Exuberant and Uninformed: How Financial Markets (Mis-)Allocate Capital during Booms

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Abstract

I develop a macroeconomic model of information production in financial markets during asset price booms and busts. Agents acquire information to decide which firms to fund. In the aggregate, more precise information leads to less capital misallocation. The source of booms and busts determines their effect on information production. Productivity booms increase information production and are amplified by a fall in misallocation. Sentiment booms and busts *crowd out* information as private information becomes less likely to change the trading decision. Information production is constrained inefficient in the competitive equilibrium for two reasons. First, each trader produces information to extract rents from others. Second, traders fail to internalize that their information production improves the capital allocation. Looking through the lens of the model, the US dot-com boom of the late 1990s appears to have been driven by productivity, whereas the US housing boom of the mid 2000s was driven by sentiment. As an application, I show that asset purchase programs can be an effective way to address financial market inefficiencies.

Keywords: Financial Markets; Information Production; Misallocation; Macroeconomics; Booms
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1 Introduction

Financial markets play a central role in allocating capital to its most productive uses. Yet, they do not always fulfill this role well. The last three decades, for instance, have been characterized by successive booms and busts in financial markets.[1] These cycles have been difficult to justify on fundamental grounds alone (Martin and Ventura, 2018). Against this backdrop, there are growing concerns that such booms lead to the deterioration of capital allocation, ultimately reducing aggregate productivity.[2] The general narrative is as follows: during booms, the perception is that all investments perform well. As a result, agents are less prone to produce information about specific investments, and markets eventually become less informative, thereby worsening the allocation of resources in the economy.[3] Though suggestive, this narrative is loose and cannot be fully evaluated without a theory of information production and its macroeconomic effects. The goal of this paper is to provide such a theory.

In this paper, I develop a tractable macroeconomic framework in which financial markets play the key role of aggregating information that is dispersed among economic agents. The framework's central feature is that information is endogenous, in a sense that agents can decide to engage in costly information production. The framework's novelty is to study the two-way feedback between macroeconomic conditions and agents' incentives to produce information.

I model a dynamic economy populated by firms with heterogeneous productivity and households, which consist of many traders. Households decide on borrowing and saving. Traders decide which firms to invest in, but they have imperfect information about firm productivity. To make their investment decisions, traders combine their private information with a public signal provided by financial markets, which effectively aggregates all traders' information.

The model is based on two core assumptions. First, traders agree on realizations of aggregate shocks but disagree about the distribution of firm productivity. Whereas the former part is for simplicity, the latter is central for motivating trade. In particular, traders' private information features both idiosyncratic and correlated noise. The idiosyncratic noise captures trader-specific information and drives disagreement. In contrast, the correlated noise stands

[1] For example, the dot-com bubble in the US and the housing bubbles in the US and Southern Europe.

[2] For instance, Gopinath et al. (2017) and García-Santana et al. (2020) have found that the credit and asset price boom in Southern Europe, preceding the global financial crisis, had coincided with a rise in capital misallocation; Doerr (2018) provides such evidence for the US. Relatedly, Borio et al. (2015) show that credit booms tend to also coincide with growing misallocation of labor.

[3] There are several studies that point to a decline in information production and quality in explaining a worsening of investment efficiency, for example, Asea and Blomberg (1998), Keys et al. (2010), and Becker, Bos, and Roszbach (2020).

for a common "sentiment" across traders.[4] Second, to incentivize information production in equilibrium by avoiding the well-known Grossman-Stiglitz paradox (see Grossman and Stiglitz, 1980),[5] traders suffer from *correlation neglect*. Formally, each trader believes the noise in her private information to be entirely idiosyncratic, allowing her to exploit mispricing due to sentiment. In a nutshell, each trader believes that she is not prone to sentiments even though she understands that everyone else is.

I find that information production crucially depends on the state of the economy. In particular, I study how information production reacts to two types of macroeconomic shocks: sentiment and productivity. Sentiment shocks, defined as waves of optimism or pessimism, formally drive the correlated noise in traders' private information. Sentiment and productivity shocks lead to similar co-movements in output, investment, and asset prices. However, they affect information production differently. Information is central to this model, as more precise information strengthens the correlation between the size of a firm and its productivity, thereby raising allocative efficiency. Consequently, an economy with higher information production allocates more capital to more productive firms and has higher aggregate productivity.

In particular, information production increases in productivity but is non-monotonic in sentiment. Productivity increases information production due to a scale effect. Since high productivity raises the optimal size of a firm, it also boosts the benefits of producing more precise information about it. From the viewpoint of an individual trader, producing more information is valuable if it significantly impacts the trader's investment decisions. However, if sentiment regarding a specific firm is too high or too low, producing more information is likely to not yield much. In particular, even without precise information, a trader knows not to invest in firms where sentiment is high (i.e., firms that are "overvalued") and to invest in firms where sentiment is low (i.e., firms that are "undervalued"). Thus, extreme sentiments discourage the production of information.

Finally, while productivity booms are endogenously amplified by information production's effect on capital allocation, sentiment booms may be dampened. Productivity booms *crowd in* information and improve allocative efficiency, thereby further increasing productivity.

[4] From an economic standpoint, this sentiment is meant to capture a range of phenomena that drive asset prices away from their fundamental value, such as herding, network effects, social learning, extrapolative expectations, or bubbles (see Kindleberger and Aliber, 2015; Shiller, 2015, 2017).

[5] The Grossman-Stiglitz paradox states that no equilibrium exists in models of financial markets with costly information production without noise that keeps prices from being perfectly revealing. If prices reveal all information, traders have no reason to produce costly information. However, prices cannot be informative if traders do not produce information. Therefore, no equilibrium exists. Many models of informative financial markets (Grossman and Stiglitz, 1980; Kyle, 1985; Albagli, Hellwig, and Tsyvinski, 2021) circumvent this problem by introducing so-called noise traders. These agents are non-optimizing, which makes them difficult to embed in a general equilibrium model.

Sentiment booms, however, *crowd out* information and worsen allocative efficiency, thereby decreasing productivity. My finding is consistent with the empirical evidence that booms can fuel resource misallocation (e.g., Gopinath et al., 2017; Doerr, 2018), suggesting that such booms are driven by sentiment. It also captures a dichotomy of booms as in Gorton and Ordoñez (2020) but stresses that the source of booms is the essential factor.

On the normative front, information production is too high or too low in the competitive equilibrium due to the presence of two externalities. First, there can be too much information because traders produce information to gain at the expense of other traders (*rent-extracting behavior*). Second, there can also be too little information because traders do not reap the benefits of improved capital allocation through collective information production, a *pecuniary externality* similar to Pavan, Sundaresan, and Vives (2022). Which effect dominates depends on whether the allocation of capital is important for aggregate productivity. For example, if firms produce similar goods, allocating capital to the most productive firms becomes exceedingly important. Yet, this is exactly when the competitive equilibrium features little information production.

Moreover, the model sheds light on two current policy debates. First, it suggests that policymakers should tax investment during sentiment-driven booms, which increasingly synchronous asset price movements can identify. This policy prescription of "leaning against the wind" is often criticized on informational grounds:[6] namely, it requires the policymaker to be able to distinguish sentiment- from productivity-driven booms in real-time (e.g., Mishkin et al., 2011). My model suggests that, although they look similar in many respects, both types of booms can be distinguished through their effects on information production. In particular, less informative asset prices display more synchronous movements, which can identify sentiment booms. In contrast, productivity booms lead to more asynchronous asset price movements. Looking through the lense of the model, the late 1990s US dot-com boom seemed to be driven by productivity, whereas the US housing boom was driven by sentiment.

A second policy debate refers to the effects of large-scale asset purchases by central banks. There is the widespread perception that asset purchases can distort prices and worsen the allocation of resources.[7] My model yields a simple yet robust insight: whether this concern is justified depends on whether asset purchases reduce or aggravate the aggregate mispricing of assets. By reducing the asset supply in the hands of traders, asset purchases change the marginal trader's identity and thus raise equilibrium prices. If asset prices were ini-

[6] See Cecchetti et al. (2000).

[7] See da Silva and Rungcharoenkitkul (2017), DNB (2017), Fernandez, Bortz, and Zeolla (2018), Borio and Zabai (2018), Acharya et al. (2019), and Kurtzman and Zeke (2020). In particular, the Dutch central bank argues in their 2016 annual report (DNB, 2017): "*The large-scale purchase programmes and the flood of liquid assets has set the risk compass in financial markets spinning, with misallocations as a result.*"

tially depressed due to low productivity, the common perception laid out above is correct. By distorting prices upward, asset purchases discourage information production and thus worsen the allocation of capital. However, if asset prices were initially depressed due to negative sentiment, asset purchases *reduce* aggregate mispricing. Indeed, by undoing the effects of negative sentiment, asset purchases fuel information production, thereby improving the resource allocation.

Finally, the paper makes a methodological contribution by providing a tractable macroeconomic model of information production and aggregation, where financial market informativeness plays an important role for macroeconomic dynamics. With a few exceptions,[8] the role of financial markets as aggregators of dispersed information has received little attention in macroeconomics. The primary reason is that most standard models of informative financial markets rely on non-optimizing agents, such as noise traders, which are not straightforward to reconcile with general-equilibrium analysis. Instead, this model relies on a small behavioral deviation – correlation neglect – which means that traders do not adequately perceive the idiosyncratic and correlated components in their signals. This misperception motivates them to produce costly information as they believe in having an informational edge over the market. This simple assumption is grounded on empirical evidence,[9] and it avoids the Grossman-Stiglitz paradox.

1.1 Literature Review

This paper most closely relates to the recent literature that studies the link between information production and the business cycle (Veldkamp, 2005; Ambrocio, 2020; Farboodi and Kondor, 2020; Chousakos, Gorton, and Ordoñez, 2020; Gorton and Ordoñez, 2020; Asriyan, Laeven, and Martin, 2021).[10] Similar to Asriyan, Laeven, and Martin (2021) and in contrast to Gorton and Ordoñez (2020), the source of booms is important to understand the effect on information production. This paper adds to the literature by studying the role of limits to arbitrage in declining information production during sentiment booms. Sufficiently convex adjustment costs are sufficient to generate a decline in the information-sensitivity of trading in the presence of severe market mispricing.

The paper's core is a noisy rational expectations model (Grossman and Stiglitz, 1980;

[8]For example, see Peress (2014), David, Hopenhayn, and Venkateswaran (2016), Straub and Ulbricht (2018), and Bostancı and Ordonez (2020).

[9]See Eyster et al. (2018), Grimm and Mengel (2020), Enke and Zimmermann (2019), and Chandrasekhar, Larreguy, and Xandri (2020).

[10]See also Van Nieuwerburgh and Veldkamp (2006), Angeletos, Lorenzoni, and Pavan (2010), Ordoñez (2013), Gorton and Ordoñez (2014), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), and Straub and Ulbricht (2018) for further related work.

Kyle, 1985; Vives, 2008; Albagli, Hellwig, and Tsyvinski, 2021). In this literature, limits to arbitrage keep arbitrageurs from fully eliminating mispricing and, therefore, incentives to trade and produce information persist in equilibrium. The model is most similar to Albagli, Hellwig, and Tsyvinski (2021). Different from Albagli, Hellwig, and Tsyvinski (2021), all traders suffer from correlation neglect instead of combining both rational and noise traders, allowing to embed the financial market model in a straightforward way in a standard macroeconomic model. Whereas the work-horse model in this literature builds on risk aversion and linear demand functions for tractability, the main result of this paper builds on non-linear (concave) demand functions which arise with sufficiently convex adjustment costs. This paper focuses on the case with exogenous position limits to regain tractability and derive analytical solutions.

The literature on real feedback (as surveyed in Bond, Edmans, and Goldstein, 2012) studies the role of asset prices in informing economic decisions. For example, secondary markets can be sources of information for managers (Holmström and Tirole, 1993; Dow and Gorton, 1997). Different from these papers, information in this model directly determines the allocation of capital instead of managers learning from the price. Similar to Dow, Goldstein, and Guembel (2017), I study the two-way feedback between the financial and real economy when traders produce information endogenously. A number of papers has brought this paradigm to macroeconomics (Peress, 2014; David, Hopenhayn, and Venkateswaran, 2016; Albagli, Hellwig, and Tsyvinski, 2017; Straub and Ulbricht, 2018; Asriyan, 2021). Of these papers, the most closely related is Peress (2014). In contrast Peress (2014), this paper studies the role of aggregate shocks on information production and does not rely on a linear approximation. Moreover, I conduct a normative analysis and show under which conditions information production is likely to be too high or too low in the competitive equilibrium.

There is ample empirical evidence that asset prices are indeed informative. See Morck, Yeung, and Yu (2013) for a survey on the literature that uses "non-synchronicity" as a measure of price-informativeness. Morck, Yeung, and Yu (2000) found that more developed countries have stock markets that are more informative. Focusing instead on the cross-section of firms, Durnev, Morck, and Yeung (2004) found that non-synchronicity is positively related to the efficiency of corporate investment. More recently, Bai, Philippon, and Savov (2016) and Farboodi et al. (2020) have shown that prices have become better predictors of corporate earnings in the US since the 1960s. The latter emphasize that this has been mainly the case for large growth firms. Bennett, Stulz, and Wang (2020) provide evidence that price informativeness increases firm productivity. Recently, Dávila and Parlatore (2022) provide an identified measure of relative price informativeness following a structural approach. Their measure if closely related to information production in this paper.

In this model, traders suffer from correlation neglect. This bias has been intensively studied and documented repeatedly in labs (Brandts, Giritligil, and Weber, 2015; Eyster et al., 2018; Enke and Zimmermann, 2019; Grimm and Mengel, 2020) and in field experiments (Chandrasekhar, Larreguy, and Xandri, 2020). Broadly, this bias arises whenever information from multiple sources is processed without acknowledging that this information may have correlated noise (e.g., through a common source). Following the terminology in Eyster, Rabin, and Vayanos (2019), correlation neglect leads to *overconfidence* (Glaser and Weber, 2010; Daniel and Hirshleifer, 2015), as traders feel overly informed after reading repetitions of the same story. Correlation neglect is different from overconfidence as in Scheinkman and Xiong (2003), as traders correctly assess the precision of other traders' information. Unlike Eyster and Rabin (2005) and Mondria, Vives, and Yang (2022), traders learn correctly from prices. Similar to these approaches, correlation neglect leads to an underweighting of information contained in the price as the flipside of the overweighting of private information. Finally, in this paper, correlation neglect does not follow from wishful thinking as in Banerjee, Davis, and Gondhi (2019), but is instead assumed as a primitive. Instead, correlation neglect arises from an imperfect understanding about how different information sources build on each other.

Finally, a broad literature studies the role of sentiments in macroeconomics (for a survey, see Nowzohour and Stracca, 2020). There are different definitions of sentiments, ranging from self-fulfilling beliefs (Martin and Ventura, 2018; Asriyan, Fuchs, and Green, 2019) to news and noise shocks (Angeletos, Lorenzoni, and Pavan, 2010; Schmitt-Grohé and Uribe, 2012). In this paper, sentiments are waves of non-fundamental optimism or pessimism. When a positive sentiment shock hits, agents become optimistic about productivity and vice versa.

2 General Model

Before going to the model with a real economy, I show that traders produce less precise information when markets are severely mispriced in a partial equilibrium setting with convex trading or adjustment costs. These costs capture limits to arbitrage due to monetary costs or risk aversion. For example, mobilizing additional capital may require selling illiquid assets at a discount, or borrowing can increase risk-premia. Alternatively, traders may be unwilling to take on large amounts of risk. In either case, sufficiently convex adjustment costs limit the traders' ability to take large positions. As a result, traders react less sensitively to information once they already have a large position, making information acquisition less attractive ex-ante when such large positions are expected (e.g., buying during a depression). This section seeks to capture this intuition in a simplified yet general model.

The analysis focuses on a setting with a single trader who can buy a divisible asset with payoff $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$. Furthermore, the trader takes the price function $P = \frac{\beta z + \beta\sigma_\varepsilon^{-2}\left(z - \frac{\bar{\varepsilon}}{\sqrt{\beta}}\right)}{\sigma_\theta^{-2} + \beta + \beta\sigma_\varepsilon^{-2}}$ and the price signal $z = \theta + \frac{\varepsilon}{\sqrt{\beta}}$ as given, where $\varepsilon \sim \mathcal{N}(\bar{\varepsilon}, \sigma_\varepsilon^2)$ can be interpreted as sentiment or noise trading.[11] The trader receives a private signal $s_i = \theta + \frac{\eta_i}{\sqrt{\beta_i}}$, where $\eta_i \sim \mathcal{N}(0, 1)$ is idiosyncratic noise. Importantly, the trader decides on her information precision β_i subject to a convex cost function $\mathcal{C}(\beta_i)$ before going to the market. The informativeness of the price signal $\beta\sigma_\varepsilon^{-2}$ is taken as given. Finally, mispricing is captured by $\bar{\varepsilon}$, as the price P deviates far from the dividend θ.

Solving the trader's problem taking the information precision β_i as given, the maximization problem is

$$\max_{d_i} d_i(\mathbb{E}(\theta|s_i, z) - P) - a\frac{d_i^{2b}}{2b}, \tag{1}$$

with $b \in \mathbb{N}$. The adjustment cost $a\frac{d_i^{2b}}{2b}$ can be either interpreted as a monetary cost of mobilizing the necessary capital or as a proxy for risk aversion. The resulting demand function is

$$d_i = \left(\frac{\mathbb{E}(\theta|s_i, z) - P}{a}\right)^{\frac{1}{2b-1}}. \tag{2}$$

Setting $b = 1$ yields the canonical linear demand function, which is necessary to derive a linear equilibrium such as in Grossman (1976). With $a = \gamma Var(\theta|s_i, z)$, the demand function of a trader with CARA-utility is recovered. These assumptions guarantee that the price is a linear function of normally-distributed variables, leading to a normal posterior for traders and ensuring tractability. In contrast, demand functions are concave with $b > 1$, such that each unit of expected return $\mathbb{E}(\theta|s_i, z) - P$ leads to ever smaller increases in demand d_i. In other words, financial constraints become tighter as b increases.

One indicator for the value of information is how strongly the trader's demand reacts to changes in the private signal s_i,

$$\frac{\partial d_i}{\partial s_i} = \frac{\omega_{i,s}}{a(2b-1)d_i^{2b-2}}, \tag{3}$$

where $\omega_{i,s} = \frac{\partial \mathbb{E}(\theta|s_i,z)}{\partial s_i} = \frac{\beta_i}{\sigma_\theta^{-2} + \beta_i + \beta\sigma_\varepsilon^{-2}}$ is the weight on private signal s_i when forming expectations about the payoff θ that follows from Bayesian learning. The sensitivity of demand to the signal s_i depends on two components: (i) the information learned from s_i captured through $\omega_{i,s}$ and (ii) the position size d_i (for $b > 1$). The second channel plays an important role in the presence of mispricing. The trader expects to take large positions $|d_i|$ when also the anticipated mispricing $|\bar{\varepsilon}|$ is large. Therefore, the trader's sensitivity to the signal s_i

[11] This price function and signal can be derived in equilibrium in Albagli, Hellwig, and Tsyvinski (2021).

decreases in the presence of mispricing as captured by (3). As a result, traders decide to acquire less information when faced with a severely distorted market, which decreases price informativeness in equilibrium and increases capital misallocation in the main model.

This section showed that the paper's main result, i.e., a reduction in information production during bubbles and depressions, arises in a simple setting with sufficiently convex adjustment costs. The remainder of the paper derives analytical results for the tractable case with $b \to \infty$, which leads to exogenous position limits as in Albagli, Hellwig, and Tsyvinski (2021). The paper's main contribution is to provide micro-foundations and embed a noisy financial market in a macro-setting. In particular, traders with dispersed information trade shares of firms with heterogeneous productivities. Firms use the proceeds from selling their equity to invest in firm capital. Here, the role of financial markets is to aggregate the investors' dispersed information and allocate more capital to more productive firms. As a result, when financial markets are inefficient in the aggregate, investors produce less information, which makes financial markets also inefficient in the cross-section. In other words, more capital flows to relatively unproductive firms, which drags down aggregate productivity.

3 Main Model

3.1 Households and Traders

The model is populated by overlapping generations of households indexed by $i \in [0,1]$. As is common in the New Keynesian literature, I assume that each household i consists of a unit mass of traders indexed by $ij \in [0,1] \times [0,1]$ (for example, see Blanchard and Galí, 2010). Households pool resources, borrow on behalf of traders, and distribute consumption equally, whereas traders individually maximize the utility for the household given by

$$U_{it} = C_{it,t} + \delta \mathbb{E}\left\{C_{it,t+1}\right\} - \int_0^1 \mathcal{C}(\beta_{ijt}) dj, \tag{4}$$

where $C_{it,t}$ is youth consumption, $C_{it,t+1}$ is old age consumption, $\delta \in (0,1)$ is the discount factor, and $\int_0^1 \mathcal{C}(\beta_{ijt}) dj$ are information production costs, which are introduced in more detail later.

When young, traders each supply one unit of labor inelastically, receive wage W_t and buy shares of intermediate good firms in a competitive financial market. To avoid unbounded demands by risk-neutral traders, demand for each stock is limited to the interval $[\kappa_L, \kappa_H]$ where $\kappa_L \leq 0$ and $\kappa_H > 1$.[12] Traders also choose the precision β_{ijt} of a noisy signal of firm

[12] Exogenous position limits are not necessary to derive the main result as shown in section 2 and instead constitute a tractable limit case of convex adjustment costs. See Dow, Goldstein, and Guembel (e.g., 2017)

productivity to inform their trading decision subject to a utility cost $\mathcal{C}(\beta_{ijt})$. Finally, the household lends and borrows through risk-free bonds with return R_{t+1}.

3.2 Technologies

3.2.1 Final Good Sector

There are many identical final good firms owned by households. The production function for the final good, which also serves as the numéraire, is Cobb-Douglas over labor and a CES-aggregate of intermediate goods. Aggregate output is

$$Y_t = L^{1-\alpha} \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}}, \tag{5}$$

where $\theta \in (0, \infty)$ is the elasticity of substitution between varieties and α is the share of intermediate goods. Y_{jt} is an intermediate good produced by firm j. The final good can be consumed or invested in firm capital. The labor supply L is normalized to one.

3.2.2 Intermediate Good Sector

For each generation, there is a unit mass of intermediate good firms $j \in [0, 1]$ with production function

$$Y_{jt} = A_{jt-1}^{\frac{\theta}{\theta-1}} K_{jt}, \tag{6}$$

where K_{jt} is firm capital and $\ln(A_{jt-1}) \overset{iid}{\sim} \mathcal{N}(a_{t-1}, \sigma_a^2)$ is firm productivity. Note that time subscript $t-1$ is used as agents learn about firm productivity in the period prior to production. Capital takes time to build, such that investment takes place in t but production in $t+1$, and depreciates fully after production. Each firm sells a unit mass of claims to total firm-revenue to households and finances capital investment with the proceeds,

$$P_{jt} = K_{jt+1}. \tag{7}$$

3.2.3 Information Structure

Trader ij is only active in the market for shares of firm j, for which she is an expert as she receives the signal

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt} + \varepsilon_{jt}}{\sqrt{\beta_{ijt}}}, \tag{8}$$

and Albagli, Hellwig, and Tsyvinski (2021) for similar approaches.

where $a_{jt} \overset{iid}{\sim} \mathcal{N}\left(a_t, \sigma_a^2\right)$ is firm productivity, $\eta_{ijt} \overset{iid}{\sim} \mathcal{N}\left(0, 1\right)$ is idiosyncratic noise, $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ is correlated noise, interpreted as *sentiment* and may stem from traders using the same information sources, and β_{ijt} is a information precision parameter chosen by trader ij. Both idiosyncratic and correlated noise are *iid* over time and across markets; idiosyncratic noise is also *iid* between traders. A high realization of η_{ijt} means that trader ij is optimistic about firm j relative to other traders in the same market. Similarly, a high realization of ε_{jt} means that all traders in market j are too optimistic.

Assumption 1 (***Correlation Neglect***)**.** *Trader ij believes the information structure to be*

$$s_{ijt} = a_{jt} + \frac{\eta_{ijt}}{\sqrt{\beta_{ijt}}} \tag{9}$$

$$s_{-ijt} = a_{jt} + \frac{\eta_{-ijt} + \varepsilon_{jt}}{\sqrt{\beta_{-ijt}}}. \tag{10}$$

Following Assumption 1, traders believe that sentiment ε_{jt} drives the beliefs of all traders but not their own. This correlation neglect leads effectively to *overconfidence* and motivates costly information production to exploit mispricing due to sentiment.[13] Finally, trader ij chooses the precision β_{ijt} of the perceived signal in (9) subject to a convex cost function $\mathcal{C}\left(\beta_{ijt}\right)$ with standard properties $\mathcal{C}(0) = 0$, $\mathcal{C}'(0) = 0$, $\mathcal{C}''\left(\cdot\right) > 0$. The marginal cost is denoted as $\mathcal{MC}(\cdot)$.

3.2.4 Aggregate Shocks

Two classes of shocks drive the economy. *Aggregate productivity shocks* move the mean of the distribution of firm-specific productivity shocks, $a_{jt} \sim \mathcal{N}\left(a_t, \sigma_a^2\right)$, and *aggregate sentiment shocks* drive the mean of firm-specific sentiment shocks, $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$, similar to Angeletos, Lorenzoni, and Pavan (2010). The sentiment shock ε_t is meant to capture a range of phenomena that lead to non-fundamental price movements in financial markets, e.g., herding, informational cascades, social learning, bubbles, liquidity trading (see Kindleberger and Aliber, 2015; Shiller, 2015, 2017). I study economy-wide sentiment shocks as they affect cross-sectional misallocation of capital only through their effect on information production to avoid the direct misallocation through sectoral sentiment shocks.

To affect the information production decision, aggregate shocks need to be at least imperfectly anticipated by traders. To make the analysis as clear as possible, traders *perfectly*

[13]This assumption avoids the Grossman-Stiglitz paradox (Grossman and Stiglitz, 1980). It states that informationally efficient markets are impossible in the absence of noise when information is costly. In that case, markets would already reveal all information and, therefore, destroy the incentive to produce costly information in the first place.

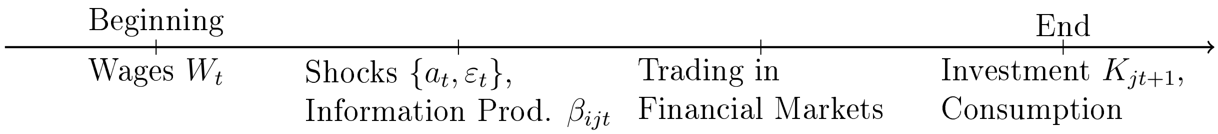


Figure 1: Intraperiod Timing.

observe aggregate shocks $\{a_t, \varepsilon_t\}$, but firm-specific shocks $\{a_{jt}, \varepsilon_{jt}\}$ need to be learned.[14] The laws of motion for the aggregate shocks are irrelevant for the main result, as the dynamic model is a repetition of static problems. It follows that the information set of trader ij consists of the private signal s_{ijt}, share prices $\{P_{jt}\}$ for all markets $j \in [0, 1]$, and the mean and variances of firm-specific shocks $\{a_t, \varepsilon_t\}$, i.e., $\mathcal{I}_{ij} = \{s_{ijt}, \{P_{jt}\}, a_t, \varepsilon_t\}$. In other words, traders have rational beliefs about aggregates but disagree about the productivity of intermediate firms based on public information in the forms of prices and private signals.

3.3 Timing

The timing is laid out in Figure 1. At the beginning of each period, young traders work in the final good sector and receive wage W_t. Then, traders choose the precision of their signal and the financial market opens. At the end of the period, both investment and consumption take place.

3.4 Notation

Traders think that their private signals do not contain correlated noise ε_{jt} as in Assumption 1. Therefore, expectations that condition on private signals are distorted and denoted by $\tilde{\mathbb{E}}\{\cdot\}$ and objective expectations are denoted by $\mathbb{E}\{\cdot\}$.

The determinants of functions are usually omitted to save on notation. For example, firm j's revenue is denoted by Π_{jt+1} instead of $\Pi(A_{jt}, K_{jt+1}, Y_{t+1})$. Moreover, A_{jt} is indexed by t instead of $t+1$, as traders can learn about firm productivity in period t.

3.5 The Household's Problem and The Trader's Problem

The main role of households is to coordinate borrowing and lending for its traders. Therefore, households share the correlation neglect of Assumption 1, in that each household believes that all its traders indeed have signals free of sentiment. However, households do not observe the private signals of traders to avoid conflicting information about aggregates $\{a_t, \varepsilon_t\}$. The

[14] The case with imperfect knowledge about aggregate shocks is studied in section 7.4

household's problem is taking interest rate R_{t+1} as given,

$$\max_{B_{it+1}} \quad C_{it,t} + \delta \tilde{\mathbb{E}}_t \left\{ C_{it,t+1} \right\} - \int_0^1 \mathcal{C}\left(\beta_{ijt}\right) dj \tag{P1.1}$$

$$s.t. \quad C_{it,t} = W_t - \int_0^1 x_{ijt} P_{jt} dj - B_{it+1} \tag{11}$$

$$C_{it,t+1} = \int_0^1 x_{ijt} \Pi_{jt+1} dj + R_{t+1} B_{it+1} \tag{12}$$

$$C_{it,t}, C_{it,t+1} \geq 0. \tag{13}$$

Households optimally choose how much to lend or borrow subject to budget constraints. The first constraint (11) states that consumption during youth is equal to wages W_t minus the costs of buying stocks $\int_0^1 x_{ijt} P_{jt} dj$ and saving through the bond market B_{it+1}. Constraint (12) states that old age consumption is equal to revenue $\int_0^1 x_{ijt} \Pi_{jt+1} dj$ plus income from lending on the bond market $R_{t+1} B_{it+1}$. Although household i is overly optimistic about the return of its portfolio due to correlation neglect, each household correctly values the portfolio of all other households. Therefore, limiting borrowing by the natural borrowing constraint as in (13) rules out default on bonds.

Household i's optimal saving decision is given by

$$B_{it+1} \begin{cases} = -\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}} & \text{if } R_{t+1} < \frac{1}{\delta} \\ \in \left[-\frac{\int_0^1 x_{ijt} \Pi_{jt+1} dj}{R_{t+1}}, W_t - \int_0^1 x_{ijt} P_{jt} dj \right] - & \text{if } R_{t+1} = \frac{1}{\delta} \\ = W_t - \int_0^1 x_{ijt} P_{jt} dj & \text{if } R_{t+1} > \frac{1}{\delta} \end{cases}. \tag{14}$$

If the interest rate R_{t+1} is below $\frac{1}{\delta}$, it is optimal to borrow as much as possible. If the interest is equal to $\frac{1}{\delta}$, household i is indifferent between borrowing and saving. Finally, if the interest rate is above $\frac{1}{\delta}$, then it is optimal to save as much as possible. Plugging (11) and (12) into P1.1 and using the solution for the saving decision (14) yields trader ij's problem

$$\max_{\beta_{ijt}} \quad \tilde{\mathbb{E}}_t \left\{ \lambda_t \max_{x_{ijt}} \tilde{\mathbb{E}} \left\{ x_{ijt} \left(\frac{1}{R_{t+1}} \Pi_{jt+1} - P_{jt} \right) | s_{ijt}, P_{jt} \right\} \right\} - \mathcal{C}\left(\beta_{ijt}\right) \tag{P1.2}$$

$$s.t. \quad x_{ijt} \in \left[\kappa_L, \kappa_H\right] \tag{15}$$

$$\beta_{ijt} \geq 0, \tag{16}$$

where $\lambda_t = \max\left\{1, \delta R_{t+1}\right\}$ and terms that do not depend on the decision by trader ij were dropped. The problem is split into two parts, which are solved in reverse chronological order. Given information production β_{ijt} and realizations of the private signal s_{ijt} and price

P_{jt}, trader ij chooses demand x_{ijt} for share j subject to the position limits (15). Using the solution to the trading problem, trader ij decides on the information precision β_{ijt} to increase the likelihood of trading profitably subject to a non-negativity constraint. Trader ij can use household i's pooled resources and borrow through the household for trading. The term λ_t reflects that the value of an additional unit of wealth during youth may be above one if the interest rate R_{t+1} is sufficienlty large.

4 Equilibrium Characterization

4.1 Input Markets

Wages and intermediate good prices are determined competitively,

$$W_t = \frac{\partial Y_t}{\partial L} = (1-\alpha) Y_t \tag{17}$$

$$\rho_{jt} = \frac{\partial Y_t}{\partial Y_{jt}} = \alpha Y_t^{\alpha_Y} Y_{jt}^{-\frac{1}{\theta}}, \tag{18}$$

where $\alpha_Y = \frac{\alpha\theta-\theta+1}{\alpha\theta}$. Wages are equal to a share $(1-\alpha)$ of output. The price for intermediate good j is downward sloping in the quantity produced of the same good. Finally, the revenue of intermediate good firm j is given by

$$\Pi_{jt+1} = \rho_{jt+1} Y_{jt+1}. \tag{19}$$

4.2 Trader's Decisions

Trading If price P_{jt} exceeds expectations of discounted revenue $R_{t+1}^{-1}\Pi_{jt+1}$ using the interest rate on bonds as the benchmark rate, trader ij sells $-\kappa_L$ shares; trader ij is indifferent between buying and selling when these values are equal. When expectations exceed the price, trader ij buys κ_H shares:

$$x(s_{ijt}, P_{jt}) = \begin{cases} \kappa_L & \text{if } R_{t+1}^{-1}\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt}, P_{jt}\} < P_{jt} \\ \in [\kappa_L, \kappa_H] & \text{if } R_{t+1}^{-1}\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt}, P_{jt}\} = P_{jt} \\ \kappa_H & \text{if } R_{t+1}^{-1}\tilde{\mathbb{E}}\{\Pi_{jt+1}|s_{ijt}, P_{jt}\} > P_{jt} \end{cases}. \tag{20}$$

Information Production As laid out in (20), the trading decision is driven by the realization of the private signal s_{ijt} relative to price P_{jt}. Consequently, trader ij chooses information precision β_{ijt} to improve her ability to identify profitable trading opportunities.

A central object in this context is the subjective probability of buying conditional on shocks $\{a_{jt}, \varepsilon_{jt}\}$ and choices $\{\beta_{ijt}, \beta_{jt}\}$, where β_{jt} stands for the symmetric information choice of all other traders in market j. Taking expectations with respect to the realizations of idiosyncratic noise, η_{ijt}, yields the probability of buying,[15]

$$\mathcal{P}\left\{x_{ijt} = \kappa_H\right\} = \mathbb{E}\left\{R_{t+1}^{-1}\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}, P_{jt}\right\} > P_{jt}\right\}. \tag{21}$$

The first-order condition for the information production decision is obtained after plugging (20) into (P1.2). Evaluating the expectations with respect to the realizations of the idiosyncratic noise η_{ijt} and taking the symmetric information production decisions of all other traders as given ($\beta_{-ijt} = \beta_{jt}$), leads to the first-order condition:

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right) = \lambda_t\left(\kappa_H - \kappa_L\right)\tilde{\mathbb{E}}_t\left\{\underbrace{\frac{\partial \mathcal{P}\left\{x_{ijt} = \kappa_H\right\}}{\partial \beta_{ijt}}}_{\text{Change in the Probability of Buying}} \underbrace{\left(R_{t+1}^{-1}\Pi_{jt+1} - P_{jt}\right)}_{\text{Rents}}\right\} = \mathcal{MC}\left(\beta_{ijt}\right). \tag{22}$$

The marginal benefit of producing information consists of two parts. First, the probability of buying in state $\{a_{jt}, \varepsilon_{jt}\}$ given information choices $\{\beta_{ijt}, \beta_{jt}\}$. Second, trading rents given by the difference between the net present value of firm revenue minus the stock price. When additional information makes buying more likely when rents are high, then the marginal benefit of information production is high.

4.3 Financial Market

After solving the traders' problem, I turn now to the equilibrium in the financial market.

Market-Clearing At the symmetric equilibrium ($\forall j : \beta_{ijt} = \beta_{jt}$), traders buy κ_H shares whenever their private signals are above some threshold, $\hat{s}\left(P_{jt}\right)$, are indifferent between buying and selling when their private signals coincide with the threshold, and sell otherwise. After normalizing the supply of shares in each market j to one, the market-clearing condition becomes

$$\kappa_H\left(1 - \Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right)\right) - \kappa_L\Phi\left(\sqrt{\beta_{jt}}\left(\hat{s}\left(P_{jt}\right) - a_{jt}\right) - \varepsilon_{jt}\right) = 1, \tag{23}$$

[15] The ex-ante probability of buying depends on the realizations of firm-specific shocks and the information choises of trader ij and all other traders in market j. These variables are suppressed to save on notation. A more detailed derivation can be found in section A.1.

where $\Phi(\cdot)$ is the standard normal cdf. The threshold $\hat{s}(P_{jt})$ can be solved for directly,

$$\hat{s}(P_{jt}) = a_{jt} + \frac{\varepsilon_{jt} + \Phi^{-1}\left(\frac{\kappa_H - 1}{\kappa_H + \kappa_L}\right)}{\sqrt{\beta_{jt}}}. \tag{24}$$

Price Signal Traders learn from prices, which is equivalent to observing a noisy signal of the form

$$z_{jt} = \hat{s}(P_{jt}) - \frac{\Phi^{-1}\left(\frac{\kappa_H - 1}{\kappa_H + \kappa_L}\right)}{\sqrt{\beta_{jt}}} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}. \tag{25}$$

When the price P_{jt} is high, traders realize that this can be due to two reasons: either firm j is productive (high a_{jt}) or other traders are very optimistic (high ε_{jt}). Therefore, prices are a noisy signal of firm productivity. The combination of dispersed information and position limits for asset demand ensure that the signal is normally distributed as $z_{jt} \sim \mathcal{N}(a_{jt}, \sigma_\varepsilon^2/\beta_{ijt})$ for all values of κ_L and κ_H. I call z_{jt} the *price signal* and expectations condition on z_{jt} instead of P_{jt} as they are equivalent.

As can be seen from (25) the values of κ_H and κ_L do not matter for the price signal z_{jt}. Taking the limit $b \to \infty$ to the general model in section 2 and adding the unit net supply to both limits leads to position limits $x_{ijt} \in [0, 2]$, which also avoids a bias in the price through the choice of κ_H and κ_L. For instance, choosing a larger κ_H means that relatively optimistic traders can clear the market, which increases the price.

Proposition 1. *Observing P_{jt} is equivalent to observing the signal (25) whenever K_{jt+1} is non-decreasing in z_{jt}. In the unique equilibrium, in which demand $x(s_{ijt}, P_{jt})$ is non-increasing in P_{jt}, the price is equal to the valuation of the trader with the private signal $s_{ijt} = z_{jt}$,*

$$P(z_{jt}) = \frac{1}{R_{t+1}} \tilde{\mathbb{E}}\left\{\Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{26}$$

This proposition shows that the described equilibrium is unique. By definition, the price P_{jt} is equal to the valuation of the *marginal trader* who is just indifferent between buying or not buying and observed the private signal $s_{ijt} = z_{jt}$. Any trader who is more optimistic than the marginal trader ($s_{ijt} > z_{jt}$) buys two shares, whereas more pessimistic traders buy nothing.

4.4 Bond and Capital Market

The net supply of bonds is equal to zero, $\int_0^1 B_{it+1} di = 0$. Moreover, as all households are ex-ante identical, positions in bond markets are zero for all households, $\forall i : B_{it+1} = 0$. There is no excess demand or supply for bonds whenever the return on bonds R_{t+1} is equal to the return that traders expect to earn on the stock market. This is the case whenever

$$R_{t+1} = \frac{\int_0^1 \tilde{\mathbb{E}} \left\{ \Pi_{jt+1} | s_{ijt} = z_{jt}, z_{jt} \right\} dj}{\int_0^1 P_{jt} dj}, \tag{27}$$

which follows from integrating (26) on both sides.

Finally, the aggregate value of the stock market is equal to the aggregate capital stock as all revenue from financial markets is invested by firms as follows from aggregating (7),

$$\int_0^1 P_{jt} dj = K_{t+1}. \tag{28}$$

4.5 Equilibrium Definition

In equilibrium, all traders choose the same information precision for all markets ($\forall ij : \beta_{ijt} = \beta_t$) and expect all other traders to choose the same.

Definition 1. *A competitive equilibrium consists of prices* $\{W_t, \rho_{jt+1}, P_{jt}, R_{t+1}\}$ *and allocations* $\{B_{it+1}, x_{ijt}, \beta_{ijt}, K_{jt+1}\}$ *such that:*

1. *Given prices* $\{W_t, \rho_{jt+1}, P_{jt}, R_{t+1}\}$ *and allocations* $\{x_{ijt}, \beta_{ijt}\}$, B_{it+1} *solves the household's problem P1.1.*

2. *Given prices* $\{P_{jt}, R_{t+1}\}$ *and allocations* $\{B_{it+1}, \beta_{jt}, K_{jt+1}\}$, $\{x_{ijt}, \beta_{ijt}\}$ *solve the trader's problem P1.2.*

3. *Prices are such that markets for labor, intermediate goods, shares, bonds, and capital clear, i.e., (17), (18), (23), (27)* and *(28) hold.*

4.6 Capital Allocation and TFP

The previous results can be combined to derive the allocation of capital and total factor productivity in equilibrium as captured in the following proposition.

Proposition 2 (***Market Allocation***)**.** *Under the market allocation,*

(i) firm capital is given by

$$K_{jt+1} = \frac{\tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^{\theta} dj} K_{t+1}. \tag{29}$$

(ii) the aggregate production function is

$$Y_t = A\left(a_{t-1}, \beta_{t-1}\right) K_t^{\alpha} \tag{30}$$

with total factor productivity

$$\ln A\left(a_{t-1}, \beta_{t-1}\right) = \underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1} + \frac{\sigma_a^2}{2}\right)}_{exogenous} + \underbrace{\kappa^a\left(\beta_{t-1}\right)\sigma_a^2 - \kappa^{\varepsilon}\left(\beta_{t-1}\right)\sigma_{\varepsilon}^2}_{allocative\ efficiency}, \tag{31}$$

where $\kappa^a(\beta_{t-1})$ is increasing in β_{t-1} and $\kappa^{\varepsilon}(\beta_{t-1})$ is hump-shaped in β_{t-1}.
(iii) $A\left(a_{t-1}, \beta_{t-1}\right)$ is taking its minimum for some $\beta_{t-1} > 0$ if $\sigma_{\varepsilon}^2 > 1$.
(iv) $A\left(a_{t-1}, \beta_{t-1}\right)$ is monotonically increasing in β_{t-1} if $\sigma_{\varepsilon}^2 \leq 1$.

The proposition's first part highlights that more capital is allocated to firms with higher realizations of the price signal z_{jt} irrespective of whether it is driven by sentiment or productivity. Moreover, firm capital for all firms is proportional to aggregate investment K_{t+1}. Consequently, total factor productivity (TFP) has both an exogenous and endogenous component. The exogenous component is solely related to the realization of the aggregate productivity shock a_t, which mechanically increases the productivity of all firms. The endogenous component captures instead the allocational efficiency of financial markets, i.e., the ability of the stock market to allocate more capital to more productive firms, which is determined by aggregate information production β_t.

The more surprising part is (*iii*), as it captures the inefficient use of dispersed information through financial markets. Here, this inefficiency is transparent and follows from the behavioral Assumption 1. As traders suffer from correlation neglect, they fail to acknowledge that the market aggregates the traders' collective information perfectly and instead still rely on their private signal, leading to an excess sensitivity of prices to the price signal z_{jt}. However, this distortion does not solely arise due to correlation neglect, but is generally a result of imperfect information aggregation in financial markets with dispersed information as shown in a setting with noise and rational traders in Albagli, Hellwig, and Tsyvinski (2021), where this distortion is studied in greater detail.

As this direct distortionary effect is not the center of this paper, the following analysis focuses on the case where the effect of the distortion is limited and better information leads also to better economic outcomes.

Assumption 2. *The variance of correlated noise is sufficiently low ($\sigma_{\varepsilon}^2 \leq 1$), such that TFP* (31) *is increasing in information production β_t.*

4.7 Aggregate Investment

Aggregate investment is in one of two regions. In the first region, traders consume during youth and investment is pinned down by $R_{t+1} = \frac{1}{\delta}$. In the second region, the interest rate is so high ($R_{t+1} > \frac{1}{\delta}$) that traders exhaust their wages for investment. Finally, $R_{t+1} < \frac{1}{\delta}$ cannot arise in equilibrium as investment would collapse to zero and the interest rate R_{t+1} would go to infinity.[16] Taken together, aggregate investment is equal to

$$K_{t+1} = \min\left\{ \left(\alpha\delta A_t^{\alpha_Y} \left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^{\theta} dj \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\alpha}}, W_t \right\}. \tag{32}$$

Aggregate shocks and information production determine investment in the elastic region. Aggregate productivity and sentiment shocks increase investment, as traders expect all firms to be more productive.

5 Main Results

The following section focuses on how aggregate productivity and sentiment shocks affect investment and the incentive to produce information. The main result will be that productivity shocks crowd in information, whereas sentiment shocks crowd out information and increase capital misallocation.

5.1 Aggregate Shocks and Information Acquisition

Recent experiences during stock and credit booms have raised concerns about increasing capital misallocation during these episodes (Gopinath et al., 2017; Doerr, 2018; Gorton and Ordoñez, 2020). This model can be used as a laboratory to think about the effects of productivity and sentiment shocks that may drive booms and their effects on the incentive to produce information, thereby affecting allocative efficiency. Note that aggregate shocks affect neither price informativeness nor capital misallocation directly but only through information production, highlighting the endogenous response of traders.

Sentiment Shocks The following proposition starts with the effect of aggregate sentiment shocks.

Proposition 3. *There exists a threshold $\bar{\varepsilon}$, such that*

(i) information production is increasing in the sentiment shock if $\varepsilon_t < \bar{\varepsilon}$,

[16] The equilibrium with zero investment can be ruled out by assuming that firms produce an infinitesimal amount even without investment from traders, avoiding that their share price collapses to zero.

(ii) information production is decreasing in the sentiment shock if $\varepsilon_t > \bar{\varepsilon}$,

(iii) the threshold $\bar{\varepsilon}$ is negative for $\theta > \frac{1}{1-\alpha}$ and positive for $\theta < \frac{1}{1-\alpha}$.

Proposition 3 shows that the effect of small sentiment shocks ($\varepsilon_t \approx 0$) on information production is ambiguous and depends on the parameters of the model. However, sentiment shocks always crowd out information production once they are sufficiently large in absolute terms.

At first, it may seem surprising that *aggregate* sentiment shocks crowd out information production, especially as in this model, *firm-specific* sentiment shocks incentivize information production in the first place. The difference arises because *aggregate* sentiment shocks are assumed to be *anticipated*, which changes the incentive to produce *firm-specific* information ex-ante. Therefore, the result carries through even if a sentiment shock is anticipated on the firm level, but fails if an aggregate sentiment shock is entirely unanticipated.

To further build intuition for this result, I use (29) in (22) to rewrite the marginal benefit of information production evaluated at the symmetric equilibrium ($\beta_{ijt} = \beta_t$),

$$\widetilde{MB} \propto \tilde{\mathbb{E}} \left\{ \overbrace{\underbrace{\frac{\partial \mathcal{P}\left\{x_{ijt} = 2\right\}}{\partial \beta_{ijt}} \bigg|_{\beta_{ijt}=\beta_{jt}}}_{\text{Information-Sensitivity}} \left(\frac{K_{jt+1}}{K_{t+1}}\right)^{\frac{\theta-1}{\theta}}}^{\text{Relative Size}} \underbrace{K_{t+1}^{\alpha}}_{\text{Absolute Size}} \left(A_{jt} - \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt} = z_{jt}, z_{jt}\right\}\right) \right\}. \tag{33}$$

As can be seen from (33), three channels can be isolated through which anticipated, aggregated sentiment shocks affect the incentive to produce information. I explain each in detail in the following.

The *information-sensitivity* channel materializes through the interaction of the change in the buying probability with the distribution of firm-specific sentiment shocks ε_{jt}. In the symmetric equilibrium ($\beta_{ijt} = \beta_{jt}$), traders expect to buy whenever they are more optimistic than the marginal trader, i.e., $s_{ijt} \geq z_{jt} \iff \eta_{ijt} \geq \varepsilon_{jt}$. The resulting probability of buying is $\Phi(-\varepsilon_{jt})$ where $\Phi(\cdot)$ is the standard-normal cdf. Consequently, the derivative of the buying probability with respect to ε_{jt} can be interpreted as the *information-sensitivity* of the buying decision. The *information-sensitivity* decreases as ε_{jt} takes more extreme values as is formally shown in

$$\phi\left(\varepsilon_{jt}\right) f\left(\varepsilon_{jt}\right) \propto \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\} \tilde{f}(\varepsilon_{jt}), \tag{34}$$

where $\phi(\cdot)$ is the standard-normal pdf, $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ and $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of ε_{jt} as if its distribution was $\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$.

The *information-sensitivity* channel is captured by the term $\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}$, which is

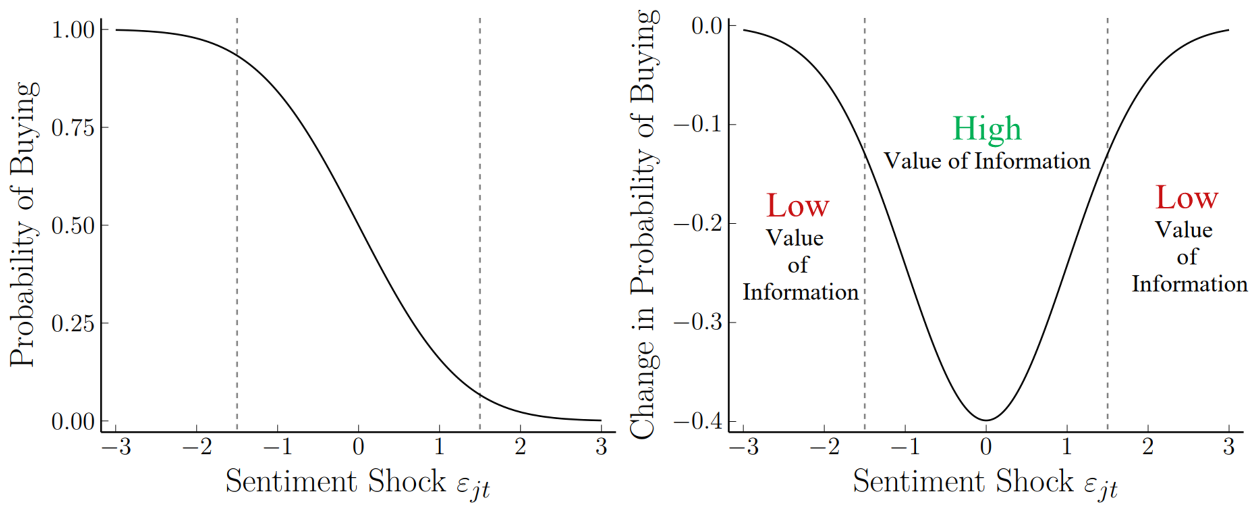


Figure 2: Probability of Buying and Sentiment Shocks.

Notes: Left panel: The probability of buying depending on the realization of the firm-specific sentiment shock ε_{jt}. Right panel: The derivative of the probability of buying. The trading decision is most information-sensitive, i.e., varies most with the realization of the sentiment shock ε_{jt}, around $\varepsilon_{jt} = 0$.

symmetrically decreasing around zero. Surprisingly, the decline in *information-sensitivity* does not depend on the actual pass-through of sentiment shocks to expectations. The reason can be found in the trading decision, which also does not depend on the mispricing. Therefore, aggregate sentiment shocks can discourage information production, even if they do not significantly affect actual prizes.

Figure 2 captures this result, as the trading decision is most elastic for realizations of the firm-specific sentiment shock ε_{jt} around zero. However, aggregate sentiment shocks push the distribution of ε_{jt} to the more inelastic regions toward the extremes.

The decline in *information-sensitivity* is amplified by a change in the *relative size* of firms, for which information remains valuable. As seen in Figure 2 before, the trading decision is most elastic for $\varepsilon_{jt} \approx 0$. Firms that remain in this region must have received *lower than average* sentiment shocks, making them less able to attract capital. This effect is captured by

$$\int_0^1 \tilde{f}(\varepsilon_{jt}) \left(\frac{K_{jt+1}}{K_{t+1}} \right)^{\frac{\theta-1}{\theta}} dj \propto \exp\left\{ -\left(\theta - 1\right) \omega_{s\varepsilon} \varepsilon_t \right\}, \tag{35}$$

where $\omega_{s\varepsilon} = \frac{\sqrt{\beta_t}}{\sigma_a^{-2} + \beta_t \left(1 + \sigma_\varepsilon^{-2}\right)}$ and $\frac{K_{jt+1}}{K_{t+1}}$ is the relative firm size. For a positive sentiment shock, information production becomes effectively directed toward smaller firms, weakening the incentive to produce information. This channel is formally captured by the term $\exp\left\{ -\left(\theta - 1\right) \omega_{s\varepsilon} \varepsilon_t \right\}$.

The *relative size* effect is increasing in the elasticity of substitution and in the pass-through of aggregate sentiment shocks $\omega_{s\varepsilon}$, which is hump-shaped in information production β_t. If intermediate goods are close substitutes, firms that traders perceive as unproductive attract very little capital. Moreover, if aggregate sentiment shocks affect expectations strongly, underpriced firms will be even smaller, making information production even less attractive.

Finally, the *absolute size* effect is captured by changes in aggregate investment. Restricting our attention to shocks for which $K_{t+1} < W_t$ leads to

$$K_{t+1}^{\alpha} \propto \exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{36}$$

As long as traders do not fully invest their wages, the *absolute size* effect can be captured by the term $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Intuitively, the effect on investment is stronger when α and, therefore, the returns to scale increase. A further increase in the sentiment shock is ineffectual for the *absolute size* channel once traders fully invest their wages but incentivizes nonetheless more information production through an increase in the value of resources, as captured by $\lambda_t = \max\{1, R_{t+1}\delta\}$ in (P1.2).

Putting all three effects together allows capturing the effect of sentiment shocks on the incentive to produce information at the symmetric equilibrium succinctly,

$$\widetilde{MB} \propto \exp\left\{\underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\text{Information-Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\text{Relative Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\text{Absolute Size}}\right\}. \tag{37}$$

For the empirically plausible calibration $\theta - 1 > \frac{\alpha}{1-\alpha}$, positive sentiment shocks *crowd out* information, as the increase in aggregate investment is dominated by a larger decrease in the size of fairly priced firms. Conversely, negative sentiment shocks initially *crowd in* information, as fairly priced firms turn out to be relatively large, although aggregate investment goes down. Finally, the *information-sensitivity* channel, which also arises in the partial equilibrium setting with fixed firm size, always dominates for large shocks.

Productivity Shocks Productivity shocks have quite different effects on the incentive to produce information. Whereas sentiment shocks affect trading in multiple ways, productivity shocks leave the buying decision unaffected. The reason is that traders believe that sentiment shocks affect only other traders, whereas productivity shocks affect all traders.

Productivity shocks affect information production through three channels:

$$\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\left(\underbrace{\frac{\alpha\theta-\theta+1}{(1-\alpha)\,\alpha\theta}}_{\text{Aggregate Demand}} + \underbrace{\frac{\alpha}{1-\alpha}}_{\text{Absolute Size}} + \underbrace{1}_{\text{Log-Normal Scaling}}\right) a_t\right\}. \quad (38)$$

The first effect works through TFP A_t and its effect on aggregate demand, the second through the effect on aggregate investment (*absolute size* channel), and, since productivity shocks are log-normal, a larger mean a_t also multiplies the effects of changes in firm-specific productivity a_{jt} around that mean. Of these three, only the first one is negative for $\theta > \frac{1}{1-\alpha}$. The combined effect is positive if intermediate goods for sufficiently strong complements ($\theta < \frac{1}{1-2\alpha}$).[17] This result is captured in the following Proposition.

Proposition 4. *Information production is increasing in productivity shocks for* $\theta < \frac{1}{1-2\alpha}$.

To summarize, the model provides a rationale for the different impacts of "good" and "bad" booms as in Gorton and Ordoñez (2020). Whereas productivity-driven "good" booms increase information production and improve allocative efficiency, sentiment-driven "bad" booms *crowd out* information and increase capital misallocation. The results of Propositions 3 and 4 are pictured in Figure 3. In contrast, both downturns due to negative sentiment or low productivity lead to crowding out of information production.

5.2 Amplification

Financial markets do not only react to aggregate shocks but also shape the economy's response to aggregate shocks. In the following, aggregate shocks hit an economy in steady state. Whether shocks amplify or dampen the effect of shocks on output is determined relative to an economy for which the information choice is fixed at the steady state value β^*.

With fixed information production β^*, aggregate shocks only affect TFP and investment directly. Positive shocks increase investment, and productivity shocks also affect TFP. Indirect effects work through the information choice β_t, which has two channels. First, information production changes TFP. Higher TFP makes investment more attractive. Second, changes to β_t also change the pass-through of sentiment shocks to investment. Intuitively, sentiment plays no role in traders' beliefs when β_t is either zero or infinite. Therefore, some

[17] In the standard Dixit-Stiglitz economy, intermediate goods are complements and an increase in aggregate demand Y_t always increases the demand for each intermediate good. Here, the final good sector combines labor and a CES-aggregate of intermediate goods, leading to decreasing marginal returns to intermediate good production. Therefore, intermediate goods need to be sufficiently close substitutes to retain an overall positive effect of productivity shocks on information production.

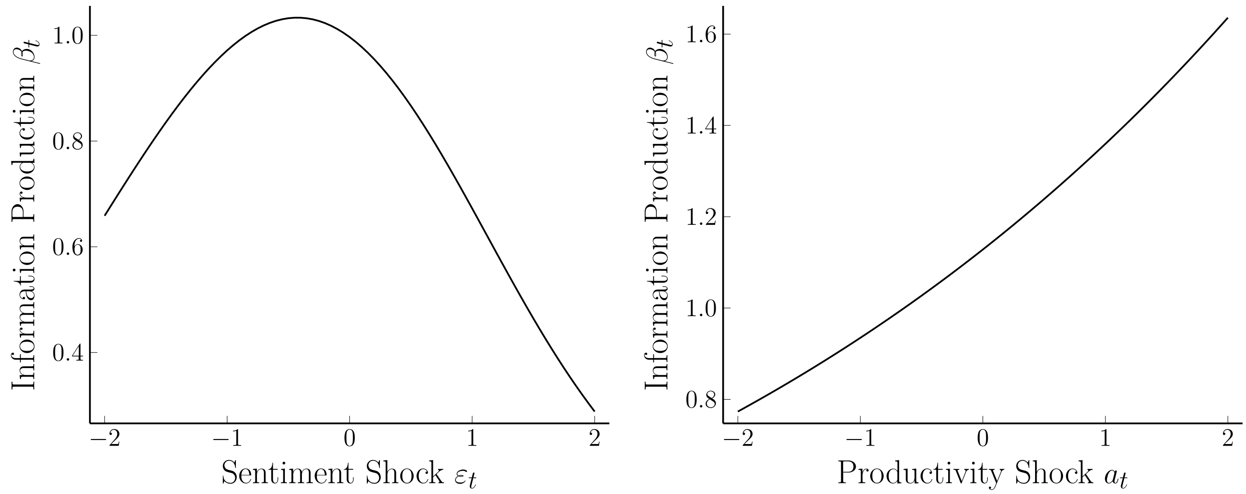


Figure 3: Information Production and Aggregate Shocks.

Notes: Information production is non-monotonic in the sentiment shock, with the peak $\bar{\varepsilon}$ being negative for $\theta > \frac{1}{1-\alpha}$. Information production is monotonically increasing in the productivity shock.

interior value of β_t must maximize the pass-through of sentiment shocks, such that changes to β_t can increase or decrease the pass-through of sentiment shocks, depending on the value of β^*.

The following Proposition focuses on the case when β_t is relatively low, and lower information production further dampens sentiment shocks.

Proposition 5. *(i) Positive sentiment shocks are dampened through endogenous information production when $\theta > \frac{1}{1-\alpha}$ and $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$. (ii) Large positive sentiment shocks eventually lead to a decrease in aggregate investment if* $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$.

These results may initially seem counterintuitive since sufficiently large positive sentiment shocks decrease prices and output. However, the decrease in information production must eventually outweigh the expansionary effect of sentiment shocks as the pass-through of sentiment shocks goes to zero. Moreover, note that this section studies only *anticipated* sentiment shocks. Positive sentiment shocks would unambiguously increase investment if the same shock were unknown prior to the information production decision or information precision was exogenous.

Similar forces are relevant for negative shocks with the exception that negative sentiment shocks initially crowd in information production if the elasticity of substitution is large enough ($\theta > \frac{1}{1-\alpha}$). If strong enough, this indirect effect can even lead to small negative sentiment shocks being expansionary. In contrast, if the elasticity of substitution is relatively small ($\theta < \frac{1}{1-\alpha}$), then negative sentiment shocks always crowd out information production and are,

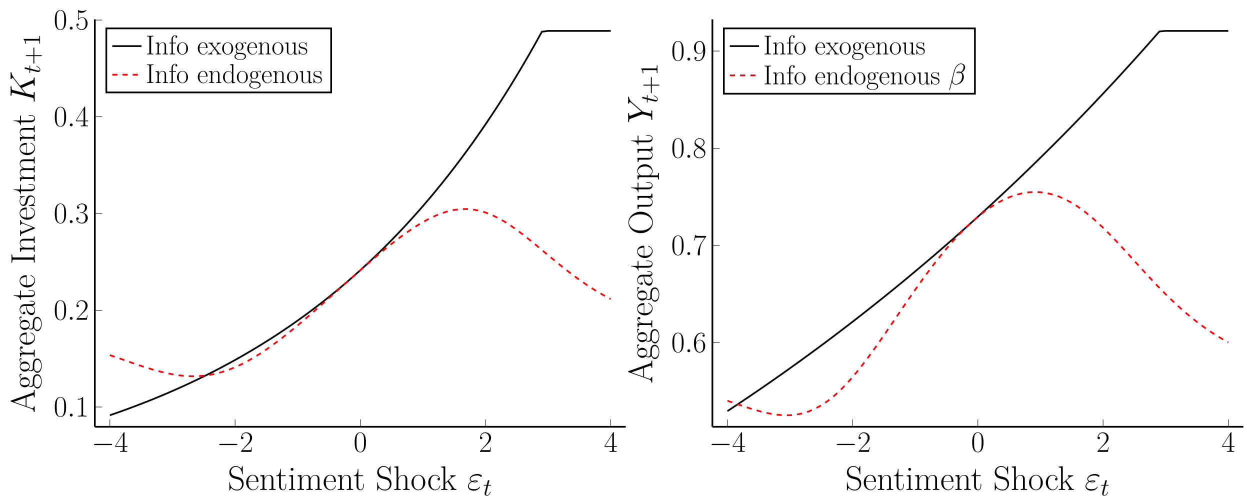


Figure 4: Amplification and Dampening for Sentiment Shocks.

Notes: Whether information production dampens or amplifies sentiment shocks depends on the size of the shock and the parameters. As information production affects both allocative efficiency and the pass-through of sentiment shocks, large sentiment shocks eventually drive information production so low that investment and output decrease.

therefore, initially amplified. These results are also captured in Figure 4.

Like the previous section, the indirect effect of productivity shocks leads to amplification. As follows from Proposition 4, positive productivity shocks crowd in information production, leading to an improvement in capital allocation and incentivizing additional investment. Therefore, compared to the economy with fixed information precision, the reaction of both output and investment to a productivity shock is larger if information precision is allowed to adjust, as seen in Figure 5.

Proposition 6. *Productivity shocks are amplified through endogenous information production for* $\theta < \frac{1}{1-2\alpha}$.

5.3 Numerical Simulation

This section provides a numerical illustration of booms driven by productivity and sentiment shocks, focusing on the region of parameters and shocks for which sentiment shocks are expansionary and dampened by information production. To capture the notion of booms, aggregate shocks build up over time according to the auto-regressive process

$$\chi_t = \begin{cases} \rho \chi_{t-1} + \zeta & t \in [0, B] \\ 0 & \text{otherwise} \end{cases}, \tag{39}$$

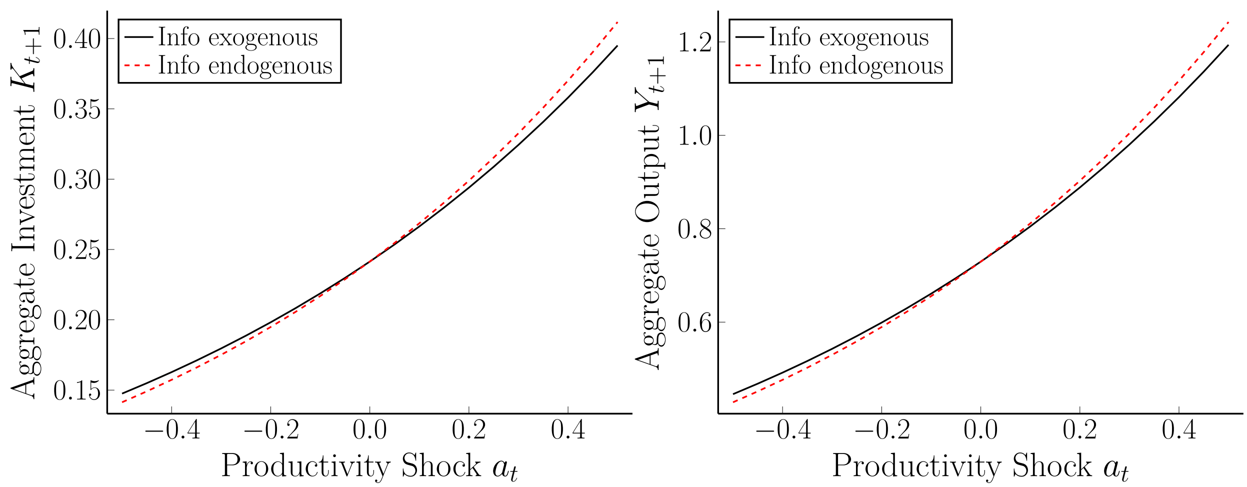


Figure 5: Amplification of Productivity Shocks.

Notes: Productivity shocks crowd in information production, leading to an additional increase in TFP and Aggregate Investment K_{t+1}. As a result, productivity shocks are amplified.

where $\chi_t \in \{a_t, \varepsilon_t\}$ is the aggregate shock, $\zeta > 0$ is a constant innovation, $\rho \in (0, 1)$ is the persistence, and B captures the duration of the boom. After the boom ends, the aggregate shock returns to a neutral stance.

The expansionary effect of sentiment shocks is dampened, as shown in Figure 6. Optimistic expectations lead to an increase in investment, but traders decide to cut back on information production, which decreases the allocative efficiency of financial markets. In total, output still increases because the sentiment shock leads to an offsetting increase in investment. In this case, the endogenous response of traders dampens the effect of a positive sentiment shock.

In contrast, productivity-driven booms are generally amplified by an increase in information production, as seen in Figure 7, mirroring the result from Figure 5 and Proposition 6. Expectations of higher productivity tomorrow cause an increase in investment today, which triggers more information production. As a result, the endogenous response of traders amplifies the effect of productivity shocks. In times of high productivity, financial markets allocate capital efficiently.

6 Is there a Role for Policy?

After studying the positive properties of the model, I turn now to the normative implications. There are two sources of inefficiencies. First, traders' correlation neglect distorts the allocation of capital among firms and lets anticipated aggregate sentiment shocks drive investment.

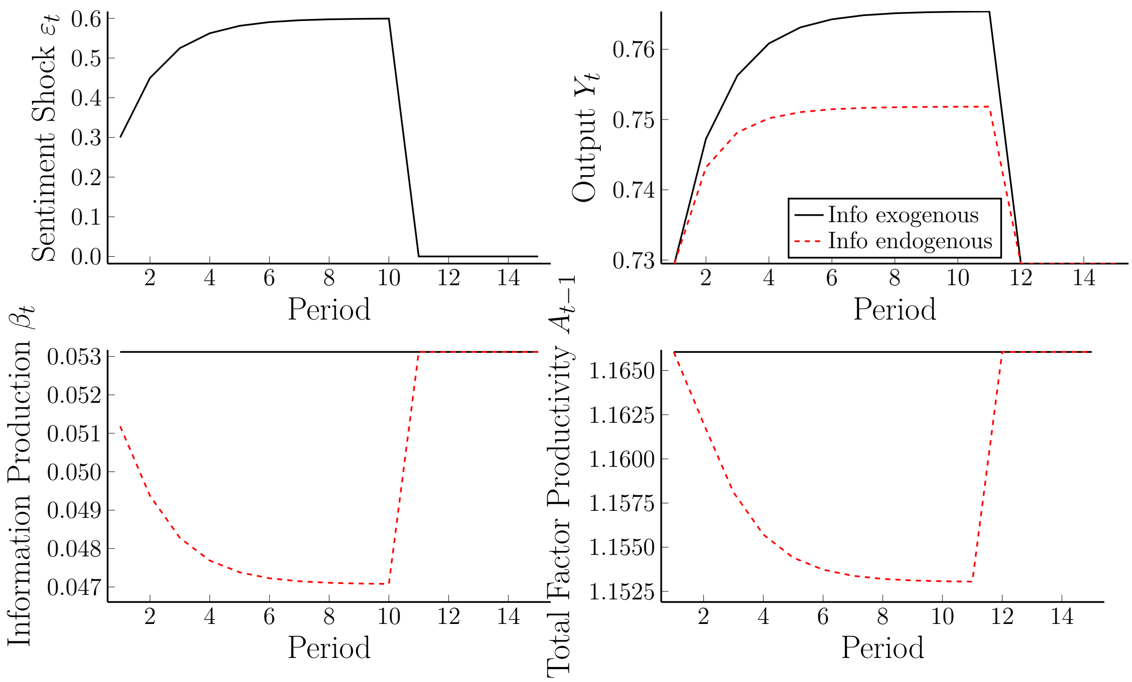


Figure 6: Numerical Simulation Sentiment-Driven Boom.

Notes: Sentiment-driven booms are dampened by information production.

A state- and price-dependent tax/subsidy on dividends is sufficient to fix this distortion. Second, there are two externalities concerning the information production decision that work in opposite directions. On the one hand, traders produce information to extract rents from other traders and ignore the negative effects they impose on others. On the other hand, traders do not internalize the positive effects of information production on the allocation of capital. The second effect is similar to the *pecuniary externality* in Pavan, Sundaresan, and Vives (2022). Whether information production is inefficiently high or low depends on the strength of the rent-stealing motive relative to the usefulness of information in allocating capital.

I abstract from well-known inter-generational trade-offs for the following welfare analysis using a two-period model. Traders are born with an endowment, produce information, and buy shares. Production takes place in the second period, and the final good sector combines intermediate goods into the final good without labor,

$$Y_1 = \left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}}. \tag{40}$$

The setup is otherwise identical to the main model.

The remaining section explains the inefficiencies in more detail before moving on to the

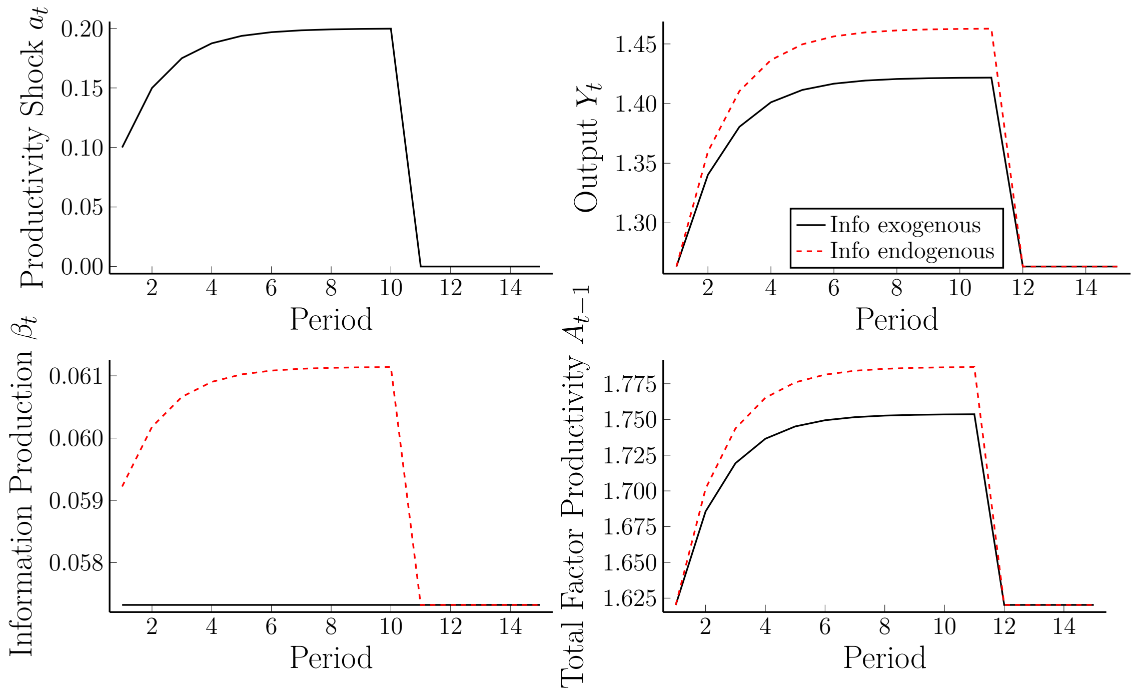


Figure 7: Numerical Simulation Productivity-Driven Boom.

Notes: Productivity-driven booms are amplified by information production.

social planner's problem.

6.1 Imperfect Information Aggregation

It comes as no surprise that the traders' correlation neglect in Assumption 1 distorts the way traders process information. Since the market perfectly reveals all private information that traders collectively possess, traders should optimally discard their private signal similar to Grossman (1976), leading to the in-existence of equilibria as captured in Grossman and Stiglitz (1980). I avoid these complications by assuming that traders suffer from correlation neglect, yielding the same equilibrium as in Albagli, Hellwig, and Tsyvinski (2021). The price overreacts to information since traders put too much weight on the private signal. In other words, P_{jt} behaves as if the precision of the market signal z_{jt} was $\beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}$, although its true precision is $\beta_{jt}\sigma_\varepsilon^{-2}$. As another consequence of correlation neglect, even if traders anticipate a sentiment shock, they fail to clean their private signal from sentiment.[18]

[18] Both imperfections happen in a setting with rational and noise traders as well. The overreaction of prices has intensively been studied in Albagli, Hellwig, and Tsyvinski (2011, 2015, 2021) and is named the "information aggregation wedge." Generally, it arises in any informative financial market model in which traders learn from both a heterogeneous private signal and the price. It does not arise when traders observe the same signal (Grossman and Stiglitz, 1980) or when traders submit market orders (Kyle, 1985). In both cases, traders do not learn from the price. See also Vives (2017) for an in-depth analysis in a linear setting.

6.2 Information Production, Rent Stealing, and Pecuniary Externality

Information is valuable as it is used within the financial market to allocate capital among firms with heterogeneous productivity. However, information production is at an inefficient level due to the presence of two externalities in opposite directions. First, traders produce information to exploit sentiment-driven mispricing. However, such rent-stealing directly substracts from any rent earned from other traders and is, therefore, socially wasteful. This negative *rent-stealing externality* drives information production too high. Second, similar to Pavan, Sundaresan, and Vives (2022), traders do not internalize the positive effects of information production on capital allocation. This positive *pecuniary externality* may lead to inefficiently low information production.

Proposition 7. *Information production is inefficiently high or low in the competitive equilibrium.*

Whether information production is too high or too low in the competitive equilibrium depends on the strength of the rent-stealing motive relative to the usefulness of information in allocating capital. Particularly, this depends on the elasticity of substitution θ. As intermediate goods become more substitutable, the allocation of capital to the most productive firm becomes all the more important, and information production is likely to be insufficient. As intermediate goods become perfect complements ($\theta \to 0$), an equal allocation of capital becomes optimal, and information production is excessive in the competitive equilibrium. Therefore, information production is too low, particularly when the allocation of capital becomes important. This result is captured in Figure 8.

6.3 Social Planner's Problem

In the following, I study the full social planner's problem. Note that the social planner does not possess superior information relative to any agent in the economy. All traders agree that correlation neglect is pervasive, and sentiment shocks are best eliminated to improve the

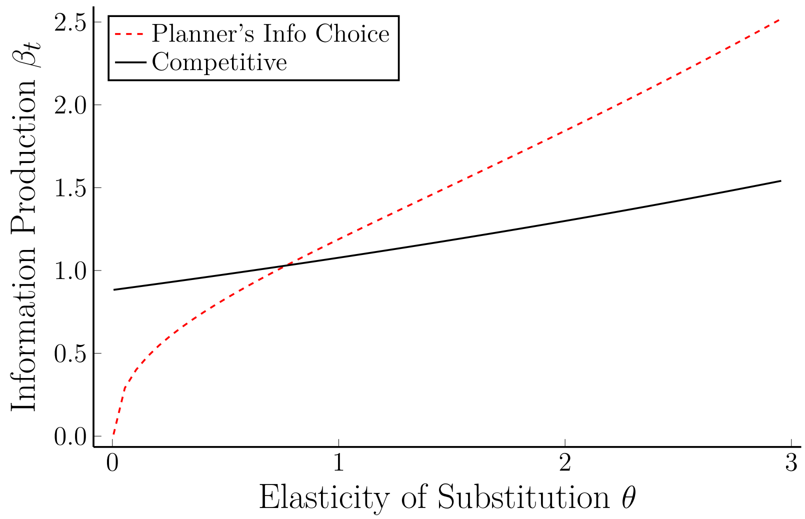


Figure 8: Planner's and Market's Information Production and Elasticity of Substitution in the Competitive Equilibrium.

Notes: Low elasticity of substitution: Too much information production. High elasticity of substitution: Too little information production.

allocation of capital. The social planner's problem is

$$\max_{K_{j1},C_0,C_1,\beta_{j0}} \quad C_0 + \delta \mathbb{E}_0 \{C_1\} - \int_0^1 IA(\beta_{j0})\, dj \tag{SP}$$

$$s.t. \quad K_1 = W_0 - C_0 \tag{41}$$

$$C_1 \leq Y_1(\{K_{j1}\}, \{\beta_{j0}\}) \tag{42}$$

$$C_0 \leq W_0 \tag{43}$$

$$K_{j1}, C_0, C_1, \beta_{j0} \geq 0. \tag{44}$$

Constraint (41) states that aggregate capital in period 1 is equal to endowments W_0 minus youth consumption C_0. Resource constraints for consumption are given in (42) and (43). Finally, non-negativity constraints on consumption, information production and capital are given in (44). The solution to the social planner's problem is given in the following Proposition.

Proposition 8. *The social planner's allocation under perfect information about aggregate shocks* $\{a_0, \varepsilon_0\}$ *is given by* $\left\{C_0^{SP}, K_{j1}^{SP}, K_1^{SP}, \beta_0^{SP}\right\}$, *where*

$$K_{j1}^{SP} = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}^{\theta}}{\int_0^1 \mathbb{E}\{A_{j0}|z_{j0}\}^{\theta} dj} K_1^{SP} \quad \textit{with } K_1^{SP} = \min\left\{\left(\alpha\delta A_0^{SP}\right)^{\frac{1}{1-\alpha}}, W_0\right\}, \tag{45}$$

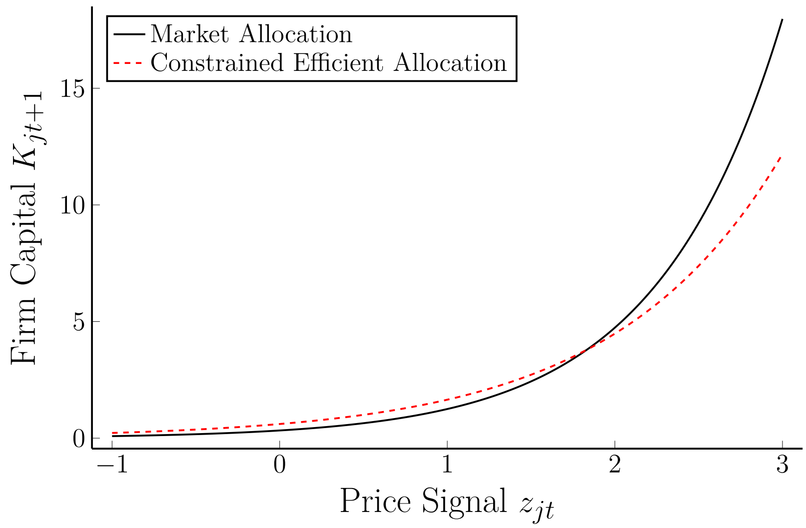


Figure 9: Market and Efficient Capital Allocation.

Notes: Market allocation of capital K_{jt} as in (29) and the efficient allocation K_{jt}^{SP} as in (45) keeping aggregate investment and information production constant.

leading to aggregate output

$$Y_1^{SP} = A_0^{SP} \left(K_1^{SP}\right)^{\alpha} \quad \text{with } A_0^{SP} = \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{\alpha}{\theta-1}}, \tag{46}$$

where A^{SP} is monotonically increasing in information production. The symmetric information production choice is

$$\text{for all } \beta_{j0} = \beta_0^{SP} : \delta \left.\frac{\partial A_0^{SP}}{\partial \beta_0}\right|_{\beta_0 = \beta_0^{SP}} \left(K_1^{SP}\right)^{\alpha} = \mathcal{MC}(\beta_0^{SP}). \tag{47}$$

The social planner fixes the two inefficiencies mentioned above. First, the social planner distributes capital optimally by attributing the correct precision to the price signal z_{jt} as in (45). As a result, ex-ante marginal products of capital are equalized between firms. This reallocation of capital leads to an increase in TFP compared to the competitive allocation. The difference between the overreacting market price and the efficient capital allocation is illustrated in Figure 9. Since the social planner is perfectly informed about aggregate shocks, the social planner does not increase investment during sentiment shocks. Second, the social planner chooses information production β_0^{SP} to increase TFP A_0^{SP} instead of trading rents.

6.4 Implementation

The social planner can achieve the constrained-efficient capital allocation by using a state- and price-dependent tax on dividends.

Proposition 9. *The dividend tax $\tau(z_{j0})$ with*

$$\Pi_{j1}^{DE} = \tau(z_{j0})\,\Pi_{j1}, \quad \text{where } \tau(z_{j0}) = \frac{\mathbb{E}\{A_{j0}|z_{j0}\}}{\tilde{\mathbb{E}}\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\}}. \tag{48}$$

implements the constrained efficient capital allocation (45).

As seen in Figure 9, $\tau(z_{j0})$ is a subsidy on dividends whenever $K_{j1}^{eff} < K_{j1}$. If the social planner has information about aggregate shocks, the tax/subsidy corrects also for aggregate sentiment shocks through the marginal trader's expectations $\tilde{\mathbb{E}}\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\}$. A tax (subsidy) can lower (increase) investment in response to a positive (negative) sentiment shock.

The social planner can similarly tax or subsidize information production if the level of information production is contractable. When this is not possible, the social planner can redistribute dividends to make investment more or less risky, affecting the incentives for information production. Intuitively, if the trade is riskier, the incentive to produce more information is also greater.

As an illustration, the following combination of a tax $\tau(a_{j0}, z_{j0})$ and a lump-sum transfer $T(a_{j0}, z_{j0})$ encourage information production, where I assume $a_0 = -\frac{\sigma_a^2}{2}$ as a normalization,

$$\tau(a_{j0}, z_{j0}) = \begin{cases} 0 & a_{j0} < \omega_a z_{j0} \\ 1 & a_{j0} \geq \omega_a z_{j0} \end{cases} \tag{49}$$

$$T(a_{j0}, z_{j0}) = \begin{cases} 0 & a_{j0} < \omega_a z_{j0} \\ \tilde{\mathbb{E}}\{\Pi_{j1}|a_{j0} < z_{j0}, s_{ij0} = z_{j0}, z_{j0}\} & a_{j0} \geq \omega_a z_{j0} \end{cases}, \tag{50}$$

where $\omega_a = \frac{\beta_0(1+\sigma_\varepsilon^{-2})}{\sigma_a^{-2}+\beta_0(1+\sigma_\varepsilon^{-2})}$ and the post-tax dividend payment is

$$\hat{\Pi}(a_{j0}, z_{j0}) = \tau(a_{j0}, z_{j0})\,\Pi_{j1} + T(a_{j0}, z_{j0}). \tag{51}$$

The tax is confiscatory if the realization of the productivity shock a_{j0} is below the mean expectation of the marginal trader $\omega_a z_{j0}$, i.e., the firm disappoints market expectations. The expected tax revenue from the perspective of the marginal trader is transferred to buyers if the realization of a_{j0} is above $\omega_a z_{j0}$, i.e., the firm exceeds market expectations. A tax schedule that incentivizes information production increases both the potential downsides and upsides

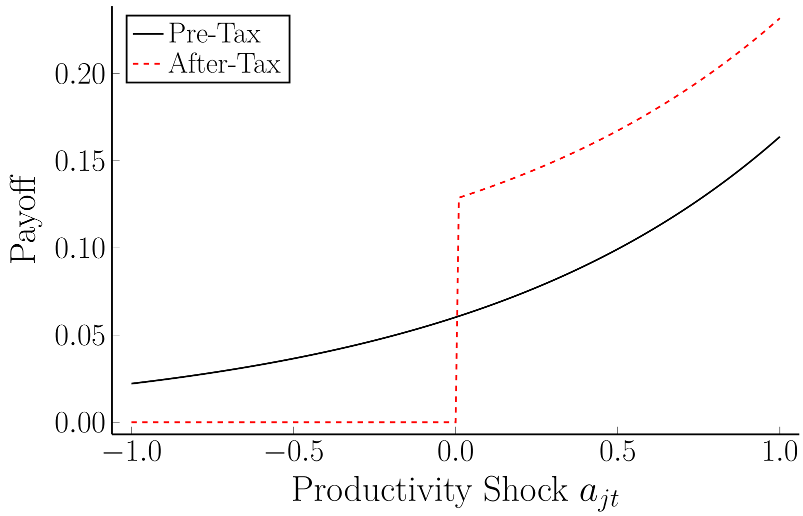


Figure 10: Information Production Incentivizing Tax Schedule.

of any trade. The before- and after-tax dividend schedule is shown in Figure 10 for the case with $z_{j0} = 0$.

While examples of tax schedules are rare that both amplify the down- and upsides of investments, tax schedules that discourage information production are the standard. In particular, a tax schedule that taxes gains and allows to carry losses forward flattens the payoff structure and discourages information production.

7 Discussion

This section explores several extensions: risk aversion, bank lending, asset purchases, and aggregate uncertainty.

7.1 Risk Aversion

Anticipated mispricings can affect the incentive to produce information through two channels in the presence of risk aversion and an exogenous dividend schedule.

First, as laid out in section 2, concave demand schedules can limit the sensitivity of the trader's demand to her private information. Whereas mean-variance preferences or CARA-utility lead to linear demand functions, preferences that put extra weight on tail-risk can recover such concave demand functions, e.g., mean-kurtosis preferences.

Second, risk aversion leads to decreasing marginal utility. At the same time, sentiment shocks lead to expected profits, driving down the utility enjoyed from additional profits. If information production costs are not in terms of future consumption, the incentive to produce precise information may decrease as the marginal utility decreases. This discussion is connected to Peress (2014), where the effect of an increase in wealth on the incentive to acquire information depends on the complementarity between leisure and consumption. For wealthy traders, leisure is valuable, decreasing the incentive to acquire additional information on their own time. However, the time cost of choosing asset managers might be small. In that case, the incentive for information acquisition through asset managers may even increase as the invested wealth increases (Mihet, 2020). In reality, wealthier households achieve higher returns on their investment (Xavier, 2021), pointing to more extensive information production in response to larger investments.

7.2 Bank Lending

The described channel is not limited to stock and debt markets but can be extended in a straightforward way to the case of bank lending. While financial markets are essential in financing real investment, bank lending dominates in many countries, especially for small firms. Therefore, the interaction of sentiment shocks and information production can have pervasive effects throughout the economy.

Banks are less incentivized to produce information if market-wide lending conditions are overly lax or strict. To make this point more formally, assume that banks decide whether to fund risky projects which pay $\Pi > 1$ with probability $\Phi(a) \in (0, 1)$ where $a \sim \mathcal{N}(\bar{a}, \sigma_a^2)$ and $\Phi(\cdot)$ is the standard-normal cdf. One unit of investment is necessary to fund the project. Banks offer a loan contract that specifies a repayment of $R_i > 1$ upon success and nothing in case of failure. Moreover, banks can costlessly verify success, and the entrepreneur cannot run away with the project's proceeds. Banks can inform their lending decision by hiring analysts that study the entrepreneur's business plan, which results in a signal $s_i = a + \frac{\eta_i}{\sqrt{\beta_i}}$ where $\eta_i \sim \mathcal{N}(0, 1)$ is noise and β_i is the information precision.

Importantly, banks face competition and are aware of the loan conditions other banks would offer. This leads to a situation where the bank cannot charge an interest rate higher than R. For instance, the entrepreneur might have a standing offer from a competitor bank with an interest rate of R or expect a higher value from negotiating with another bank rather than accepting any offer with $R_i > R$. Finally, banks believe that other banks' analysts are prone to sentiment and, therefore, misjudge project riskiness.[19] Without fully specifying how R is determined, assume that $\frac{1}{R} = \Phi(\frac{\beta}{c}\bar{a} + \frac{\sqrt{\beta}}{c}\bar{\varepsilon})$ where β is the information precision chosen by other banks and $c = \sqrt{(\sigma_a^{-2} + \beta)(1 + \sigma_a^{-2} + \beta)}$

[19] Ma, Paligorova, and Peydro (2021) find substantial disagreement among banks about macroeconomic conditions like the development of the house price index or unemployment rate. Such disagreement may lead to a situation in which banks would assess the riskiness of projects differently.

In this case, the bank will only decide to extend a loan at interest rate $R_i = R$ whenever $\mathbb{E}\{\Phi(a)|s_i\} > \frac{1}{R}$ leading to the probability of extending a loan $P(\mathbb{E}\{\Phi(a)|s_i\} > \frac{1}{R})$. Taking the derivative of the probability with respect to β_i and evaluating at the symmetric equilibrium $\beta_i = \beta$ leads to

$$\left.\frac{\partial P(\mathbb{E}\{\Phi(a)|s_i\} > \frac{1}{R})}{\partial \beta}\right|_{\beta=\beta'} \propto \exp\left\{-\frac{\bar{\varepsilon}}{2}\right\}, \tag{52}$$

which replicates the information-sensitivity channel from (37), showing that both positive and negative anticipated sentiment shocks discourage information production in this stylized bank lending problem.

7.3 Asset Purchases

During the last decade, central banks have repeatedly used asset purchases to stabilize financial markets and accelerate economic growth and price inflation (for a brief overview, see Gagnon and Sack, 2018). These interventions were accompanied by concerns that asset purchases might harm market efficiency and increase capital misallocation.[20] Although the presented model is too stylized to give a full assessment of asset purchases, it can be used to shed light on the effect of asset purchases on information production in financial markets.[21]

In the model, asset purchases have real effects by exploiting that information is dispersed among traders. Intuitively, since each trader can only buy a limited number of shares, a reduction in the number of shares in the hands of traders must lead to a concentration of stock ownership among optimists. Moreover, announced asset purchases affect information production. Traders anticipate that the reduction in asset supply distorts prices upward, discouraging information production like a positive sentiment shock. This effect provides a rationale for the concerns about asset purchases and declines in market efficiency.

However, asset purchases can also reduce distortions in financial markets, for example, through negative sentiment shocks. When a sufficiently large negative sentiment shock hits the economy, traders anticipate that prices will be depressed, discouraging information production as trading becomes less information-sensitive. The central bank can offset the downward bias on asset prices by purchasing assets. Remarkably, asset purchases restore asset prices to unbiased levels *and* the incentives to produce information. This result is captured

[20]See da Silva and Rungcharoenkitkul (2017), DNB (2017), Fernandez, Bortz, and Zeolla (2018), Borio and Zabai (2018), Acharya et al. (2019), and Kurtzman and Zeke (2020). In particular, the Dutch central bank argues in their 2016 annual report (DNB, 2017): *"The large-scale purchase programmes and the flood of liquid assets has set the risk compass in financial markets spinning, with misallocations as a result."*

[21]While most central banks focused on buying government bonds, purchases of corporate bonds were also common (Gagnon and Sack, 2018). The presented analysis directly carries over to this case as firms can equally issue debt instead of equity. Moreover, the Bank of Japan bought shares directly in stock ETFs (Okimoto, 2019), which maps directly to asset purchases in this model.

in the following Proposition.

Proposition 10. *Let the social planner acquire $d^{SP} \in (-1,1)$ units of assets, such that $1-d^{SP}$ shares are left for traders. Then, asset purchases $d^{SP} = 2\Phi(-\varepsilon_0)-1$ undo sentiment shocks both in terms of investment and information production.*

To understand the effectiveness of asset purchases, use an investment subsidy as a comparison. When facing a negative sentiment shock, the government can subsidize investment and restore unbiased asset prices. However, traders still think that all other traders are depressed, making trading less information-sensitive. Instead, asset purchases reduce the asset float and effectively price out the traders with the most negative beliefs. Now, buying traders have no longer depressed beliefs on average, and trading has become more information sensitive again.

In other words, asset purchases and sales can *increase* market efficiency by countering sentiment shocks. This finding is relevant for central banks in deciding when to start shrinking the size of their balance sheets. Central banks can avoid the adverse effects of asset sales by waiting until sentiment has reached a more neutral level.

7.4 Uncertainty

Trading under Uncertainty The analysis so far assumed that traders observed aggregate states perfectly before deciding on information precision. This assumption is not crucial for the results, which also holds when traders only have imperfect information about aggregate states before making their information production decision. Nonetheless, traders or policymakers do not have perfect knowledge about the current aggregate state in reality

To make the role of imperfect information about the aggregate shock clear, it is useful to decompose sentiment into two components, such that total sentiment is $\varepsilon_{jt} + \bar{\varepsilon}_t$ where $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\bar{\varepsilon}_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\bar{\varepsilon}}^2)$. If traders do not receive additional information about the aggregate sentiment shock $\bar{\varepsilon}$, then these do not affect information production and only move aggregate investment. Assume instead that traders do receive a signal of the form

$$s_t = \bar{\varepsilon}_t + \xi_t \tag{53}$$

where $\xi_t \sim \mathcal{N}(0, \sigma_\xi^2)$. The resulting expectation about total sentiment is

$$\varepsilon_{jt} + \bar{\varepsilon}_t | s_t \sim \mathcal{N}\left(\frac{\sigma_\xi^{-2}}{\sigma_{\bar{\varepsilon}}^{-2} + \sigma_\xi^{-2}} s_t, \sigma_\varepsilon^2 + \frac{1}{\sigma_{\bar{\varepsilon}}^{-2} + \sigma_\xi^{-2}}\right) \tag{54}$$

In the case with only one asset, the case with uncertainty about $\bar{\varepsilon}_t$ is isomorphic to case with perfect knowledge about $\bar{\varepsilon}_t$ in which the variance of sentiment shocks is larger and the mean sentiment shock is replaced with $\frac{\sigma_\xi^{-2}}{\sigma_{\bar{\varepsilon}}^{-2}+\sigma_\xi^{-2}} s_t$. Note that the realization of the sentiment shock $\bar{\varepsilon}_t$ is unimportant, and instead, the *expectations* about sentiment drive the main result.

Policy under Uncertainty The policy analysis is not substantially changed under aggregate uncertainty. Indeed, negative effects of sentiment shocks on information production can be offset by conditioning on the *expectations* of aggregate sentiment. The social planner can collect information about traders' expectations through surveys to elicit s_t. This information can be used to guide asset purchases to offset the effect of anticipated sentiment shocks on information production.

If the policymaker does not observe s_t, then multiple indicators can be used by the social planner to identify whether a boom is driven by sentiment or productivity. A sentiment-driven boom crowds out information production and decreases the variance of prices, leading all firms to look more alike. In contrast, a productivity-driven boom crowds in information, leading to more dispersion in asset prices and firm capital. The same applies to the dispersion of returns, which was a common measure of information in financial markets Roll (1984) and Morck, Yeung, and Yu (2000, 2013).

If asset prices increase across the board and the dispersion in asset prices or returns between firms shrinks, the social planner wants to lower investment and increase information production. In contrast, if there are winners and losers, even as asset prices are booming, price discovery still occurs, and traders are producing information. Using dispersion in asset prices and returns is attractive as asset prices are available continuously and can potentially inform the social planner in real time. This result is captured in the following Proposition.

Proposition 11. *For $\sigma_\varepsilon^2 \leq 1$,*

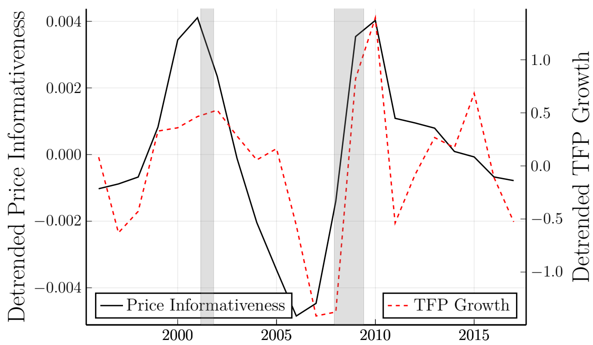
(i) the cross-sectional variance in asset prices is increasing in β_t.

(ii) the cross-sectional variance in asset price returns is increasing in β_t.

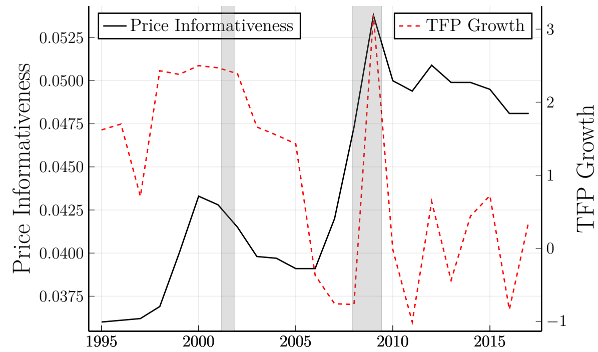
Note that these measures cannot be used to distinguish fundamental recessions from depressions driven by sentiment, as both crowd out information and lead to a decrease of dispersion in asset prices and returns.

8 Price Informativeness in the Data

There are several approaches to estimating the information that asset prices convey. Early approaches such as Roll (1988) and Morck, Yeung, and Yu (2000, 2013) employ *return non-synchronicity* to measure new firm-specific information being incorporated in asset prices. Hou, Peng, and Xiong (2013) casts doubt on this measure as it lacks a direct structural



(a) Detrended



(b) Non-Detrended

Figure 11: Price Informativeness and TFP Growth.

Notes: Relative price informativeness (black) as measured in Dávila and Parlatore (2022) and utilization-adjusted TFP growth (red) taken from the Federal Reserve of San Francisco following Basu, Fernald, and Kimball (2006). Grey bars indicate NBER recession dates. The time series have been detrended using a cubic time trend and smoothed with a two-year moving average in the left panel.

mapping to a model and several empirical studies fail to link it to other measures of price informativeness. Bai, Philippon, and Savov (2016) use a cross-sectional approach to the ability of asset prices to forecast earnings but also does not provide an identification result. In this context, Dávila and Parlatore (2022) provide an approach to estimate an identified firm-level measure of *relative price informativeness.* Their measure captures the Kalman-gain that an uninformed trader applies to the information contained in the asset price, which corresponds exactly to $\frac{\beta_{jt}}{\sigma_a^{-2}+\beta_{jt}}$ in this model.

I use the measure of relative price informativeness from Dávila and Parlatore (2022) in combination with a measure of TFP growth for the US economy following Basu, Fernald, and Kimball (2006) to study the two most-recent booms in US history: the dot-com boom of the late 1990s and the housing boom leading up to the great financial crisis of 2008-2009. Since the model focuses on business cycles instead of long-run developments, both time series are detrended using a cubic time trend between 1995 and 2017 and smoothed with a two-year moving average. The resulting time series are shown in Figure 11a, whereas the raw time series are in Figure 11b. Both graphs have gray bars that indicate NBER recession dates.

The first observation is that relative price informativeness exhibits variation at business cycle frequency. A rapid increase between 1995 and 2000 is followed by a decline until 2006 when the housing bubble peaked, and a rapid further increase until 2009. At the same time, relative price informativeness is not just anti- or procyclical but instead increased during the dot-com boom and declined during the housing boom. Through the lens of the model, this suggests that the dotcom boom was driven by expectations of higher productivity, whereas the US housing boom was predominantly driven by exuberant sentiment (see also Borio et al., 2015; Doerr, 2018). The different growth rates further support this observation in TFP, which was accelerating during the dot-com boom and decreasing rapidly during the housing boom.

9 Conclusion

I developed a tractable framework to study information production in financial markets embedded in a standard macroeconomic model. In such a model, total factor productivity has an endogenous component that depends on the traders' decentralized information production. When asset prices are more informative, more capital is allocated to the most productive firms and total factor productivity increases. I add to the literature by studying the effect of aggregate shocks on information production.

I prove that sentiment shocks, defined as waves of non-fundamental optimism or pessimism, crowd out information production as trading becomes less information-sensitive. Although such optimism increases investment, it also worsens the allocation of capital. This result rationalizes the empirical finding that credit booms often worsen aggregate productivity (Borio et al., 2015; Gopinath et al., 2017; Doerr, 2018; Gorton and Ordoñez, 2020) through a novel information mechanism. In contrast, expectations of heightened productivity crowd in information, thereby improving capital allocation and aggregate productivity beyond the initial shock. This dichotomy mirrors the "good" and "bad" booms of Gorton and Ordoñez (2020). The model suggests that "good" booms are driven by productivity, whereas "bad" booms are driven by sentiment.

From a normative perspective, I show that information production is too high or too low in the competitive equilibrium. There are two externalities with opposing effects. On the one hand, traders produce information to increase trading rents at the expense of other traders. This rent-extracting behavior can lead to excessive information production. On the other hand, traders do not reap the benefits of improving the capital allocation through *collective* information production. This information spillover can lead to information production being too low. Generally, information production is too low in the competitive equilibrium exactly when the allocation of capital matters the most and, hence, information is most valuable.

Finally, I apply the model to evaluate the effect of large-scale asset purchase programs. I show that asset purchases can discourage information production. This finding confirms the concerns of policymakers about such programs (e.g., DNB, 2017). However, asset purchases can also improve capital allocation by leaning against sentiment. Therefore, policymakers need to know which force is currently driving the cycle to react appropriately. The model suggests booms with low return dispersion are likely driven by sentiment. Using the measure of relative price informativeness of Dávila and Parlatore (2022) suggests that the US dot-com boom was driven by expectations of high productivity. In contrast, the US housing boom was driven by exuberant sentiment.

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A Proofs and Derivations

A.1 Derivations

Every household i consists of many traders indexed by $ij \in [0,1]$. The information set of each trader consists of $\{s_{ijt}, \{z_{jt}\}, a_t, \varepsilon_t\}$, i.e., traders observe their private signal, all public signals and the aggregate states. This setting allows that traders have rational expectations about aggregates, but still disagree about firm-specific variables, which motivates trade. I impose that $\kappa_H = 2$ and $\kappa_L = 0$ to avoid distortions in asset prices that stem from the choice of position limits.

The beliefs of traders about firm productivity A_{jt} are relevant for their trading decision. Trader ij's beliefs are given by

$$\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}, z_{jt}\right\} = \exp\left\{\omega_{p,ijt}a_t + \omega_{s,ijt}s_{ijt} + \omega_{z,ijt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{ijt}\right\}. \tag{55}$$

Similarly, the beliefs of the marginal trader are

$$\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \exp\left\{\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right) + \frac{1}{2}\mathbb{V}_{jt}\right\}, \tag{56}$$

where $a_{jt} \overset{iid}{\sim} \mathcal{N}\left(a_t, \sigma_a^2\right)$, $\varepsilon_{jt} \overset{iid}{\sim} \mathcal{N}\left(\varepsilon_t, \sigma_\varepsilon^2\right)$ and ω-terms are the corresponding Bayesian weights,

$$\omega_{z,ijt} = \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{z,jt} = \frac{\beta_{jt}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \tag{57}$$

$$\omega_{s,ijt} = \frac{\beta_{ijt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{s,jt} = \frac{\beta_{jt}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}} \tag{58}$$

$$\omega_{p,ijt} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \omega_{p,jt} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \tag{59}$$

and $\{\mathbb{V}_{jt}, \mathbb{V}_{ijt}\}$ stand for posterior uncertainty

$$\mathbb{V}_{ijt} = \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt}\sigma_\varepsilon^{-2}}, \quad \mathbb{V}_{jt} = \frac{1}{\sigma_a^{-2} + \beta_{jt} + \beta_{jt}\sigma_\varepsilon^{-2}}. \tag{60}$$

The private information precision β_{ijt} is highlighted in blue and is part of the information production decision. Alternatively, the beliefs of the marginal trader who observed $s_{ijt} = z_{jt}$ can be expressed as a function of shocks,

$$\ln\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} = \omega_{p,jt}a_t + \omega_{s\varepsilon,jt}\varepsilon_t + \omega_{a,jt}a_{jt} + \frac{\omega_{a,jt}}{\sqrt{\beta_{jt}}}\left(\varepsilon_{jt} - \varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{jt}, \tag{61}$$

where the corresponding Bayesian weights are

$$\omega_{a,jt} = \omega_{z,jt} + \omega_{s,jt} \tag{62}$$
$$\omega_{\varepsilon,jt} = \omega_{a,jt}/\sqrt{\beta_{jt}} \tag{63}$$
$$\omega_{s\varepsilon,jt} = \omega_{s,jt}/\sqrt{\beta_{jt}}. \tag{64}$$

Trader ij buys shares of firm j whenever

$$\tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt}, z_{jt}\right\} \geq \tilde{\mathbb{E}}\left\{\Pi_{jt+1}|s_{ijt} = z_{jt}, z_{jt}\right\} \tag{65}$$
$$\iff \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}, z_{jt}\right\} \geq \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\}. \tag{66}$$

The inequality can be expressed as a cutoff for the idiosyncratic noise,

$$\begin{aligned}\eta_{ijt} \geq{}& \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t + \omega_{z,ijt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{ijt}}{2}\right) + \sqrt{\beta_{ijt}}a_{jt} \\ &- \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{\mathbb{V}_{jt}}{2}\right).\end{aligned} \tag{67}$$

Since η_{ijt} is standard-normal, the perceived probability of buying is

$$\begin{aligned}\mathcal{P}\left\{x_{ijt} = 2\right\} = \Phi\Bigg[&-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t + \omega_{z,ijt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{ijt}\right) + \sqrt{\beta_{ijt}}a_{jt} \\ &- \frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t + \omega_{s,jt}z_{jt} + \omega_{z,jt}\left(z_{jt} - \frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right) + \frac{1}{2}\mathbb{V}_{jt}\right)\Bigg],\end{aligned} \tag{68}$$

where $\Phi(\cdot)$ is the standard-normal cdf. For a symmetric information choice ($\beta_{ijt} = \beta_{jt}$), the buying probability can be simplified to

$$\mathcal{P}\left\{x_{ijt} = 2|a_{jt}, \varepsilon_{jt}, \beta_{ijt}, \beta_{jt}\right\}\big|_{\beta_{ijt}=\beta_{jt}} = \Phi\left(-\varepsilon_{jt}\right). \tag{69}$$

Traders think they are more likely to buy shares when the realization of the sentiment shock is relatively low, and shares are therefore cheap relative to their fundamental value.

Finally, traders choose their information precision, taking all other traders' symmetric choice as given. The derivative of the probability of buying with respect to β_{ijt} is

$$\begin{aligned}\frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}} &= \phi\left[-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,ijt}a_t+\omega_{z,ijt}\left(z_{jt}-\frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{ijt}\right)+\sqrt{\beta_{ijt}}a_{jt}\right.\\ &\quad\left.-\frac{\sqrt{\beta_{ijt}}}{\omega_{s,ijt}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{1}{\sqrt{\beta_{jt}}}\varepsilon_t\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right]\\ &\quad*\left[-\frac{1}{2\beta_{ijt}^{3/2}}\left(\sigma_a^{-2}a_t+\beta_{jt}\sigma_\varepsilon^{-2}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\right)+\frac{a_{jt}}{2\sqrt{\beta_{ijt}}}\right.\\ &\quad\left.-\left(\frac{1}{\sqrt{\beta_{ijt}}}-\frac{1}{2\beta_{ijt}^{3/2}}\left(\mathbb{V}_{ijt}\right)^{-1}\right)*\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right] \quad (70)\end{aligned}$$

where $\phi(\cdot)$ is the standard normal pdf. For a symmetric information choice ($\beta_{ijt}=\beta_{jt}$) this expression can be simplified to

$$\begin{aligned}&\left.\frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}} =\\ &\phi\left(\varepsilon_{jt}\right)\left[\frac{1}{2\sqrt{\beta}_{jt}}\left(a_{jt}+z_{jt}\right)-\frac{1}{\sqrt{\beta_{jt}}}\left(\omega_{p,jt}a_t+\omega_{s,jt}z_{jt}+\omega_{z,jt}\left(z_{jt}-\frac{\varepsilon_t}{\sqrt{\beta_{jt}}}\right)+\frac{1}{2}\mathbb{V}_{jt}\right)\right]. \quad (71)\end{aligned}$$

A.2 Auxiliary Results

Lemma 1 (Auxiliary Results Market Allocation)**.** *Denote the Bayesian weights*

$$\omega_a=\frac{\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)}, \quad (72)$$

$$\omega_\varepsilon=\frac{\sqrt{\beta_{jt}}\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_{jt}\left(1+\sigma_\varepsilon^{-2}\right)} \quad (73)$$

$$\omega_{z\varepsilon}=\frac{\sqrt{\beta_{jt}}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\sqrt{\beta_{jt}}\left(1+\sigma_\varepsilon^{-2}\right)}, \quad (74)$$

and posterior uncertainty

$$\mathbb{V}=\frac{1}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)} \quad (75)$$

then

(i) $\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}+\mathbb{V}=\sigma_a^2$,

(ii) $\omega_a^2\sigma_a^2+\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}=\sigma_a^2-\mathbb{V}=\omega_a\sigma_a^2$,

(iii) $\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2}=\omega_{z\varepsilon}$.

Proof. (i)

$$
\begin{aligned}
\omega_a^2\sigma_a^2 + \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} + \mathbb{V} &= \frac{\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} + \frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&+ \frac{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-2}+2\beta\left(1+\sigma_\varepsilon^{-2}\right)+\sigma_a^2\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\sigma_a^{-4}+2\sigma_a^{-2}\beta\left(1+\sigma_\varepsilon^{-2}\right)+\beta^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \frac{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}{\left(\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2 \\
&= \sigma_a^2
\end{aligned}
\tag{76}
$$

(ii) The first equality follows from (i). Then

$$
\begin{aligned}
\sigma_a^2 - \mathbb{V} &= \sigma_a^2 - \frac{1}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\sigma_a^2\sigma_a^{-2}+\sigma_a^2\beta\left(1+\sigma_\varepsilon^{-2}\right)-1}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)} \\
&= \frac{\beta\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta\left(1+\sigma_\varepsilon^{-2}\right)}\sigma_a^2 \\
&= \omega_a\sigma_a^2.
\end{aligned}
\tag{77}
$$

(iii)

$$
\frac{\omega_\varepsilon}{1+\sigma_\varepsilon^2} = \frac{\omega_\varepsilon}{\sigma_\varepsilon^2\left(1+\sigma_\varepsilon^{-2}\right)} = \frac{\omega_\varepsilon\sigma_\varepsilon^{-2}}{\left(1+\sigma_\varepsilon^{-2}\right)} = \omega_{z\varepsilon}. \tag{78}
$$

□

Lemma 2. *Denote the efficient Bayesian weights*

$$
\omega_a^{eff} = \frac{\beta_t\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}} \tag{79}
$$

$$
\omega_\varepsilon^{eff} = \frac{\sqrt{\beta_t}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}} \tag{80}
$$

and posterior uncertainty

$$
\mathbb{V}^{eff} = \frac{1}{\sigma_a^{-2}+\beta_t\sigma_\varepsilon^{-2}}. \tag{81}
$$

These are the weights that a rational uninformed observer would use after observing the price *signal z_{jt}, in contrast to the Bayesian weights that traders that suffer from correlation neglect use as introduced in section A.1. Then the following relationships hold:*

(i)

$$\underbrace{\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2}_{=Var(\mathbb{E}\{a_{jt}|z_{jt}\})} + \underbrace{\mathbb{V}^{eff}}_{Var(a_{jt}|z_{jt})} = \underbrace{\sigma_a^2}_{Var(a_{jt})} . \tag{82}$$

(ii)

$$\sigma_a^2 - \mathbb{V}^{eff} = \omega_a^{eff} \sigma_a^2 \tag{83}$$

(iii)

$$\theta \omega_a^{eff} \sigma_a^2 + \mathbb{V}^{eff} = (\theta - 1)\, \omega_a^{eff} \sigma_a^2 + \sigma_a^2 \tag{84}$$

Proof. (i):

$$\begin{aligned}
\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \mathbb{V}^{eff} &= \frac{\beta_t^2 \sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2} \sigma_a^2 + \frac{\beta_t \sigma_\varepsilon^{-4}}{\left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2} \sigma_\varepsilon^2 \\
&\quad + \frac{1}{\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}} \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + 2\beta_t \sigma_\varepsilon^{-2} + \beta_t^2 \sigma_\varepsilon^{-4} \sigma_a^2\right) \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-4} + 2\beta_t \sigma_\varepsilon^{-2} \sigma_a^{-2} + \beta_t^2 \sigma_\varepsilon^{-4}\right) \sigma_a^2 \\
&= \left(\mathbb{V}^{eff}\right)^2 \left(\sigma_a^{-2} + \beta_t \sigma_\varepsilon^{-2}\right)^2 \sigma_a^2 \\
&= \sigma_a^2.
\end{aligned} \tag{85}$$

(ii):

$$\begin{aligned}
\sigma_a^2 - \mathbb{V}^{eff} &= \sigma_a^2 - \frac{1}{\sigma_a^2 + \beta \sigma_\varepsilon^{-2}} \\
&= \frac{\sigma_a^2 \sigma_a^{-2} + \sigma_a^2 \beta \sigma_\varepsilon^{-2} - 1}{\sigma_a^{-2} + \beta \sigma_\varepsilon^{-2}} \\
&= \frac{\beta \sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta \sigma_\varepsilon^{-2}} \sigma_a^2 \\
&= \omega_a^{eff} \sigma_a^2
\end{aligned} \tag{86}$$

(iii):

$$\begin{aligned}
\theta\omega_a^{eff}\sigma_a^2+\mathbb{V}^{eff} &= \frac{\theta\beta_{t-1}\sigma_\varepsilon^{-2}\sigma_a^2}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}+\frac{1}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}} \\
&= \frac{\theta\beta_{t-1}\sigma_\varepsilon^{-2}+\sigma_a^{-2}}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}\sigma_a^2 \\
\text{Add and subtract} &= \frac{\theta\beta_{t-1}\sigma_\varepsilon^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}-\beta_{t-1}\sigma_\varepsilon^{-2}+\sigma_a^{-2}}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}\sigma_a^2 \\
\text{Split} &= (\theta-1)\frac{\beta_{t-1}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}\sigma_a^2+\frac{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}{\sigma_a^{-2}+\beta_{t-1}\sigma_\varepsilon^{-2}}\sigma_a^2 \\
&= (\theta-1)\,\omega_a^{eff}\sigma_a^2+\sigma_a^2 \qquad (87)
\end{aligned}$$

□

Lemma 3 (Joining two Normal PDFs). *Let* $f(\varepsilon_{jt})$ *be the pdf of* $\varepsilon_{jt}\sim\mathcal{N}(\varepsilon_t,\sigma_\varepsilon^2)$ *and* $\phi(\cdot)$ *the standard-normal pdf. Then*

$$f(\varepsilon_{jt})\,\phi(\varepsilon_{jt})=\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}\sqrt{\frac{1}{2\pi(1+\sigma_\varepsilon^2)}}\tilde{f}(\varepsilon_{jt}) \qquad (88)$$

where $\tilde{f}(\varepsilon_{jt})$ *is the transformed pdf of* $\varepsilon_{jt}\sim\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2},\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$.

Proof. Write out the pdfs explicitly,

$$\phi(\varepsilon_{jt})=\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}\right\} \qquad (89)$$

$$f(\varepsilon_{jt})=\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\exp\left\{-\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{2\sigma_\varepsilon^2}\right\} \qquad (90)$$

$$f(\varepsilon_{jt})\,\phi(\varepsilon_{jt})=\frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}^2}{2}-\frac{(\varepsilon_{jt}-\varepsilon_t)^2}{2\sigma_\varepsilon^2}\right\}. \qquad (91)$$

Rearranging the term inside the exponential function,

$$
\begin{aligned}
\frac{\left(\varepsilon_{jt}-\varepsilon_t\right)^2}{\sigma_\varepsilon^2}+\varepsilon_{jt}^2 &= \frac{\varepsilon_{jt}^2-2\varepsilon_{jt}\varepsilon_t+\varepsilon_t^2}{\sigma_\varepsilon^2}+\varepsilon_{jt}^2 \\
\text{join fractions} &= \frac{\left(1+\sigma_\varepsilon^2\right)\varepsilon_{jt}^2-2\varepsilon_t\varepsilon_{jt}+\varepsilon_t^2}{\sigma_\varepsilon^2} \\
\text{divide by } \left(1+\sigma_\varepsilon^2\right) &= \frac{\varepsilon_{jt}^2-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} \\
\text{add and subtract} &= \frac{\varepsilon_{jt}^2-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} \\
\text{exchange terms} &= \frac{\varepsilon_{jt}^2-2\varepsilon_{jt}\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}+\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} \\
\text{binomial} &= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}-\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} \\
&= \frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}+\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2} \qquad (92)
\end{aligned}
$$

This allows to write

$$
\begin{aligned}
f\left(\varepsilon_{jt}\right)\phi\left(\varepsilon_{jt}\right) &= \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{\varepsilon_{jt}}{2}-\frac{\left(\varepsilon_{jt}-\varepsilon_t\right)^2}{2\sigma_\varepsilon^2}\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\}\exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \frac{1}{\sqrt{\sigma_\varepsilon^2}}\sqrt{\frac{\sigma_\varepsilon^2}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\frac{1}{\sqrt{2\pi\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}\exp\left\{-\frac{1}{2}\frac{\left(\varepsilon_{jt}-\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}\right)^2}{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\right\} \\
&* \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_t^2}{1+\sigma_\varepsilon^2}\right)\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}), \qquad (93)
\end{aligned}
$$

where $\tilde{f}\left(\varepsilon_{jt}\right)$ is the transformed pdf of $\varepsilon_{jt}\sim\mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2},\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. □

A.3 Main Proofs

Proof of Proposition 1. This proof follows the same steps as the proof for Proposition 1 in Albagli, Hellwig, and Tsyvinski (2017), since the financial market in this model is isomorphic to their model. Their proof is repeated here for completeness. The only difference is that K_{jt+1} depends on the price signal z_{jt}, whereas k in Albagli, Hellwig, and Tsyvinski (2017) is determined before trading takes place. Therefore, it is necessary to assume that $K_{jt+1}(z_{jt})$ is non-decreasing in z_{jt} as the price might otherwise be not invertible, which is confirmed ex-post. The proof begins in the following.

There must be a threshold $\hat{s}(P_{jt})$ such that all households with $s_{ijt} \geq \hat{s}(P_{jt})$ find it profitable to buy two units of share j and otherwise abstain from trading. It follows that the price must be equal to the valuation of the trader who is merely indifferent between buying and not buying,

$$P_{jt} = \frac{1}{R_{t+1}} \tilde{\mathbb{E}} \left\{ \Pi \left(A_{jt}, K_{jt+1} \right) | s_{ijt} = \hat{s}(P_{jt}), P_{jt} \right\}. \tag{94}$$

This monotone demand schedule leads to total demand

$$D(\theta, \varepsilon, P) = 2 \left(1 - \Phi \left(\sqrt{\beta_{jt}} \left(\hat{s}(P_{jt}) - a_{jt} \right) - \varepsilon_{jt} \right) \right). \tag{95}$$

Equalizing total demand with a normalized supply of one leads to the market-clearing condition

$$2 \left(1 - \Phi \left(\sqrt{\beta_{jt}} \left(\hat{s}(P_{jt}) - a_{jt} \right) - \varepsilon_{jt} \right) \right) = 1, \tag{96}$$

with the unique solution $\hat{s}(P_{jt}) = z_{jt} = a_{jt} + \frac{\varepsilon_{jt}}{\sqrt{\beta_{jt}}}$. If P_{jt} is pinned down by z_{jt}, then P_{jt} is invertible, given that K_{jt+1} is non-decreasing in z_{jt}. It follows, then, that observing P_{jt} is equivalent to observing $z_{jt} \sim \mathcal{N}\left(a_{jt}, \beta_{jt}^{-1} \sigma_{\varepsilon}^2\right)$. Traders treat the signal z_{jt} and their private signal $s_{ijt} \sim \mathcal{N}\left(a_{jt}, \beta_{ijt}^{-1}\right)$ as mutually independent. Using this result, the price can be restated as

$$P\left(z_{jt}, K_{jt+1}\right) = \frac{1}{R_{t+1}} \tilde{\mathbb{E}} \left\{ \Pi \left(A_{jt}, K_{jt+1} \right) | s_{ijt} = z_{jt}, z_{jt} \right\}, \tag{97}$$

where posterior expectations of trader ij are given by

$$a_{jt} | s_{ijt}, z_{jt} \sim \mathcal{N} \left(\frac{\sigma_a^{-2} a_t + \beta_{ijt} s_{ijt} + \beta_{jt} \sigma_{\varepsilon}^{-2} z_{jt}}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt} \sigma_{\varepsilon}^{-2}}, \frac{1}{\sigma_a^{-2} + \beta_{ijt} + \beta_{jt} \sigma_{\varepsilon}^{-2}} \right). \tag{98}$$

Using the result that the firm invests all proceeds into capital ($K_{jt+1} = P_{jt}$), it follows indeed that K_{jt+1} is non-decreasing in z_{jt} and P_{jt} is an invertible function of the price signal z_{jt}. It remains to show the uniqueness of the above solution. Begin with the assumption that demand $x(s_{ijt}, P_{jt})$ is non-increasing in P_{jt}. It follows that $\hat{s}(P_{jt})$ is non-decreasing in P_{jt}.

There are two cases to differentiate. First, if $\hat{s}(P_{jt})$ is strictly increasing in P_{jt}, then the price is indeed uniquely pinned-down by z_{jt} and invertible; it can be expressed like above. Secondly, assume that the threshold is flat over some interval, such that $\hat{s}(P_{jt}) = \hat{s}$ over some interval $P_{jt} \in (P', P'')$ for $P' \neq P''$. Furthermore, choose $\epsilon > 0$ small enough such that $\hat{s}(P_{jt})$ is increasing to the left and the right of the interval, i.e., over $P_{jt} \in (P' - \epsilon, P')$ and $P_{jt} \in (P'', P'' + \epsilon)$. In these regions, $\hat{s}(P_{jt})$ is monotonically increasing in P_{jt}, which is uniquely pinned down by z_{jt} and invertible; observing the price is equivalent to observing the signal z_{jt}. In this case the price can be expressed as before for $z_{jt} \in (\hat{s}(P' - \varepsilon), \hat{s})$ and $z_{jt} \in (\hat{s}, \hat{s}(P'' + \epsilon))$. This leads to a contradiction in the assumption that $P' \neq P''$, because $P(z_{jt}, K_{jt+1})$ is both continuous and monotonically increasing in z_{jt}. Therefore, $\hat{s}(P_{jt})$ cannot be flat, and the above solution is indeed unique. $\square$

Proof of Proposition 2. (i) Using (7) in (26) leads to the expression for firm capital

$$K_{jt+1} = \left(\frac{\alpha Y_{t+1}^{\alpha_Y}}{R_{t+1}} \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\} \right)^{\theta}. \tag{99}$$

Plugging R_{t+1} from (27) into (26) using (99) leads to

$$K_{jt+1} = \frac{\tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^{\theta}}{\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt} | s_{ijt} = z_{jt}, z_{jt} \right\}^{\theta} dj} K_{t+1}, \tag{100}$$

which finishes the proof.

(ii) Plugging the above expression for firm capital (29) into the aggregate production function (5) leads to

$$\begin{aligned}
Y_t &= \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \left(\int_0^1 A_{jt-1} K_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} \\
&= \frac{\left(\int_0^1 A_{jt-1} \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta-1} dj \right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}} \left\{ A_{jt-1} | s_{ijt-1} = z_{jt-1}, z_{jt-1} \right\}^{\theta} dj \right)^{\alpha}} K_t^{\alpha} \\
&= A_{t-1} L^{1-\alpha} K_t^{\alpha}
\end{aligned} \tag{101}$$

where total factor productivity is

$$
\begin{aligned}
A_{t-1} &= \frac{\left(\int_0^1 A_{jt-1}\tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1}=z_{jt-1},z_{jt-1}\right\}^{\theta-1}dj\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt-1}|s_{ijt-1}=z_{jt-1},z_{jt-1}\right\}^{\theta}dj\right)^{\alpha}} \\
&= \frac{\exp\left\{\theta a_{t-1}+\left(\left(\theta-1\right)\omega_a+1\right)^2\frac{\sigma_a^2}{2}+\left(\theta-1\right)^2\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}+\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_{t-1}+\frac{(\theta-1)}{2}\mathbb{V}\right\}^{\frac{\alpha\theta}{\theta-1}}}{\exp\left\{\theta a_{t-1}+\theta^2\omega_a^2\frac{\sigma_a^2}{2}+\theta^2\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}+\theta\omega_{s\varepsilon}\varepsilon_{t-1}+\frac{\theta}{2}\mathbb{V}\right\}^{\alpha}} \\
&= \exp\left(\frac{\alpha\theta}{\theta-1}a_{t-1}+\left(\frac{\alpha\theta}{\theta-1}\left(\left(\theta-1\right)\omega_a+1\right)^2-\alpha\theta^2\omega_a^2\right)\frac{\sigma_a^2}{2}+\left(\alpha\theta\left(\theta-1\right)-\alpha\theta^2\right)\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{\alpha\theta}{\theta-1}a_{t-1}+\alpha\theta\left(\left(\theta-1\right)\omega_a^2+2\omega_a+\frac{1}{\theta-1}-\theta\omega_a^2\right)\frac{\sigma_a^2}{2}-\alpha\theta\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}\right) \\
&= \exp\left(\frac{1}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right)+\omega_a\left(2-\omega_a\right)\frac{\sigma_a^2}{2}-\omega_\varepsilon^2\frac{\sigma_\varepsilon^2}{2}\right)^{\alpha\theta}.
\end{aligned}
\tag{102}
$$

The weights $\{\omega_a,\omega_\varepsilon,\omega_{s\varepsilon}\}$ and $\mathbb{V}$ are derived in section A.1. Finally, total factor productivity can be expressed as

$$
\ln A_{t-1}\left(a_{t-1},\beta_{t-1}\right)=\underbrace{\frac{\alpha\theta}{\theta-1}\left(a_{t-1}+\frac{\sigma_a^2}{2}\right)}_{\text{exogenous}}+\underbrace{\kappa^a\left(\beta_{t-1}\right)\sigma_a^2-\kappa^\varepsilon\left(\beta_{t-1}\right)\sigma_\varepsilon^2}_{\text{allocative efficiency}}
\tag{103}
$$

where $\kappa^a\left(\beta_{t-1}\right)=\frac{\alpha\theta}{2}\omega_a\left(2-\omega_a\right)$ and $\kappa^\varepsilon\left(\beta_{t-1}\right)=\frac{\alpha\theta}{2}\omega_\varepsilon^2$.

(iii) I will show that the allocative efficiency component of TFP takes its minimum for $\beta_{t-1}>0$ if $\sigma_\varepsilon^2>1$. The allocational efficiency component is proportional to

$$
\begin{aligned}
&\omega_a\left(2-\omega_a\right)\sigma_a^2-\omega_\varepsilon^2\sigma_\varepsilon^2 \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)}{\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)}\sigma_a^2-\frac{\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_a^2-\frac{\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\sigma_\varepsilon^2 \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)+2\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2-\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2-\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{2\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)+\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2-\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2} \\
&= \frac{\beta_{t-1}^2\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^2+\beta_{t-1}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2},
\end{aligned}
\tag{104}
$$

which is weakly positive for all values of β_{t-1} if $\sigma_\varepsilon^2<1$. Since the allocative efficiency component is zero for $\beta_{t-1}=0$, it must be that the minimum is attained for some $\beta_{t-1}>0$ when $\sigma_\varepsilon^2>1$.

(iv) Using the previous result, it remains to take the derivative of (104) with respect to β_{t-1}. Denote

$$a = \beta_{t-1}^2\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^2 + \beta_{t-1}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) \tag{105}$$

$$b = \left(\sigma_a^{-2} + \beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2 \tag{106}$$

After some algebra,

$$\begin{aligned}\frac{\partial a}{\partial \beta_{t-1}}b = {} & 2\beta_{t-1}^3\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^4 + 4\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^3 + 2\beta_{t-1}\sigma_a^{-2}\left(1+\sigma_\varepsilon^{-2}\right)^2 \\ & + \beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) + 2\beta_{t-1}\sigma_a^{-2}\left(1+\sigma_\varepsilon^{-2}\right)\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right) \\ & + \sigma_a^{-4}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)\end{aligned} \tag{107}$$

and

$$\begin{aligned}\frac{\partial b}{\partial \beta_{t-1}}a = {} & 2\beta_{t-1}^3\sigma_a^2\left(1+\sigma_\varepsilon^{-2}\right)^4 + 2\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^3 \\ & + 2\beta_{t-1}^2\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)\left(1+\sigma_\varepsilon^{-2}\right)^2 + 2\beta_{t-1}\sigma_a^{-2}\left(\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2\right)\left(1+\sigma_\varepsilon^{-2}\right)\end{aligned} \tag{108}$$

Dropping the positive denominator, the derivative of (104) is $\frac{\partial a}{\partial \beta_{t-1}}b - \frac{\partial b}{\partial \beta_{t-1}}a$, which is after dividing through $\left(1+\sigma_\varepsilon^{-2}\right)^2$,

$$2\beta_{t-1}^2 + \beta_{t-1}^2\sigma_\varepsilon^{-2} + 2\beta_{t-1}\sigma_a^{-2} + \sigma_a^{-4}\frac{\sigma_\varepsilon^{-2}-\sigma_\varepsilon^2}{\left(1+\sigma_\varepsilon^{-2}\right)^2} + \beta_{t-1}^2\sigma_\varepsilon^2, \tag{109}$$

which is positive for all values of β_{t-1} if $\sigma_\varepsilon^2<1$. □

Lemma 4. *In the symmetric equilibrium with $\beta_{ijt}=\beta_{jt}$ and $K_{t+1}<W_t$,*

(i) Sentiment shocks ε_t affect the marginal benefit of information production through three channels,

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\textit{Information-Sensitivity}}\ \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\textit{Relative Size}} + \underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\textit{Absolute Size}}\right\}. \tag{110}$$

(ii) Productivity shocks a_t affect the marginal benefit of information production through

three channels,

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\left(\underbrace{\frac{\alpha\theta-\theta+1}{(1-\alpha)\,\alpha\theta}}_{\textit{Aggregate Demand}} + \underbrace{\frac{\alpha}{1-\alpha}}_{\textit{Absolute Size}} + \underbrace{1}_{\textit{Log-Normal Scaling}}\right)a_t\right\}, \tag{111}$$

with an overall positive effect for $\theta < \frac{1}{1-2\alpha}$.

Proof of Lemma 4. *(i)* Assume $a_t = 0$ without loss of generality. The marginal benefit to increasing β_{ijt} is

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} = \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} f\left(\varepsilon_{jt}\right) \frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}}\Big|_{\beta_{ijt}=\beta_{jt}} \alpha A_t^{\alpha_Y} \tag{112}$$
$$* \frac{\left(A_{jt} - \mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta} dj\right)^{\frac{\theta-1}{\theta}}} K_{t+1}^{\alpha} d\varepsilon_{jt} da_{jt},$$

where $g\left(a_{jt}\right)$ is the pdf of $a_{jt} \sim \mathcal{N}\left(0,\sigma_a^2\right)$ and $f\left(\varepsilon_{jt}\right)$ is the pdf of $\varepsilon_{jt} \sim \mathcal{N}\left(\varepsilon_t,\sigma_\varepsilon^2\right)$. The most immediate effect comes from changes to aggregate investment K_{t+1}^{α}. For $\delta R_{t+1} = 1$,

$$K_{t+1}^{\alpha} = \left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta} dj\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}} \propto \exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}. \tag{113}$$

The *Absolute Size* channel is summarized by $\exp\left\{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right\}$. Next, the derivative of the probability of buying at $\beta_{ijt} = \beta_{jt}$ is

$$\frac{\partial \mathcal{P}\left\{x_{ijt}=2\right\}}{\partial \beta_{ijt}}\Big|_{\beta_{ijt}=\beta_{jt}} = \phi\left(\varepsilon_{jt}\right)\left(\frac{\omega_{p,jt}}{\sqrt{\beta_{jt}}}a_{jt} + \frac{\varepsilon_{jt}}{2\beta_{jt}} - \frac{\omega_{\varepsilon,jt}\varepsilon_{jt} - \omega_{z\varepsilon,jt}\varepsilon_t}{\sqrt{\beta_{jt}}} + \frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}\right), \tag{114}$$

where $\phi(\cdot)$ is the standard-normal pdf. Combine $f\left(\varepsilon_{jt}\right)$ with $\phi\left(\varepsilon_{jt}\right)$ using Lemma 3,

$$\phi\left(\varepsilon_{jt}\right) f\left(\varepsilon_{jt}\right) = \exp\left\{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}\right\}\sqrt{\frac{1}{2\pi\left(1+\sigma_\varepsilon^2\right)}}\tilde{f}(\varepsilon_{jt}), \tag{115}$$

where $\tilde{f}\left(\varepsilon_{jt}\right)$ is the pdf of a fictional variable $\varepsilon_{jt} \sim \mathcal{N}\left(\frac{\varepsilon_t}{1+\sigma_\varepsilon^2}, \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)$. The *Information-*

Sensitivity channel is summarized by $\exp\left\{-\frac{\varepsilon_t^2}{2(1+\sigma_\varepsilon^2)}\right\}$. For the rest of the proof, use

$$\varepsilon_{jt} = \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{\varepsilon_t}{1+\sigma_\varepsilon^2} \tag{116}$$

$$d\varepsilon_{jt} = \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}dx. \tag{117}$$

Substitute ε_{jt} out of the terms in parenthesis for $\frac{\partial \mathcal{P}\{x_{ijt}=2\}}{\partial \beta_{ijt}}\Big|_{\beta_{ijt}=\beta_{jt}}$ leads to

$$\frac{\omega_{p,jt}}{\sqrt{\beta_{jt}}}a_{jt} + \left(\frac{1}{2\beta_{jt}} - \frac{\omega_{\varepsilon,jt}}{\sqrt{\beta_{jt}}}\right)\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{\varepsilon_t}{2\beta_{jt}\left(1+\sigma_\varepsilon^2\right)} + \frac{1}{2}\frac{\mathbb{V}_{jt}}{\sqrt{\beta_{jt}}}. \tag{118}$$

Substituting ε_{jt} out of $\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}$,

$$\begin{aligned}\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\} &= \exp\left\{\omega_a a_{jt} + \omega_\varepsilon \varepsilon_{jt} - \omega_{z\varepsilon}\varepsilon_t + \frac{1}{2}\mathbb{V}\right\} \\ &= \exp\left\{\omega_a a_{jt} + \omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \cancel{\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2} - \omega_{z\varepsilon}\varepsilon_t} + \frac{1}{2}\mathbb{V}\right\} \\ &\overset{\text{Lemma 1 (iii)}}{=} \exp\left\{\omega_a a_{jt} + \omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + \frac{1}{2}\mathbb{V}\right\}.\end{aligned} \tag{119}$$

Substitute ε_{jt} out of the firm-specific multiplier for firm capital,

$$\begin{aligned}&\frac{\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta} dj\right)^{\frac{\theta-1}{\theta}}} \\ &= \exp\left\{(\theta-1)\,\omega_a a_{jt} + (\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x + (\theta-1)\,\omega_\varepsilon\frac{\varepsilon_t}{1+\sigma_\varepsilon^2} - (\theta-1)\,\omega_\varepsilon\varepsilon_t - \frac{(\theta-1)\,\theta}{2}\left(\omega_a^2\sigma_a^2 + \omega_\varepsilon^2\sigma_\varepsilon^2\right)\right\} \\ &\propto \exp\left\{(\theta-1)\,\omega_a a_{jt} + (\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x - (\theta-1)\,\omega_\varepsilon\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\varepsilon_t\right\} \\ &= \exp\left\{(\theta-1)\,\omega_a a_{jt} + (\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x - (\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\} \\ &= \exp\left\{(\theta-1)\,\omega_a a_{jt} + (\theta-1)\,\omega_\varepsilon\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}x\right\}\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\},\end{aligned} \tag{120}$$

where I used Lemma 1 (iii) repeatedly. The *Relative Size* channel is summarized through $\exp\left\{-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t\right\}$. It remains to show that there are no other terms in $\widetilde{MB}\left(\beta_{ijt}, \beta_{jt}\right)$ that depend on ε_t. It is sufficient to show that

$$\int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \tilde{f}\left(\varepsilon_{jt}\right) \frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt} | s_{ijt}=z_{jt} z_{jt}\right\}\right) \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}^\theta dj\right)^{\frac{\theta-1}{\theta}}} d\varepsilon_{jt} da_{jt} \stackrel{!}{=} 0. \quad (121)$$

Substituting ε_{jt} out leads to

$$\begin{aligned}
&\int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \cancel{\frac{1}{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}}} \phi(x) \frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt} | s_{ijt}=z_{jt} z_{jt}\right\}\right) \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta-1}}{\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}^\theta dj\right)^{\frac{\theta-1}{\theta}}} \cancel{\sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}} dx da_{jt} \\
&\propto \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi(x)\left(A_{jt} \tilde{\mathbb{E}}\left\{A_{jt} | s_{ijt}=z_{jt}, z_{jt}\right\}^{\theta-1}-\mathbb{E}\left\{A_{jt} | s_{ijt}=z_{jt} z_{jt}\right\}^\theta\right) dx da_{jt} \\
&=\int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi(x)\left(\exp \left\{\left((\theta-1) \omega_a+1\right) a_{jt}+(\theta-1) \omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x+\frac{(\theta-1)}{2} \mathbb{V}\right\}\right. \\
&\quad \left.-\exp \left\{\theta \omega_a a_{jt}+\theta \omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x+\frac{\theta}{2} \mathbb{V}\right\}\right) dx da_{jt} \\
&\propto \int_{-\infty}^{\infty} g\left(a_{jt}\right) \int_{-\infty}^{\infty} \phi(x)\left(\exp \left\{\left((\theta-1) \omega_a+1\right) a_{jt}+(\theta-1) \omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x-\frac{\mathbb{V}}{2}\right\}\right. \\
&\quad \left.-\exp \left\{\theta \omega_a a_{jt}+\theta \omega_\varepsilon \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}} x\right\}\right) dx da_{jt} \\
&=\int_{-\infty}^{\infty} g\left(a_{jt}\right)\left(\exp \left\{\left((\theta-1) \omega_a+1\right) a_{jt}+\frac{(\theta-1)^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\frac{\mathbb{V}}{2}\right\}-\exp \left\{\theta \omega_a a_{jt}+\frac{\theta^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}\right) da_{jt} \\
&=\exp \left\{\frac{\left((\theta-1) \omega_a+1\right)^2}{2} \sigma_a^2+\frac{(\theta-1)^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\frac{\mathbb{V}}{2}\right\}-\exp \left\{\frac{\theta^2 \omega_a^2}{2} \sigma_a^2+\frac{\theta^2 \omega_\varepsilon^2}{2} \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right\}. \quad (122)
\end{aligned}$$

It remains to show that

$$\left((\theta-1) \omega_a+1\right)^2 \sigma_a^2+(\theta-1)^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}-\mathbb{V} \stackrel{!}{=} \theta^2 \omega_a^2 \sigma_a^2+\theta^2 \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}. \quad (123)$$

Using Lemma 1 (i), the LHS is equal to

$$\begin{aligned}
&\left(\left(\theta^2-2\theta+1\right) \omega_a^2+2(\theta-1) \omega_a \cancel{+1}\right) \sigma_a^2+\left(\theta^2-2\theta+1\right) \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \cancel{-\sigma_a^2}+\omega_a^2 \sigma_a^2+\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \\
&=\left(\left(\theta^2-2\theta+2\right) \omega_a^2+2(\theta-1) \omega_a\right) \sigma_a^2+\left(\theta^2-2\theta+2\right) \omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2} \\
&=2(\theta-1) \omega_a \sigma_a^2+\left(\theta^2+2(1-\theta)\right)\left(\omega_a^2 \sigma_a^2+\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right) \\
&\stackrel{\text{Lemma 1 (ii)}}{=} 2(\theta-1) \omega_a \sigma_a^2+\left(\theta^2+2(1-\theta)\right)\left(\omega_a \sigma_a^2\right) \\
&=\theta^2 \omega_a \sigma_a^2. \quad (124)
\end{aligned}$$

Using Lemma 1 (ii), the RHS is equal to

$$\theta^2\left(\omega_a^2 \sigma_a^2+\omega_\varepsilon^2 \frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}\right)=\theta^2 \omega_a \sigma_a^2. \quad (125)$$

Combining both confirms the conjecture. The marginal benefit of information production depends on ε_t only through the multiplicative effects in (113), (115) and (120), such that

$$\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\Big|_{\beta_{ijt}=\beta_{jt}} \propto \exp\left\{\underbrace{\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t}_{\text{Absolute Size}} \underbrace{-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}}_{\text{Information Sensitivity}} \underbrace{-\left(\theta-1\right)\omega_{s\varepsilon}\varepsilon_t}_{\text{Relative Size}}\right\}. \tag{126}$$

(ii) Follow the same strategy as in *(i)* and use the same expression for the marginal benefit of information production (112). Start with the expressions for aggregate investment, K_{t+1}^{α}, and productivity $A_t^{\alpha_Y}$ in (112). For $\delta R_{t+1}=1$, they are equal to

$$A_t^{\alpha_Y}K_{t+1}^{\alpha} = A_t^{\alpha_Y}\left(\alpha\delta A_t^{\alpha_Y}\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{1}{\theta}}\right)^{\frac{\alpha}{1-\alpha}}, \tag{127}$$

Recall that $\alpha_Y=\frac{\alpha\theta-\theta+1}{\alpha\theta}$. Then

$$A_t^{\alpha_Y} \propto \exp\left\{\frac{\alpha\theta-\theta+1}{\alpha\theta}a_t\right\} \tag{128}$$

$$\left(\int_0^1 \tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{1}{\theta}} \propto \exp\left\{a_t\right\}. \tag{129}$$

Putting both together yields

$$\begin{aligned} A_t^{\alpha_Y}K_{t+1}^{\alpha} &\propto \exp\left\{\frac{\alpha\theta-\theta+1}{\alpha\theta}\left(1+\frac{\alpha}{1-\alpha}\right)a_t+\frac{\alpha}{1-\alpha}a_t\right\} \\ &= \exp\left\{\left(\frac{\alpha\theta-\theta+1}{\left(1-\alpha\right)\alpha\theta}+\frac{\alpha}{1-\alpha}\right)a_t\right\}. \end{aligned} \tag{130}$$

Again, using substitution with

$$a_{jt} = \sqrt{\sigma_a^2}y+a_t \tag{131}$$

$$da_{jt} = \sqrt{\sigma_a^2}dy \tag{132}$$

it follows that

$$A_{jt} \propto \exp\left\{a_t\right\} \tag{133}$$

$$\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\} \propto \exp\left\{a_t\right\}, \tag{134}$$

which yields

$$\frac{\left(A_{jt}-\mathbb{E}\left\{A_{jt}|s_{ijt}=z_{jt}z_{jt}\right\}\right)\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta-1}}{\left(\int_0^1\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}^{\theta}dj\right)^{\frac{\theta-1}{\theta}}}\propto\exp\left\{a_t\right\}. \tag{135}$$

The change in the buying probability does *not* depend on a_t

$$\left.\frac{\partial\mathcal{P}\left\{x_{ijt}=2\right\}}{\partial\beta_{ijt}}\right|_{\beta_{ijt}=\beta_{jt}}=\phi\left(\varepsilon_{jt}\right)\left(\frac{\omega_p}{\sqrt{\beta_{jt}}}\sqrt{\sigma_a^2}y+\frac{\varepsilon_{jt}}{2\beta_{jt}}-\frac{\omega_\varepsilon\varepsilon_{jt}-\omega_{z\varepsilon}\varepsilon_t}{\sqrt{\beta_{jt}}}+\frac{\mathbb{V}_{jt}}{2\sqrt{\beta_{jt}}}\right). \tag{136}$$

Since no other terms depend on a_t and I substituted a_{jt} out, the individual terms can be added up,

$$\frac{\alpha\theta-\theta+1}{(1-\alpha)\alpha\theta}+\frac{\alpha}{1-\alpha}+1=\frac{2\alpha\theta-\theta+1}{(1-\alpha)\alpha\theta}, \tag{137}$$

such that the marginal benefit of information production is

$$\left.\widetilde{MB}\left(\beta_{ijt},\beta_{jt}\right)\right|_{\beta_{ijt}=\beta_{jt}}\propto\exp\left\{\left(\frac{2\alpha\theta-\theta+1}{(1-\alpha)\alpha\theta}\right)a_t\right\}, \tag{138}$$

and the term inside the parenthesis is positive whenever $\theta<\frac{1}{1-2\alpha}$. □

Proof of Proposition 3. The cutoff can be derived by using the result from Lemma 4 (i) and taking the derivative with respect to ε_t of the following expression,

$$\frac{\partial}{\partial\varepsilon_t}\left(-\frac{\varepsilon_t^2}{2\left(1+\sigma_\varepsilon^2\right)}-(\theta-1)\,\omega_{s\varepsilon}\varepsilon_t+\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}\varepsilon_t\right)\overset{!}{=}0. \tag{139}$$

Denote $\bar{\varepsilon}$ as the value of ε_t for which the above expression is maximized. Then,

$$\begin{aligned}-\frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2}-(\theta-1)\,\omega_{s\varepsilon}+\frac{\alpha}{1-\alpha}\omega_{s\varepsilon}=0&\\ \iff\frac{\bar{\varepsilon}}{1+\sigma_\varepsilon^2}=\left(\frac{1}{1-\alpha}-\theta\right)\omega_{s\varepsilon}&\\ \iff\bar{\varepsilon}=\left(1+\sigma_\varepsilon^2\right)\left(\frac{1}{1-\alpha}-\theta\right)\omega_{s\varepsilon},&\end{aligned} \tag{140}$$

where $\omega_{s\varepsilon}=\frac{\sqrt{\beta_t}}{\sigma_a^{-2}+\beta_t\left(1+\sigma_\varepsilon^{-2}\right)}$. For $\varepsilon_t<\bar{\varepsilon}$, information production β_t is increasing in ε_t. For $\varepsilon_t>\bar{\varepsilon}$, information production β_t is decreasing in ε_t. □

Proof of Proposition 4. Follows from Lemma 4 (ii). □

Proof of Proposition 5. *(i)* Using the result from Proposition 3 (ii) and the assumption that $\theta > \frac{1}{1-\alpha}$, it must be that positive sentiment shocks crowd out information production. Moreover, as $\beta^* < \frac{\sigma_a^{-2}}{1+\sigma_\varepsilon^{-2}}$, it must be that the pass-through of aggregate sentiment shocks $\omega_{s\varepsilon}$ decreases in information production. Therefore, the pass-through is smaller when information is endogenous than if information production is fixed at β^*. As a result, sentiment shocks are dampened by information production in financial markets, as less precise information by itself leads to less investment and lowers the pass-through of sentiment shocks.

(ii) $\lim_{\varepsilon_t \to \infty} \sqrt{\beta_t(\varepsilon_t)}\varepsilon_t = 0$ guarantees that the pass-through of sentiment shocks goes faster to zero than the sentiment shock goes to infinity, i.e., the direct effect of sentiment shocks on investment disappears as shocks become arbitrarily large. Moreover, Lemma 4 (i) shows that through the information-sensitivity effect $\lim_{\varepsilon_t \to \infty} \beta_t(\varepsilon_t) = 0$. □

Proof of Proposition 6. Follows from Proposition 4 and Assumption 2. An increase in aggregate productivity encourages more information production, leading to more investment. As a result, productivity shocks are amplified. □

Proof of Proposition 7. Since $\widetilde{MB}(\beta_{ijt}, \beta_{jt}) \propto \kappa_H - \kappa_L$ whereas $MB^{SP}(\beta_t)$ is not a function of position limits $\{\kappa_H, \kappa_L\}$, it must that $\widetilde{MB}(\beta_{ijt}, \beta_{jt})\Big|_{\beta_{ijt}=\beta_{jt}=\beta_t} \neq MB^{SP}(\beta_t)$ for almost all values of β_t. Therefore, the information production in the competitive economy and social planner allocation do not coincide almost everywhere. □

Proof of Proposition 8. The social planner's allocation is given by equalizing the marginal products of capital for each firm given the market signals $\{z_{jt}\}$. The maximization problem of the social planner for firm capital allocation is

$$\max_{K_{j1}} \mathbb{E}\left\{ \left(\int_0^1 Y_{j1}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\alpha\theta}{\theta-1}} |z_{jt} \right\} - R_1^{SP} K_{j1}, \tag{141}$$

for some interest rate R_1^{SP}. The resulting first-order condition for firm capital is

$$K_{j1}^{SP} = \left(\alpha Y_1^{\alpha_Y} \frac{\mathbb{E}\{A_{j0}|z_{j0}\}}{R_1^{SP}} \right)^{\theta}. \tag{142}$$

Integrating on both sides yields

$$R_1^{SP} = \alpha Y_1^{\alpha_Y} \left(\int_0^1 \mathbb{E}\{A_{j0}|z_{j0}\}^{\theta} dj \right)^{\frac{1}{\theta}} \left(K_1^{SP}\right)^{-\frac{1}{\theta}}. \tag{143}$$

Substituting R_1^{SP} out of K_{j1}^{SP} yields (45). Plugging (45) into (5) leads to

$$Y_1^{SP} = A_0^{SP} K_1^{\alpha} \tag{144}$$

where

$$A_0^{SP} = \frac{\left(\int_0^1 A_0 \mathbb{E}\left\{A_0|z_0\right\}^{\theta-1} dj\right)^{\frac{\alpha\theta}{\theta-1}}}{\left(\int_0^1 \mathbb{E}\left\{A_0|z_0\right\}^{\theta} dj\right)^{\alpha}} \overset{\text{L.I.E.}}{=} \left(\int_0^1 \mathbb{E}\left\{A_0|z_0\right\}^{\theta} dj\right)^{\frac{\alpha}{\theta-1}}. \tag{145}$$

The analytical expression can be obtained by evaluating the conditional expectations and using the constrained efficient Bayesian weights and posterior uncertainty,

$$\omega_p^{eff} = \frac{\sigma_a^{-2}}{\sigma_a^{-2} + \beta_0 \sigma_\varepsilon^{-2}}, \quad \omega_a^{eff} = \frac{\beta_0 \sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_0 \sigma_\varepsilon^{-2}} \tag{146}$$

$$\omega_\varepsilon^{eff} = \frac{\sqrt{\beta_0}\sigma_\varepsilon^{-2}}{\sigma_a^{-2} + \beta_0 \sigma_\varepsilon^{-2}}, \quad \mathbb{V}^{eff} = \frac{1}{\sigma_a^{-2} + \beta_0 \sigma_\varepsilon^{-2}} \tag{147}$$

leading to

$$\begin{aligned}
A_0^{SP} &= \left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{\alpha}{\theta-1}} \\
&= \left(\int_0^1 \left\{\exp\left\{\theta\omega_p^{eff} a_0 + \theta\omega_a^{eff} a_{j0} + \theta\omega_\varepsilon^{eff}(\varepsilon_{j0} - \varepsilon_0) + \theta\frac{\mathbb{V}^{eff}}{2}\right\}\right\} dj\right)^{\frac{\alpha}{\theta-1}} \\
&= \exp\left\{\theta\omega_p^{eff} a_0 + \theta\omega_a^{eff} a_0 + \frac{\theta^2}{2}\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \frac{\theta^2}{2}\left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \theta\frac{\mathbb{V}^{eff}}{2}\right\}^{\frac{\alpha}{\theta-1}} \\
&= \exp\left\{a_0 + \frac{\theta}{2}\left(\omega_a^{eff}\right)^2 \sigma_a^2 + \frac{\theta}{2}\left(\omega_\varepsilon^{eff}\right)^2 \sigma_\varepsilon^2 + \frac{\mathbb{V}^{eff}}{2}\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&\overset{\text{Lemma 2 (i) and (ii)}}{=} \exp\left\{a_0 + \frac{1}{2}\left(\theta\omega_a^{eff}\sigma_a^2 + \mathbb{V}^{eff}\right)\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&\overset{\text{Lemma 2 (iii)}}{=} \exp\left\{a_0 + \frac{1}{2}\left(\sigma_a^2 + (\theta-1)\,\omega_a^{eff}\sigma_a^2\right)\right\}^{\frac{\alpha\theta}{\theta-1}} \\
&= \exp\left\{\frac{1}{1-\theta}\left(a_0 + \frac{\sigma_a^2}{2}\right) + \omega_a^{eff}\frac{\sigma_a^2}{2}\right\}^{\alpha\theta}.
\end{aligned} \tag{148}$$

TFP under the efficient allocation of capital can be decomposed into two expressions,

$$\ln A_0^{SP} = \underbrace{\frac{\alpha\theta}{\theta-1}\left(a_0 + \frac{\sigma_a^2}{2}\right)}_{\text{exogenous}} + \underbrace{\alpha\theta\omega_a^{eff}\frac{\sigma_a^2}{2}}_{\text{allocative efficiency}}. \tag{149}$$

It follows that

$$\frac{\partial\omega_a^{eff}}{\partial\beta_0} > 0 \Rightarrow \frac{\partial A_0^{SP}}{\partial\beta_0} > 0. \tag{150}$$

Substituting Y_1^{SP} out of the expression for R_1^{SP} then leads to the interest rate

$$R_1^{SP} = \alpha A_0^{SP} \left(K_1^{SP}\right)^{\alpha-1}, \tag{151}$$

and aggregate investment

$$K_1^{SP} = \min\left\{\left(\alpha\delta A_0^{SP}\right)^{\frac{1}{1-\alpha}}, W_0\right\}. \tag{152}$$

Consumption follows using (152) in (41). Finally, taking K_1^{SP} as given and plugging aggregate capital investment in in Y_1^{SP} in (46) in (SP), (47) follows after taking the derivative of (46) with respect to β_0. □

Proof of Proposition 9. I will show that the decentralized allocations coincide with the social planner's allocations. Households receive from firm j the dividend

$$\hat{\Pi}_{j1} = \tau\left(z_{j0}\right)\Pi_{j1} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}}{\tilde{\mathbb{E}}\left\{A_{j0}|s_{ij0} = z_{j0}, z_{j0}\right\}}\alpha Y_1^{\alpha_Y} A_{j0} K_{j1}^{\frac{\theta-1}{\theta}} \tag{153}$$

and the marginal trader expects the dividend to be

$$\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} = \alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\} K_{j1}^{\frac{\theta-1}{\theta}}. \tag{154}$$

The price is using $P_{j0} = K_{j1}$,

$$P_{j0} = \frac{1}{R_1}\tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} = \left(\frac{1}{R_1}\alpha Y_1^{\alpha_Y}\mathbb{E}\left\{A_{j0}|z_{j0}\right\}\right)^{\theta}. \tag{155}$$

Integrating on both sides yields the interest rate

$$R_1 = \frac{\int_0^1 \tilde{\mathbb{E}}\left\{\hat{\Pi}_{j1}|s_{ij0} = z_{j0}, z_{j0}\right\} dj}{\int_0^1 P_{j0} dj} = \alpha Y_1^{\alpha_Y}\left(\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj\right)^{\frac{1}{\theta}} K_1^{-\frac{1}{\theta}}, \tag{156}$$

plugging in the interest rate in the expression for the price yields

$$K_{j1}^{DE} = \frac{\mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta}}{\int_0^1 \mathbb{E}\left\{A_{j0}|z_{j0}\right\}^{\theta} dj} K_1 = K_{j1}^{SP}, \tag{157}$$

Finally, following the same steps as in Proposition 8 shows that the interest rate and aggregate investment replicate the social planner's allocation for a given information precision β_0.

□

Proof of Proposition 10. Let the social planner buy $d^{SP} \in (-1,1)$ units of shares in all markets. The market-clearing condition for market j becomes

$$2\left(1-\Phi\left(\sqrt{\beta_{j0}}\left(\hat{s}\left(P_{j0}\right)-a_{j0}\right)-\varepsilon_{j0}\right)\right)=1-d^{SP}. \tag{158}$$

Keeping position limits fixed, the social planner's demand d^{SP} changes the identity of the marginal trader. The marginal trader becomes more optimistic on average if the social planner purchases more assets. The threshold signal becomes,

$$\hat{s}\left(P_{j0},d^{SP}\right)=a_{j0}+\frac{\varepsilon_{j0}+\Phi^{-1}\left(\frac{1+d^{SP}}{2}\right)}{\sqrt{\beta_{j0}}}. \tag{159}$$

It follows immediately that asset purchases or sales with $d^{SP}=2\Phi\left(-\varepsilon_0\right)-1$ ensure that the marginal trader holds unbiased beliefs,

$$\hat{s}\left(P_{j0},d^{SP}\right)=a_{j0}+\frac{\varepsilon_{j0}-\varepsilon_0}{\sqrt{\beta_{j0}}}. \tag{160}$$

It follows that prices are unbiased, and aggregate investment is as if the sentiment shock was absent.

Traders expect to buy in equilibrium whenever $s_{ijt}>\hat{s}\left(P_{j0},d^{SP}\right)$. Asset purchases/sales set the threshold $\hat{s}\left(P_{j0},d^{SP}\right)$ at a level as if the aggregate sentiment shock was $\varepsilon_0=0$, effectively undoing any change to the incentive to produce information. Because the trader thinks that she is unaffected by the sentiment shock and asset purchases / sales correct the sentiment shock, information production reverts to the previous level □

Proof of Proposition 11. (i) Denote $k_{jt+1}=\ln K_{jt+1}$. Using (29) allows writing the variance of the log of firm capital stocks as

$$Var\left(k_{jt+1}\right)=\theta^2 Var\left(\ln\tilde{\mathbb{E}}\left\{A_{jt}|s_{ijt}=z_{jt},z_{jt}\right\}\right)=\frac{\theta^2}{2}\left(\omega_a^2\sigma_a^2+\omega_\varepsilon^2\sigma_\varepsilon^2\right). \tag{161}$$

Which can be expressed as

$$\begin{aligned}\omega_a^2\sigma_a^2+\omega_\varepsilon^2\sigma_\varepsilon^2&=\frac{\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_a^2+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)^2\sigma_\varepsilon^2}{\left(\sigma_a^{-2}+\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)\right)^2}\\&\propto\frac{\beta_{t-1}^2\sigma_a^2+\beta_{t-1}\sigma_\varepsilon^2}{\sigma_a^{-4}+2\sigma_a^{-2}\beta_{t-1}\left(1+\sigma_\varepsilon^{-2}\right)+\beta_{t-1}^2\left(1+\sigma_\varepsilon^{-2}\right)^2}.\end{aligned} \tag{162}$$

Taking the derivative with respect to β_{t-1} and dropping the denominator leads to the simplified expression

$$2\beta_{t-1}\sigma_a^{-2} + \sigma_a^{-4}\sigma_\varepsilon^2 + \beta_{t-1}^2 \left(1 - \sigma_\varepsilon^2\right) \left(1 + \sigma_\varepsilon^{-2}\right) \tag{163}$$

which is positive for all values of β_{t-1} for $\sigma_\varepsilon^2 \leq 1$.

(ii) Denote $\Delta k_{jt+1} = k_{jt+1} - k_{jt}$. Then

$$\Delta k_{jt+1} = \Delta\theta \ln \tilde{\mathbb{E}} \left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} + \Delta \ln \int_0^1 \tilde{\mathbb{E}} \left\{A_{jt}|s_{ijt} = z_{jt}, z_{jt}\right\} dj + \Delta K_{t+1}. \tag{164}$$

Deriving the variance of Δk_{jt+1} across firms yields

$$Var\left(\Delta k_{jt+1}\right) = \theta^2 \left(\omega_a^2 \sigma_a^2 + \omega_\varepsilon^2 \sigma_\varepsilon^2\right) \tag{165}$$

which is monotonically increasing in β_{t-1} for $\sigma_\varepsilon^2 \leq 1$ as in (i). □