See also Van Nieuwerburgh and Veldkamp (2006), Angeletos, Lorenzoni, and Pavan (2010), Ordoñez (2013), Gorton and Ordoñez (2014), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), and Straub and Ulbricht (2018) for related work.

fore, incentives to trade and produce information persist in equilibrium. My model's market microstructure is similar to Albagli, Hellwig, and Tsyvinski (2021). Whereas Albagli, Hellwig, and Tsyvinski (2021) use noise traders to keep prices from being fully revealing, I make the model more tractable by assuming instead that traders are overconfident. As I show, this innovation allows me to embed a noisy financial market into an otherwise standard macroeconomic model.

A strand of the literature uses the insight that prices can be informative to study the role of this information in economic decisions, as surveyed in Bond, Edmans, and Goldstein (2012). For example, secondary markets can be sources of information for managers (Holmström and Tirole, 1993; Dow and Gorton, 1997). Information is important in my model as a measure of allocative efficiency without any firms actively learning from prices. Similar to Dow, Goldstein, and Guembel (2017), I study the two-way feedback between the financial and real economy when traders produce information endogenously. A number of papers has brought this paradigm to macroeconomics (Peress, 2014; David, Hopenhayn, and Venkateswaran, 2016; Albagli, Hellwig, and Tsyvinski, 2017; Straub and Ulbricht, 2018; Asriyan, 2021). My contribution is to study the effects of aggregate shocks on information production and the allocation of capital. From a normative perspective, I show under which conditions information production is likely to be too high or too low in the competitive equilibrium.

There is ample empirical evidence that asset prices are indeed informative. See Morck, Yeung, and Yu (2013) for a survey on the literature that uses "non-synchronicity" as a measure of price-informativeness.[13] Morck, Yeung, and Yu (2000) found that more developed countries have stock markets that are more informative. Focusing instead on the cross-section of firms, Durnev, Morck, and Yeung (2004) found that non-synchronicity is positively related to the efficiency of corporate investment. More recently, Bai, Philippon, and Savov (2016) and Farboodi et al. (2020) have shown that prices have become better predictors of corporate earnings in the US since the 1960s. The latter emphasize that this has been mainly the case for large growth firms. Finally, Bennett, Stulz, and Wang (2020) provide evidence that price informativeness increases firm productivity. Price informativeness is closely related to the allocative efficiency of financial markets in my model.

The results of my model are broadly consistent with empirical evidence on how price informativeness varies over the business cycle. Dávila and Parlatore (2021) proposed an identification procedure to estimate price informativeness from price and earnings data, which is closely related to information production in my model.[14] Comparing fluctuations around the

[13]Non-synchronicity has been suggested by Roll (1988) as a measure of firm-specific information in asset prices. The main idea is that as the volatility of asset prices increasingly relates to firm-specific factors, prices also become increasingly informative about firms.

[14]Intuitively, relative price informativeness is the weight an otherwise uninformed observer puts on the

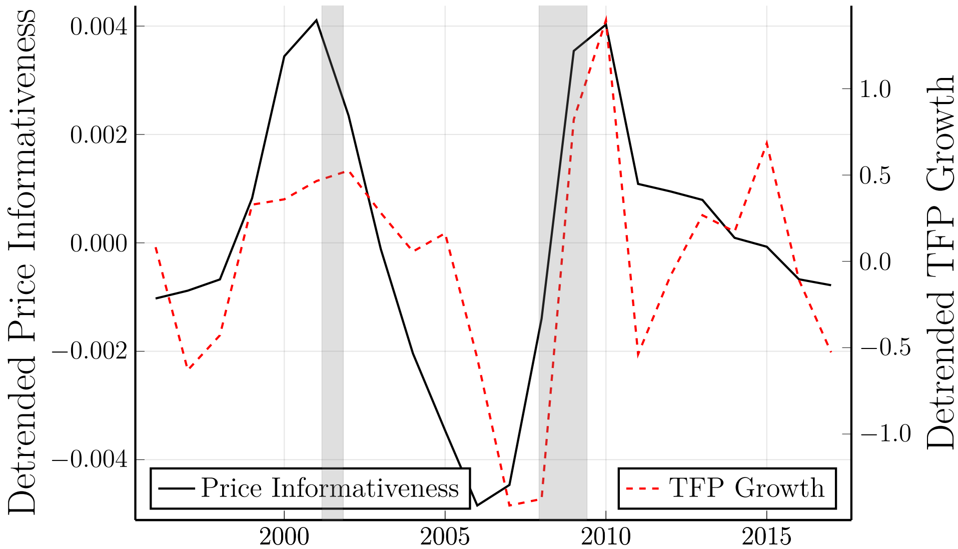

Figure 1: Detrended Price Informativeness and TFP Growth.

Figure 2: *

Notes: Price informativeness (black) as measured in Dávila and Parlatore (2021) and utilization-adjusted TFP growth (red) taken from the Federal Reserve of San Francisco following Basu, Fernald, and Kimball (2006). Grey bars indicate recessions following the NBER dating methodology. The time series have been detrended using a cubic time trend and smoothed with a two-year moving average. Through the lens of the model, productivity drove the expansion until 2001, as indicated by a rise in information and TFP growth. In contrast, sentiment drove the expansion from 2002 to 2008 as indicated by the decline in information and TFP growth.

corresponding trends for the US reveals a highly positive correlation between price informativeness and TFP growth, as can be seen in Figure 1. From 1995 to 2001, price informativeness and TFP growth were increasing, pointing to a productivity-driven expansion. In contrast, the housing boom from 2002 to 2008 eventually even led to a decline in TFP and a steep fall in price informativeness relative to trend, which indicates a sentiment-driven boom during these years. This interpretation is in line with Borio et al. (2015), who suggested that TFP growth slowed between 2002 and 2008 *because of the financial boom*, not despite it.

In my model, traders suffer from correlation neglect. This bias has been studied in the literature and documented repeatedly in experimental settings (Brandts, Giritligil, and Weber, 2015; Eyster et al., 2018; Enke and Zimmermann, 2019; Grimm and Mengel, 2020; Chandrasekhar, Larreguy, and Xandri, 2020). When receiving information from multiple sources, neglecting correlated noise in the signals can lead to an overly precise posterior. Therefore,

information embodied in the price relative to her prior. In my model, the weight only varies due to changes in the information production by traders.

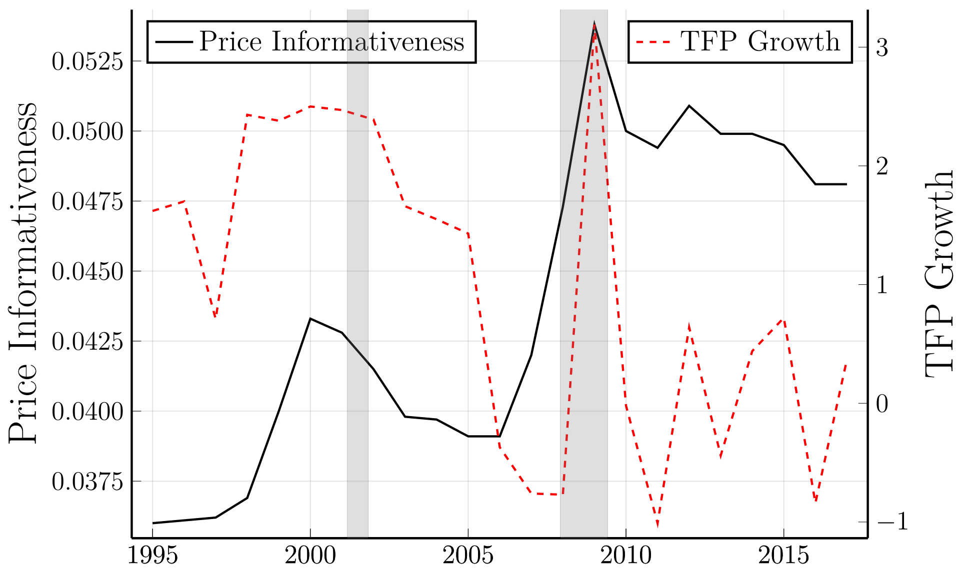

Figure 3: Price Informativeness and TFP Growth.

Figure 4: *
Notes: Price informativeness (black) as measured in Dávila and Parlatore (2021) and utilization-adjusted TFP growth (red) following Basu, Fernald, and Kimball (2006) taken from the Federal Reserve of San Francisco. Grey bars indicate recessions following the NBER dating methodology. Raw time series.

correlation neglect leads to overconfidence, which plays a central role in the literature on behavioral biases, especially in relation to financial markets (Glaser and Weber, 2010; Daniel and Hirshleifer, 2015).

Finally, a broad literature studies the role of sentiments in macroeconomics (for a survey, see Nowzohour and Stracca, 2020). There are different definitions of sentiments, ranging from self-fulfilling beliefs (Martin and Ventura, 2018; Asriyan, Fuchs, and Green, 2019) to news and noise shocks (Angeletos, Lorenzoni, and Pavan, 2010; Schmitt-Grohé and Uribe, 2012). In my model, sentiments are waves of non-fundamental optimism or pessimism. When a positive sentiment shock hits, agents become optimistic about productivity and vice versa.

2 Model

2.1 Households and Traders

The model is populated by overlapping generations of households indexed by $i \in [0,1]$. As is common in the New Keynesian literature, I assume that each household i consists of a unit mass of traders indexed by $ij \in [0,1] \times [0,1]$ (for example, see Blanchard and Galí, 2010). Households pool resources, borrow on behalf of traders, and distribute consumption equally, whereas traders individually maximize the utility for the household given by

$$U_{it} = C_{it,t} + \delta \mathbb{E}\left\{C_{it,t+1}\right\} - \int_0^1 IA(\beta_{ijt})dj, \tag{1}$$

where $C_{it,t}$ is youth consumption, $C_{it,t+1}$ is old age consumption, $\delta \in (0,1)$ is the discount factor, and $\int_0^1 IA(\beta_{ijt})dj$ are information production costs, which are introduced in more detail in a later section.

When young, traders each supply one unit of labor inelastically, receive wage W_t and buy shares of intermediate good firms in a competitive financial market. To avoid unbounded demands by risk-neutral traders, demand for each stock is limited to the interval $[\kappa_L, \kappa_H]$ where $\kappa_L \leq 0$ and $\kappa_H > 1$.[15] Traders also choose the precision β_{ijt} of a noisy signal of firm productivity to inform their trading decision subject to a utility cost $IA\left(\beta_{ijt}\right)$. Finally, the household lends and borrows through risk-free bonds with return R_{t+1}.

[15]See Dow, Goldstein, and Guembel (e.g., 2017) and Albagli, Hellwig, and Tsyvinski (2021) for similar approaches and Appendix B for a further elaboration.

2.2 Technologies

2.2.1 Final Good Sector

There are many identical final good firms owned by households. The production function for the final good, which also serves as the numéraire, is Cobb-Douglas over labor and a CES-aggregate of intermediate goods. Aggregate output is

$$Y_t = L^{1-\alpha}\left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}}dj\right)^{\frac{\alpha\theta}{\theta-1}}, \tag{2}$$

where $\theta \in (0,\infty)$ is the elasticity of substitution between varieties and α is the share of intermediate goods. Y_{jt} is an intermediate good produced by firm j. The final good can be consumed or invested in firm capital. L the labor supply and normalized to one.

2.2.2 Intermediate Good Sector

For each generation, there is a unit mass of intermediate good firms $j \in [0,1]$ with production function

$$Y_{jt} = A_{jt-1}^{\frac{\theta}{\theta-1}}K_{jt}, \tag{3}$$

where K_{jt} is firm capital and $\ln(A_{jt-1}) \overset{iid}{\sim} \mathcal{N}(a_{t-1},\sigma_a^2)$ is firm productivity. Note that time subscript $t-1$ is used as agents learn about firm productivity in the period prior to production. Capital takes time to build, such that investment takes place in t but production in $t+1$, and depreciates fully after production. Each firm sells a unit mass of claims to total firm-revenue to households and finances capital investment with the proceeds:[16]

$$P_{jt} = K_{jt+1}. \tag{4}$$

2.2.3 Information Structure

Trader ij is only active in the market for shares of firm j, for which she is an expert as she receives the signal

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt}+\varepsilon_{jt}}{\sqrt{\beta_{ijt}}}, \tag{5}$$

where $a_{jt} \overset{iid}{\sim} \mathcal{N}(a_t,\sigma_a^2)$ is firm productivity, $\eta_{ijt} \overset{iid}{\sim} \mathcal{N}(0,1)$ is idiosyncratic noise, $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}(\varepsilon_t,\sigma_\varepsilon^2)$ is correlated noise, interpreted as *sentiment*, and β_{ijt} is a information precision parameter chosen by trader ij.[17] Both idiosyncratic and correlated noise are *iid* over time

[16] See Appendix C for a micro-foundation and further discussion.
[17] See section 7.2 for the effect of uncertainty about aggregate shocks.

and across markets; idiosyncratic noise is also *iid* between traders. A high realization of η_{ijt} means that trader ij is optimistic about firm j relative to other traders in the same market. Similarly, a high realization of ε_{jt} means that all traders in market j are too optimistic.

Assumption 1 (***Overconfidence***)**.** *Trader ij believes the information structure to be*

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt}}{\sqrt{\beta_{ijt}}}$$
$$s_{-ijt} = a_{jt} + \frac{\eta_{-ijt} + \varepsilon_{jt}}{\sqrt{\beta_{-ijt}}}.$$

Following Assumption 1, traders believe that sentiment ε_{jt} drives the beliefs of all traders but not their own beliefs. As a result, traders are overconfident and willing to produce costly information to exploit mispricing induced through sentiment shocks ε_{jt}.[18] Finally, trader ij chooses the precision of her private signal β_{ijt} subject to a convex cost function $IA(\beta_{ijt})$ with standard properties $IA(0) = 0$, $IA'(0) = 0$, $IA''(\cdot) > 0$.

2.2.4 Aggregate Shocks

Two classes of shocks drive the economy. *Aggregate productivity shocks* move the mean of the distribution of firm-specific productivity shocks, $a_{jt} \sim \mathcal{N}(a_t, \sigma_a^2)$, and *aggregate sentiment shocks* drive the mean of firm-specific sentiment shocks, $\varepsilon_{jt} \sim \mathcal{N}(\varepsilon_t, \sigma_\varepsilon^2)$, similar to Angeletos, Lorenzoni, and Pavan (2010). The sentiment shock ε_t is meant to capture a range of phenomena that lead to non-fundamental price movements in financial markets, e.g., herding, informational cascades, social learning, bubbles, liquidity trading (see Kindleberger and Aliber, 2015; Shiller, 2015, 2017). I study economy-wide sentiment shocks as they affect cross-sectional misallocation of capital only through their effect on information production.[19]

For simplicity, traders perfectly observe aggregate shocks $\{a_t, \varepsilon_t\}$ before their information production decision, but firm-specific shocks $\{a_{jt}, \varepsilon_{jt}\}$ need to be learned. The laws of motion for the aggregate shocks are irrelevant for this setup, as the dynamic model is a repetition of static problems. It follows that the information set of trader ij consists of the private signal s_{ijt}, share prices $\{P_{jt}\}$ for all markets $j \in [0, 1]$, and the mean and variances of firm-specific shocks $\{a_t, \varepsilon_t\}$, i.e., $\mathcal{I}_{ij} = \{s_{ijt}, \{P_{jt}\}, a_t, \varepsilon_t\}$. In other words, traders have rational beliefs

[18] This assumption is necessary to avoid the Grossman-Stiglitz paradox (Grossman and Stiglitz, 1980). It states that informationally efficient markets are impossible in the absence of noise when information is costly. In that case, markets would already reveal all information and, therefore, destroy the incentive to produce costly information in the first place.

[19] Sector-specific sentiment shocks lead directly to an increase in capital misallocation, as aggregate output could be increased by reallocating capital away from the shocked sector. In this case, the results still go through on the sector level, as a sector-specific shock leads to an increase of capital misallocation inside the shocked sector. A more detailed analysis can be found in Appendix D.

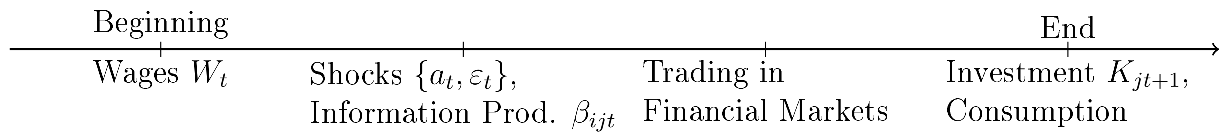

Figure 5: Intraperiod Timing.

about aggregates, but disagree about the productivity of intermediate firms based on public information in the forms of prices and private signals.

2.3 Timing

The timing is laid out in Figure 5. At the beginning of each period, young traders work in the final good sector and receive wage W_t. Then, traders choose the precision of their signal and the financial market opens. At the end of the period, both investment and consumption take place.

2.4 Notation

Traders think that their private signals do not contain correlated noise ε_{jt} as in Assumption 1. Therefore, expectations that condition on private signals are distorted and denoted by $\tilde{\mathbb{E}}(\cdot)$.

The determinants of functions are usually omitted to save on notation. For example, firm j's revenue is denoted by Π_{jt+1} instead of $\Pi\left(A_{jt}, K_{jt+1}, Y_{t+1}\right)$. Moreover, A_{jt} is indexed by t instead of $t+1$, as traders can learn about firm productivity in period t.

2.5 The Household's Problem and The Trader's Problem

Household i takes interest rate R_{t+1} as given and decides how much to borrow or lend. Furthermore, households are also prone to the behavioral bias of Assumption 1, in the that they each household believes that all its traders indeed have signals that are free of sentiment. However, households do not observe the private signals of traders. The household's problem is

$$
\max_{B_{it+1}} \quad C_{it,t} + \delta \tilde{\mathbb{E}}_t \left\{ C_{it,t+1} \right\} - \int_0^1 IA\left(\beta_{ijt}\right) dj \tag{P1.1}
$$

$$
s.t. \quad C_{it,t} = W_t - \int_0^1 x_{ijt} P_{jt} dj - B_{it+1} \tag{6}
$$

$$
C_{it,t+1} = \int_0^1 x_{ijt} \Pi_{jt+1} dj + R_{t+1} B_{it+1} \tag{7}
$$

$$
C_{it,t}, C_{it,t+1} \geq 0. \tag{8}
$$

Households optimally choose how to much lend or borrow subject to the budget constraints. The first constraint (6) states that consumption during youth is equal to wages W_t minus the costs of buying stocks $\int_0^1 x_{ijt} P_{jt} dj$ and saving through the bond market B_{it+1}. Constraint (7) states that old age consumption is equal to revenue $\int_0^1 x_{ijt} \Pi_{jt+1} dj$ plus income from lending on the bond market $R_{t+1} B_{it+1}$. Although household i is overly optimistic about the return of its portfolio due to overconfidence, each household correctly values the portfolio of all other households. Therefore, limiting borrowing by the natural borrowing constraint as in (8) rules out defaulting on any borrowing through bonds.

Household i's optimal saving decision is given by

$$
B_{it+1} \begin{cases} = -\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}} & \text{if } R_{t+1} < \frac{1}{\delta} \\ \in \left[-\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}}, W_t - \int_0^1 x_{ijt} P_{jt} dj \right] - & \text{if } R_{t+1} = \frac{1}{\delta} \\ = W_t - \int_0^1 x_{ijt} P_{jt} dj & \text{if } R_{t+1} > \frac{1}{\delta} \end{cases} . \tag{9}
$$

If the interest rate R_{t+1} is below $\frac{1}{\delta}$, it is optimal to borrow as much as possible. If the interest is equal to $\frac{1}{\delta}$, household i is indifferent between borrowing and saving. Finally, if the interest rate is above $\frac{1}{\delta}$, then it is optimal to save as much as possible. Plugging (6) and (7) into (P1.1) and using the solution for the saving decision (9) yields trader ij's problem

$$
\max_{\beta_{ijt}} \quad \tilde{\mathbb{E}}_t \left\{ \lambda_t \max_{x_{ijt}} \tilde{\mathbb{E}} \left\{ x_{ijt} \left(\frac{1}{R_{t+1}} \Pi_{jt+1} - P_{jt} \right) | s_{ijt}, P_{jt} \right\} \right\} - IA\left(\beta_{ijt}\right) \tag{P1.2}
$$

$$
s.t. \quad x_{ijt} \in \left[\kappa_L, \kappa_H\right] \tag{10}
$$

$$
\beta_{ijt} \geq 0, \tag{11}
$$

where $\lambda_t = \max\left\{1, \delta R_{t+1}\right\}$ and terms that do not depend on the decision by trader ij were dropped. The problem is split into two parts, which are solved in reverse chronological order. Given information production β_{ijt} and realizations of the private signal s_{ijt} and price

P_{jt}, trader ij chooses demand x_{ijt} for share j subject to the position limits (10). Using the solution to the trading problem, trader ij decides on the information precision β_{ijt} to increase the likelihood of trading profitably subject to a non-negativity constraint. Trader ij can use the household i's pooled resources and borrow through the household for trading. The term λ_t reflects that the value of an additional unit of wealth during youth may be above one.

3 Equilibrium Characterization

3.1 Input Markets

Wages and intermediate good prices are determined competitively,

$$W_t = \frac{\partial Y_t}{\partial L} = (1-\alpha)\, Y_t \tag{12}$$

$$\rho_{jt} = \frac{\partial Y_t}{\partial Y_{jt}} = \alpha Y_t^{\alpha_Y} Y_{jt}^{-\frac{1}{\theta}}, \tag{13}$$

where $\alpha_Y = \frac{\alpha\theta-\theta+1}{\alpha\theta}$. Wages are equal to a share $(1-\alpha)$ of output. The price for intermediate good j is downward sloping in the quantity produced of the same good. Finally, the revenue of intermediate good firm j is given by

$$\Pi_{jt+1} = \rho_{jt+1} Y_{jt+1}. \tag{14}$$

3.2 Trader's Decisions

Trading If price P_{jt} exceeds expectations of revenue Π_{jt+1} using the interest rate on bonds R_{t+1} as the benchmark rate, trader ij sells $-\kappa_L$ shares; when these values coincide trader ij is indifferent between buying and selling. When expectations exceed the price, trader ij buys κ_H shares:

$$x\left(s_{ijt}, P_{jt}\right) = \begin{cases} \kappa_L & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} < P_{jt} \\ \in [\kappa_L, \kappa_H] & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} = P_{jt} \\ \kappa_H & \text{if } \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt}, P_{jt}\right\} > P_{jt} \end{cases}. \tag{15}$$

Information Production As laid out in (15), the trading decision is driven by the realization of the private signal s_{ijt} relative to price P_{jt}. Consequently, trader ij chooses information precision β_{ijt} to improve her ability to identify profitable trading opportunities. A central object in this context is the subjective probability of buying conditional on realizations of productivity a_{jt} and sentiment ε_{jt}, trader ij's information choice β_{ijt}, and the symmetric choice of all other traders in the market β_{jt}. Taking expectations with respect to the realizations of idiosyncratic noise, η_{ijt}, yields the probability of buying,

$$\mathcal{P}\left\{x_{ijt}=\kappa_H|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}=\int_{-\infty}^{\infty}\phi\left(\eta_{ijt}\right)1_{\frac{1}{R_{t+1}}\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt},P_{jt}\}>P_{jt}}d\eta_{ijt}, \tag{16}$$

where $\phi(\cdot)$ is the standard-normal pdf.[20]

The first-order condition for the information production decision is obtained after plugging (15) into (P1.2). Evaluating the expectations with respect to the realizations of the idiosyncratic noise η_{ijt} and taking the symmetric information production decisions of all other traders as given ($\beta_{-ijt}=\beta_{jt}$), leads to the first-order condition:

$$\begin{aligned}\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)&=\lambda_t\tilde{\mathbb{E}}_t\left\{\left(\kappa_H-\kappa_L\right)\underbrace{\frac{\partial\mathcal{P}\left\{x_{ijt}=\kappa_H|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}}{\partial\beta_{ijt}}}_{\textit{Change in the Probability of Buying}}\underbrace{\left(\frac{1}{R_{t+1}}\Pi_{jt+1}-P_{jt}\right)}_{\textit{Rents}}\right\}\\&=IA'\left(\beta_{ijt}\right).\end{aligned} \tag{17}$$

The marginal benefit of producing information consists of two parts. First, the probability of buying in state $(a_{jt},\varepsilon_{jt})$ given information choices (β_{ijt},β_{jt}). Second, trading rents given by the difference between the net present value of firm revenue minus the price of the stock.

3.3 Financial Market

Market-Clearing At the symmetric equilibrium ($\forall j:\beta_{ijt}=\beta_{jt}$), traders buy κ_H shares whenever their private signals are above some threshold, $\hat{s}\left(P_{jt}\right)$, are indifferent between buying and selling when their private signals coincide with the threshold, and sell otherwise. After normalizing the supply of shares in each market j to one, the market-clearing condition becomes

$$\kappa_H\left(1-\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right)-a_{jt}\right)-\varepsilon_{jt}\right)\right)-\kappa_L\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right)-a_{jt}\right)-\varepsilon_{jt}\right)=1, \tag{18}$$

where $\Phi(\cdot)$ is the standard normal cdf. The threshold $\hat{s}\left(P_{jt}\right)$ can be solved for directly,

$$\hat{s}\left(P_{jt}\right)=a_{jt}+\frac{\varepsilon_{jt}+\Phi^{-1}\left(\frac{\kappa_H-1}{\kappa_H+\kappa_L}\right)}{\sqrt{\beta_{jt}}}. \tag{19}$$

[20] A more detailed derivation can be found in Appendix A.

Price Signal Traders learn from prices, which is equivalent to observing a noisy signal of the form

$$z_{jt} = \hat{s}\left(P_{jt}\right) - \frac{\Phi^{-1}\left(\frac{\kappa_H - 1}{\kappa_H + \kappa_L}\right)}{\sqrt{\beta_{jt}}} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}. \tag{20}$$

When the price P_{jt} is high, traders realize that this can be due to two reasons: either firm j is productive (high a_{jt}) or other traders are very optimistic (high ε_{jt}). Therefore, prices are a noisy signal of firm productivity. The combination of dispersed information and position limits for asset demand ensure that the signal is normally distributed as $z_{jt} \sim \mathcal{N}(a_{jt}, \sigma_\varepsilon^2/\beta_{ijt})$ for all values of κ_L and κ_H. I call z_{jt} the *price signal* and expectations condition on z_{jt} instead of P_{jt}.

A crucial object in my analysis is the precision of the price signal $\beta_{jt}\sigma_\varepsilon^{-2}$, also referred to as *price informativeness* in the literature. If $\beta_{jt}\sigma_\varepsilon^{-2}$ is high, financial markets efficiently aggregate information and asset prices are informative about firm productivity. As a result, productive firms receive on average more capital, which improves the capital allocation through financial markets. I focus on the endogenous component β_{jt}.

As is evident now, the values of κ_H and κ_L do not matter for the price signal z_{jt}. They only pin down the identity of the marginal trader, which has a predictable effect on the price. For instance, the marginal trader is relatively optimistic for $\kappa_H - \kappa_L > 2$, which means that the price is set by a trader who received a private signal with positive idiosyncratic noise ($\eta_{ijt} > 0$). As a result, the price would be upward biased.[21] Choosing $\kappa_H = 2$ and $\kappa_L = 0$ ensures that the choice of position limits does not introduce a bias in share prices as the marginal trader has unbiased beliefs ($\eta_{ijt} = 0$).

The following proposition shows that the described equilibrium is unique. Moreover, the price P_{jt} is equal to the valuation of the *marginal trader* who is just indifferent between buying or not buying and who observed the private signal $s_{ijt} = z_{jt}$. Any trader who is more optimistic than the marginal trader ($s_{ijt} > z_{jt}$) buys two shares, whereas more pessimistic traders buy nothing.

Proposition 1. *Observing P_{jt} is equivalent to observing the signal (20) whenever K_{jt+1} is non-decreasing in z_{jt}. In the unique equilibrium, in which demand $x\left(s_{ijt}, P_{jt}\right)$ is non-increasing in P_{jt}, the price is equal to the valuation of the trader with the private signal $s_{ijt} = z_{jt}$,*

$$P\left(z_{jt}\right) = \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{21}$$

[21] This mechanism plays an important role in Fostel and Geanakoplos (2012) and Simsek (2013) and is treated more in-depth in Appendix B.

3.4 Bond and Capital Market

The net supply of bonds is equal to zero, $\int_0^1 B_{it+1} di = 0$. Moreover, as all households are ex-ante identical, positions in bond markets are zero for all households, $\forall i : B_{it+1} = 0$. There is no excess demand or supply for bonds whenever the return on bonds R_{t+1} is equal to the return that traders expect to earn on the stock market. This is the case whenever

$$R_{t+1} = \frac{\int_0^1 \tilde{\mathbb{E}} \left\{ \Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt} \right\} dj}{\int_0^1 P_{jt} dj}, \tag{22}$$

which is derived by integrating (21) on both sides.

The aggregate value of the stock market is equal to the aggregate capital stock as all revenue from financial markets is invested by firms as follows from aggregating (4),

$$\int_0^1 P_{jt} dj = K_{t+1}. \tag{23}$$

3.5 Equilibrium Definition

In equilibrium, all traders choose the same information precision for all markets ($\forall ij : \beta_{ijt} = \beta_t$) and expect all other traders to choose the same.

Definition 1. *A competitive equilibrium consists of prices $\{W_t, \rho_{jt+1}, P_{jt}, R_{t+1}\}$ and allocations $\{B_{it+1}, x_{ijt}, \beta_{ijt}, K_{jt+1}\}$ such that:*

1. *Given prices $\{W_t, \rho_{jt+1}, P_{jt}, R_{t+1}\}$ and allocations $\{x_{ijt}, \beta_{ijt}\}$, B_{it+1} solves the household's problem P1.1.*

2. *Given prices $\{P_{jt}, R_{t+1}\}$ and allocations $\{B_{it+1}, \beta_{jt}, K_{jt+1}\}$, $\{x_{ijt}, \beta_{ijt}\}$ solve the trader's problem P1.2.*

3. *Prices are such that markets for labor, intermediate goods, shares, bonds, and capital clear, i.e., (12), (13), (18), (22) and (23) hold.*

4 Properties of the Equilibrium

In the following, I work out the properties of the equilibrium abstracting from the information production decision until the next section. I focus on how the allocation of capital can be expressed in terms of beliefs of the marginal trader and how these beliefs respond to both idiosyncratic and aggregate shocks. Next, I demonstrate how the allocation of capital through the stock market determines total factor productivity, which depends on the information choice. Finally, I show that the market allocation is distorted and derive the constrained efficient allocation.

As shown in (21), the beliefs of the marginal trader determine share prices. Therefore, they play a central role for the allocation of capital both in the cross-section and aggregate. The marginal trader's expectations are a weighted sum of the realization of both idiosyncratic and aggregate productivity and sentiment shocks,

$$\begin{aligned}\ln \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\} = {} & \omega_p\left(\beta_{jt}\right)a_t+\omega_a\left(\beta_{jt}\right)a_{jt}+\omega_\varepsilon\left(\beta_{jt}\right)\left(\varepsilon_{jt}-\varepsilon_t\right)+\omega_{s\varepsilon}\left(\beta_{jt}\right)\varepsilon_t \\ & +\frac{1}{2}\mathbb{V}_{jt}. \end{aligned} \tag{24}$$

The weights $\{\omega_p\left(\beta_{jt}\right),\omega_a\left(\beta_{jt}\right),\omega_\varepsilon\left(\beta_{jt}\right),\omega_{s\varepsilon}\left(\beta_{jt}\right)\}$ depend on information production β_{jt}. $\mathbb{V}_{jt}$ is posterior uncertainty of the marginal trader.[22]

The first two terms capture the effect of aggregate and idiosyncratic productivity shocks. If traders do not produce information ($\beta_{jt}=0$), traders rely solely upon their prior a_t ($\omega_p(0)=1$ and $\omega_a(0)=0$). As traders produce more information, they shift weight from their prior to the realization of firm productivity ($\lim_{\beta_{jt}\to\infty}\omega_a\left(\beta_{jt}\right)=1$). This leads to a higher sensitivity of the allocation of capital to firm-specific productivity shocks and improves the allocative efficiency of financial markets.

In contrast to the weights on productivity shocks, the weights on sentiment shocks are hump-shaped in β_{jt}. If traders do not produce information, they do not have a signal to learn from and, therefore, their expectations cannot be moved by noise ($\omega_\varepsilon(0)=\omega_{s\varepsilon}(0)=0$). For perfect information, traders receive signals that do not contain noise in the first place ($\lim_{\beta_{jt}\to\infty}\omega_\varepsilon(\beta_{jt})=\omega_{s\varepsilon}(\beta_{jt})=0$). If β_{jt} goes to either extreme, both idiosyncratic and aggregate sentiment shocks do not affect the beliefs of traders.

The aggregate sentiment shock ε_t moves the beliefs of traders although ε_t is common knowledge. This effect stems from the behavioral bias in Assumption 1. Traders correct the price signal z_{jt} for the aggregate sentiment shock but mistakenly believe that their private signal s_{ijt} is unaffected by sentiment and, therefore, do not correct their private signal in a similar way.

4.1 Capital Allocation and TFP

The results so far can be combined to derive the allocation of capital and total factor productivity in equilibrium as captured in the following proposition.

[22]See Appendix A for derivations.

Proposition 2 (***Market Allocation***)**.** *Under the market allocation,*

(i) firm capital is given by

$$K_{jt+1} = \frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj} K_{t+1}. \tag{25}$$

(ii) the aggregate production function is

$$Y_t = A\left(a_{t-1}, \beta_{t-1}\right) K_t^{\alpha} \tag{26}$$

with total factor productivity

$$\ln A\left(a_{t-1}, \beta_{t-1}\right) = \underbrace{\frac{\alpha\theta}{\theta - 1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right)}_{exogenous} + \underbrace{\kappa^a\left(\beta_{t-1}\right)\sigma_a^2 - \kappa^{\varepsilon}\left(\beta_{t-1}\right)\sigma_{\varepsilon}^2}_{allocative\ efficiency}, \tag{27}$$

where $\kappa^a(\beta_{t-1})$ is increasing in β_{t-1} and $\kappa^{\varepsilon}(\beta_{t-1})$ is hump-shaped in β_{t-1}.

(iii) $A\left(a_{t-1}, \beta_{t-1}\right)$ is taking its minimum for some $\beta_{t-1} > 0$ if $\sigma_{\varepsilon}^2 > 1$.

(iv) $A\left(a_{t-1}, \beta_{t-1}\right)$ is monotonically increasing in β_{t-1} if $\sigma_{\varepsilon} \leq 1$.

The proposition's first part highlights that more capital is allocated to firms with higher realizations of the price signal z_{jt} whether it is driven by sentiment or productivity. Moreover, firm capital for all firms is proportional to aggregate investment K_{t+1}. Consequently, total factor productivity (TFP) has both an exogenous and endogenous component. The exogenous component is related to the realization of the aggregate productivity shock a_t, which mechanically increases the productivity of all firms. The endogenous component captures instead the allocational efficiency of financial markets, which is determined by aggregate information production β_t.

However, the market does not allocate capital efficiently given the available information. As traders are overconfident, expectations in (25) condition also on the private signal s_{ijt}, although it is uninformative after observing z_{jt}. In other words, P_{jt} behaves as if the precision of the market signal z_{jt} was $\beta_{jt} + \beta_{jt}\sigma_{\varepsilon}^{-2}$, although its true precision is $\beta_{jt}\sigma_{\varepsilon}^{-2}$. Therefore, the price *overreacts* to the price signal z_{jt}.[23]

[23] This distortion has been studied intensively in Albagli, Hellwig, and Tsyvinski (2011a, 2015, 2021) and is called the "information aggregation wedge." Its general equilibrium implications are studied in Albagli, Hellwig, and Tsyvinski (2017). In contrast to this paper, their model features a combination of rational and noise traders. Therefore, the information aggregation wedge does not require a behavioral price-setting traders. Furthermore, it arises in any informative financial market model in which traders learn from both a heterogeneous private signal and the price. It does not arise in models in which the information set of informed agents is homogeneous (Grossman and Stiglitz, 1980) or in models where traders do not observe the price before submitting market orders (Kyle, 1985). In the former case, informed agents cannot learn

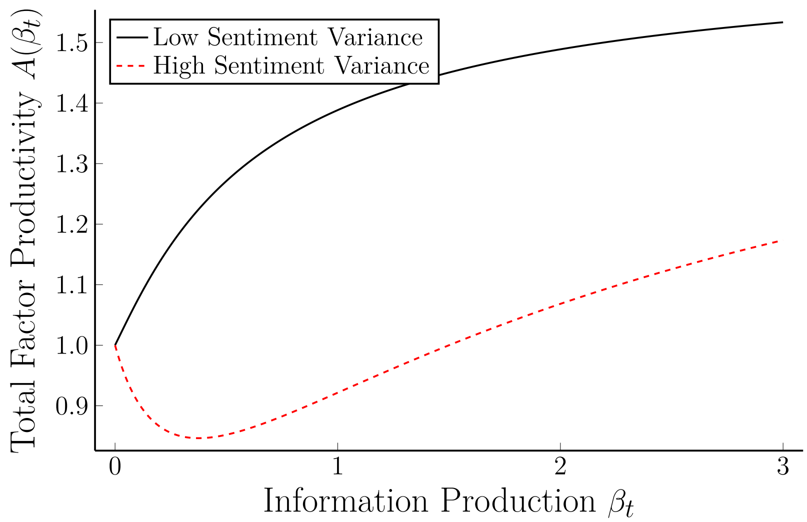

Figure 6: Total Factor Productivity and Information Production.

Figure 7: *

Notes: Total factor productivity as defined in (27). If the variance of sentiment shocks σ_ε^2 is sufficiently large, financial markets may worsen allocative efficiency relative to the case in which capital is equally distributed between firms ($\beta_t = 0$).

This distortion can be so severe that an increase in information production β_t leads to a decrease in TFP, as stated in Proposition 2 (iii) and seen in Figure 6. As traders produce more precise information, they also wrongly put more weight on their private signal. The overall effect on TFP depends on the balance between the beneficial effect of an increase in price informativeness $\beta_t \sigma_\varepsilon^{-2}$ and an increased weight on the private signal.

This price distortion leads to ex-ante misallocation of capital, i.e., output can be increased by reallocating capital between firms given the same publicly available information $\{z_{jt}\}$. A social planner would use the available information efficiently, leading to the *constrained efficient allocation* summarized in the following proposition.

Proposition 3 (***Constrained Efficient Allocation***)**.** *Under the constrained efficient allocation,*

(i) firm capital is

$$K_{jt+1}^{eff} = \frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}^\theta}{\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^\theta dj} K_{t+1}. \tag{28}$$

anything from the price, and in the latter, it is not possible to learn from the price before trading. Both of these models restrict the analysis to linear models, whereas non-linearity arises naturally in macroeconomic models; therefore, a different model is used here. See also Vives (2017) for an in-depth analysis in a linear setting.

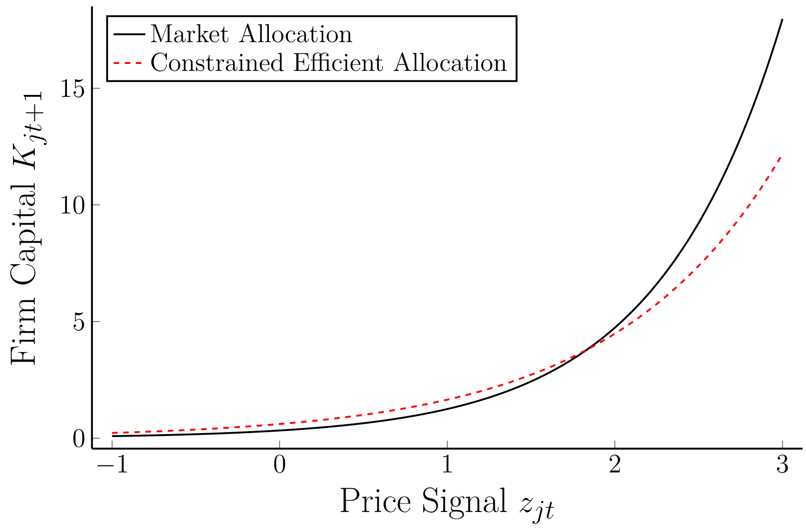

Figure 8: Market and Constrained Efficient Capital Allocation.

Figure 9: *

Notes: Market allocation of capital K_{jt} as in (25) and the constrained efficient allocation K_{jt}^{eff} as in (28).

(ii) total factor productivity is

$$\ln A_{t-1}^{eff} = \ln \left(\int_0^1 \mathbb{E} \left\{ A_{jt-1} | z_{jt-1} \right\}^\theta dj \right)^{\frac{\alpha}{\theta - 1}} = \underbrace{\frac{\alpha\theta}{\theta - 1} \left(a_{t-1} + \frac{\sigma_a^2}{2} \right)}_{exogenous} + \underbrace{\frac{\alpha\theta}{2} \omega_a^{eff} \sigma_a^2}_{allocative\ efficiency}, \tag{29}$$

where $\omega_a^{eff} = \frac{\beta_{t-1}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}$. A_{t-1}^{eff} *is monotonically increasing in* β_{t-1}.
(iii) $A^{eff}(a_{t-1}, \beta_{t-1}) \geq A(a_{t-1}, \beta_{t-1})$, *with strict inequality for* $\beta_{t-1} \in (0, \infty)$.

The *constrained efficient allocation* assigns the correct precision $\beta_t \sigma_\varepsilon^{-2}$ to the price signal z_{jt}. Relative to the market allocation, the constrained efficient allocation redistributes capital from firms that were previously too large to firms that were too small, as seen in Figure 9. Moreover, total factor productivity $A^{eff}(a_{t-1}, \beta_{t-1})$ is monotonically increasing in aggregate information production β_{t-1} under the constrained efficient allocation, because the distortion due to traders' overconfidence is removed.

The following corollary provides conditions under which the market and constrained efficient allocation coincide.

Corollary 1. *The market allocation and constrained efficient allocation* ($K_{jt} = K_{jt}^{eff}$) *coincide if*

(i) symmetric information production β_t *goes to zero or infinity.*

(ii) the variance of firm-specific productivity shocks σ_a^2 goes to zero or infinity.

(iii) the variance of firm-specific sentiment shocks σ_ε^2 goes to zero.

As Corollary 1 shows, the behavioral bias disappears both when households have perfect information or when households have no information at all ($\beta_{jt} \in \{0, \infty\}$), as in both cases traders put zero weight on their private signal. There is also no distortion if the prior is arbitrarily noisy ($\sigma_a^2 \to \infty$), as in that case both the market and the efficient allocation put full weight on the price signal z_{jt}. If the prior is arbitrarily precise ($\sigma_a^2 \to 0$), the weight is zero for both. Finally, if the variance of sentiment shocks goes to zero, financial markets perfectly aggregate information as the price signal z_{jt} converges to firm productivity a_{jt} and private signals become irrelevant.

4.2 Aggregate Investment

Aggregate investment is in one of two regions. In the first region, traders consume during youth and investment is pinned down by $R_{t+1} = \frac{1}{\delta}$. In the second region, the interest rate is so high ($R_{t+1} > \frac{1}{\delta}$) that traders exhaust their wages for investment. Finally, $R_{t+1} < \frac{1}{\delta}$ cannot arise in equilibrium as investment would collapse to zero and the interest rate R_{t+1} would go to infinity. Taken together, aggregate investment is equal to

$$K_{t+1} = \min \left\{ \left(\alpha \delta A_t^{\alpha_Y} \left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^\theta dj \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\alpha}}, W_t \right\}. \tag{30}$$

Aggregate shocks and information production determine investment in the elastic region. Aggregate productivity and sentiment shocks increase investment, as traders expect all firms to be more productive. An increase in aggregate information production β_t has ambivalent effects, as it may increase or decrease TFP A_t and the average expectations of firm productivity $\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^\theta dj$ may be hump-shaped in β_t.

5 Main Results

As laid out in the prior section, the model has several sources of non-monotonicity. Not only may TFP be locally decreasing in aggregate information production β_t, but also aggregate investment K_{t+1} may be non-monotonic in β_t. These pathological cases are not due to a friction that can easily be removed but, rather, arise through the imperfect aggregation of information in a market with dispersed information.

Economic intuition tells us that better information usually leads to better economic outcomes. Indeed, the model allows for this intuition to hold by restricting the parameter space. As Corollary 1 shows, the distortion vanishes as the variance of firm-specific sentiment shocks σ_ε^2 goes to zero. As follows from Propositions 2 (iv) and later from the proof of Proposition 10, total factor productivity $A(a_t, \beta_t)$ and aggregate investment K_{t+1} are increasing in β_t for a neutral stance of sentiment ($\varepsilon_t = 0$) when $\sigma_\varepsilon^2 \leq 1$.[24] For the following analysis, I assume that more information production has beneficial effects as captured in the following Assumption.

Assumption 2. $\sigma_\varepsilon^2 \leq 1$, *such that*

(i) $\frac{\partial A(a_t,\beta_t)}{\partial \beta_t} \geq 0$.

(ii) $\left.\frac{\partial K_{t+1}(\beta_t)}{\partial \beta_t}\right|_{\varepsilon_t=0} \geq 0$.

5.1 Aggregate Shocks and Information Acquisition

Recent experiences during stock and credit booms have raised concerns about increasing capital misallocation during these episodes (Gopinath et al., 2017; Doerr, 2018; Gorton and Ordoñez, 2020). My model can be used as a laboratory to think about the effects of productivity and sentiment shocks that may drive booms and their effects on the incentive to produce information, thereby affecting allocative efficiency.

Sentiment Shocks The following proposition starts with the effect of aggregate sentiment shocks.

Proposition 4. *There exists a threshold $\bar{\varepsilon}$, such that*

(i) information production is increasing in the sentiment shock if $\varepsilon_t < \bar{\varepsilon}$,

(ii) information production is decreasing in the sentiment shock if $\varepsilon_t > \bar{\varepsilon}$,

(iii) the threshold $\bar{\varepsilon}$ is negative for $\theta > \frac{1}{1-\alpha}$ and positive for $\theta < \frac{1}{1-\alpha}$.

Proposition 4 shows that the effect of small sentiment shocks ($\varepsilon_t \approx 0$) on information production is ambiguous and depends on the parameters of the model. However, sentiment shocks always crowd out information production once they are sufficiently large. Moreover, note that aggregate sentiment shocks do not affect price informativeness directly but only through information production.

At first, it may seem surprising that *aggregate* sentiment shocks crowd out information production, especially as in my model, *firm-specific* sentiment shocks incentivize information production in the first place. This is because knowledge about an *aggregate* sentiment shock changes the incentive to produce *firm-specific* information. In particular, there are two direct channels through which sentiment shocks affect the incentive to produce information.

[24] This threshold coincides with traders putting at most the same weight on their private and public signal. For $\sigma_\varepsilon^2 < 1$, traders put strictly more weight on the public signal than their private signal.

1. Sentiment shocks make valuations more extreme. As a result, trading becomes less *information-sensitive*. A relatively imprecise yet unbiased signal is sufficient to identify grossly mispriced firms and trade accordingly. Moreover, sentiment shocks make subtle mispricing rarer, for which precise information is helpful as shown in Figure 11. This effect *crowds out* information production for positive and negative sentiment shocks equally. Moreover, firms with such subtle mispricing must appear relatively unproductive in an otherwise exuberant market and consequently attract less capital, as in Figure 13. This *relative size* effect *crowds out* information production for positive sentiment shocks, as learning about smaller firms is unattractive.

2. Aggregate sentiment shocks increase aggregate investment K_{t+1}, which leads to an increase in the *absolute size* of all firms, encouraging more information production.

To further build intuition for this result, I use (25) in (17) to rewrite the marginal benefit of information production evaluated at the symmetric equilibrium,

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto$$

$$\tilde{\mathbb{E}}\left\{\underbrace{\underbrace{\frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}}\Bigg|_{\beta_{ijt}=\beta_{jt}}}_{\text{Information-Sensitivity}}\left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}}}_{\text{Relative Size}} \underbrace{K_{t+1}^{\alpha}}_{\text{Absolute Size}} \left(A_{jt}-\tilde{\mathbb{E}}\left\{A_{jt} \mid s_{ijt}=z_{jt}, z_{jt}\right\}\right)\right\}. \tag{31}$$

The *information sensitivity* channel materializes through the interaction of the change in the buying probability with the distribution of firm-specific sentiment shocks ε_{jt}. In the symmetric equilibrium ($\beta_{ijt}=\beta_{jt}$), traders expect to buy whenever they are more optimistic than the marginal trader, i.e., $s_{ijt} \geq z_{jt} \iff \eta_{ijt} \geq \varepsilon_{jt}$. The resulting probability of buying is $\Phi(-\varepsilon_{jt})$ where $\Phi(\cdot)$ is the standard-normal cdf. Consequently, the derivative of the buying probability with respect to the realization of the firm-specific sentiment shock ε_{jt} is $-\phi(\varepsilon_{jt})$ where $\phi(\cdot)$ is the standard-normal pdf. As shown in Figure 11, the trading decision is most elastic for relatively small realizations of the firm-specific sentiment shock ε_{jt}. However, aggregate sentiment shocks push the distribution of ε_{jt} to the more inelastic regions toward the extremes.

Formally, this effect can be captured by multiplying the change in the buying probability

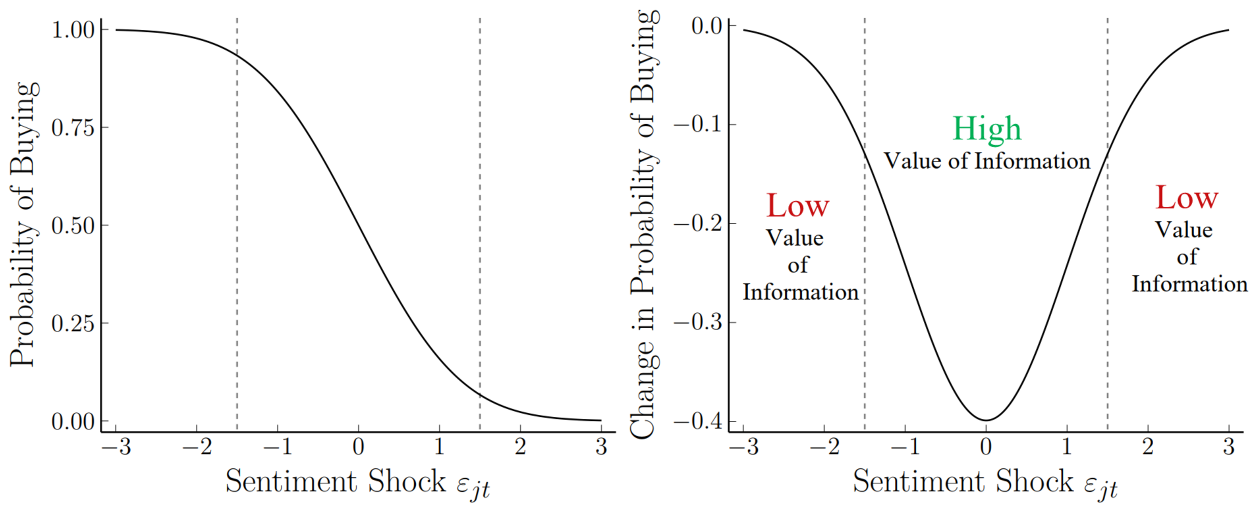

Figure 10: Probability of Buying and Sentiment Shocks.

Figure 11: *

Notes: Left panel: The probability of buying depending on the realization of the firm-specific sentiment shock ε_{jt}. Right panel: The derivative of the probability of buying. The trading decision is most information-sensitive, i.e., varies most with the realization of the sentiment shock ε_{jt}, around $\varepsilon_{jt} = 0$.

with the distribution of sentiment shocks,

$$\phi\left(\varepsilon_{jt}\right) f\left(\varepsilon_{jt}\right) \propto \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\} \tilde{f}(\varepsilon_{jt}), \tag{32}$$

where $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ and $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of ε_{jt} as if its distribution was $\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. The *information sensitivity* channel is captured by the term $\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}$, which is symmetrically decreasing around zero. Somewhat surprisingly, the decline in *information sensitivity* does not depend on the actual pass-through of sentiment shocks to expectations. The reason can be found in the trading decision, which does not depend on the actual mispricing caused by sentiment shocks but only on the realization of the firm-specific sentiment shock ε_{jt}. Therefore, aggregate sentiment shocks can discourage information production, even if they do not significantly affect actual prizes.

The additional effect of a decline in information sensitivity on the *relative size* of firms, for which information remains valuable, is captured by taking expectations of the relative firm-size $\left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}}$ with the density $\tilde{f}\left(\varepsilon_{jt}\right)$,

$$\int_0^1 \tilde{f}(\varepsilon_{jt}) \left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}} dj \propto \exp\left\{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t\right\}, \tag{33}$$

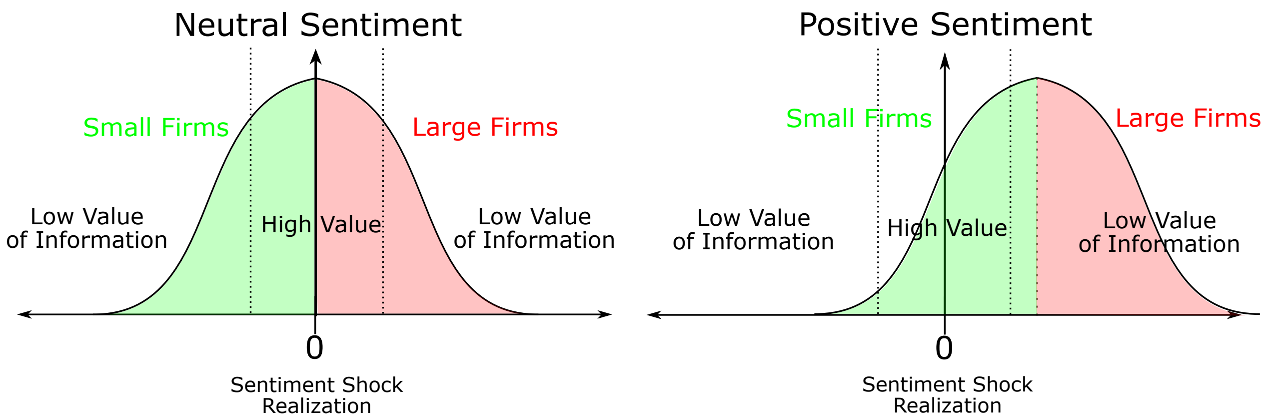

Figure 12: Illustration Size Channel.

Figure 13: *

Notes: Firms that are fairly priced and for which information is valuable are in the center of the firm-size distribution under neutral sentiment ($\varepsilon_t = 0$). In contrast, for positive sentiment shocks, the same firms are in the left part of the firm-size distribution as they appear to be unproductive relative to other firms.

where $\omega_{s\varepsilon} = \frac{\sqrt{\beta_t}}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$. For a positive sentiment shock, information production becomes effectively directed toward smaller firms, weakening the incentive to produce information. This channel is illustrated in Figure 13 and formally captured by the term $\exp\left\{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t\right\}$.

The *relative size* effect is increasing in the elasticity of substitution and in the pass-through of aggregate sentiment shocks $\omega_{s\varepsilon}$, which is non-monotonic in information production β_t. If intermediate goods are close substitutes, firms that are perceived as unproductive attract very little capital. Moreover, if aggregate sentiment shocks have a large effect on expectations, underpriced firms will be even smaller, making information production even less attractive.

The *absolute size* effect is captured by changes in aggregate investment. Restricting our attention to shocks for which $K_{t+1} < W_t$ leads to

$$K_{t+1}^{\alpha} \propto \exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{34}$$

As long as traders do not fully invest their wages, the *absolute size* effect can be captured by the term $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Intuitively, the effect on investment is stronger when α and, therefore, the returns to scale increase. A further increase in the sentiment shock is ineffectual for the *absolute size* channel once traders fully invest their wages but incentivizes nonetheless more information production through an increase in the value of resources, as captured by $\lambda_t = \max\left\{1, R_{t+1}\delta\right\}$ in (P1.2).

Putting all three effects together yields the marginal benefit of information production for a given symmetric information production choice ($\beta_{ijt} = \beta_{jt}$) as

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information-Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}}\right\}. \tag{35}$$

For the empirically plausible calibration $\theta - 1 > \frac{\alpha}{1-\alpha}$, positive sentiment shocks always *crowd out* information, as the increase in aggregate investment is dominated by a larger decrease in size of fairly priced firms. Conversely, negative sentiment shocks initially *crowd in* information, as fairly priced firms turn out to be relatively large although aggregate investment goes down. Finally, the *information sensitivity* channel always dominates for large shocks.

Productivity Shocks Productivity shocks have quite different effects on the incentive to produce information. Whereas sentiment shocks affect trading in multiple ways, productivity shocks leave the buying decisions unaffected. The reason is that traders believe that sentiment shocks affect only other traders, whereas productivity shocks affect all traders. The only channel through which productivity shocks change the incentive to produce information is through an increase in aggregate investment (*absolute size* channel) and dividends for all firms. This result is captured in the following proposition.

Proposition 5. *Positive (negative) productivity shocks crowd in (out) information.*

The model provides a rationale for the different impact of "good" and "bad" booms as in Gorton and Ordoñez (2020). Whereas productivity-driven "good" booms increase information production and improve allocative efficiency, sentiment-driven "bad" booms *crowd out* information and increase capital misallocation. The results of Propositions 4 and 5 are pictured in Figure 15.

5.2 Real Feedback

Financial markets do not only react to aggregate shocks, but also shape the economy's response to aggregate shocks. In the following, aggregate shocks hit an economy that is in steady state. Whether shocks amplify or dampen the effect of shocks on output is determined relative to an economy for which the information choice is fixed at the endogenous steady state information production β^*.

In the economy with fixed information production β^*, the only effect of aggregate shocks is the direct effect on TFP and investment. Positive shocks of both types increase investment, whereas only productivity shocks also have a direct effect on TFP. The opposite is true

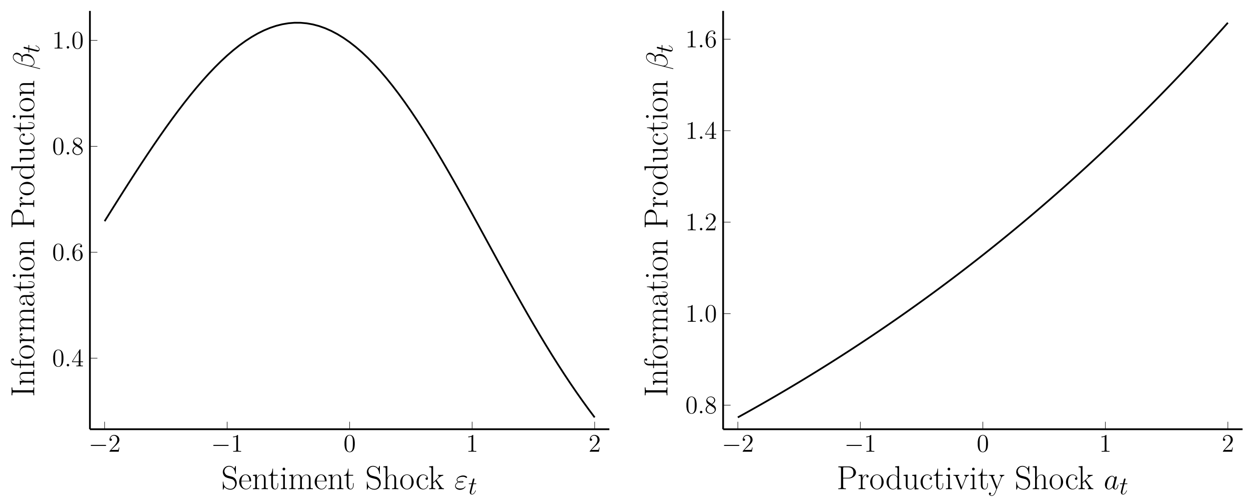

Figure 14: Information Production and Aggregate Shocks.

Figure 15: *

Notes: Information production is non-monotonic in the sentiment shock, with the peak $\bar{\varepsilon}$ being negative for $\theta > \frac{1}{1-\alpha}$. Information production is monotonically increasing in the productivity shock.

for negative shocks, which depress investment and TFP in the case of productivity shocks. Whereas the direct effect of aggregate shocks are straightforward, the indirect effects under endogenous information production are more subtle.

There are two indirect effects of sentiment shocks, both of which lead to a non-monotonic response of investment and output. First, sentiment shocks affect the allocative efficiency of financial markets through their effect on information production, which also decreases investment. The cost of misallocation through a decrease in information production depends on the elasticity of substitution between intermediate goods. If the elasticity of substitution is large, misallocation between firms is costly. Moreover, a high elasticity of substitution also leads to a stronger decrease in information production for a positive sentiment shock. In contrast, the costs of misallocating capital are low if the elasticity of substitution is small.

The second effect concerns the pass-through of sentiment shocks. Since traders are unaffected by sentiment if they produce either no or perfect information ($\beta_t \in \{0, \infty\}$), the effect of a given sentiment shock on beliefs must be maximized for an interior value of β_t. Therefore, a change in information production by traders may increase or decrease the effect of a given sentiment shock on their beliefs, which depends on whether steady state information production β^* is above or below the threshold $\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$. If β^* is above (below) the threshold, then the effect of aggregate sentiment shocks is locally increasing (decreasing) in information production. For example, a positive sentiment shock crowds out information production, leading to an amplification of the shock if the resulting precision choice β^* is still above the threshold $\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$.

The results for the case with $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$ are captured in the following proposition and visualized in Figure 17.

Proposition 6. *(i) For $\theta > \frac{1}{1-\alpha}$ and $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$, information production dampens positive sentiment shocks.*

(ii) Large positive sentiment shocks eventually lead to a decrease in aggregate investment if $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$.

The second result of Proposition 6 captures that the costs of misallocation must be eventually so large that they outweigh the investment-stimulating effect of sentiment shocks. Moreover, the direct effect of sentiment shocks vanishes as sentiment shocks grow large, as long as information production declines fast enough. This result is captured in the following corollary.

Corollary 2. *If information production declines fast enough as sentiment shocks grow large, then aggregate investment approaches its level without information production $\beta_t = 0$. Formally,*

$$\lim_{\varepsilon_t \to \pm\infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0 \Rightarrow \lim_{\varepsilon_t \to \pm\infty} K(\beta_t(\varepsilon_t), \varepsilon_t) = K(0, \varepsilon_t). \tag{36}$$

These results may initially seem counterintuitive, since sufficiently large positive sentiment shocks possibly decrease prices and output. However, the decrease in information production must eventually outweigh the expansionary effect of sentiment shocks as the pass-through of sentiment shocks goes to zero. Moreover, this section studies only *anticipated* sentiment shocks. If the same shock was unknown prior to the information production decision, positive sentiment shocks would unambiguously increase investment as in the economy with exogenous information precision.

Similar forces are active for negative shocks with the exception that negative sentiment shocks initially crowd in information production if the elasticity of substitution is large enough ($\theta > \frac{1}{1-\alpha}$). If strong enough, this indirect effect can even lead to negative sentiment shocks being initially expansionary. In contrast, if the elasticity of substitution is relatively small ($\theta < \frac{1}{1-\alpha}$), then negative sentiment shocks always crowd out information production and are, therefore, initially amplified.

Similar to the previous section, the indirect effect of productivity shocks leads generally to amplification. As follows from Proposition 5, positive productivity shocks crowd in information production, leading to an improvement in the allocation of capital and incentivizes additional investment. Therefore, compared to the economy with fixed information precision,

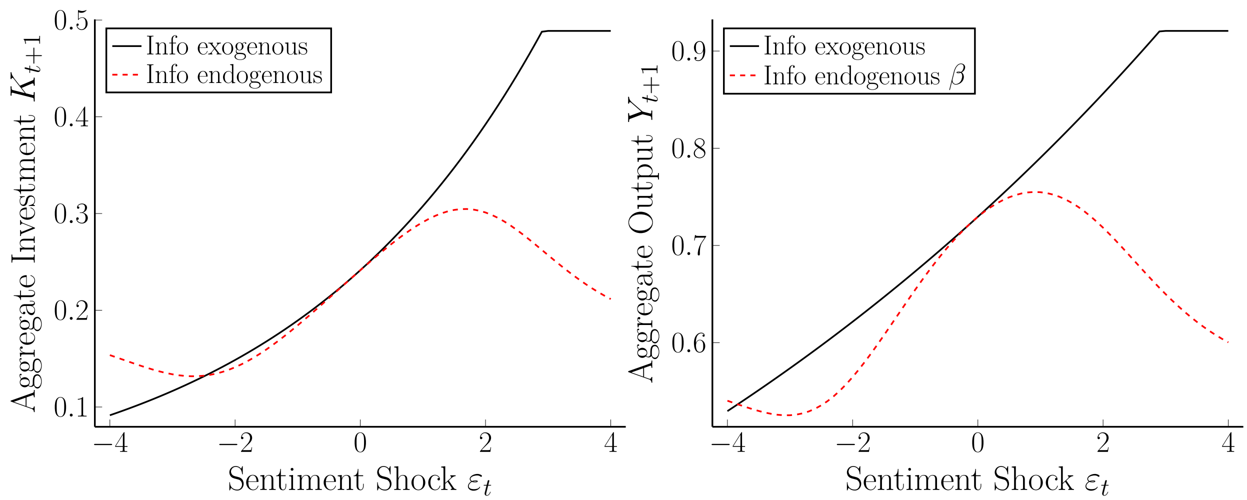

Figure 16: Amplification and Dampening for Sentiment Shocks.

Figure 17: *

Notes: Whether information production dampens or amplifies sentiment shocks depends on the size of the shock and the parameters. As information production affects both allocative efficiency and the pass-through of sentiment shocks, large sentiment shocks eventually drive information so low that investment and output decrease.

the reaction of both output and investment to a productivity shock are larger if information precision is allowed to adjust, as can be seen in Figure 18. This result is captured in the following proposition.

Proposition 7. *Information production amplifies productivity shocks.*

5.2.1 Numerical Illustration

This section provides a numerical illustration of booms driven by productivity and sentiment shocks, focusing on the region of parameters and shocks for which sentiment shocks are expansionary and dampened by information production. To capture the notion of booms, aggregate shocks build up over time according to the auto-regressive process

$$y_t = \begin{cases} \rho y_{t-1} + \zeta & t \in [0, B] \\ 0 & \text{otherwise} \end{cases}, \tag{37}$$

where $y_t \in \{a_t, \varepsilon_t\}$ is the aggregate shock, ζ is a constant innovation, $\rho \in (0, 1)$ is the persistence, and B captures the duration of the boom. After the boom is over, the aggregate shock returns to a neutral stance and remains there.

The expansionary effect of sentiment shocks is dampened, as can be seen in Figure 21.

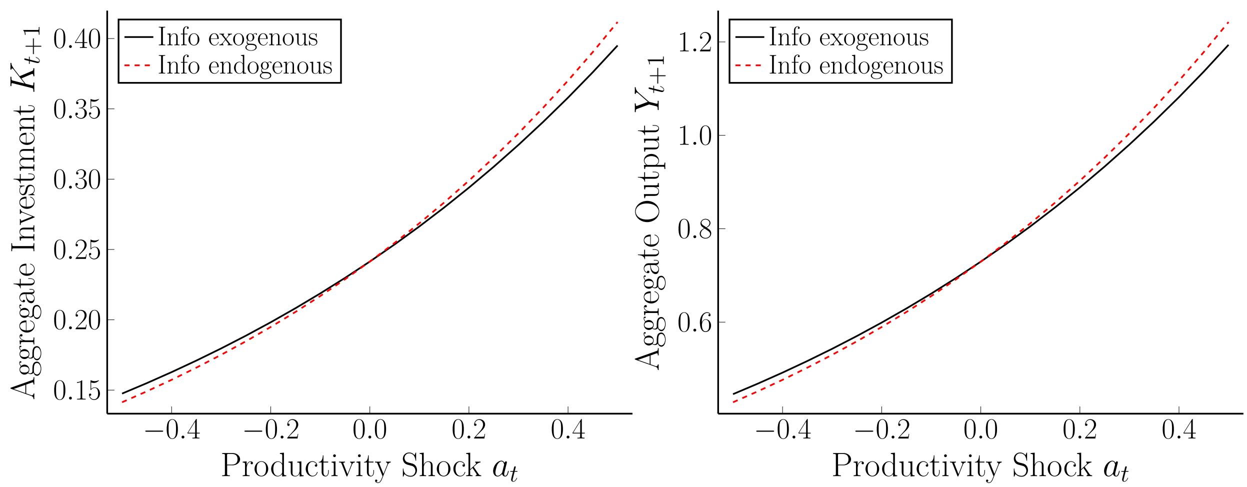

Figure 18: Amplification of Productivity Shocks.

Figure 19: *

Notes: Productivity shocks crowd in information production, leading to an additional increase in TFP and Aggregate Investment K_{t+1}. As a result, the effect of productivity shocks is amplified.

Optimistic expectations lead to an increase in investment, but traders decide to cut back on information production, which decreases the allocative efficiency of financial markets. In total, output still increases because the sentiment shock leads to an offsetting increase in investment. In this case, the endogenous response of traders dampens the effect of a positive sentiment shock.

In contrast, productivity-driven booms are generally amplified by an increase in information production, as seen in Figure 23, mirroring the result from Figure 18 and Proposition 7. Expectations of higher productivity tomorrow cause an increase in investment today, which triggers more information production. As a result, the endogenous response of traders amplifies the effect of productivity shocks. Times of high productivity are also times in which financial markets allocate capital efficiently.

6 Is there a Role for Policy?

After studying the positive properties of the model, I turn now to the normative implications. There are two sources of inefficiency in my model. First, there are two externalities with respect to the information production decision that work in opposite directions. On the one hand, traders produce information to extract rents from other traders and ignore the negative effects they impose on others. On the other hand, traders do not reap the benefits

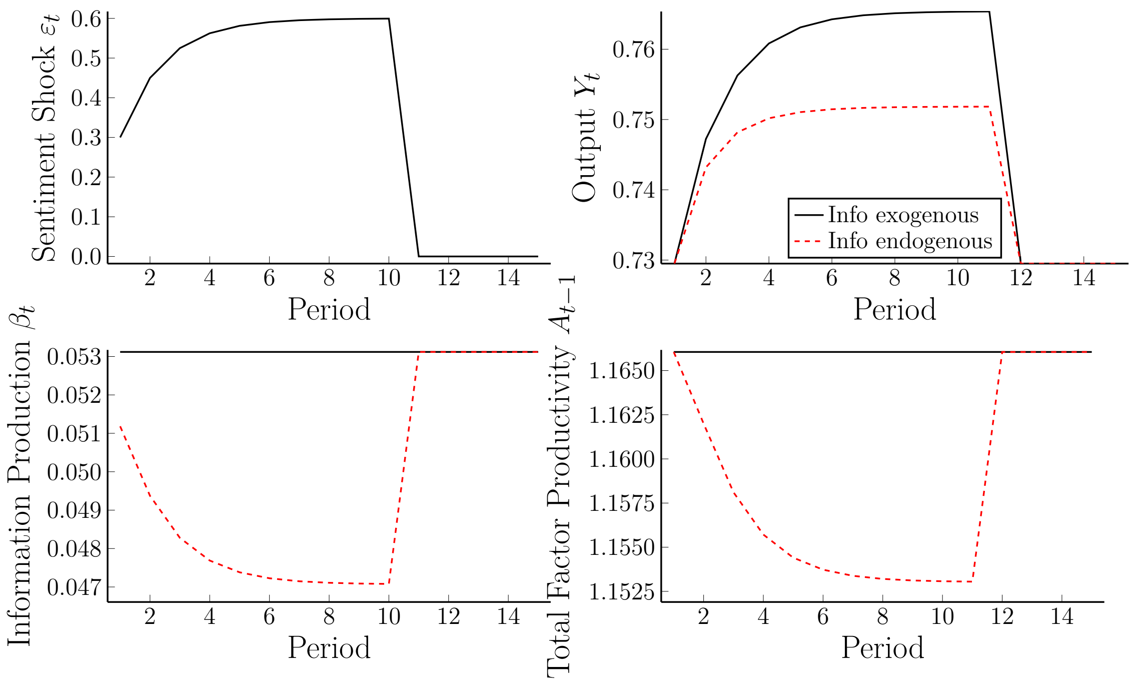

Figure 20: Numerical Simulation Sentiment-Driven Boom.

Figure 21: *

Notes: Sentiment-driven booms are dampened by information production.

of an improving capital allocation due to information production and, therefore, do not ignore this positive externality of information production. Whether information production is inefficiently high or low depends on the strength of the rent-stealing motive relative to the usefulness for information in allocating capital.

Second, traders' overconfidence distorts the allocation of capital between firms as described in section 4.1, and lets aggregate sentiment shocks drive investment. A state- and price-dependent tax/subsidy on dividends is sufficient to fix this distortion. The formal analysis has been delegated to Appendix E as the focus of this paper is on information production.

For the following welfare analysis, I abstract from well-known inter-generational trade-offs using a two-period model. Traders are born with an endowment, produce information, and buy shares. Production takes place in the second period and the final good sector combines intermediate goods into the final good without labor,

$$Y_1 = \left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}}. \tag{38}$$

The setup is otherwise identical to the main model.

The section proceeds in the following steps. First, I explain in detail why information

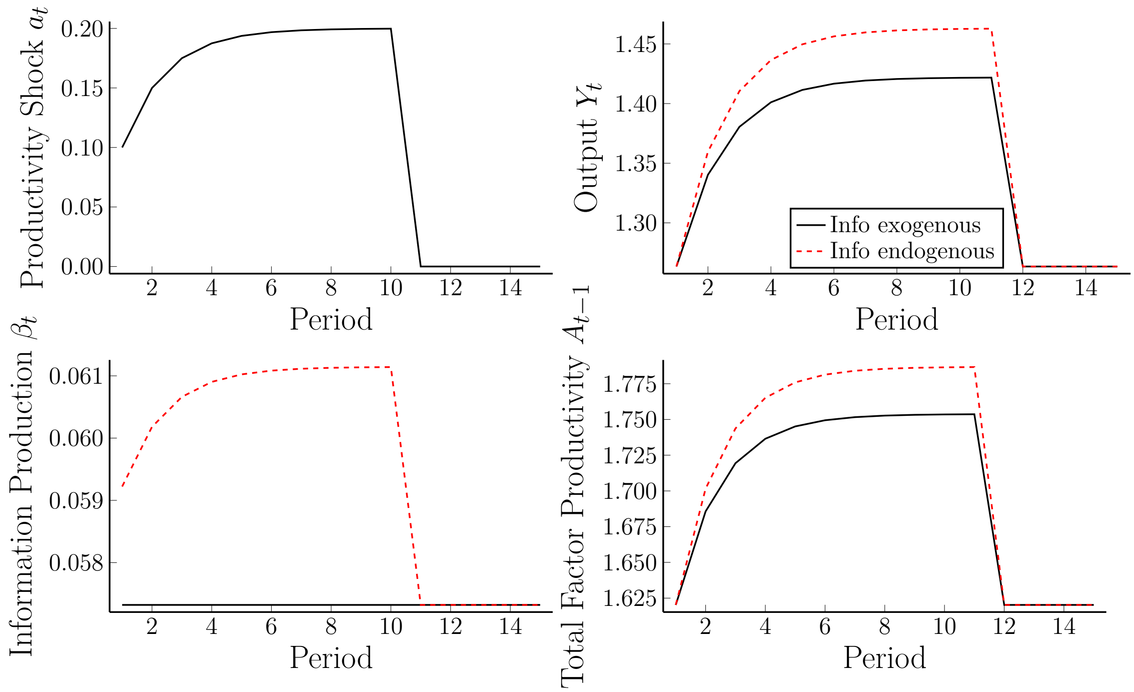

Figure 22: Numerical Simulation Productivity-Driven Boom.

Figure 23: *
Notes: Productivity-driven booms are amplified by information production.

production is inefficient in the competitive equilibrium and for which parameters information production is likely to be either too high or too low. Next, I consider the optimal intervention if the social planner can only steer the information choice, but cannot decide on aggregate investment. Finally, I propose an implementation for a policy that incentivizes or discourages information production.

6.1 Static Information Choice

Endow a social planner with the ability to dictate a level of information production β_{ij0} to each trader, but households autonomously decide on consumption and investment.[25] Moreover, the social planner observes aggregate shocks $\{a_0, \varepsilon_0\}$ before taking her decision. The

[25]The full planner's problem is covered in the Appendix E.

corresponding maximization problem is

$$\max_{\{\beta_{ij0}\}} \quad C_0 + \delta C_1 - \int_0^1 IA\left(\beta_{ij0}\right) dj \tag{SP1.1}$$

$$s.t. \quad C_1 = A_0\left(\{\beta_{ij0}\}\right) K_1^{\alpha} \tag{39}$$

$$C_0 = W_0 - K_1 \tag{40}$$

$$K_1 = \min\left\{ \left(\alpha\delta A_t^{\alpha_Y} \left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\alpha}}, W_t \right\} \tag{41}$$

$$\beta_{ij0} \geq 0. \tag{42}$$

The social planner maximizes welfare subject to a number of constraints. Old age consumption is equal to aggregate production as in (39), for which total factor productivity $A\left(\{\beta_{ij0}\}\right)$ depends on information production. Youth consumption as in (40) depends on aggregate investment, which also depends implicitly on information production, as seen in (41). Finally, (42) is a non-negativity constraint on information production.

Since all traders and firms are ex-ante homogeneous, the social planner chooses the same level of information precision $\beta_0 = \beta_{ij0}$ for all traders and markets. The marginal benefit of increasing β_0 for the social planner is

$$MB^{SP}\left(\beta_0\right) = \delta \underbrace{\frac{\partial A_0\left(\beta_0\right)}{\partial \beta_0}}_{\textit{Change in TFP}} K_1\left(\beta_0\right)^{\alpha} + \left(\delta\alpha A_0\left(\beta_0\right) K_1\left(\beta_0\right)^{\alpha-1} - 1\right) \underbrace{\frac{\partial K_1\left(\beta_0\right)}{\partial \beta_0}}_{\textit{Change in Investment}} . \tag{43}$$

The social planner targets both TFP $A_0\left(\beta_0\right)$ and aggregate investment $K_1\left(\beta_0\right)$. Note that the latter effect is only relevant if aggregate investment is inefficiently high or low, which is generally the case due to the price distortion described in section 4.1 and aggregate sentiment shocks.

The first observation is that (43) does not coincide with the marginal benefit in (17). Moreover, the difference cannot be expressed in the form of a simple wedge. This finding leads directly to the following proposition.

Proposition 8. *Information production is inefficiently high or low in the competitive equilibrium.*

The reason for this result is that the information production decision is subject to two externalities with opposing effects. First, traders produce information to extract rents from other traders, i.e., they ignore a negative externality. In other words, traders seek to get a larger piece of a fixed pie of trading profits. Second, as atomistic traders take prices as given, they do not take into account the allocation-improving effect of *collective* information production, i.e., they do not take into account the positive *spillover* of information production. If all traders produce more precise information, the allocation of capital improves and aggregate productivity increases. Both externalities are explained in more detail in what follows.

Traders think that their information allows them to systematically buy undervalued shares, thus earning a rent. Producing more precise information allows them to better identify profitable trading opportunities. However, if trader ij decides to buy shares, these shares cannot be bought by another trader. Consequently, any rent that accrues to trader ij must be subtracted from rents that are earned by other traders. Although this rent-extracting behavior drives information production in the first place, it can also lead to inefficiently high information production.

In contrast, the social benefit of information production stems from an improvement in the allocation of capital. However, this effect only arises if traders *collectively* produce more precise information . In contrast, individual information production and trading have only infinitesimal effects on prices, which are ignored by price-taking traders in their information production decision. Therefore, information production has a positive *spillover*, which can lead to information production being too low in the competitive equilibrium.

Two simple examples can be constructed to showcase situations in which information production is unambiguously too high or too low in equilibrium. First, assume that the social planner confiscates rents and redistributes them equally. Traders have no incentive to produce information, but the social planner still values information for its effect on the allocation of capital. In this case, information production is inefficiently low. Second, let firm output be given exogenously, such that $Y_{jt} = A_{jt}$. Traders can still make bets on firm revenue by trading shares. However, information production has no social value as production is given exogenously. In this case, information production is inefficiently high.

Cases with too much and too little information can be produced in the clearest way by varying the elasticity of substitution, which captures the importance of capital allocation for aggregate productivity. The comparison between the planner's choice and the competitive outcome is shown in Figure 25. First, consider the case of no substitution with ($\theta \to 0$). In that case, every intermediate good is necessary to produce the final good and the necessary mix is pinned down by firm productivities. It follows that an equal distribution of capital becomes optimal and information about firm productivity has no social value since it no longer aids the optimal allocation. In other words, TFP becomes flat in information. Nonetheless, traders find it profitable to produce information as firm revenue still depends on the realization of firm productivity.

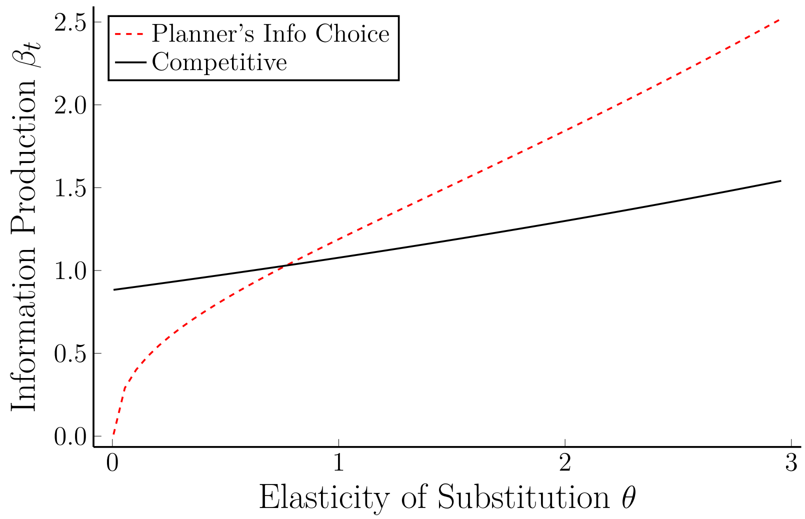

Figure 24: Planner's and Market's Information Production and Elasticity of Substitution.

Figure 25: *

Notes: Low elasticity of substitution: Too much information production. High elasticity of substitution: Too little information production.

Second, if the elasticity of substitution grows arbitrarily large ($\theta \to \infty$), intermediate goods become increasingly substitutable and the allocation of capital more important. In contrast, traders find it at some point unattractive to produce information as most firms will be unable to attract capital, and only the firm with the highest combination of productivity *and* sentiment shock receives the economy's capital stock. As a result, the planner's information precision choice is eventually above the outcome in the competitive equilibrium. The market underproduces information exactly when it is most valuable.

6.2 Responding to Aggregate Shocks

The social planner increases information production in response to both negative and positive sentiment shocks for two reasons. First, traders expect that trading becomes less information-sensitive when a sentiment shock hits the economy. However, the value of information for the allocation of capital is only affected insofar as aggregate investment changes. Second, the social planner also seeks to steer investment through the information production decision. For example, when a positive sentiment shock hits the economy, then producing more precise information eventually dampens the impact of the sentiment shock. The resulting response is asymmetric for positive and negative shocks, as positive shocks increase investment, which makes information more valuable, whereas negative shocks lower investment.

In contrast, the social planner's choice in response to productivity shocks is pro-cyclical.

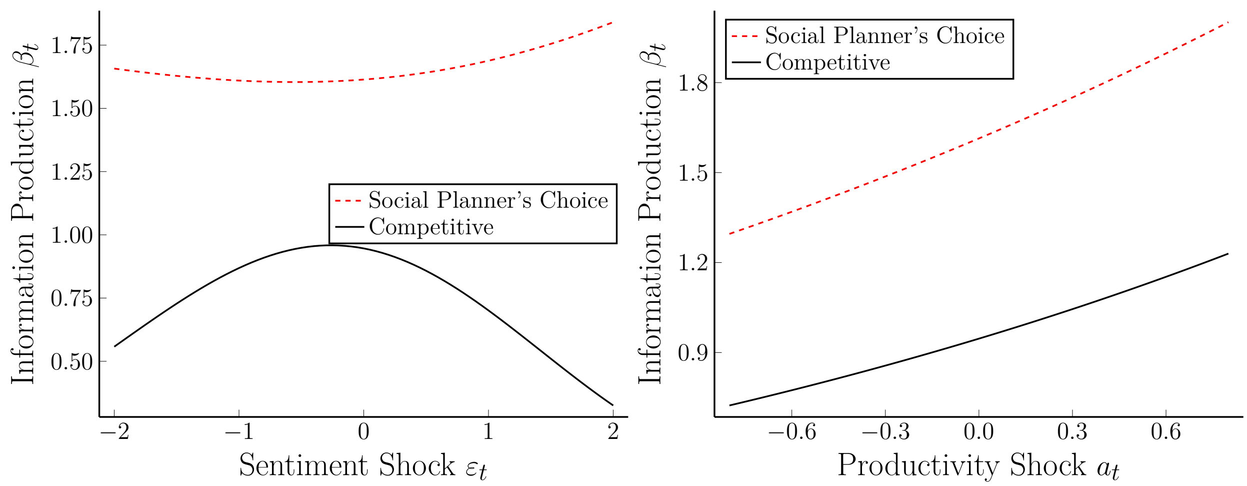

Figure 26: Planner's and Market's Information Production and Aggregate Shocks.

An increase in exogenous productivity a_0 incentivizes information production in two ways. First, note that TFP can be decomposed into two parts, $A_0 = A_0(a_0) A_0(\beta_0)$, where the first is exogenously driven by a_0 and the second is related to allocative efficiency through β_0. Therefore, an increase in a_0 amplifies the improvement in the allocative efficiency through an increase in β_0. Second, positive productivity shocks lead to an increase in investment which additionally incentivizes information production.

The social planner's choice is shown in comparison to the competitive equilibrium in Figure 26. For the chosen parameters, the social planner chooses generally more precise information than traders choose in the competitive equilibrium. Sentiment shocks widen the difference between the social planner's choice and the competitive outcome, whereas productivity shocks leave the gap largely unchanged. How the social planner can implement this policy is discussed in the following section.

6.3 Implementation

Traders are taking a gamble when they decide to buy shares in a given asset. The social planner can incentivize information production by increasing the stakes for each trade. This idea can be implemented through a redistribution of dividends between over- and underperforming firms as shown in the following corollary.

Corollary 3. *A state-dependent tax/subsidy $\tau(a_{jt}, z_{jt})$ on dividends with the properties,*

(i) No price distortions: $\tilde{\mathbb{E}}\{\tau(a_{j0}, z_{j0})\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\} = \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0} = z_{j0}, z_{j0}\}$

(ii) Monotonicity of beliefs: $\frac{\partial \tilde{\mathbb{E}}\{\tau(a_{j0},z_{j0})\Pi_{j1}|s_{ij0},z_{j0}\}}{\partial s_{ij0}} > 0$

(iii) Monotonicity of prices: $\frac{\partial \tilde{\mathbb{E}}\{\tau(a_{j0},z_{j0})\Pi_{j1}|s_{ij0}=z_{j0},z_{j0}\}}{\partial z_{j0}} > 0$

encourages (discourages) information productionwhen

$$\tau(a_{j0},z_{j0}) \geq (\leq)\, 1 \iff \Pi_{j1} \geq \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0}=z_{j0},z_{j0}\} \ \textit{and}\ \frac{\partial \mathcal{P}\{x_{ij0}=2\}}{\partial \beta_{ij0}} \geq 0$$
$$\tau(a_{j0},z_{j0}) \leq (\geq)\, 1 \iff \Pi_{j1} \leq \tilde{\mathbb{E}}\{\Pi_{j1}|s_{ij0}=z_{j0},z_{j0}\} \ \textit{and}\ \frac{\partial \mathcal{P}\{x_{ij0}=2\}}{\partial \beta_{ij0}} \leq 0$$

and the inequalities are strict for at least some realizations of $\{a_{j0},z_{j0}\}$*.*

Intuitively, the social planner can make assets more or less risky by taxing/subsidizing dividends depending on realized productivity and market expectations. For example, subsidizing dividend payments of over-performing firms and taxing under-performing firms makes any investment riskier and information production more attractive. To avoid distorting prices, subsidies and taxes must offset each other in expectations.

As an illustration, the following combination of a tax $\tau(a_{j0},z_{j0})$ and a lump-sum transfer $T(a_{j0},z_{j0})$ encourage information production, where I assume $a_0 = -\frac{\sigma_a^2}{2}$ as a normalization,

$$\tau(a_{j0},z_{j0}) = \begin{cases} 0 & a_{j0} < \omega_a z_{j0} \\ 1 & a_{j0} \geq \omega_a z_{j0} \end{cases} \tag{44}$$

$$T(a_{j0},z_{j0}) = \begin{cases} 0 & a_{j0} < \omega_a z_{j0} \\ \tilde{\mathbb{E}}\{\Pi_{j1}|a_{j0} < z_{j0}, s_{ij0}=z_{j0},z_{j0}\} & a_{j0} \geq \omega_a z_{j0} \end{cases}, \tag{45}$$

where $\omega_a = \frac{\beta_0(1+\sigma_\varepsilon^{-2})}{\sigma_a^{-2}+\beta_0(1+\sigma_\varepsilon^{-2})}$ and the post-tax dividend payment is

$$\hat{\Pi}(a_{j0},z_{j0}) = \tau(a_{j0},z_{j0})\,\Pi_{j1} + T(a_{j0},z_{j0}). \tag{46}$$

The tax is confiscatory if the realization of the productivity shock a_{j0} is below the mean expectation of the marginal trader $\omega_a z_{j0}$, i.e., the firm disappoints market expectations. The expected tax revenue from the perspective of the marginal trader is transferred to buyers if the realization of a_{j0} is above $\omega_a z_{j0}$, i.e., the firm exceeds market expectations. A tax schedule that incentivizes information production, therefore, increases both the potential downsides and upsides of any trade. The before- and after-tax dividend schedule is shown in Figure 27 for the case with $z_{j0} = 0$.

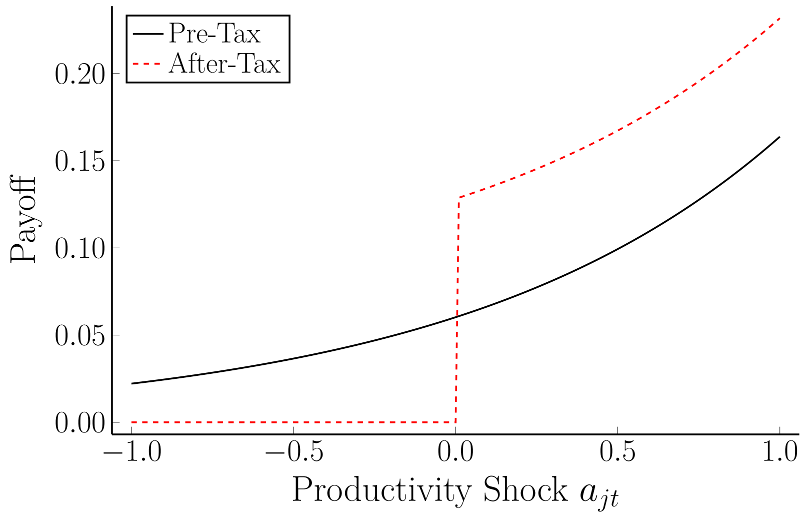

Figure 27: Information Production Incentivizing Tax Schedule.

Information production can be discouraged by flattening the dividend function instead. A straightforward and common implementation is through a progressive dividend tax in combination with the deduction of losses from realized gains, effectively offsetting part of the incurred losses by reducing the tax owed. In the model, the social planner can completely crowd out information production by buying all shares and selling shares that are claims on aggregate output. As there is no aggregate uncertainty, such shares pay a deterministic dividend and traders do not produce information.

To recapitulate, the social planner generally chooses a level of information production that deviates from the competitive equilibrium. If the efficient allocation of capital is sufficiently important, e.g., due to a high elasticity of substitution, then the social planner chooses a higher level of information production than would arise in the competitive equilibrium. Moreover, the social planner increases information production in response to both negative and positive sentiment shocks. In contrast, information production increases with the productivity shock. Finally, taxes and subsidies that increase the exposure to risk stemming from firm productivity increase the incentive to produce information.

7 Discussion

7.1 Asset Purchases

During the last decade, central banks have repeatedly used asset purchases to stabilize financial markets and accelerate economic growth and price inflation (for a brief overview, see Gagnon and Sack, 2018). These interventions were accompanied by concerns that asset purchases might harm market efficiency and lead to an increase in capital misallocation.[26] Although my model is too stylized to give a full assessment of asset purchases, it can be used to shed light on the effect of asset purchases on information production in financial markets.[27]

In my model, asset purchases have real effects by exploiting that information is dispersed between traders. The mechanism works as follows: Asset purchases reduce the number of shares in the hands of traders, leading to an upward shift in the identity of the marginal trader. The marginal traders turn out to be more optimistic than in absence of asset purchases, and consequently, asset prices increase. Additionally, announced asset purchases affect information production. Traders anticipate the reduction in asset supply distorts prices upward, discouraging information production similar to a positive sentiment shock. Therefore, my model can provide a rationale for the concerns about asset purchases and declines in market efficiency.

However, asset purchases can also be used to reduce distortions in financial markets, for example, through negative sentiment shocks. When a sufficiently large negative sentiment shock hits the economy, traders anticipate that prices will be depressed, which discourages information production as trading becomes less information-sensitive. The central bank can offset the downward bias on asset prices by purchasing assets. This counter-measure can lead to unbiased prices, which restore the incentive to produce information for traders at the same time as increasing asset prices. This logic is captured in the following proposition and is visualized in Figure 28.

Proposition 9. *Let the social planner acquire $d^{SP} \in (-1, 1)$ units of assets, such that $1 - d^{SP}$ shares are left for traders. Then,*

(i) asset purchases ($d^{SP} > 0$) undo negative sentiment shocks both in terms of investment and information production.

(ii) asset sales ($d^{SP} < 0$) undo positive sentiment shocks both in terms of investment and information production.

In other words, asset purchases and sales can *increase* market efficiency by countering sentiment shocks. This finding is relevant for central banks in deciding when to start shrinking the size of their balance sheets. Central banks can avoid the adverse effects of asset sales by

[26] See da Silva and Rungcharoenkitkul (2017), DNB (2017), Fernandez, Bortz, and Zeolla (2018), Borio and Zabai (2018), Acharya et al. (2019), and Kurtzman and Zeke (2020). In particular, the Dutch central bank argues in their 2016 annual report (DNB, 2017): *"The large-scale purchase programmes and the flood of liquid assets has set the risk compass in financial markets spinning, with misallocations as a result."*

[27] Although most central banks focused on buying government bonds as a form of quantitative easing, also interventions in corporate bond markets were common, which can be interpreted through the lens of my model (Gagnon and Sack, 2018). Moreover, the Bank of Japan bought directly shares in stock ETFs (Okimoto, 2019).

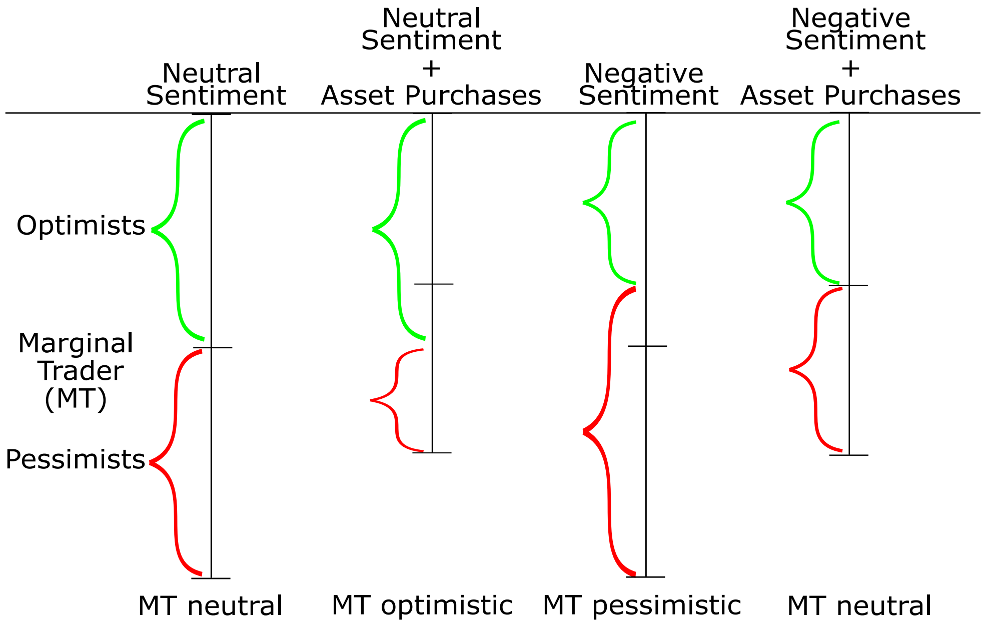

Figure 28: Asset Purchases Counter Negative Sentiment Shocks.

waiting until sentiment has reached a more neutral level. A reduction in asset holdings can then even increase information production and market efficiency.

7.2 Uncertainty

7.2.1 Traders have Imperfect Information about Aggregate Shocks

The analysis so far assumed that traders observed aggregate states perfectly before deciding on information precision. This assumption is not crucial for the results, which also hold when traders have only imperfect information about aggregate states before they make their information production decision. Nonetheless, in reality, traders or policymakers do not have perfect knowledge about the current aggregate state.

The simplest setting to think about the effects of uncertainty is to reveal aggregate shocks after the information production decision but before trading. Furthermore, assume that aggregate productivity and sentiment shocks are auto-correlated.[28] Then, the laws of motion

[28] An alternative would be not to reveal aggregate shocks before trading takes place. In this setting traders learn from private and public signals also about aggregate states. Similarly, the social planner can use publicly available information to guide her interventions. The insights are broadly the same as in the case when aggregate shocks are revealed after the information production decision.

for aggregate shocks are given by

$$a_t = \rho_a a_{t-1} + \xi_t^a \tag{47}$$

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \xi_t^\varepsilon, \tag{48}$$

where $\rho_a \in (0,1)$ and $\rho_\varepsilon \in (0,1)$ capture the persistence of aggregate shocks and $\xi_t^a \sim \mathcal{N}\left(0, \sigma_{\xi a}^2\right)$ and $\xi_t^\varepsilon \sim \mathcal{N}\left(0, \sigma_{\xi\varepsilon}^2\right)$ are the corresponding innovations. Traders can learn about past aggregate states by observing past aggregate investment K_t and output Y_t. Whereas K_t is moved by both productivity and sentiment, output Y_t reacts only to productivity after controlling for K_t^α. For example, if investment was high, but output was disappointing, investment must have been driven by a positive sentiment shock. The prior for traders about aggregate states is then given by

$$a_t | a_{t-1} \sim \mathcal{N}\left(\rho_a a_{t-1}, \sigma_{\xi a}^2\right) \tag{49}$$

$$\varepsilon_t | \varepsilon_{t-1} \sim \mathcal{N}\left(\rho_\varepsilon \varepsilon_{t-1}, \sigma_{\xi\varepsilon}^2\right). \tag{50}$$

In this setting, past sentiment shocks generate expectations about future sentiment shocks. The analysis of Proposition 4 still applies, as traders evaluate the value of information for different realizations of the sentiment shock ε_t.

7.2.2 Policy under Uncertainty

The policy analysis is not substantially changed under aggregate uncertainty if the social planner has to take her decision before aggregate shocks are revealed. Indeed, negative effects of sentiment shocks on information production can be offset without knowing the exact realization, as only *expectations* of sentiment shocks affect information production. The social planner can collect information about traders' expectations of sentiment through surveys. This information can be used to implement a policy that offsets the effect of anticipated sentiment shocks on information production.

The effect of uncertainty is more subtle when the social planner also tries to steer investment. In this case, realizations of sentiment shocks matter. Therefore, any intervention that does not explicitly condition on the realizations of sentiment shocks has to weigh the costs and benefits of taxes or subsidies on investment in different states. Increasing information production can diminish the impact of sentiment shocks for all realizations.

A special case arises when traders are informed about aggregate shocks, but the social planner is not. In this case, multiple indicators can be used by the social planner to identify whether a boom is driven by sentiment or productivity. A sentiment-driven boom crowds out information production and decreases the variance of prices, leading all firms to look more alike. In contrast, a productivity-driven boom crowds in information, leading to more dispersion in asset prices and firm capital. For example, if asset prices increase across the board and the dispersion in asset prices or returns between firms shrinks, the social planner wants to lower investment and increase information production. Instead, if there are winners and losers even as asset prices are booming, price discovery still occurs, and traders are producing information. Using dispersion in asset prices and returns is more attractive than measuring information production directly, as to whether asset prices reflect fundamentals can only be backed out after production happened. However, asset prices are available continuously. This result is captured in the following proposition.

Proposition 10. *For* $\sigma_\varepsilon^2 \leq 1$*,*

(i) the cross-sectional variance in asset prices is increasing in β_t*.*

(ii) the cross-sectional variance in asset price returns is increasing in β_t*.*

Finally, if policymakers need to commit to interventions before prices form and aggregate shocks are persistent, past realizations of price-earnings ratios can also be informative regarding future aggregate shocks. For example, if investment was high, but output was relatively low, then investment must have been driven by sentiment, and future investment is also likely to be driven by sentiment.

7.3 Empirical Evidence

Many measures seek to capture a notion of information in financial markets. However, the literature so far has not converged on any single measure. Roll (1988) suggested a measure that attributes movements in asset prices that are uncorrelated with the market or industry portfolio with new firm-specific information. However, firm-specific variance can also stem from firm-specific noise (for an overview, see Ningning and Hongquan, 2014). Chousakos, Gorton, and Ordoñez (2020) employed a measure that follows a similar idea.

In contrast, Bai, Philippon, and Savov (2016) and Farboodi et al. (2020) suggested a measure that uses asset prices to forecast earnings. According to this measure, financial markets are informative if firms with higher earnings also have a higher market capitalization. The downside of their approach is that it implicitly assumes that the data generating processes for earnings and prices are identical between firms, as they run regressions for cross-sections of firms.

Dávila and Parlatore (2021) avoided these objections by providing a micro-founded procedure to estimate (relative) price informativeness at the firm level, allowing for different data generating processes for each firm. Relative price informativeness captures a notion of how precise the price signal is relative to prior uncertainty, which corresponds exactly to $\frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_{jt}\sigma_\varepsilon^{-2}}$ in my model. Their measure can be used to provide suggestive evidence that information precision indeed depends on the cycle. I use an estimate of utilization-adjusted TFP following Basu, Fernald, and Kimball (2006) from the San Francisco Fed to verify the connection between information and aggregate productivity and as an indicator for the type of shock that drives the cycle.

Using data from the US between 1995 and 2017, Figure 1 provides suggestive evidence that information in financial markets varies depending on what type of shock drives the cycle. Because the model focuses on cycles instead of long-run developments, both time series are detrended using a cubic time trend between 1995 and 2017 and smoothed with a two-year moving average. The resulting time series is shown in the left graph, whereas the original can be seen on the right. Both graphs have gray bars that indicate recessions following the methodology of the NBER for dating recessions. The first striking observation is that the cyclical components of price informativeness and TFP growth are positively related. As so far as cyclical movements in TFP growth capture changes to allocational efficiency, this provides evidence that information in financial markets indeed impacts TFP.

A second exercise allows us to back out which type of shocks drove the expansions up to 2001 and from 2002 to 2008. The period between 1995 and 2001 was marked by an acceleration in TFP growth, accompanied by an increase in price informativeness. This increase suggests that the dot-com boom was driven by technological innovations, for example, the introduction of advanced information technologies. In contrast, the expansion between 2002 and 2008 was marked by a sharp decline in TFP growth into negative territory and a fall in price informativeness. Through the lens of the model, an expansion accompanied by a decline in TFP signals a sentiment boom (see also Borio et al., 2015; Doerr, 2018). The finding that price informativeness was also declining verifies the model's prediction that information production declines during sentiment booms.

This narrative is also supported when using return non-synchronicity as a measure of price informativeness as Roll (1988). I use the database *CRSP* to compute the standard deviation in monthly stock returns for the US. Following standard practices, I drop the financial sector with four digit SICC codes 6xxx and firms with market caps in the bottom 20 percent, for which I take breaking points from Kenneth R. French's website. Similarly, I only include ordinary common shares (share code ten and eleven) which are traded on the NYSE, NYSE American and NASDAQ (exchange code one, two, and three).

Similarly to price informativeness as in Figure 1, the dispersion of monthly returns increases during the dot-com boom and decreases during the subsequent housing boom, as can

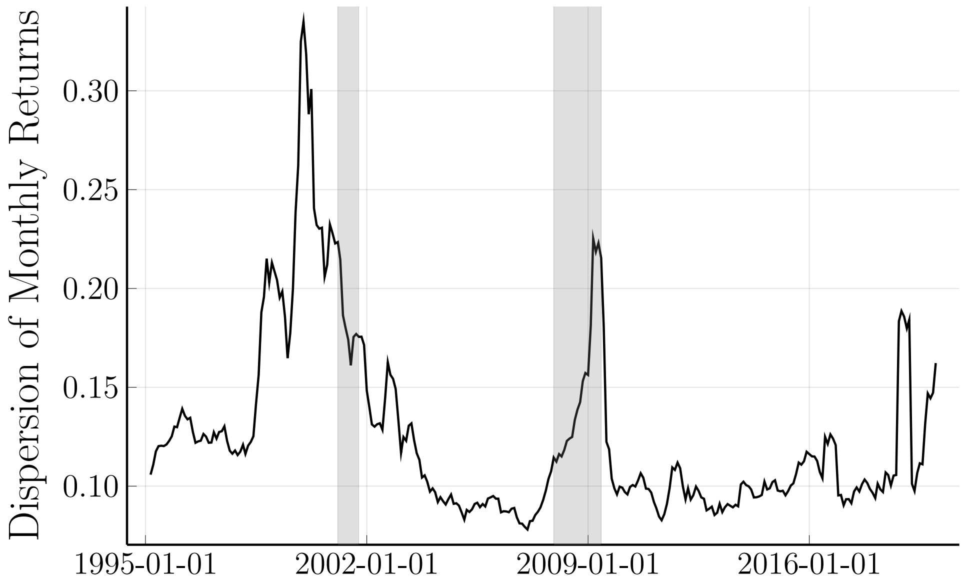

Figure 29: Return Dispersion.

Figure 30: *

Notes: Return dispersion was high during the dot-com boom leading up to 2001, but decreased substantially during the housing boom leading up to the Great Financial Crisis.

be seen in Figure 30. Viewed through the lens of the model, this suggests that the dot-com boom has been driven by productivity, whereas the housing boom has been driven by sentiment. Different to Figure 3, the dispersion in monthly returns stays relatively low after the Great Financial Crisis, whereas price informativeness following Dávila and Parlatore (2021) decreases but stays historically high. A possible explanation is that return dispersion also captures changes in the variance of fundamentals and noise, whereas the measure of Dávila and Parlatore (2021) aims to correct for changes in variances. Therefore, volatility may have remained low due to a decrease in variances, whereas informativeness remained high as information production decreased by less.

8 Conclusion

I develop a tractable framework to study information production in financial markets embedded in a standard macroeconomic model. In such a model, total factor productivity has an endogenous component that depends on the traders' decentralized information production. When asset prices are more informative, more capital is allocated to the most productive firms, and total factor productivity increases. I add to the literature by studying the effect of aggregate shocks on information production.

I prove that sentiment shocks, defined as waves of non-fundamental optimism or pessimism, crowd out information production as trading becomes less information-sensitive. Although such optimism increases investment, it also worsens the allocation of capital. This result rationalizes the empirical finding that credit booms often worsen aggregate productivity (Borio et al., 2015; Gopinath et al., 2017; Doerr, 2018; Gorton and Ordoñez, 2020) through a novel information mechanism. In contrast, expectations of heightened productivity crowd in information, thereby improving capital allocation and aggregate productivity beyond the initial shock. This dichotomy mirrors the "good" and "bad" booms of Gorton and Ordoñez (2020). My model suggests that "good" booms are driven by productivity, whereas "bad" booms are driven by sentiment.

From a normative perspective, I show that information production is too high or too low in the competitive equilibrium. There are two externalities with opposing effects. On the one hand, traders produce information to increase trading rents at the expense of other traders. This rent-extracting behavior can lead to excessive information production. On the other hand, traders do not reap the benefits of improving the capital allocation through *collective* information production. This information spillover can lead to information production being too low. Generally, information production is too low in the competitive equilibrium exactly when the allocation of capital matters the most and, hence, information is most valuable.

Finally, I apply the model to evaluate the effect of large-scale asset purchase programs. I show that asset purchases can discourage information production. This finding confirms the concerns of policymakers about such programs (e.g., DNB, 2017). However, asset purchases can also improve capital allocation if they effectively reduce aggregate mispricing of assets. Therefore, policymakers need to know which force is currently driving the cycle to react appropriately. The model suggests that dispersion in asset prices and returns identify the source of fluctuations in real-time. For example, sentiment booms decrease information production, which lowers the dispersion in asset prices and returns and lets firms appear more alike.

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A Trading

Every household i consists of many traders indexed by $ij \in [0,1]$. The information set of each trader consists of $\{s_{ijt}, \{z_{jt}\}, a_t, \varepsilon_t\}$, i.e., traders observe their private signal, all public signals and the aggregate states. This setting allows that traders have rational expectations about aggregates, but still disagree about firm-specific variables, which motivates trade. I impose that $\kappa_H = 2$ and $\kappa_L = 0$ to avoid distortions in asset prices that stem from the choice of position limits.

The beliefs of traders about firm productivity A_{jt} are relevant for their trading decision. Trader ij's beliefs are given by

$$\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}, z_{jt}\right\} = \exp\left\{\omega_{p,ijt}a_t + \omega_{s,ijt}s_{ijt} + \omega_{z,ijt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{ijt}\right\}. \tag{51}$$

Similarly, the beliefs of the marginal trader are

$$\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \exp\left\{\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{jt}\right\}, \tag{52}$$

where $a_{jt} \overset{iid}{\sim} \mathcal{N}\left(a_t, \sigma_a^2\right)$, $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ and ω-terms are the corresponding Bayesian weights,

$$\omega_{z,ijt} = \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{z,jt} = \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \tag{53}$$

$$\omega_{s,ijt} = \frac{\beta_{ijt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{s,jt} = \frac{\beta_{jt}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \tag{54}$$

$$\omega_{p,ijt} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{p,jt} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \tag{55}$$

and $\{\mathbb{V}_{jt}, \mathbb{V}_{ijt}\}$ stand for posterior uncertainty

$$\mathbb{V}_{ijt} = \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \mathbb{V}_{jt} = \frac{1}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}}. \tag{56}$$

The private information precision β_{ijt} is highlighted in blue and is part of the information acquisition decision. Alternatively, the beliefs of the marginal trader who observed $s_{ijt} = z_{jt}$ can be expressed as a function of shocks,

$$\ln\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \omega_{p,jt}a_t + \omega_{s\varepsilon,jt}\varepsilon_t + \omega_{a,jt}a_{jt} + \frac{\omega_{a,jt}}{\sqrt{\beta_{jt}}}\left(\varepsilon_{jt} - \varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{jt}, \tag{57}$$

where the corresponding Bayesian weights are

$$\omega_{a,jt} = \omega_{z,jt} + \omega_{s,jt} \tag{58}$$

$$\omega_{\varepsilon,jt} = \omega_{a,jt}/\sqrt{\beta_{jt}} \tag{59}$$

$$\omega_{s\varepsilon,jt} = \omega_{s,jt}/\sqrt{\beta_{jt}}. \tag{60}$$

Trader ij buys shares of firm j whenever

$$\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}, z_{jt}\right\} \geq \tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt} = z_{jt}, z_{jt}\right\} \tag{61}$$

$$\iff \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}, z_{jt}\right\} \geq \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{62}$$

The above inequality is equivalent to

$$\omega_{p,ijt}a_t + \omega_{s,ijt}s_{ijt} + \omega_{z,ijt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{ijt}}{2} \geq \tag{63}$$

$$\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{jt}}{2}. \tag{64}$$

The inequality can be expressed as a cutoff for the idiosyncratic noise,

$$\begin{aligned}\eta_{ijt} \geq\ & \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t + \omega_{z,ijt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{ijt}}{2}\right) + \sqrt{\beta_{ijt}}a_{jt} \\ & - \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{jt}}{2}\right).\end{aligned} \tag{65}$$

Since η_{ijt} is standard-normally distributed, the perceived probability of buying can be written in closed form

$$\begin{aligned}&\mathcal{P}\left\{x_{ijt} = 2|a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\right\} \\ &= \Phi\left(-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t + \omega_{z,ijt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{ijt}\right) + \sqrt{\beta_{ijt}}a_{jt}\right. \\ &\left. - \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{jt}\right)\right),\end{aligned} \tag{66}$$

where $\Phi(\cdot)$ is the standard-normal cdf. For a symmetric information choice ($\beta_{ijt} = \beta_{jt}$), the buying probability can be simplified to

$$\mathcal{P}\left\{x_{ijt} = 2|a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\right\}\big|_{\beta_{ijt}=\beta_{jt}} = \Phi\left(-\varepsilon_{jt}\right). \tag{67}$$

Traders think that they are more likely to buy shares when the realization of the sentiment shock is relatively low and shares are therefore cheap relative to their fundamental value.

Finally, traders choose their information precision taking the symmetric choice of all other traders as given. The derivative of the probability of buying with respect to β_{ijt} is

$$\frac{\partial \mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}}{\partial \beta_{ijt}} \tag{68}$$

$$\begin{aligned}
&= \phi\left(-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{ijt}\right)+\sqrt{\beta_{ijt}}a_{jt}\right.\\
&-\left.\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right)\\
&* \left(-\frac{1}{2\beta_{ijt}^{3/2}}\left(\sigma_a^{-2}a_t+\beta_{jt}\sigma_\varepsilon^{-2}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\right)+\frac{a_{jt}}{2\sqrt{\beta_{ijt}}}\right.\\
&-\left(\frac{1}{\sqrt{\beta_{ijt}}}-\frac{1}{2\beta_{ijt}^{3/2}}\left(\mathbb{V}_{ijt}\right)^{-1}\right)\\
&* \left.\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right)
\end{aligned} \tag{69}$$

where $\phi(\cdot)$ is the standard normal pdf. For a symmetric information choice ($\beta_{ijt}=\beta_{jt}$) this expression can be simplified to

$$\left.\frac{\partial \mathcal{P}\left\{x_{ijt}=2|a_{jt},\varepsilon_{jt},\beta_{ijt},\beta_{jt}\right\}}{\partial \beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} =$$

$$\phi\left(\varepsilon_{jt}\right)\left[\frac{1}{2\sqrt{\beta}_{jt}}\left(a_{jt}+z_{jt}\right)-\frac{1}{\sqrt{\beta_{jt}}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right]. \tag{70}$$

B Position Limits

B.1 Exogenous Position Limits

In the main text, I have assumed that traders can buy up to two units of each stock and normalized the total asset supply to one. Assume now that traders' position limits are given by $x_{ijt} \in [0, \kappa_H]$. Consider first some special cases.

Let $\kappa_H \in [0, 1)$. In this case, demand by traders is insufficient to buy the total asset supply. The result is that the stock price collapses to zero, all traders buy κ_H units of firm j's stock, and the price is uninformative because it does not vary according to firm productivity. Similarly, if $\kappa_H = 1$, traders can clear the market, but the same outcome arises.

In contrast, if there are no upper limits to how much traders can buy ($\kappa_H = \infty$), the most optimistic trader alone can clear the whole market. Expectations about dividends and the interest rate R_{t+1} go to infinity, but prices are remain finite. Information becomes useless for traders because the probability of buying in any given market is zero.

To avoid these edge cases, I focus on position limits for which the market clearing condition gives an interior solution for the threshold, i.e., $\kappa_H \in (1, \infty)$. The market-clearing condition

$$\kappa_H \left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right) = 1 \tag{71}$$

leads to the threshold

$$\hat{s}\left(P_{jt}\right) = a_{jt} + \frac{\varepsilon_{jt} + \Phi^{-1}\left(1 - \frac{1}{\kappa_H}\right)}{\sqrt{\beta_{jt}}}. \tag{72}$$

The resulting expectations of dividends can be expressed by multiplying the price when traders can buy up to two units with a factor related to κ_H,

$$\tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = \hat{s}\left(P_{jt}\right), z_{jt}\right\} = \underbrace{\exp\left\{\Phi^{-1}\left(1 - \frac{1}{\kappa_H}\right) / \sqrt{\beta_{jt}}\right\}^{\theta}}_{\textit{Bias through Choice of Position Limits}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{73}$$

As is now evident, the bias is equal to zero for $\kappa_H = 2$. In other words, the marginal trader is neither an optimist nor pessimist. Consequently, the interest rate is also distorted,

$$R_{t+1} = \exp\left\{\Phi^{-1}\left(1 - \frac{1}{\kappa_H}\right) / \sqrt{\beta_{jt}}\right\}^{\theta} \frac{\int_0^1 \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\} dj}{K_{t+1}}. \tag{74}$$

Holding K_{t+1} constant leads to an allocation of capital that is independent of the position limit κ_H,

$$\begin{aligned} K_{jt+1} &= \frac{\tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = \hat{s}\left(P_{jt}\right), z_{jt}\right\}}{R_{t+1}} \\ &= \frac{\tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj} K_{t+1}. \end{aligned} \tag{75}$$

Still, a different interest rate will affect aggregate investment through (30). If buyers are relatively optimistic ($\kappa_H > 2$), then the interest rate and aggregate investment increase. Setting $\kappa_H = 2$ is for the model with exogenously given information inconsequential and only avoids introducing a multiplicative factor for expectations of dividends.

For the information production decision, the choice of position limits has similar effects as aggregate sentiment shocks or reductions in asset supply. The main idea is the same: when the aggregate sentiment shock is positive, traders expect the trading decision to become less information-sensitive, making information less valuable. The same effect is present when setting a higher position limit $\kappa_H > 2$. In this case, however, it is counteracted by an increase in the traders' buying capacity. Depending on which effect dominates, the highest information choice is achieved for $\kappa_H < 2$ or $\kappa_H > 2$.

Position limits affect the analysis for sentiment shocks and revert the logic outlined in the main text. For example, assume that $\kappa_H = 1 + \eta$, where $\eta > 0$ is a small number. Then almost all traders need to buy shares to clear the market. It follows that trading is fully information-insensitive because all traders expect to buy κ_H units of nearly all firms irrespective of the private signal. Different from the intuition before, a positive sentiment shock makes traders think that the trading decision becomes *more information-sensitive.* Recall that the trading decision is most information-sensitive if the ex-ante probability of buying is 50%. As the increase in the sentiment shock pushes the ex-ante probability of trading towards 50% from below, a sentiment shock can make the trading decision more information-sensitive and encourage information production.

The choice of $\kappa_H = 2$ in the main text guarantees that the marginal trader is, on average, neither optimistic nor pessimistic absent aggregate sentiment shocks. Moreover, considering aspects outside of the model, excess or lack of demand can lead to the entry or exit of traders because firms are predictably under- or overpriced. It can also lead to the additional entry or exit of firms for the same reason. Both forces tend to undo the effects of too much or too little demand. Finally, denoting position limits in units of shares is mainly an analytical simplification when including risk-neutral traders in the financial markets. A formulation where position limits depend on the trader's wealth is investigated in Chapter **??**.

B.2 Short-Sales

Short-sales were ruled out in the main text for analytical convenience, but its presence would not affect the main results of the model. Assume that traders can take also negative positions, such that $x_{ijt} \in [-2, 2]$. The market-clearing condition becomes

$$\underbrace{2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right)}_{buying} - \underbrace{2\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)}_{selling} = 1, \tag{76}$$

with the threshold

$$\hat{s}\left(P_{jt}\right) = a_{jt} + \frac{\varepsilon_{jt} + \Phi^{-1}\left(\frac{1}{4}\right)}{\sqrt{\beta_{jt}}}. \tag{77}$$

In contrast to before, more traders need to buy to clear the market, because previously inactive traders now short shares and thereby increase the asset supply. Therefore, allowing short-sales leads to a lower price because the marginal trader will be more pessimistic than before. The bias can be avoided by imposing asymmetric position limits, e.g., $x_{ijt} \in [-2, 4]$, in which case the marginal trader still is identified by the signal $\hat{s}(P_{jt}) = a_{jt} + \varepsilon_{jt}/\sqrt{\beta_{jt}}$.

Similar to the previous section, the choice of position limits can change the effect of sentiment shocks on the incentive to produce information by introducing a bias to the asset price. This issue is avoided by ruling out short-sales and setting $\kappa_H = 2$.

B.3 Endogenous Position Limits

Finally, let traders choose position limits $x_{ijt} \in [\kappa_L, \kappa_H]$ subject to cost $c^L(\kappa_L)$ and $c^H(\kappa_H)$ before trading takes place. One interpretation is that funds and credit lines have to be allocated between markets, which can be costly. Such shifting of funds can be valuable if traders expect that some markets are under- or overpriced. For example, if market j is hit by a positive sentiment shock, traders may want to extend their ability to short shares in this market while reducing their ability to buy, by shifting collateral towards this market and cash towards other markets. Generally, endogenously choosing position limits will tend to counteract the effects of sentiment shocks.

The effect on the information production decision is more subtle. Consider as a partial equilibrium example that trader ij received private information that shares of firm j will be underpriced. In anticipation of a depressed market, trader ij extends her ability to buy but completely forgoes short-sales. The opportunity cost of buying when prices are too high is proportional to $\kappa_H - \kappa_L$. Therefore, the value of information is increasing in $\kappa_H - \kappa_L$. Whether the adjustment of position limits increases information production depends, therefore, on whether $\kappa_H - \kappa_L$ is increased as a result.

More formally, the expected trading rents can be written as

$$\widetilde{EU}(\beta_{ijt}, \beta_{jt}) = \tilde{\mathbb{E}}\left\{(\kappa_H \mathcal{P}\{x_{ijt} = \kappa_H\} + \kappa_L \mathcal{P}\{x_{ijt} = \kappa_L\})\left(\frac{1}{R_{t+1}}\Pi_{jt+1} - P_j t\right)\right\}, \tag{78}$$

where profits are equal to the dividend earned minus the opportunity cost of the price paid. Because there are no trading costs, it must be that $\mathcal{P}\{x_{ijt} = \kappa_L\} = 1 - \mathcal{P}\{x_{ijt} = \kappa_H\}$:

$$\begin{aligned}\widetilde{EU}(\beta_{ijt}, \beta_{jt}) = {} & \tilde{\mathbb{E}}\left\{(\kappa_H - \kappa_L)\mathcal{P}\{x_{ijt} = \kappa_H\}\left(\frac{1}{R_{t+1}}\Pi_{jt+1} - P_j t\right)\right\} \\ & - \kappa_L \tilde{\mathbb{E}}\left\{\frac{1}{R_{t+1}}\Pi_{jt+1} - P_{jt}\right\}.\end{aligned} \tag{79}$$

Taking the derivative with respect to β_{ijt} yields

$$\widetilde{MB}(\beta_{ijt}, \beta_{jt}) = \tilde{\mathbb{E}}\left\{(\kappa_H - \kappa_L)\frac{\partial \mathcal{P}\{x_{ijt} = \kappa_H\}}{\partial \beta_{ijt}}\left(\frac{1}{R_{t+1}}\Pi_{jt+1} - P_{jt}\right)\right\}. \tag{80}$$

The marginal benefit of producing information is proportional to $\kappa_H - \kappa_L$. Therefore, if traders decide to expand $\kappa_H - \kappa_L$ in response to a shock, the endogenous choice of position limits will tend to increase information production.

C Intermediate Good Firms

C.1 Micro-Foundation

Intermediate good firms sell their whole revenue stream to traders to focus the analysis on information frictions. This assumption can be micro-founded by assuming that there are at least two entrepreneurs without private wealth for each variety j. Entrepreneurs need to turn to financial markets to fund their projects, but the market is competitive in the sense that, at most, one entrepreneur for each variety j can sell her shares to traders. A mechanism chooses the entrepreneur who promises the highest rate of return on her shares. If there is a tie, the successful entrepreneur is chosen randomly among the entrepreneurs who offer the highest return.

Formally, the entrepreneur's problem is

$$\max_{K_{jkt+1}, D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right)} C_{jkt} + \delta \mathbb{E}\left\{C_{jkt+1}\right\} \tag{81}$$

$$s.t. \quad K_{jkt+1} + C_{jkt} \leq P_{jkt}. \tag{82}$$

$$C_{jkt+1} \leq \Pi_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) - D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) \tag{83}$$

$$C_{jkt}, C_{jkt+1}, K_{jkt+1}, D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) \geq 0 \tag{84}$$

where

$$P_{jkt} = \begin{cases} 0 & \text{if } \exists k' \neq k : R_{jkt+1} < R_{jk't+1} \\ \begin{cases} 0 & \text{w.p. } 1 - \frac{1}{|k''|} \\ \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) | s_{ijt} = z_{jt}, z_{jt}\right\} & \text{w.p. } \frac{1}{|k''|} \end{cases} & \text{if } \substack{\exists k'' \neq k : R_{jkt+1} = R_{jk''t+1} \\ \forall k' \neq k : R_{jkt+1} \geq R_{jk't+1}} \\ \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) | s_{ijt} = z_{jt}, z_{jt}\right\} & \text{if } \forall k' \neq k : R_{jkt+1} > R_{jk't+1} \end{cases} . \tag{85}$$

The entrepreneur maximizes her utility over consumption today and tomorrow using the same utility function as households.[29] When young, entrepreneurs can either consume or invest in their firm. When old, entrepreneurs pay out a dividend D_{jkt+1} and consume what remains of revenue Π_{jkt+1}.

[29]Entrepreneurs can only raise funds by selling claims to revenue and cannot otherwise borrow or lend. This setting guarantees that asset prices are an invertible function of z_{jt}, a noisy signal of firm productivity, without which the equilibrium in the financial market does not exist. See Albagli, Hellwig, and Tsyvinski (2011b, 2017) for a discussion of this issue.

The entrepreneur is only able to sell her shares at a positive price if she offers the highest return in market j. If the entrepreneur promises a lower rate of return R_{jkt+1} than some other entrepreneur k', she will not be able to sell her shares and will raise nothing. If she promises the highest rate of return in the economy, but another entrepreneur promises the same return, she will be able to sell her shares with probability $1/|k''|$ where $|k''|$ is the number of entrepreneurs who also promise the highest return. If only she promises the highest return, she will be able to sell her shares with probability one. Finally, the expected rate of return is given by

$$R_{jkt+1} = \frac{\mathbb{E}\left\{D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right)\right\}}{P_{jkt+1}}. \tag{86}$$

There is perfect competition between entrepreneurs because productivity A_{jt} is attached to variety j. Therefore, all entrepreneurs sell their goods at the same price ρ_{jt+1} . The only equilibrium is one in which at least two entrepreneurs choose

$$D_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) = \Pi_{jkt+1}\left(A_{jt}, K_{jkt+1}\right) \tag{87}$$

$$K_{jkt+1} = P_{jkt}. \tag{88}$$

It show first that the choice above is an equilibrium and then I show that it is the only equilibrium. Any entrepreneur k who chooses (87) and (88) can only deviate by either investing less or paying a lower dividend. In either case, another entrepreneur exists who promises a higher return on investment and entrepreneur k is unable to sell her shares. Similarly, any entrepreneur who does not choose (87) and (88) does not have a profitable deviation either. Choosing to invest less or promising a lower dividend leads to no change, as the rate of return is only further depressed. Investing more or promising a higher dividend is similarly inconsequential as long as the entrepreneur does not choose at least (87) and (88). If she chooses to deviate to (87) and (88), she still earns zero profits but gets to produce with positive probability. Therefore, (87) and (88) are an equilibrium.

To show that at least two entrepreneurs choosing (87) and (88) is the only equilibrium, it is necessary to show that profitable deviations exist for all other choices of investment and dividends. First, consider that only one entrepreneur k chooses (87) and (88) and that all others either invest strictly less or pay a lower dividend in some states. Then entrepreneur k can raise her profits by either investing less or paying a lower dividend while still promising the highest rate of return. Second, assume that all entrepreneurs choose an investment and dividend policy that leads to positive profits in at least some states. In this case, there is a profitable deviation for any entrepreneur. Entrepreneur k can invest more or pay a larger dividend to promise the highest rate of return while still keeping positive profits. Therefore, the only equilibrium is given by at least two entrepreneurs choosing (87) and (88).

C.2 Entrepreneurs with Market Power: Equity

Alternatively, assume that there is only one entrepreneur per variety. Furthermore, assume that entrepreneurs are patient and restricted to selling equity contracts as captured in the following Assumption.

Assumption 3 (***Equity Contracts***)**.** *Entrepreneurs can only sell claims to a fraction $\lambda(P_{jt}, P_t) \in [0,1]$ of firm revenue.*

The share of revenue that is sold to the market is allowed to depend on the price P_{jt} and on the aggregate value of the stock market P_t. The entrepreneur's maximization problem is

$$\max_{\lambda(P_{jt},P_t),K_{jt+1}} \quad \mathbb{E}\left\{C_{jt+1}|P_{jt}\right\} \tag{89}$$

$$C_{jt+1} \leq \Pi\left(A_{jt}, K_{jt+1}\right) - D\left(A_{jt}, K_{jt+1}\right) \tag{90}$$

$$D\left(A_{jt}, K_{jt+1}\right) = \lambda\left(P_{jt}, P_t\right)\Pi\left(A_{jt}, K_{jt+1}\right) \tag{91}$$

$$0 \leq K_{jt+1} \leq P_{jt}. \tag{92}$$

The entrepreneur maximizes her old age consumption, consisting of firm revenue $\Pi\left(A_{jt}, K_{jt+1}\right)$ after paying dividends $D\left(A_{jt}, K_{jt+1}\right)$ subject to constraints (90), (91) and (92). The first constraint states that consumption cannot be negative. The second constraint follows from Assumption 3. The final constraint imposes non-negativity on investment and states that entrepreneurs cannot borrow additional funds from other sources. Plugging in the constraints into the objective yields the simplified problem

$$\max_{\lambda(P_{jt},P_t),K_{jt+1}} \quad \mathbb{E}\left\{\Pi_{jt+1} - \lambda\left(P_{jt}, P_t\right)\Pi_{jt+1}|z_{jt}\right\} \tag{93}$$

$$0 \leq K_{jt+1} \leq P_{jt}. \tag{94}$$

Following the same steps as in the main text, the asset price P_{jt} can be expressed as

$$P_{jt} = \alpha\frac{\lambda\left(P_{jt}, P_t\right)}{R_{t+1}}Y_t^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}K_{jt+1}^{\frac{\theta-1}{\theta}}. \tag{95}$$

It is optimal for the entrepreneur to invest everything she raises, which allows firm capital to be written as

$$P_{jt} = K_{jt+1} = \left(\alpha\frac{\lambda\left(P_{jt}, P_t\right)}{R_{t+1}}Y_t^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}\right)^{\theta}. \tag{96}$$

Plugging the expression for capital back into the entrepreneur's problem leads to the final problem

$$\max_{\lambda(P_{jt},P_t)} \quad (1-\lambda(P_{jt},P_t))\,\lambda(P_{jt},P_t)^{\theta-1}\,c \tag{97}$$

$$c = \left(1-\alpha^{\theta-1}\right)\mathbb{E}\left\{A_{jt}|z_{jt}\right\}\left(\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}\frac{Y_t^{\alpha_Y}}{R_{t+1}}\right)^{\theta-1}. \tag{98}$$

The first-order condition to the simplified problem is

$$\begin{aligned}
&\frac{\partial}{\partial\lambda_{jt}}(1-\lambda(P_{jt},P_t))\,\lambda(P_{jt},P_t)^{\theta-1}=0\\
\iff\ &(\theta-1)\,\lambda(P_{jt},P_t)^{\theta-2}-\theta\lambda(P_{jt},P_t)^{\theta-1}=0\\
&\iff \forall j,t: \quad \lambda_{jt}=\frac{\theta-1}{\theta}
\end{aligned} \tag{99}$$

Therefore, all entrepreneurs irrespective of (P_{jt},P_t) sell a constant fraction $\lambda_{jt}=\frac{\theta-1}{\theta}$ of revenue to the financial market. The resulting dividend per share is

$$D_{jt+1}=\frac{\theta-1}{\theta}\alpha Y_{t+1}^{\alpha_Y}A_{jt}K_{jt+1}^{\frac{\theta-1}{\theta}}. \tag{100}$$

Assigning market power to entrepreneurs, therefore, effectively leads to a markup on the price of the intermediate good as traders only receive a fraction $\frac{\theta-1}{\theta}$ of firm revenue for completely funding firm investment. The effect is to depress investment, which can be undone through an ad-valorem subsidy of $\tau=\frac{\theta}{\theta-1}$ in the market for intermediate goods.

C.3 Entrepreneurs with Market Power: Credit Markets

The main focus of this paper is to study booms that are caused by productivity or sentiment. The literature extensively studies such booms in credit markets. The model can be extended to cover debt securities that are centrally traded instead of stock markets. Assume that the entrepreneur's technology is given by

$$Y_{jt}=\begin{cases}A^{\frac{\theta-1}{\theta}}K_{jt} & \text{w.p. } \pi_{jt}\\ 0 & \text{w.p. } 1-\pi_{jt}\end{cases}. \tag{101}$$

In the main text, entrepreneurs were sure to succeed , but their productivity was uncertain. Now, assume that entrepreneurs run projects that are either successful and give a certain payoff or fail and produce nothing. Success or failure is determined by the realization of a normally distributed variable,

$$\mathcal{P}\left(Y_{jt}>0\right)=P\left(a_{jt}>\bar{a}\right)=\Phi\left(\frac{a_t-\bar{a}}{\sqrt{\sigma_a^2}}\right)=\pi_t. \tag{102}$$

The entrepreneur's project succeeds whenever $a_{jt}\sim\mathcal{N}\left(a_t,\sigma_a^2\right)$ has a realization above the threshold $\bar{a}$. Households have dispersed information about the firm-specific shock $s_{ijt}=a_{jt}+\frac{\eta_{ijt}+\varepsilon_{jt}}{\sqrt{\beta_{ijt}}}$ where η_{ijt} is idiosyncratic noise and ε_{jt} is correlated noise. Same as before, traders suffer from correlation neglect and perceive only their own signal to be $s_{ijt}=a_{jt}+\eta_{ijt}/\sqrt{\beta_{ijt}}$. The household's problem is the same as in the main model.

To finance their projects, entrepreneurs issue a unit mass of debt securities with the payoff

$$X_{jt}=\begin{cases}\lambda_{jt} & \text{if } Y_{jt}>0\\ 0 & \text{otherwise}\end{cases}. \tag{103}$$

The security pays an amount λ_{jt} when the project succeeds and pays zero otherwise.[30] The entrepreneur maximizes the revenue that she can keep in case of success after repaying her debt obligations

$$\max_{\lambda_{jt},K_{jt+1}}\quad \rho_{jt+1}Y_{jt+1}\left(a_{jt},K_{jt+1}\right)-X_{jt}\left(a_{jt},\lambda_{jt}\right) \tag{104}$$

$$0\le K_{jt+1}\le P_{jt}. \tag{105}$$

The entrepreneur invests all raised funds, $K_{jt+1}=P_{jt}$. According to the beliefs of the marginal trader, the price of debt and firm capital can then be written as

$$K_{jt+1}=P_{jt}=\frac{\lambda_{jt}}{R_{t+1}}\Phi\left(\frac{\tilde{\mathbb{E}}\left\{a_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}-\bar{a}}{\sqrt{\mathbb{V}}}\right), \tag{106}$$

where $\mathbb{V}=\left(\sigma_a^{-2}+\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)\right)^{-1}$is the posterior uncertainty. The solution to the entrepreneur's problem is

$$\lambda_{jt}=\left(\frac{\theta-1}{\theta}\alpha Y_{t+1}^{\alpha_Y}A\right)^{\theta}\frac{\Phi\left(\frac{\tilde{\mathbb{E}}\left\{a_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}-\bar{a}}{\sqrt{\mathbb{V}}}\right)^{\theta-1}}{\left(R_{t+1}\right)^{\theta-1}}, \tag{107}$$

which depends on the market valuation of debt or equivalently the interest rate that en-

[30] Quantity and payoffs can be interchanged by denoting the mass of securities by λ_{jt} so the payoff in the good state is normalized to one. Instead, the quantity is normalized to one and the payoff is allowed to vary.

trepreneur j faces. Using (106) and (107) in the expression for firm-revenue allows the entrepreneur's decision to be expressed as a fraction of output in the case of success,

$$\frac{\lambda_{jt}}{\rho_{jt+1} Y_{jt+1}\left(a_{jt}, K_{jt+1}\right)} = \frac{\theta - 1}{\theta}. \tag{108}$$

This result recovers the optimal equity contract from section C.2.

In contrast to the model with equity, there is an additional channel through which shocks affect information production. The binary payoff function introduces changes in the variance of outcomes for firms driven by productivity and sentiment shocks. The variance of outcomes is captured by $\pi_{jt}\left(1-\pi_{jt}\right)$, whereas riskiness normally would only be captured by the probability of failure, $1-\pi_{jt}$. Intuitively, a project is entirely safe whenever the probability of success, π_{jt}, is equal to one. In this case, learning about the firm-specific shock, a_{jt}, is inconsequential. The same reasoning applies if the project is sure to fail ($\pi_{jt}=0$). Therefore, the effect of changes to a_t is ambiguous. Positive shocks to a_t trigger additional information production only when π_{jt} was low before, but they crowd out information when debt becomes safe as a consequence. Therefore, aggregate shocks affect the (perceived) riskiness of debt.[31]

Although my model abstracts from banks and credit intermediation, it replicates the main stylized facts of credit booms before financial crises. First, credit booms are episodes of sharp increases in lending and economic activity (Jordà, Schularick, and Taylor, 2011). This is the case in the model presented here, as the volume of credit increases in response to a positive aggregate shock. As a result, investment and economic activity increase. Second, credit becomes riskier as lending standards are relaxed, i.e., riskier firms get access to credit (Keys et al., 2010). In response to a sentiment shock, all firms are considered to be safer than they actually are. Because there is more scope for a change in beliefs for relatively risky firms, the sentiment shock leads to a disproportionate increase in funding for risky firms (low realization of a_{jt}). Third, credit spreads decrease in the boom phase before a financial crisis (Krishnamurthy and Muir, 2017). In the case of sentiment-driven booms, all firms are perceived to be safer than they actually are and, therefore, spreads are low.

[31] An alternative interpretation is that productivity shocks affect productivity conditional on success, A. In this case, productivity shocks would have no effect on the riskiness of debt and would behave similarly to a productivity shock in the model with equity markets.

D Multi-Sector /-Country Model and Sector-/ Country-Specific Shocks

Let the economy consist of $N \in \mathbb{N}$ sectors or countries. Each consists of a unit mass of firms indexed by $nj \in N \times [0,1]$. Similarly, each household now has one trader for each firm in each sector or country. The aggregate production function becomes

$$Y_t = L^{1-\alpha} \left[\sum_{n \in N} \left(\int_0^1 Y_{njt}^{\frac{\theta_n - 1}{\theta_n}} dnj \right)^{\frac{\theta_n}{\theta_n - 1} \frac{\theta - 1}{\theta}} \right]^{\frac{\alpha\theta}{\theta - 1}}. \tag{109}$$

where θ_n is the elasticity substitution inside sector or country $n \in N$. Productivity and sentiment shocks can now also be sector-specific, such that $a_{njt} \sim \mathcal{N}\left(a_{nt}, \sigma_{a,n}^2\right)$, $a_{nt} \sim \mathcal{N}\left(a_t, \sigma_a^2\right)$ and $\varepsilon_{njt} \sim \mathcal{N}\left(\varepsilon_{nt}, \sigma_{\varepsilon,n}^2\right)$, $\varepsilon_{nt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_a^2\right)$. Sector-specific and aggregate shocks are observable.

In reality, booms rarely affect the whole economy equally. For example, the dot-com boom of the late 1990s was mainly about information technology and the emerging internet. Similarly, the housing boom in the 2000s concentrated in the financial and construction sector. In contrast to the economy-wide sentiment shock studied in the paper's main body, sector-specific sentiment shocks lead directly to an increase in capital misallocation as the marginal product of capital declines in the shocked sector.

Nonetheless, the main finding leads then to an additional insight: sector-specific sentiment shocks lead to an increase in capital misallocation inside the shocked sector. Not only is there too much investment in a specific sector, but this investment also flows to increasingly unproductive firms, thus amplifying the welfare costs of a sentiment boom. This result is captured in the following corollary analogously to Proposition 4.

Corollary 4. *A sector / country-specific positive sentiment shock can lead to an increase in capital misallocation inside the sector or country.*

At the same time, the redirection of capital investment towards the positively shocked sector can hurt non-shocked sectors, leading to a negative spillover of positive shocks across sectors. This is the case whenever aggregate investment is fixed ($\delta \to \infty$) or goods from different sectors are close substitutes ($\theta \to \infty$). This analysis also extends to a multi-country setting with free capital flows, in which a sentiment boom in one country leads to an increase in capital misallocation in both countries.

Corollary 5. *If aggregate investment is fixed ($\delta \to \infty$) or sector-/country-specific goods are close substitutes ($\theta \to \infty$), a sector- / country-specific positive shock leads to an increase in capital misallocation also in all other sectors / countries.*

E Full Social Planner Problem

In the main text, the social planner could only intervene by choosing information production. Now, the social planner can also choose consumption and investment in the aggregate and cross-section to maximize social welfare, defined as the sum of the utilities of all traders. Therefore, the social planner is able to achieve the second best by fixing all inefficiencies. The maximization problem is

$$\max_{K_{j1},C_0,C_1,\beta_{j0}} \quad C_0 + \delta \mathbb{E}_0 \left\{ C_1 \right\} - \int_0^1 IA\left(\beta_{j0}\right) dj \tag{SPFull}$$

$$s.t. \quad K_1 = W_0 - C_0 \tag{110}$$

$$C_1 \leq Y_1 \left(\left\{ K_{j1} \right\}, \left\{ \beta_{j0} \right\} \right) \tag{111}$$

$$C_0 \leq W_0 \tag{112}$$

$$K_{j1}, C_0, C_1, \beta_{j0} \geq 0. \tag{113}$$

Constraint (110) states that aggregate capital in period 1 is equal to endowments W_0 minus youth consumption C_0. Resource constraints for consumption are given in (111) and (112). Finally, non-negativity constraints on consumption, information production and capital are given in (113). The solution to the social planner's problem is given in the following proposition.

Proposition 11. *The social planner's allocation under perfect information about aggregate shocks $\{a_0, \varepsilon_0\}$ is given by $\left\{ C_0^{SP}, K_{j1}^{SP}, K_1^{SP}, \beta_0^{SP} \right\}$, where*

$$K_{j1}^{SP} = \frac{\mathbb{E}\left\{ A_{j0} | z_{j0} \right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{ A_{j0} | z_{j0} \right\}^{\theta} dj} K_1^{SP} \tag{114}$$

leading to aggregate output

$$Y_1^{SP} = A_0^{SP} \left(K_1^{SP} \right)^{\alpha} \quad \text{with } A_0^{SP} = \left(\int_0^1 \mathbb{E}\left\{ A_{j0} | z_{j0} \right\}^{\theta} dj \right)^{\frac{\alpha}{\theta - 1}}. \tag{115}$$

The interest rate is

$$R_1^{SP} = \alpha A_0^{SP} \left(K_1^{SP} \right)^{\alpha - 1}, \tag{116}$$

leading to aggregate investment

$$K_1^{SP} = \min \left\{ \left(\alpha \delta A_0^{SP} \right)^{\frac{1}{1-\alpha}}, W_0 \right\}. \tag{117}$$

The symmetric information production choice is

$$\text{for all } \beta_{j0} = \beta_0^{SP} : \delta \left. \frac{\partial A_0^{SP}}{\partial \beta_0} \right|_{\beta_0 = \beta_0^{SP}} \left(K_1^{SP}\right)^{\alpha} = \left. \frac{\partial IA_0}{\partial \beta_0} \right|_{\beta_0 = \beta_0^{SP}} . \tag{118}$$

The social planner fixes the two aforementioned inefficiencies. First, the social planner distributes capital optimally by attributing the correct precision to the price signal z_{jt} as in (28) and (114). As a result, ex-ante marginal products of capital are equalized between firms. This reallocation of capital leads to an increase in TFP as in Proposition 3 compared to the competitive allocation. Second, the social planner chooses information production β_0^{SP} to increase TFP instead of trading rents. Given that the social planner optimally distributes capital between firms as in (114), an increase in β_0^{SP} always benefits aggregate productivity A_0^{SP}.

E.1 Implementation

In this section, I investigate how the social planner can implement the centralized allocation through the use of taxes and subsidies. Net proceeds and costs of taxes and subsidies are distributed lump-sum between old traders.

The social planner can apply a tax/subsidy on dividend income to achieve the constrained efficient allocation of capital. Under this state-dependent tax/subsidy, traders receive

$$\Pi_{j1}^{DE} = \tau^{Bias}(z_{j0}) \Pi_{j1}, \quad \text{where } \tau^{Bias}(z_{j0}) = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}}{\tilde{\mathbb{E}}\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\}}. \tag{119}$$

As seen in Figure 9, $\tau^{Bias}(z_{j0})$ is a subsidy on dividends whenever $K_{j1}^{eff} < K_{j1}$. If the social planner has information about aggregate shocks, the tax/subsidy corrects also for aggregate sentiment shocks through the marginal trader's expectations $\tilde{\mathbb{E}}\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\}$. A tax (subsidy) can lower (increase) investment in response to a positive (negative) sentiment shock.

Moreover, a tax/subsidy $\tau^{Info}(\beta_{ij0})$ on information production is sufficient to induce the socially optimal level,

$$\frac{\partial IA^{DE}(\beta_{ij0})}{\partial \beta_{ij0}} = \tau^{Info}(\beta_{ij0}) \frac{\partial IA(\beta_{ij0})}{\partial \beta_{ij0}}, \quad \tau^{Info}(\beta_{ij0}) = \frac{\left. \widetilde{MB}(\beta_{ij0}, \beta_{j0}) \right|_{\beta_{ij0} = \beta_{j0}}}{\delta \frac{\partial A_0}{\partial \beta_0} K_1^{\alpha}}. \tag{120}$$

Applying the after-tax marginal cost leads directly to the first-order condition as in (118). The results are summarized in the following proposition:

Proposition 12. *The social planner's allocation* $\left\{K_1^{SP}, K_{j1}^{SP}, \beta_0^{SP}\right\}$ *can be implemented through* *taxes/subsidies (119) and (120).*

Alternatively, the social planner can use transaction taxes to implement the optimal capital allocation. Since Tobin (1972), financial transaction taxes have been discussed with the objective of reducing volatility by making short-term speculation less profitable. This perspective cannot be studied here as assets are short-lived and only traded once. Nonetheless, a transaction tax can be used to drive a wedge between how much traders pay for shares and how much is invested in capital. The following proposition shows how such a transaction tax can be used to stabilize investment against sentiment shocks and reallocate capital across firms.

Corollary 6. *(i) Aggregate investment can be stabilized with respect to sentiment shocks through a transaction tax,*

$$K_{j1}^{DE} = \tau^{Trans}\left(\varepsilon_0\right) P_{j0}, \quad \tau^{Trans}\left(\varepsilon_0\right) = \exp\left\{-\omega_{s\varepsilon}\varepsilon_0\right\}, \quad \omega_{s\varepsilon} = \frac{\sqrt{\beta_0}}{\sigma_a^{-2} + \beta_0\left(1+\sigma_\varepsilon^{-2}\right)}. \tag{121}$$

(ii) The dividend tax/subsidy $\tau^{Bias}\left(z_{j0}\right)$ (119) can be substituted by a state-dependent transaction tax,

$$K_{j1}^{DE} = \tau^{Trans}\left(P_{j0}\right) P_{j0}, \quad \tau^{Trans}\left(P_{j0}\right) = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0}=z_{j0}, z_{j0}\right\}}. \tag{122}$$

F Information Structure

I assume that traders are overconfident in that they wrongly believe that sentiment drives the beliefs of all other traders but does not drive their own beliefs. Whereas it is empirically reasonable to assume that behavioral biases play a role in financial markets, I chose this approach for tractability. Avoiding the introduction of non-optimizing agents greatly simplifies embedding a model of informative financial markets in a macro setting and facilitates the welfare analysis. Moreover, overconfidence is sufficient to motivate costly information production and to avoid the Grossman-Stiglitz paradox. This assumption is not necessary for deriving the main result that sentiment shocks crowd out information and can identically be derived with noise trades in partial equilibrium.

In the following, I walk through different assumptions for the information structure and their relationships to information aggregation and production.

Exogenous Public Signals

The simplest case is one in which traders do not have private signals but instead observe public signals of the form $z_{jt} = a_{jt} + \varepsilon_{jt}/\sqrt{\beta}$. This setting mirrors the allocation in Proposition 3. However, it has nothing to say about the origin of the signal. How does it come about, and what determines its precision?

Heterogeneous Private Signals

To say something about the aggregation of information, endow traders with heterogeneous private signals as in (5). Following the same steps as in section 3.3 leads to the market equilibrium.

Under rationality, the constrained efficient capital allocation as in Proposition 3 is achieved, but information production is ruled out. As in the model with overconfidence, observing the asset price is isomorphic to observing $z_{jt} = \int s_{ijt} dj$. Rational traders realize that they have nothing to learn from their private signal after observing the public signal z_{jt}. However, setting up this equilibrium requires that traders use their private signals to make the buying decision. In this setting, traders are indifferent between buying and not buying, as they all share the same information set. Therefore, the indifference can be broken in favor of buying whenever $s_{ijt} \geq z_{jt}$.[32]

The main drawback of this approach is that it rules out costly information production. The private signal s_{ijt} becomes fully uninformative after observing the public signal z_{jt} in equilibrium ($\forall i : \beta_{ijt} = \beta_{jt}$). In this case, the trader finds it optimal to deviate to $\beta_{ijt} < \beta_{jt}$, which guarantees an informative private signal. However, since traders are ex-ante homogeneous, asymmetric equilibria cannot exist. There is no equilibrium with costly information production and rationality, similar to the result in Grossman and Stiglitz (1980).

To overcome this problem, I assume that traders think that their private signal does not contain sentiment. Therefore, they do not discard their private signal s_{ijt} after observing the price signal z_{jt}. The posterior of trader ij becomes

$$a_{jt} | s_{ijt}, z_{jt} \sim \mathcal{N}\left(\frac{\beta_{ijt} s_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2} z_{jt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}\right), \tag{123}$$

where I have marked in blue terms that follow from the overconfidence assumption. It follows that traders have posteriors that are too precise, as they think that their private signals remain informative after observing z_{jt}. This misperception motivates traders to invest in

[32]This class of equilibria is referred to as "fully revealing rational expectations equilibria" and is studied in Grossman (1976).

their private signal with the anticipation of trading rents. Finally, this bias leads to an overreaction of prices to the price signal z_{jt} as described in section 4.1. This price distortion appears also in models with rational and noise traders, if rational traders learn from prices and have heterogeneous private signals.[33] The main focus of this paper, however, is not on the price distortion, but rather on time-varying price informativeness and the allocational efficiency of financial markets.

G Proofs

Proof of Proposition 1. This proof follows the same steps as the proof for Proposition 1 in Albagli, Hellwig, and Tsyvinski (2017), since the financial market in my model is isomorphic to their model. Their proof is repeated here for completeness. The only difference is that K_{jt+1} depends on the price signal z_{jt}, whereas k in Albagli, Hellwig, and Tsyvinski (2017) is determined before trading takes place. Therefore, it is necessary to assume that $K_{jt+1}(z_{jt})$ is non-decreasing in z_{jt} as the price might otherwise be not invertible, which is confirmed ex-post. The proof begins in the following.

There must be a threshold $\hat{s}(P_{jt})$ such that all households with $s_{ijt} \geq \hat{s}(P_{jt})$ find it profitable to buy two units of share j and otherwise abstain from trading. It follows that the price must be equal to the valuation of the trader who is merely indifferent between buying and not buying,

$$P_{jt} = \frac{1}{R_{t+1}} \tilde{\mathbb{E}} \left\{ D\left(A_{jt}, K_{jt+1}\right) | s_{ijt} = \hat{s}\left(P_{jt}\right), P_{jt} \right\}. \tag{124}$$

This monotone demand schedule leads to total demand

$$D(\theta, \varepsilon, P) = 2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right). \tag{125}$$

Equalizing total demand with a normalized supply of one leads to the market-clearing condition

$$2\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right) = 1, \tag{126}$$

with the unique solution $\hat{s}(P_{jt}) = z_{jt} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}$. If P_{jt} is pinned down by z_{jt}, then P_{jt} is invertible, given that K_{jt+1} is non-decreasing in z_{jt}. It follows, then, that observing P_{jt} is equivalent to observing $z_{jt} \sim \mathcal{N}\left(a_{jt}, \beta_{jt}^{-1}\sigma_{\varepsilon}^{2}\right)$. Traders treat the signal z_{jt} and their private signal $s_{ijt} \sim \mathcal{N}\left(a_{jt}, \beta_{ijt}^{-1}\right)$ as mutually independent. Using this result, the price can be restated as

$$P\left(z_{jt}, K_{jt+1}\right) = \frac{1}{R_{t+1}} \tilde{\mathbb{E}} \left\{ \Pi\left(A_{jt}, K_{jt+1}\right) | s_{ijt} = z_{jt}, z_{jt} \right\}, \tag{127}$$

[33] For a more detailed discussion, see Albagli, Hellwig, and Tsyvinski (2011a, 2015, 2021).

where posterior expectations of trader ij are given by

$$a_{jt}|s_{ijt}, z_{jt} \sim \mathcal{N}\left(\frac{\sigma_a^{-2}a_t + \beta_{ijt}s_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}z_{jt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}\right). \tag{128}$$

Using the result that the firm invests all proceeds into capital ($K_{jt+1} = P_{jt}$), it follows indeed that K_{jt+1} is non-decreasing in z_{jt} and P_{jt} is an invertible function of the price signal z_{jt}. It remains to show the uniqueness of the above solution. Begin with the assumption that demand $x(s_{ijt}, P_{jt})$ is non-increasing in P_{jt}. It follows that $\hat{s}(P_{jt})$ is non-decreasing in P_{jt}. There are two cases to differentiate. First, if $\hat{s}(P_{jt})$ is strictly increasing in P_{jt}, then the price is indeed uniquely pinned-down by z_{jt} and invertible; it can be expressed like above. Secondly, assume that the threshold is flat over some interval, such that $\hat{s}(P_{jt}) = \hat{s}$ over some interval $P_{jt} \in (P', P'')$ for $P' \neq P''$. Furthermore, choose $\epsilon > 0$ small enough such that $\hat{s}(P_{jt})$ is increasing to the left and the right of the interval, i.e., over $P_{jt} \in (P' - \epsilon, P')$ and $P_{jt} \in (P'', P'' + \epsilon)$. In these regions, $\hat{s}(P_{jt})$ is monotonically increasing in P_{jt}, which is uniquely pinned down by z_{jt} and invertible; observing the price is equivalent to observing the signal z_{jt}. In this case the price can be expressed as before for $z_{jt} \in (\hat{s}(P' - \varepsilon), \hat{s})$ and $z_{jt} \in (\hat{s}, \hat{s}(P'' + \epsilon))$. This leads to a contradiction in the assumption that $P' \neq P''$, because $P(z_{jt}, K_{jt+1})$ is both continuous and monotonically increasing in z_{jt}. Therefore, $\hat{s}(P_{jt})$ cannot be flat and the above solution is indeed unique. □

Proof of Proposition 2. (i) Using (4) in (21) leads to the expression for firm capital

$$K_{jt+1} = \left(\frac{\alpha Y_{t+1}^{\alpha_Y}}{R_{t+1}}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}\right)^{\theta}. \tag{129}$$

Plugging R_{t+1} from (22) into (21) using (129) leads to

$$K_{jt+1} = \frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj}K_{t+1}, \tag{130}$$

which finishes the proof.

(ii) Plugging the above expression for firm capital (25) into the aggregate production function (2) leads to

$$
\begin{aligned}
Y_t &= \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \left(\int_0^1 A_{jt-1} K_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \frac{\left(\int_0^1 A_{jt-1} \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta-1} dj \right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta} dj \right)^{\alpha}} K_t^{\alpha} \\
&= A_{t-1} L^{1-\alpha} K_t^{\alpha} \qquad (131)
\end{aligned}
$$

where total factor productivity is

$$
\begin{aligned}
A_{t-1} &= \frac{\left(\int_0^1 A_{jt-1} \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta-1} dj \right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta} dj \right)^{\alpha}} \\
&= \exp \left\{ \theta a_{t-1} + \left((\theta - 1) \omega_a + 1 \right)^2 \frac{\sigma_a^2}{2} + (\theta - 1)^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} + (\theta - 1) \omega_{s\varepsilon} \varepsilon_{t-1} \right. \\
&\left. + \frac{(\theta - 1)}{2} \mathbb{V} \right\}^{\frac{\alpha\theta}{\theta-1}} \\
&: \exp \left\{ \theta a_{t-1} + \theta^2 \omega_a^2 \frac{\sigma_a^2}{2} + \theta^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} + \theta \omega_{s\varepsilon} \varepsilon_{t-1} + \frac{\theta}{2} \mathbb{V} \right\}^{\alpha} \\
&= \exp \left(\frac{\alpha\theta}{\theta - 1} a_{t-1} + \left(\frac{\alpha\theta}{\theta - 1} \left((\theta - 1) \omega_a + 1 \right)^2 - \alpha\theta^2 \omega_a^2 \right) \frac{\sigma_a^2}{2} \right. \\
&\left. + \left(\alpha\theta (\theta - 1) - \alpha\theta^2 \right) \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} \right) \\
&= \exp \left(\frac{\alpha\theta}{\theta - 1} a_{t-1} + \alpha\theta \left((\theta - 1) \omega_a^2 + 2\omega_a + \frac{1}{\theta - 1} - \theta \omega_a^2 \right) \frac{\sigma_a^2}{2} - \alpha\theta \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} \right) \\
&= \exp \left(\frac{1}{\theta - 1} \left(a_{t-1} + \frac{\sigma_a^2}{2} \right) + \omega_a (2 - \omega_a) \frac{\sigma_a^2}{2} - \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{2} \right)^{\alpha\theta}. \qquad (132)
\end{aligned}
$$

The weights $\{\omega_a, \omega_\varepsilon, \omega_{s\varepsilon}\}$ and $\mathbb{V}$ are derived in Appendix A. Finally, total factor productivity can be expressed as

$$
\ln A_{t-1} (a_{t-1}, \beta_{t-1}) = \underbrace{\frac{\alpha\theta}{\theta - 1} \left(a_{t-1} + \frac{\sigma_a^2}{2} \right)}_{exogenous} + \underbrace{\kappa^a (\beta_{t-1}) \sigma_a^2 - \kappa^\varepsilon (\beta_{t-1}) \sigma_\varepsilon^2}_{allocative\ efficiency} \qquad (133)
$$

where $\kappa^a (\beta_{t-1}) = \frac{\alpha\theta}{2} \omega_a (2 - \omega_a)$ and $\kappa^\varepsilon (\beta_{t-1}) = \frac{\alpha\theta}{2} \omega_\varepsilon^2$.

(iii) I will show that the allocative efficiency component of TFP takes its minimum for $\beta_{t-1} > 0$ if $\sigma_\varepsilon^2 > 1$. The allocational efficiency component is proportional to

$$\begin{aligned}
&\omega_a \left(2-\omega_a\right)\sigma_a^2 - \omega_\varepsilon^2 \sigma_\varepsilon^2 \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)}\sigma_a^2 - \frac{\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 - \frac{\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_\varepsilon^2 \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)+2\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2 - \beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2 - \beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)+\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2 - \beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\beta_{t-1}^2\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^2+\beta_{t-1}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}, \qquad (134)
\end{aligned}$$

which is weakly positive for all values of β_{t-1} if $\sigma_\varepsilon^2 < 1$.

(iv) Using the previous result, it remains to take the derivative of (134) with respect to β_{t-1}. Denote

$$a = \beta_{t-1}^2\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^2+\beta_{t-1}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) \qquad (135)$$

$$b = \left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2 \qquad (136)$$

After some algebra,

$$\begin{aligned}
\frac{\partial a}{\partial \beta_{t-1}}b &= 2\beta_{t-1}^3\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^4+4\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^3+2\beta_{t-1}\sigma_a^{-2}\left(1+\sigma_\varepsilon^{-2}\right)^2 \\
&+\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)+2\beta_{t-1}\sigma_a^{-2}\left(1+\sigma_\varepsilon^{-2}\right)\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) \\
&+\sigma_a^{-4}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) \qquad (137)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial b}{\partial \beta_{t-1}}a &= 2\beta_{t-1}^3\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^4+2\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^3 \\
&+2\beta_{t-1}^2\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)\left(1+\sigma_\varepsilon^{-2}\right)^2+2\beta_{t-1}\sigma_a^{-2}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)\left(1+\sigma_\varepsilon^{-2}\right) \qquad (138)
\end{aligned}$$

Dropping the positive denominator, the derivative of (134) is $\frac{\partial a}{\partial \beta_{t-1}}b - \frac{\partial b}{\partial \beta_{t-1}}a$, which is after dividing through $\left(1+\sigma_{\varepsilon}^{-2}\right)^{2}$,

$$2\beta_{t-1}^{2}+\beta_{t-1}^{2}\sigma_{\varepsilon}^{-2}+2\beta_{t-1}\sigma_{a}^{-2}+\sigma_{a}^{-4}\frac{\sigma_{\varepsilon}^{-2}-\sigma_{\varepsilon}^{2}}{\left(1+\sigma_{\varepsilon}^{-2}\right)^{2}}+\beta_{t-1}^{2}\sigma_{\varepsilon}^{2}, \tag{139}$$

which is positive for all values of β_{t-1} if $\sigma_{\varepsilon}^{2}<1$.

□

Lemma 1. *Denote the efficient Bayesian weights*

$$\omega_{a}^{eff}=\frac{\beta_{t}\sigma_{\varepsilon}^{-2}}{\sigma_{a}^{-2}+\beta_{t}\sigma_{\varepsilon}^{-2}} \tag{140}$$

$$\omega_{\varepsilon}^{eff}=\frac{\sqrt{\beta_{t}}\sigma_{\varepsilon}^{-2}}{\sigma_{a}^{-2}+\beta_{t}\sigma_{\varepsilon}^{-2}} \tag{141}$$

and posterior uncertainty

$$\mathbb{V}^{eff}=\frac{1}{\sigma_{a}^{-2}+\beta_{t}\sigma_{\varepsilon}^{-2}}, \tag{142}$$

These are the weights that a rational uninformed observer would use after observing the price signal z_{jt}, in contrast to the Bayesian weights that overconfident traders use as introduced in A. Then the following relationships hold:

(i)

$$\underbrace{\left(\omega_{a}^{eff}\right)^{2}\sigma_{a}^{2}+\left(\omega_{\varepsilon}^{eff}\right)^{2}\sigma_{\varepsilon}^{2}}_{=Var(\mathbb{E}\{a_{jt}|z_{jt}\})}+\underbrace{\mathbb{V}^{eff}}_{Var(a_{jt}|z_{jt})}=\underbrace{\sigma_{a}^{2}}_{Var(a_{jt})}. \tag{143}$$

(ii)

$$\sigma_{a}^{2}-\mathbb{V}^{eff}=\omega_{a}^{eff}\sigma_{a}^{2} \tag{144}$$

(iii)

$$\theta\omega_{a}^{eff}\sigma_{a}^{2}+\mathbb{V}^{eff}=\left(\theta-1\right)\omega_{a}^{eff}\sigma_{a}^{2}+\sigma_{a}^{2} \tag{145}$$

Proof. (i):

$$
\begin{aligned}
\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \mathbb{V}^{eff} &= \frac{\beta_t^2 \sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2} \sigma_a^2 + \frac{\beta_t \sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2} \sigma_\varepsilon^2 \\
&+ \frac{1}{\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}} \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + 2\beta_t \sigma_\varepsilon^{-2} + \beta_t^2 \sigma_\varepsilon^{-4} \sigma_a^2\right) \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-4} + 2\beta_t \sigma_\varepsilon^{-2} \sigma_a^{-2} + \beta_t^2 \sigma_\varepsilon^{-4}\right) \sigma_a^2 \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2 \sigma_a^2 \\
&= \sigma_a^2.
\end{aligned} \tag{146}
$$

(ii):

$$
\begin{aligned}
\sigma_a^2 - \mathbb{V}^{eff} &= \sigma_a^2 - \frac{1}{\sigma_a^2 + \beta \sigma_\varepsilon^{-2}} \\
&= \frac{\sigma_a^2 \sigma_a^{-2} + \sigma_a^2 \beta \sigma_\varepsilon^{-2} - 1}{\sigma_a^{-2} + \beta \sigma_\varepsilon^{-2}} \\
&= \frac{\beta \sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta \sigma_\varepsilon^{-2}} \sigma_a^2 \\
&= \omega_a^{eff} \sigma_a^2
\end{aligned} \tag{147}
$$

(iii):

$$
\begin{aligned}
\theta \omega_a^{eff} \sigma_a^2 + \mathbb{V}^{eff} &= \frac{\theta \beta_{t-1} \sigma_\varepsilon^{-2} \sigma_a^2}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} + \frac{1}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} \\
&= \frac{\theta \beta_{t-1} \sigma_\varepsilon^{-2} + \sigma_a^{-2}}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} \sigma_a^2 \\
\text{Add and subtract} &= \frac{\theta \beta_{t-1} \sigma_\varepsilon^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2} - \beta_{t-1} \sigma_\varepsilon^{-2} + \sigma_a^{-2}}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} \sigma_a^2 \\
\text{Split} &= (\theta - 1) \frac{\beta_{t-1} \sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} \sigma_a^2 + \frac{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1} \sigma_\varepsilon^{-2}} \sigma_a^2 \\
&= (\theta - 1) \omega_a^{eff} \sigma_a^2 + \sigma_a^2
\end{aligned} \tag{148}
$$

□

Proof of Proposition 3. (i) An efficient allocation of capital equalizes marginal products between firms. Demand for firm capital follows from the following maximization problem

$$
\max_{K_{jt+1}} \alpha Y_{t+1}^{\alpha_Y} \mathbb{E}\left\{A_{jt} | z_{jt}\right\} K_{jt+1}^{\frac{\theta-1}{\theta}} - R_{t+1} K_{jt+1} \tag{149}
$$

with the first-order condition

$$K_{jt+1} = \left(\frac{\theta - 1}{\theta} \frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}}{R_{t+1}} \alpha Y_{t+1}^{\alpha_Y} \right)^{\theta}. \tag{150}$$

Integrating over all firms on both sides yields

$$R_{t+1} = \left(\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta} dj \right)^{\frac{1}{\theta}} \alpha \frac{\theta - 1}{\theta} Y_{t+1}^{\alpha_Y} K_{t+1}^{-\frac{1}{\theta}}. \tag{151}$$

Plugging this expression back into the first-order condition leads to the constrained efficient allocation

$$K_{jt+1} = \frac{\mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{jt}|z_{jt}\right\}^{\theta} dj} K_{t+1}. \tag{152}$$

(ii) Plugging (28) into (2) leads to

$$Y_t = A_{t-1}^{eff} K_t^{\alpha}, \tag{153}$$

where the constrained efficient level of total factor productivity is

$$A_{t-1}^{eff} = \frac{\left(\int_0^1 A_{jt-1} \mathbb{E}\left\{A_{jt-1}|z_{jt-1}\right\}^{\theta-1} dj \right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \mathbb{E}\left\{A_{jt-1}|z_{jt-1}\right\}^{\theta} dj \right)^{\alpha}} \overset{\text{L.I.E.}}{=} \left(\int_0^1 \mathbb{E}\left\{A_{jt-1}|z_{jt-1}\right\}^{\theta} dj \right)^{\frac{\alpha}{\theta-1}}. \tag{154}$$

The analytical expression can be obtained by evaluating the conditional expectations and using the constrained efficient Bayesian weights and posterior uncertainty,

$$\omega_p^{eff} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}, \quad \omega_a^{eff} = \frac{\beta_{t-1}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}} \tag{155}$$

$$\omega_\varepsilon^{eff} = \frac{\sqrt{\beta_{t-1}}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}}, \quad \mathbb{V}^{eff} = \frac{1}{\sigma_a^{-2} + \beta_{t-1}\sigma_\varepsilon^{-2}} \tag{156}$$

leading to

$$
\begin{aligned}
A_{t-1}^{eff} &= \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{\alpha}{\theta-1}} \\
&= \left(\int_0^1 \left\{\exp\left\{\theta\omega_p^{eff} a_t + \theta\omega_a^{eff} a_{jt} + \theta\omega_\varepsilon^{eff}(\varepsilon_{jt} - \varepsilon_t) + \theta\frac{\mathbb{V}^{eff}}{2}\right\}\right\} dj\right)^{\frac{\alpha}{\theta-1}} \\
&= \exp\left\{\theta\omega_p^{eff} a_t + \theta\omega_a^{eff} a_t + \frac{\theta^2}{2}\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \frac{\theta^2}{2}\left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \theta\frac{\mathbb{V}^{eff}}{2}\right\}^{\frac{\alpha}{\theta-1}} \\
&= \exp\left\{a_t + \frac{\theta}{2}\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \frac{\theta}{2}\left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \frac{\mathbb{V}^{eff}}{2}\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&\overset{\text{Lemma 1 (i) and (ii)}}{=} \exp\left\{a_t + \frac{1}{2}\left(\theta\omega_a^{eff}\sigma_a^2 + \mathbb{V}^{eff}\right)\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&\overset{\text{Lemma 1 (iii)}}{=} \exp\left\{a_t + \frac{1}{2}\left(\sigma_a^2 + (\theta-1)\,\omega_a^{eff}\sigma_a^2\right)\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&= \exp\left\{\frac{1}{1-\theta}\left(a_t + \frac{\sigma_a^2}{2}\right) + \omega_a^{eff}\frac{\sigma_a^2}{2}\right\}^{\alpha\theta}
\end{aligned}
\tag{157}
$$

TFP under the efficient allocation of capital can be similarly decomposed into two expressions,

$$
\ln A_{t-1}^{eff} = \underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right)}_{exogenous} + \underbrace{\alpha\theta\omega_a^{eff}\frac{\sigma_a^2}{2}}_{allocative\ efficiency}. \tag{158}
$$

It follows that

$$
\frac{\partial\omega_a^{eff}}{\partial\beta_{t-1}} > 0 \Rightarrow \frac{\partial A_{t-1}^{eff}}{\partial\beta_{t-1}} > 0, \tag{159}
$$

which completes the proof.

(iii) As under both allocations capital is distributed equally between firms for $\beta_{t-1} = 0$, total factor productivity also coincides,

$$
A^{eff}(a_{t-1}, 0) = A(a_{t-1}, 0) = \exp\left(\frac{\alpha\theta}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right)\right). \tag{160}
$$

In the perfect information case ($\beta_{t-1} = \infty$) the efficient and market allocation also coincide,

$$
\lim_{\beta_{t-1}\to\infty} A^{eff}(a_{t-1}, \beta_{t-1}) = \lim_{\beta_{t-1}\to\infty} A(a_{t-1}, \beta_{t-1}) = \exp\left(\frac{1}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right) + \frac{\sigma_a^2}{2}\right)^{\alpha\theta}. \tag{161}
$$

For $\beta_t \in (0, \infty)$, note that the exogenous TFP component coincides under the efficient and market allocation. Therefore, it suffices to show that the allocative efficiency component is higher under the efficient allocation than under the market allocation,

$$\omega_a^{eff}\sigma_a^2 > \omega_a\left(2-\omega_a\right)\sigma_a^2 - \omega_\varepsilon^2\sigma_\varepsilon^2. \tag{162}$$

Using the simplification of the RHS from (134) leads to

$$\frac{\sigma_\varepsilon^{-2}\sigma_a^2}{\sigma_a^{-2}+\beta\sigma_\varepsilon^{-2}} > \frac{\sigma_\varepsilon^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2-\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}. \tag{163}$$

Multiplying on both sides by $\left(\sigma_a^{-2}+\beta\sigma_\varepsilon^{-2}\right)\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2$ and simplifying leads to

$$\begin{aligned} & 2\sigma_\varepsilon^{-2}\beta+2\sigma_\varepsilon^{-4}\beta > \beta\sigma_\varepsilon^{-4}+\beta\left(1+\sigma_\varepsilon^{-2}\right)^2-\sigma_a^{-2}\sigma_\varepsilon^2-\beta \\ \iff & 2\sigma_\varepsilon^{-2}\beta+\sigma_\varepsilon^{-4}\beta > \beta+2\beta\sigma_\varepsilon^{-2}+\beta\sigma_\varepsilon^{-4}-\sigma_a^{-2}\sigma_\varepsilon^2-\beta \\ \iff & 0 > -\sigma_a^{-2}\sigma_\varepsilon^2 \end{aligned} \tag{164}$$

Since $\sigma_a^2, \sigma_\varepsilon^2 > 0$ and all conversion were equivalent, the initial inequality holds and total factor productivity under the constrained efficient allocation is larger than under the market allocation for $\beta_{t-1} \in (0,\infty)$. $\square$

Proof of Corollary 1. The distortion due to overconfidence vanishes if the expectations of the marginal trader $\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}$ and $\mathbb{E}\left\{A_{jt}|z_{jt}\right\}$ coincide under the following conditions:

(i) when the private signal is infinitely noisy, both expectations converge to the unconditional mean,

$$\lim_{\beta\to 0}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} = \lim_{\beta\to 0}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = \mathbb{E}\left\{A_{jt}\right\}. \tag{165}$$

When the private signal is infinitely precise, both expectations converge to the actual realization of A_{jt},

$$\lim_{\beta\to\infty}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} = \lim_{\beta\to\infty}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = A_{jt} \tag{166}$$

(ii) When the variance of firm-specific productivity shocks goes to zero, i.e., the prior becomes arbitrarily precise, both expectations converge to the mean of the distribution,

$$\lim_{\sigma_a^2\to 0}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} = \lim_{\sigma_a\to 0}\mathbb{E}\left\{A_{jt}|z_{jt}\right\} = \exp\left\{a_t\right\}. \tag{167}$$

When the variance of firm-specific productivity shocks goes to infinity, i.e., the prior becomes arbitrarily noisy, both allocations coincide because they put full weight on the price signal

z_{jt},

$$\lim_{\sigma_a^{-2}\to 0} \omega_z = \lim_{\sigma_a^{-2}\to 0} \omega_z^{eff} = 1 \tag{168}$$

where

$$\omega_z = \frac{\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}, \quad \omega_z^{eff} = \frac{\beta_t\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}}. \tag{169}$$

(iii) When the variance of firm-specific sentiment shocks goes to zero, financial markets perfectly aggregate dispersed information as the precision of the price signal goes to infinity. In this case, both expectations converge to the true realization:

$$\lim_{\sigma_\varepsilon^2\to 0} \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} = \lim_{\sigma_\varepsilon\to 0} \mathbb{E}\left\{A_{jt}|z_{jt}\right\} = \exp\left\{a_{jt}\right\}. \tag{170}$$

□

Lemma 2 (Joining two Normal PDFs)**.** *Let $f\left(\varepsilon_{jt}\right)$ be the pdf of $\varepsilon_{jt}\sim\mathcal{N}\left(\varepsilon_t,\sigma_\varepsilon^2\right)$ and $\phi\left(\cdot\right)$ the standard-normal pdf. Then*

$$f\left(\varepsilon_{jt}\right)\phi\left(\varepsilon_{jt}\right) = \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}) \tag{171}$$

where $\tilde{f}\left(\varepsilon_{jt}\right)$ is the transformed pdf of $\varepsilon_{jt}\sim\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2},\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$.

Proof. Write out the pdfs explicitly,

$$\phi\left(\varepsilon_{jt}\right) = \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}\right\} \tag{172}$$

$$f\left(\varepsilon_{jt}\right) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\exp\left\{-\frac{\left(\varepsilon_{jt}-\varepsilon_t\right)^2}{2\sigma_\varepsilon^2}\right\} \tag{173}$$

$$f\left(\varepsilon_{jt}\right)\phi\left(\varepsilon_{jt}\right) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}-\frac{\left(\varepsilon_{jt}-\varepsilon_t\right)^2}{2\sigma_\varepsilon^2}\right\}. \tag{174}$$

Rearranging the term inside the exponential function,

$$
\begin{aligned}
\frac{\left(\varepsilon_{jt}-\varepsilon_{t}\right)^{2}}{\sigma_{\varepsilon}^{2}}+\varepsilon_{jt} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\varepsilon_{t}+\varepsilon_{t}^{2}}{\sigma_{\varepsilon}^{2}}+\varepsilon_{jt} \\
\text{join fractions} &= \frac{\left(1+\sigma_{\varepsilon}^{2}\right)\varepsilon_{jt}-2\varepsilon_{t}\varepsilon_{jt}+\varepsilon_{t}^{2}}{\sigma_{\varepsilon}^{2}} \\
\text{divide by } \left(1+\sigma_{\varepsilon}^{2}\right) &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}+\frac{\varepsilon_{t}^{2}}{1+\sigma_{\varepsilon}^{2}}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}} \\
\text{add and subtract} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}+\frac{\varepsilon_{t}^{2}}{1+\sigma_{\varepsilon}^{2}}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{\left(\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}-\left(\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}} \\
\text{exchange terms} &= \frac{\varepsilon_{jt}-2\varepsilon_{jt}\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}+\left(\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{\frac{\varepsilon_{t}^{2}}{1+\sigma_{\varepsilon}^{2}}-\left(\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}} \\
\text{binomial} &= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{\frac{\varepsilon_{t}^{2}}{1+\sigma_{\varepsilon}^{2}}-\left(\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}} \\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{1-\frac{1}{1+\sigma_{\varepsilon}^{2}}}{\sigma_{\varepsilon}^{2}}\varepsilon_{t}^{2} \\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{\frac{1+\sigma_{\varepsilon}^{2}-1}{1+\sigma_{\varepsilon}^{2}}}{\sigma_{\varepsilon}^{2}}\varepsilon_{t}^{2} \\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_{t}}{1+\sigma_{\varepsilon}^{2}}\right)^{2}}{\frac{\sigma_{\varepsilon}^{2}}{1+\sigma_{\varepsilon}^{2}}}+\frac{\varepsilon_{t}^{2}}{1+\sigma_{\varepsilon}^{2}}
\end{aligned}
\tag{175}
$$

This allows to write

$$
\begin{aligned}
f(\varepsilon_{jt})\,\phi(\varepsilon_{jt}) &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}}{2}-\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{2\sigma_\varepsilon^2}\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\}\exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\sqrt{\frac{\sigma_\varepsilon^2}{2\pi(1+\sigma_\varepsilon^2)}}\frac{1}{\sqrt{2\pi\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\} \\
&* \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\}\sqrt{\frac{1}{2\pi(1+\sigma_\varepsilon^2)}}\tilde{f}(\varepsilon_{jt}),
\end{aligned} \tag{176}
$$

where $\tilde{f}(\varepsilon_{jt})$ is the transformed pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. $\square$

Lemma 3 (Auxiliary Results Market Allocation)**.** *Denote the Bayesian weights*

$$\omega_a = \frac{\beta_{jt}(1+\sigma_\varepsilon^{-2})}{\sigma_a^{-2}+\beta_{jt}(1+\sigma_\varepsilon^{-2})}, \tag{177}$$

$$\omega_\varepsilon = \frac{\sqrt{\beta_{jt}}(1+\sigma_\varepsilon^{-2})}{\sigma_a^{-2}+\beta_{jt}(1+\sigma_\varepsilon^{-2})} \tag{178}$$

$$\omega_{z\varepsilon} = \frac{\sqrt{\beta_{jt}}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\sqrt{\beta_{jt}}(1+\sigma_\varepsilon^{-2})}, \tag{179}$$

and posterior uncertainty

$$\mathbb{V} = \frac{1}{\sigma_a^{-2}+\beta_t(1+\sigma_\varepsilon^{-2})} \tag{180}$$

then

(i) $\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} + \mathbb{V} = \sigma_a^2,$

(ii) $\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} = \sigma_a^2 - \mathbb{V} = \omega_a\sigma_a^2,$

(iii) $\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2} = \omega_{z\varepsilon}.$

Proof. (i)

$$\begin{aligned}
\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} + \mathbb{V} &= \frac{\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} + \frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&+ \frac{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-2}+2\beta\left(1+\sigma_\varepsilon^{-2}\right)+\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-4}+2\sigma_a^{-2}\beta\left(1+\sigma_\varepsilon^{-2}\right)+\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \frac{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \sigma_a^2 \qquad (181)
\end{aligned}$$

(ii) The first equality follows from (i). Then

$$\begin{aligned}
\sigma_a^2 - \mathbb{V} &= \sigma_a^2 - \frac{1}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\sigma_a^2\sigma_a^{-2}+\sigma_a^2\beta\left(1+\sigma_\varepsilon^{-2}\right)-1}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)}\sigma_a^2 \\
&= \omega_a\sigma_a^2. \qquad (182)
\end{aligned}$$

(iii)

$$\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2} = \frac{\omega_\varepsilon}{\sigma_\varepsilon^2\left(1+\sigma_\varepsilon^{-2}\right)} = \frac{\omega_\varepsilon\sigma_\varepsilon^{-2}}{\left(1+\sigma_\varepsilon^{-2}\right)} = \omega_{z\varepsilon}. \qquad (183)$$

□

Lemma 4. *In the symmetric equilibrium with $\beta_{ijt} = \beta_{jt}$ with $K_{t+1} < W_t$,*

(i) Sentiment shocks ε_t affect the marginal benefit of information production through three channels,

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information-Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}}\right\}. \qquad (184)$$

(ii) Productivity shocks a_t increase the marginal benefit of information production,

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\left(\frac{\alpha\theta-\theta+1}{(1-\alpha)\,\alpha\theta}+\frac{\alpha}{1-\alpha}+1\right)a_t\right\}. \tag{185}$$

Proof of Lemma 4. *(i)* Assume $a_t=0$ without loss of generality. The marginal benefit to increasing β_{ijt} is

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} = \int_{-\infty}^{\infty} g\left(a_{jt}\right)\int_{-\infty}^{\infty} f\left(\varepsilon_{jt}\right)\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}\alpha A_t^{\alpha_Y} \tag{186}$$

$$*\,\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{\theta-1}{\theta}}}K_{t+1}^{\alpha}d\varepsilon_{jt}da_{jt},$$

where $g\left(a_{jt}\right)$ is the pdf of $a_{jt}\sim\mathcal{N}\left(0,\sigma_a^2\right)$ and $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt}\sim\mathcal{N}\left(\varepsilon_t,\sigma_\varepsilon^2\right)$. The most immediate effect comes from changes to aggregate investment K_{t+1}^{α}. For $\delta R_{t+1}=1$,

$$K_{t+1}^{\alpha}=\left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}}\propto\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{187}$$

The *Absolute Size* channel is summarized by $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Next, the derivative of the probability of buying at $\beta_{ijt}=\beta_{jt}$ is

$$\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}=\phi\left(\varepsilon_{jt}\right)\left(\frac{\omega_{p,jt}}{\sqrt{\beta_{jt}}}a_{jt}+\frac{\varepsilon_{jt}}{2\beta_{jt}}-\frac{\omega_{\varepsilon,jt}\varepsilon_{jt}-\omega_{z\varepsilon,jt}\varepsilon_t}{\sqrt{\beta_{jt}}}+\frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}\right), \tag{188}$$

where $\phi\left(\cdot\right)$ is the standard-normal pdf. Combine $f\left(\varepsilon_{jt}\right)$ with $\phi\left(\varepsilon_{jt}\right)$ using Lemma 2,

$$\phi\left(\varepsilon_{jt}\right)f\left(\varepsilon_{jt}\right)=\exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}), \tag{189}$$

where $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of a fictional variable $\varepsilon_{jt}\sim\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2},\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. The *Information-Sensitivity* channel is summarized by $\exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\}$. For the rest of the proof, use

$$\varepsilon_{jt}=\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\frac{\varepsilon_t}{1+\sigma_\varepsilon^2} \tag{190}$$

$$d\varepsilon_{jt}=\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}dx. \tag{191}$$

Substitute ε_{jt} out of the terms in parenthesis for $\left.\frac{\partial \mathcal{P}\{x_{ijt}=2\}}{\partial \beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}$ leads to

$$\frac{\omega_{p,jt}}{\sqrt{\beta_{jt}}}a_{jt}+\left(\frac{1}{2\beta_{jt}}-\frac{\omega_{\varepsilon,jt}}{\sqrt{\beta_{jt}}}\right)\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\frac{\varepsilon_t}{2\beta_{jt}\left(1+\sigma_\varepsilon^2\right)}+\frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}. \tag{192}$$

Substituting ε_{jt} out of $\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}$,

$$\begin{aligned}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}&=\exp\left\{\omega_a a_{jt}+\omega_\varepsilon\varepsilon_{jt}-\omega_{z\varepsilon}\varepsilon_t+\frac{1}{2}\mathbb{V}\right\}\\&=\exp\left\{\omega_a a_{jt}+\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\cancel{\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}-\omega_{z\varepsilon}\varepsilon_t}+\frac{1}{2}\mathbb{V}\right\}\\&\overset{\text{Lemma 3 (iii)}}{=}\exp\left\{\omega_a a_{jt}+\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+\frac{1}{2}\mathbb{V}\right\}.\end{aligned} \tag{193}$$

Substitute ε_{jt} out of the firm-specific multiplier for firm capital,

$$\begin{aligned}&\frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^\theta dj\right)^{\frac{\theta-1}{\theta}}}\\&=\exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x+(\theta-1)\,\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}-(\theta-1)\,\omega_\varepsilon\varepsilon_t-\frac{(\theta-1)\,\theta}{2}\left(\omega_a^2\sigma_a^2+\omega_\varepsilon^2\sigma_\varepsilon^2\right)\right\}\\&\propto\exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x-(\theta-1)\,\omega_\varepsilon\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\varepsilon_t\right\}\\&=\exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\}\\&=\exp\left\{(\theta-1)\,\omega_a a_{jt}+(\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x\right\}\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\},\end{aligned} \tag{194}$$

where I used Lemma 3 (iii) repeatedly. The *Relative Size* channel is summarized through $\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\}$. It remains to show that there are no other terms in $\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)$ that depend on ε_t. It is sufficient to show that

$$\int_{-\infty}^{\infty}g\left(a_{jt}\right)\int_{-\infty}^{\infty}\tilde{f}\left(\varepsilon_{jt}\right)\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^\theta dj\right)^{\frac{\theta-1}{\theta}}}d\varepsilon_{jt}da_{jt}\overset{!}{=}0. \tag{195}$$

Substituting ε_{jt} out leads to

$$\begin{aligned}
&\int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \frac{1}{\cancel{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}} \phi\left(x\right) \frac{\left(A_{jt} - \mathbb{E}\left\{A_{jt} | s_{ijt} = z_{jt} z_{jt}\right\}\right) \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta} dj\right)^{\frac{\theta-1}{\theta}}} \cancel{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}} dx da_{jt} \\
&\propto \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi\left(x\right) \left(A_{jt} \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}^{\theta-1} - \mathbb{E}\left\{A_{jt} | s_{ijt} = z_{jt} z_{jt}\right\}^{\theta}\right) dx da_{jt} \\
&= \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi\left(x\right) \left(\exp\left\{\left(\left(\theta-1\right)\omega_a + 1\right) a_{jt} + \left(\theta-1\right)\omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x + \frac{\left(\theta-1\right)}{2}\mathbb{V}\right\}\right. \\
&\quad \left. - \exp\left\{\theta\omega_a a_{jt} + \theta\omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x + \frac{\theta}{2}\mathbb{V}\right\}\right) dx da_{jt} \\
&\propto \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi\left(x\right) \left(\exp\left\{\left(\left(\theta-1\right)\omega_a + 1\right) a_{jt} + \left(\theta-1\right)\omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x - \frac{\mathbb{V}}{2}\right\}\right. \\
&\quad \left. - \exp\left\{\theta\omega_a a_{jt} + \theta\omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x\right\}\right) dx da_{jt} \\
&= \int_{-\infty}^{\infty} g\left(a_{jt}\right) \left(\exp\left\{\left(\left(\theta-1\right)\omega_a + 1\right) a_{jt} + \frac{\left(\theta-1\right)^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} - \frac{\mathbb{V}}{2}\right\} - \exp\left\{\theta\omega_a a_{jt} + \frac{\theta^2\omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}\right) da_{jt} \\
&= \exp\left\{\frac{\left(\left(\theta-1\right)\omega_a + 1\right)^2}{2}\sigma_a^2 + \frac{\left(\theta-1\right)^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} - \frac{\mathbb{V}}{2}\right\} - \exp\left\{\frac{\theta^2\omega_a^2}{2}\sigma_a^2 + \frac{\theta^2\omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}.
\end{aligned} \tag{196}$$

It remains to show that

$$\left(\left(\theta-1\right)\omega_a + 1\right)^2 \sigma_a^2 + \left(\theta-1\right)^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} - \mathbb{V} \overset{!}{=} \theta^2\omega_a^2\sigma_a^2 + \theta^2\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}. \tag{197}$$

Using Lemma 3 (i), the LHS is equal to

$$\begin{aligned}
&\left(\left(\theta^2 - 2\theta + 1\right)\omega_a^2 + 2\left(\theta-1\right)\omega_a \cancel{+1}\right)\sigma_a^2 + \left(\theta^2 - 2\theta + 1\right)\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \cancel{-\sigma_a^2} + \omega_a^2\sigma_a^2 + \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \\
&= \left(\left(\theta^2 - 2\theta + 2\right)\omega_a^2 + 2\left(\theta-1\right)\omega_a\right)\sigma_a^2 + \left(\theta^2 - 2\theta + 2\right)\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \\
&= 2\left(\theta-1\right)\omega_a\sigma_a^2 + \left(\theta^2 + 2\left(1-\theta\right)\right)\left(\omega_a^2\sigma_a^2 + \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right) \\
&\overset{\text{Lemma 3 (ii)}}{=} 2\left(\theta-1\right)\omega_a\sigma_a^2 + \left(\theta^2 + 2\left(1-\theta\right)\right)\left(\omega_a\sigma_a^2\right) \\
&= \theta^2\omega_a\sigma_a^2.
\end{aligned} \tag{198}$$

Using Lemma 3 (ii), the RHS is equal to

$$\theta^2\left(\omega_a^2\sigma_a^2 + \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right) = \theta^2\omega_a\sigma_a^2. \tag{199}$$

Combining both confirms the conjecture. The marginal benefit of information production depends on ε_t only through the multiplicative effects in (187), (189) and (194), such that

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}} \quad \underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information Sensitivity}} \quad \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative Size}}\right\}. \tag{200}$$

(ii) Follow the same strategy as in *(i)* and use the same expression for the marginal benefit of information production (186). Start with the expressions for aggregate investment, K_{t+1}^{α}, and productivity $A_t^{\alpha_Y}$ in (186). For $\delta R_{t+1}=1$, they are equal to

$$A_t^{\alpha_Y}K_{t+1}^{\alpha} = A_t^{\alpha_Y}\left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{201}$$

Recall that $\alpha_Y=\frac{\alpha\theta-\theta+1}{\alpha\theta}$. Then

$$A_t^{\alpha_Y} \propto \exp\left\{\frac{\alpha\theta-\theta+1}{\alpha\theta}a_t\right\} \tag{202}$$

$$\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{1}{\theta}} \propto \exp\left\{a_t\right\}. \tag{203}$$

Putting both together yields

$$\begin{aligned}
A_t^{\alpha_Y}K_{t+1}^{\alpha} &\propto \exp\left\{\frac{\alpha\theta-\theta+1}{\alpha\theta}\left(1+\frac{\alpha}{1-\alpha}\right)a_t+\frac{\alpha}{1-\alpha}a_t\right\} \\
&= \exp\left\{\frac{\alpha\theta-\theta+1}{\left(1-\alpha\right)\alpha\theta}a_t+\frac{\alpha}{1-\alpha}a_t\right\} \\
&= \exp\left\{\left(\frac{\alpha\theta-\theta+1}{\left(1-\alpha\right)\alpha\theta}+\frac{\alpha}{1-\alpha}\right)a_t\right\}.
\end{aligned} \tag{204}$$

Again, using substitution with

$$a_{jt} = \sqrt{\sigma_a^2}y+a_t \tag{205}$$

$$da_{jt} = \sqrt{\sigma_a^2}dy \tag{206}$$

it follows that

$$A_{jt} \propto \exp\left\{a_t\right\} \tag{207}$$

$$\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\} \propto \exp\left\{a_t\right\}, \tag{208}$$

which yields

$$\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{\theta-1}{\theta}}}\propto\exp\left\{a_t\right\}. \tag{209}$$

The change in the buying probability does *not* depend on a_t

$$\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}=\phi\left(\varepsilon_{jt}\right)\left(\frac{\omega_p}{\sqrt{\beta_{jt}}}\sqrt{\sigma_a^2}y+\frac{\varepsilon_{jt}}{2\beta_{jt}}-\frac{\omega_\varepsilon\varepsilon_{jt}-\omega_{z\varepsilon}\varepsilon_t}{\sqrt{\beta_{jt}}}+\frac{\mathbb{V}_{jt}}{2\sqrt{\beta_{jt}}}\right). \tag{210}$$

Since no other terms depend on a_t and I substituted a_{jt} out, it follows that

$$\left.\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\right|_{\beta_{ijt}=\beta_{jt}}\propto\exp\left\{\left(\frac{\alpha\theta-\theta+1}{\left(1-\alpha\right)\alpha\theta}+\frac{\alpha}{1-\alpha}+1\right)a_t\right\} \tag{211}$$

□

Proof of Proposition 4. The cutoff can be derived by using the result from Lemma 4 (i) and taking the derivative with respect to ε_t to the following expression,

$$\frac{\partial}{\partial\varepsilon_t}\left(-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t+\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right)\overset{!}{=}0. \tag{212}$$

Denote $\bar{\varepsilon}$ as the value of ε_t for which the above expression is maximized. Then,

$$\begin{aligned}-\frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2}-\left(\theta-1\right)\omega_{s\varepsilon}+\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}=0&\\ \iff\frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2}=\left(\frac{1}{1-\alpha}-\theta\right)\omega_{s\varepsilon}&\\ \iff\bar{\varepsilon}=\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{1-\alpha}-\theta\right)\omega_{s\varepsilon},&\end{aligned} \tag{213}$$

where $\omega_{s\varepsilon}=\frac{\sqrt{\beta_t}}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$. For $\varepsilon_t<\bar{\varepsilon}$, information production β_t is increasing in ε_t. For $\varepsilon_t>\bar{\varepsilon}$, information production β_t is decreasing in ε_t. □

Proof of Proposition 5. Follows from Lemma 4 (ii). □

Proof of Proposition 6. *(i)* Using the result from Proposition 4 (ii) and the assumption that $\theta>\frac{1}{1-\alpha}$, it must be that positive sentiment shocks crowd out information production. Moreover, as $\beta^*<\frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$, it must be that the pass-through of aggregate sentiment shocks $\omega_{s\varepsilon}$ decreases in information production. Therefore, the pass-through is smaller when information is endogenous than if information production is fixed at β^*. As a result, sentiment shocks are dampened by information production in financial markets, as less precise information by itself leads to less investment and lowers the pass-through of sentiment shocks.

(ii) $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$ guarantees that the pass-through of sentiment shocks goes faster to zero than the sentiment shock goes to infinity, i.e., the direct effect of sentiment shocks on investment disappears as shocks become arbitrarily large. Moreover, Lemma 4 (i) shows that through the information-sensitivity effect $\lim_{\varepsilon_t \to \infty} \beta_t(\varepsilon_t) = 0$. □

Proof of Corollary 2. Follows directly from Proposition 6 (ii). □

Proof of Proposition 7. Follows from Proposition 5 and Assumption 2. An increase in aggregate productivity encourages more information production, which also leads to more investment. As a result, productivity shocks are amplified. □

Proof of Proposition 8. Since $\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right) \propto \kappa_H - \kappa_L$ whereas $MB^{SP}\left(\beta_t\right)$ is not a function of position limits $\{\kappa_H, \kappa_L\}$, it must that $\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}=\beta_t} \neq MB^{SP}\left(\beta_t\right)$ for almost all values of β_t. Therefore, the information production in the competitive economy and social planner allocation do not coincide almost everywhere. □

Proof of Corollary 3. The marginal benefit of information production after applying the tax/subsidy $\tau\left(a_{j0}, z_{j0}\right)$ is

$$\begin{aligned} &\widetilde{MB}\left(\beta_{ij0}, \beta_{j0}\right) \propto \\ &\tilde{\mathbb{E}}\left\{\frac{\partial \mathcal{P}\left\{x_{ij0}=2\right\}}{\partial \beta_{ij0}}\left(\tau\left(a_{j0}, z_{j0}\right)\Pi_{j1} - \tilde{\mathbb{E}}\left\{\tau\left(a_{j0}, z_{j0}\right)\Pi_{j1}|s_{ij0}=z_{j0}, z_{j0}\right\}\right)\right\}. \end{aligned} \tag{214}$$

Assume that a tax fulfills the following conditions

$$\tau\left(a_{j0}, z_{j0}\right) \geq (\leq)\, 1 \iff \Pi_{j1} \geq \tilde{\mathbb{E}}\left\{\Pi_{j1}|s_{ij0}=z_{j0}, z_{j0}\right\} \text{ and } \frac{\partial \mathcal{P}\left\{x_{ij0}=2\right\}}{\partial \beta_{ij0}} \geq 0 \tag{215}$$

$$\tau\left(a_{j0}, z_{j0}\right) \leq (\geq)\, 1 \iff \Pi_{j1} \leq \tilde{\mathbb{E}}\left\{\Pi_{j1}|s_{ij0}=z_{j0}, z_{j0}\right\} \text{ and } \frac{\partial \mathcal{P}\left\{x_{ij0}=2\right\}}{\partial \beta_{ij0}} \leq 0 \tag{216}$$

and the inequalities are strict for at least some $\{a_{j0}, z_{j0}\}$. In the first case, $\tau\left(a_{j0}, z_{j0}\right) \geq 1$ whenever the trading rents $\Pi_{j1} - \tilde{\mathbb{E}}\left\{\tau\left(a_{j0}, z_{j0}\right)\Pi_{j1}|s_{ij0}=z_{j0}, z_{j0}\right\}$ are positive and producing additional information leads to an increase of the probability of trading in that state. Therefore, the gains of buying when it is profitable are increased, which encourages information production. The same reasoning applies for $\tau\left(a_{j0}, z_{j0}\right) \leq 1$, as losses become more painful, increasing the incentive for information production. This set of taxes increases $\widetilde{MB}(\beta_{ij0}, \beta_{j0})$ and encourages information production.

The reverse reasoning applies when $\tau(a_{j0}, z_{j0}) \geq 1$ for losses and $\tau(a_{j0}, z_{j0}) \leq 1$ for gains, which leads to a decrease in $\widetilde{MB}(\beta_{ij0}, \beta_{j0})$. As a result, gains and losses are reduced, which discourages information production. $\square$

Proof of Proposition 9*.* Let the social planner buy $d^{SP} \in (-1, 1)$ units of shares in all markets. The market-clearing condition for market j becomes

$$2\left(1 - \Phi\left(\sqrt{\beta_{j0}}\left(\hat{s}(P_{j0}) - a_{j0}\right) - \varepsilon_{j0}\right)\right) = 1 - d^{SP}, \tag{217}$$

Keeping position limits fixed, the social planner's demand d^{SP} changes the identity of the marginal trader. If the social planner purchases more assets, the marginal trader becomes more optimistic on average. The threshold signal becomes,

$$\hat{s}\left(P_{j0}, d^{SP}\right) = a_{j0} + \frac{\varepsilon_{j0} + \Phi^{-1}\left(\frac{1+d^{SP}}{2}\right)}{\sqrt{\beta_{j0}}}. \tag{218}$$

It follows immediately that asset purchases or sales with $d^{SP} = 2\Phi(-\varepsilon_0) - 1$ ensure that the marginal trader holds unbiased beliefs,

$$\hat{s}\left(P_{j0}, d^{SP}\right) = a_{j0} + \frac{\varepsilon_{j0} - \varepsilon_0}{\sqrt{\beta_{j0}}}. \tag{219}$$

It follows that prices are unbiased and aggregate investment is at a level as if the sentiment shock was absent.

Traders expect to buy in equilibrium whenever $s_{ijt} > \hat{s}\left(P_{j0}, d^{SP}\right)$. Asset purchases/sales set the threshold $\hat{s}\left(P_{j0}, d^{SP}\right)$ at a level as if the aggregate sentiment shock was $\varepsilon_0 = 0$, effectively undoing any change to the incentive to produce information. Because the trader thinks that she is unaffected by the sentiment shock and the markets become unaffected by the sentiment shock due to asset purchases / sales d^{SP}, also the information production decision reverts to the level without the aggregate sentiment shock. $\square$

Proof of Proposition 10*.* (i) Denote $k_{jt+1} = \ln K_{jt+1}$. Using (25) allows to write the variance of the log of firm capital stocks as

$$\begin{aligned} Var\left(k_{jt+1}\right) &= \theta^2 Var\left(\ln \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}\right) \\ &= \frac{\theta^2}{2}\left(\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\sigma_\varepsilon^2\right). \end{aligned} \tag{220}$$

Which can be expressed as

$$\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\sigma_\varepsilon^2 = \frac{\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2 + \beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2} + \beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}$$
$$\propto \frac{\beta_{t-1}^2\sigma_a^2 + \beta_{t-1}\sigma_\varepsilon^2}{\sigma_a^{-4} + 2\sigma_a^{-2}\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right) + \beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2}. \tag{221}$$

Taking the derivative with respect to β_{t-1} and dropping the denominator leads to the simplified expression

$$2\beta_{t-1}\sigma_a^{-2} + \sigma_a^{-4}\sigma_\varepsilon^2 + \beta_{t-1}^2\left(1-\sigma_\varepsilon^2\right)\left(1+\sigma_\varepsilon^{-2}\right) \tag{222}$$

which is positive for all values of β_{t-1} for $\sigma_\varepsilon^2 \leq 1$.

(ii) Denote $\Delta k_{jt+1} = k_{jt+1} - k_{jt}$. Then

$$\Delta k_{jt+1} = \Delta\theta \ln \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} + \Delta \ln \int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} dj + \Delta K_{t+1}. \tag{223}$$

Deriving the variance of Δk_{jt+1} across firms yields

$$Var\left(\Delta k_{jt+1}\right) = \theta^2\left(\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\sigma_\varepsilon^2\right) \tag{224}$$

which is monotonically increasing in β_{t-1} for $\sigma_\varepsilon^2 \leq 1$ as in (i). □

Proof of Corollary 4. The derivation is analogous to Lemma 4 (i). The reduction in information-sensitivity of the trading decision dominates a possible increase in investment if the sectoral elasticity of substitution θ_n is large enough as in Proposition 4. Therefore, a positive sentiment shock can discourage information production and increase capital misallocation. □

Proof of Corollary 5. Assume first that aggregate investment is fixed ($\delta \to \infty$). Then, a positive shock in sector / country n leads to an increase in investment in sector / country n analogously to (25). Because the aggregate level of investment is fixed, it must be that investment in other sectors / countries decreases, decreasing the incentive for information production similarly to the *absolute scale* channel in Lemma 4. However, if aggregate investment is variable, then an expansion of one sector / countries shrinks investment in other sectors / countries if their respective goods are sufficiently close substitutes ($\theta \to \infty$). In this case, the positive shock discourages investment in non-shocked sectors / countries, lowering information production and increasing capital misallocation in non-shocked sectors / countries. □

Proof of Proposition 11. The social planner's allocation is given by equalizing the marginal products of capital for each firm given the market signals $\{z_{jt}\}$. The maximization problem of the social planner for firm capital allocation is therefore

$$\max_{K_{j1}} \mathbb{E}\left\{\left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\alpha\theta}{\theta-1}} |z_{jt}\right\} - R_1^{SP} K_{j1}, \tag{225}$$

for some interest rate R_1^{SP}. The resulting first-order condition for firm capital is

$$K_{j1}^{SP} = \left(\alpha Y_1^{\alpha_Y} \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{R_1^{SP}}\right)^{\theta}. \tag{226}$$

Integrating on both sides yields

$$R_1^{SP} = \alpha Y_1^{\alpha_Y} \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} \left(K_1^{SP}\right)^{-\frac{1}{\theta}}. \tag{227}$$

Substituting R_1^{SP} out of K_{j1}^{SP} yields (114). Following the same steps as in the proof for Proposition 3, leads to

$$Y_1^{SP} = A_0^{SP} K_1^{\alpha}, \quad \text{where } A_0^{SP} = \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{\alpha}{\theta-1}}, \tag{228}$$

as in (115). Substituting Y_1^{SP} out of the expression for R_1^{SP} then leads to the interest rate (116). Consumption follows using (117) in (110). Finally, taking K_1^{SP} as given and plugging aggregate capital investment in in Y_1^{SP} in (115) in (SPFull), (118) follows after taking the derivative of (115) with respect to β_0. □

Proof of Proposition 12. I will show that the decentralized allocations coincide with the social planner's allocations. The proof follows the same steps as the derivation of the equilibrium in the main section. Households receive from firm j the dividend

$$\hat{\Pi}_{j1} = \tau^{Bias}(z_{j0}) \Pi_{j1} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}} \alpha Y_1^{\alpha_Y} A_{j0} K_{j1}^{\frac{\theta-1}{\theta}} \tag{229}$$

and the marginal trader expects the dividend to be

$$\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} = \alpha Y_1^{\alpha_Y} \mathbb{E}\left\{A_{j0}|z_{j0}\right\} K_{j1}^{\frac{\theta-1}{\theta}}. \tag{230}$$

The price is using $P_{j0} = K_{j1}$,

$$P_{j0} = \frac{1}{R_1}\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} = \left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta}. \tag{231}$$

This allows to express expected dividends substituting the capital stock,

$$\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} = \left(\frac{1}{R_1}\right)^{\theta-1}\left(\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta}, \tag{232}$$

which is then used for the interest rate R_1

$$\begin{aligned} R_1 &= \frac{\int_0^1 \tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} dj}{\int_0^1 P_{j0} dj} \\ &= \left(\frac{1}{R_1}\right)^{\theta-1}\left(\alpha Y_1^{\alpha_Y}\right)^{\theta}\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj K_1^{-1} \\ \iff R_1 &= \alpha Y_1^{\alpha_Y}\left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} K_1^{-\frac{1}{\theta}}, \end{aligned} \tag{233}$$

Using this result again for the price yields

$$K_{j1}^{DE} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj} K_1 = K_{j1}^{SP}. \tag{234}$$

Plugging this into the expression for the interest rate R_1 and substituting $Y_1^{\alpha_Y}$ leads to

$$R_1^{DE} = \alpha\left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} K_1^{\alpha-1} = R_1^{SP}. \tag{235}$$

This result also leads directly to $K_1^{DE} = K_1^{SP}$. Finally, the first-order condition for information production of trader ij is

$$\begin{aligned} \widetilde{MB}\left(\beta_{ij0}, \beta_{j0}\right) &= \frac{\partial IA^{DE}}{\partial \beta_{ij0}} \\ &= \tau^{Info}\left(\beta_{ij0}, \beta_{j0}\right)\frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}} \\ &= \frac{\widetilde{MB}\left(\beta_{ij0}, \beta_{j0}\right)}{\left.\frac{\partial Y_1}{\partial \beta_0}\right|_{\beta_0=\beta_{ij0}}}\frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}} \\ \iff \left.\frac{\partial Y_1}{\partial \beta_0}\right|_{\beta_0=\beta_{ij0}} &= \frac{\partial IA\left(\beta_{ij0}\right)}{\partial \beta_{ij0}} \end{aligned} \tag{236}$$

which is the same first-order condition as in (118) and therefore $\beta_0^{DE} = \beta_0^{SP}$. $\square$

Proof of Corollary 6. *(i)* First, denote $\omega_{s\varepsilon} = \frac{\sqrt{\beta_0}}{\sigma_a^{-2}+\beta_0\left(1+\sigma_\varepsilon^{-2}\right)}$ as the weight on the correlated noise in the private signal. The transaction tax/subsidy $\tau^{Trans}(\varepsilon_0) = \exp\{-\omega_{s\varepsilon}\varepsilon_0\}$ leads to traders paying P_{j0} but only $\tau^{Trans}P_{j0}$ is collected by the firm. The transaction tax/subsidy is aimed to stabilize aggregate asset prices with respect to aggregate sentiment shocks. It is a tax when traders are exuberant and a subsidy when they are depressed. The proof follows the same steps as for Proposition 12 with the difference that $K_{j1} = \tau^{Trans}(\varepsilon_0)P_{j0}$ and therefore capital of firm j is

$$K_{j1} = \left(\frac{\tau^{Trans}(\varepsilon_0)}{R_1}\alpha Y_1^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}\right)^{\theta} \tag{237}$$

Following the same steps as before, the interest rate is

$$R_1 = \alpha Y_1^{\alpha_Y}\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} \tau^{Trans}(\varepsilon_0)K_1^{-\frac{1}{\theta}}. \tag{238}$$

Since $\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} \propto \exp\{\omega_{s\varepsilon}\varepsilon_0\}$, it follows that the transaction tax/subsidy $\tau^{Trans}(\varepsilon_0) = \exp\{-\omega_{s\varepsilon}\varepsilon_0\}$ keeps the interest rate R_1 from moving with the aggregate sentiment shock ε_0 and stabilizes, therefore, aggregate investment with respect to sentiment shocks.

(ii) Similarly, allow now the transaction tax to vary with the share price,

$$\tau^{Trans}\left(P_{j0}\right) = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}}. \tag{239}$$

Same as before, the traders pays P_{j0} but only $\tau^{Trans}\left(P_{j0}\right)P_{j0}$ is collected by the firm. Using $K_{j1} = \tau^{Trans}\left(P_{j0}\right)P_{j0}$, capital of firm j is then equal to

$$\begin{aligned} K_{j1} &= \left(\frac{\tau^{Trans}\left(P_{j0}\right)}{R_1}\alpha Y_1^{\alpha_Y}\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}\right)^{\theta} \\ &= \left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta} \end{aligned} \tag{240}$$

Integrating on both sides leads to

$$R_1 = \alpha Y_1^{\alpha_Y}\left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} K_{j1}^{-\frac{1}{\theta}} \tag{241}$$

Using this interest in (240) leads to the capital allocation

$$K_{j1} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj} K_1 \tag{242}$$

which coincides with the efficient or social planner allocation (114). It follows the transaction tax $\tau^{Trans}(P_{j0})$ corrects for the mispricing between firms and as well as stabilizes aggregate investment with respect to the aggregate sentiment shock. $\square$