Overconfidence and Information Acquisition in Financial Markets

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Abstract

I develop a model in which overconfidence in the form of correlation neglect incentivizes costly information acquisition in financial markets. Traders' information has two sources of noise, one idiosyncratic and the other correlated between traders. Traders are overconfident in that they overestimate the share of idiosyncratic noise in their private information, i.e., they partly neglect correlated noise. I find that an infinitesimal amount of overconfidence is sufficient to generate trade when the private signal is exogenous and free. However, substantial amounts of overconfidence are needed when traders acquire costly information. I show that the model can be integrated into macroeconomic models and can be used to study trader heterogeneity. Finally, I consider an extension in which traders have limited resources for trading. Such funding constraints dampen the effect of new information on the price. Moreover, disagreement can affect the price level differently depending on the relative scarcity or abundance of trading capital.

Keywords: Information Acquisition; Correlation Neglect; Overconfidence; Dispersed Information
JEL Codes: D80, G12, G14

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1 Introduction

Financial markets are among the most efficient aggregators of dispersed information in the modern economy. When traders learn new information, they seek to profit by buying when the information is positive and selling when it is negative. Such informed speculation ensures that prices quickly incorporate new information, which decreases the profitability of trading on private information in the first place. However, not all trading occurs for fundamental reasons, e.g., liquidity trading, which can help mask informed trading and maintain the incentives for costly information acquisition.[1] Although such noise trading allows financial markets to be liquid, the mechanisms behind noise trading are little understood.

In this paper, I study the role of overconfidence in the form of correlation neglect[2] for incentivizing costly information acquisition in a financial market that aggregates dispersed information. For this purpose, I develop a model along the lines of Grossman and Stiglitz (1980) and Albagli, Hellwig, and Tsyvinski (2021). In such models, privately informed arbitrageurs seek to profit from mispricing. Usually, unobservable variations in the asset supply keep prices from being fully revealing. Instead, I assume that traders are overconfident about their signals' informativeness as they perceive the signal to be overly independent of other information sources. I choose this approach for two reasons. First, overconfidence[3] and correlation neglect[4] are well-documented behavioral biases that have been observed for traders, financial managers, and experiment participants.[5] Second, assuming that traders are overconfident yields a more tractable model, addressing a significant concern in this literature. The gained tractability allows exploring additional settings while keeping the information structure normal.

The overconfidence assumption affects how traders process their private information, which contains two sources of noise. The first component is idiosyncratic, which

[1]In the absence of market-wide noise, asset prices can become fully revealing and destroy the incentive for costly information acquisition. This is called the Grossman-Stiglitz paradox (Grossman and Stiglitz 1980).

[2]Correlation neglect arises when agents learn from multiple signals but do not fully account for the signals' correlation structure. For instance, when two pieces of information are perceived as independent, although they stem from the same, possibly biased, source.

[3]See Biais et al. (2005), Allen and Evans (2005), Malmendier and Tate (2005), and Ben-David, Graham, and Harvey (2013).

[4]See Brandts, Giritligil, and Weber (2015), Eyster and Weizsäcker (2016), Eyster et al. (2018), Grimm and Mengel (2020), Enke and Zimmermann (2019), and Chandrasekhar, Larreguy, and Xandri (2020).

[5]See Glaser and Weber (2010) for a broad overview on the evidence on overconfidence and Hirshleifer (2015) for a survey on behavioral finance.

can stem from private sources of information or the imperfect understanding of public information. The second noise component is perfectly correlated across traders and can be viewed as a form of *sentiment*. Therefore, traders can be collectively optimistic because the asset's fundamental value increased or because market sentiment is exuberant. In this model, overconfidence takes the form of correlation neglect, i.e., traders believe that their signal is more idiosyncratic than it truly is. This bias leads to an overweighting of private information, which traders believe contains information not already reflected in the market.

I show that an infinitesimal amount of overconfidence is sufficient to generate trade when private signals are exogenous. Although the price aggregates all private information perfectly from the perspective of an uninformed observer, a small amount of overconfidence lets traders not discard their private information. However, when information is costly, substantial amounts of overconfidence are necessary to motivate information acquisition. The reason is that the traders' private signals serve a dual function. First, traders learn about an asset's fundamental value. Second, traders learn about correlated noise from their private signal, which is useful for filtering information learned from the asset price. As the trader exerts effort to reduce both idiosyncratic and correlated noise in her signal, she learns more about the asset's fundamental value and less about correlated noise. Therefore, traders can choose not to exert effort to maximize their information about the correlated noise, which amounts to *free-riding* on the information acquisition of other traders. For the *free-riding* strategy to be unattractive, traders must believe that their private signal is relatively uninformative about correlated noise, which is the case for strongly overconfident traders. Because free-riding cannot be an equilibrium strategy, a large amount of overconfidence is necessary for the equilibrium to exist.

I use the model to study several applications. First, the model accommodates heterogeneity in financial markets when two groups of traders differ in their information technologies and degrees of overconfidence. For example, including rational and boundedly-rational traders allows to model noise in financial markets more carefully. Second, the model's approach to noise trading is especially amenable for macroeconomic applications in which aggregate resource constraints have to be observed. The reason is that boundedly-rational traders are still utility-maximizing and observe budget constraints, whereas noise traders exogenously add and remove resources from the economy.

Finally, I study a setting in which traders' demands depend on their trading capital and the asset price instead of imposing exogenous position limits. Such funding constraints leave the ability of financial markets to aggregate information unchanged while dampening the effect of information change on the price. Because traders learn only about the next dividend payment and have limited trading capital, a higher resale price, e.g., due to lower interest rates, limits their ability to exploit their information and discourages information acquisition.

When traders face funding constraints, disagreement plays an important role in determining the asset price. If prices are high on average, most traders need to buy to clear the market, leading to a relatively pessimistic price-setting trader. Higher disagreement leads to more extreme beliefs, which lowers the price as the price-setting trader becomes more pessimistic. The reverse is true when prices are low on average, as disagreement increases asset prices by exacerbating the price-setting trader's optimism. Due to common priors, disagreement is hump-shaped in information precision. Therefore, a change in information precision also affects the *average* price level. Whether the price level increases or falls depends on the initial precision and on whether trading capital is relatively scarce or abundant.

1.1 Literature

There is a large literature studying information in financial markets that goes back to seminal papers such as Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981). The most closely related paper is Albagli, Hellwig, and Tsyvinski (2021), who provide a framework in which traders learn about a variable that summarizes the fundamental value of an asset, e.g., earnings or the probability of default, instead of learning about the dividend directly. The result is that the model allows studying assets with non-linear payoffs (e.g., equity or debt). This paper adds to the literature by considering overconfidence and correlation neglect as a source of noise trading, which yields an especially tractable model.

The paper belongs to the literature that studies the role of behavioral biases in financial markets. A standard result in the CARA-normal framework is that overconfidence, i.e., the perception that the trader's information is more accurate than it truly is, *improves* the informational efficiency of financial markets (e.g., Ko and Huang 2007; Peress 2014), as traders take more aggressive positions. In contrast, higher overconfidence *distorts* asset prices and can crowd out information acquisition in this model, worsening the efficiency of financial markets. Correlation neglect in particular has been mainly studied in experimental settings (Brandts, Giritligil, and Weber 2015; Eyster et al. 2018; Enke and Zimmermann 2019; Grimm and Mengel 2020; Chandrasekhar, Larreguy, and Xandri 2020), but received little attention as a mechanism in theoretical models. This paper shows that correlated noise or sentiments together with correlation neglect can provide incentives for information acquisition while maintaining an otherwise standard model structure as in Albagli, Hellwig, and Tsyvinski (2021).

The last part of this paper studies the role disagreement and funding constraints as in Fostel and Geanakoplos (2012) and Simsek (2013, 2021). Usually, this class of models assumes two groups of traders with exogenous information who agree to disagree, such that prices are uninformative from their perspective. This approach allows studying settings where the relative wealth of optimists and pessimists is an important determinant of the asset price. This paper adds to the literature by providing a tractable model in which overconfident traders acquire information and learn from prices, yet funding constraints can be introduced. In contrast to "cash-in-the-market"-pricing as in Allen and Gale (1994), these funding constraints can also lead to asset prices being too high if optimists buy the total asset float. Moreover, I show that the effect of disagreement on asset prices depends on the beliefs of the price-setting trader. For example, if the price-setting trader is an optimist, more disagreement makes her even more optimistic, which drives up the price.

The rest of the paper is structured as follows. In section 2 I introduce the model and elaborate its equilibrium properties in 3. Then, section 4 showcases a number of applications. Section 5 studies a setting with funding constraints in more detail. Finally, section 6 concludes.

2 Model

The financial market is populated by a unit mass of risk-neutral traders indexed by $i \in [0, 1]$. Time is static. Traders have deep pockets, and their utility function is

$$U_i = \mathbb{E}_i C_i - IA(\beta_i), \tag{1}$$

where C_i is trader i's end-of-period consumption and $IA(\beta_i)$ are convex information acquisition costs depending on signal precision β_i. Each trader can either buy up to two units of a perfectly divisible risky asset or invest in a risk-less bond with a unit return. Short-selling is ruled out.[6] Traders use their private signal and learn from the price to make their trading decision.

2.1 Assets

The risky asset has a monotonically increasing payoff function $\pi(\theta)$ where $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$ can be viewed as a sufficient statistic which pins down the asset's fundamental value (e.g., earnings or distance-to-default). For all numerical simulations $\pi(\theta) = \theta$.

2.2 Information Structure

Traders receive a private signal

$$s_i = \theta + \frac{\kappa \eta_i + \sqrt{1-\kappa^2}\varepsilon}{\sqrt{\beta_i}}, \tag{2}$$

where $\kappa \in [0, 1)$ is the share of idiosyncratic noise.[7] The signal contains two sources of noise, where $\eta_i \overset{iid}{\sim} \mathcal{N}(0, 1)$ is idiosyncratic noise and independently distributed between traders. In contrast, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is correlated noise that affects all traders equally. Traders can choose β_i subject to a convex cost $IA(\beta_i)$ to increase the information content of their private signal.

For traders to acquire costly information, they must believe in having an advantage over other traders, allowing them to recuperate the information acquisition costs. To this end, I assume that traders are overconfident.

Assumption 1 (Overconfidence)**.** *Trader i believes the information structure to be*

$$s_i = \theta + \frac{\tilde{\kappa} \eta_i + \sqrt{1-\tilde{\kappa}^2}\varepsilon}{\sqrt{\beta_i}}$$
$$s_{-i} = \theta + \frac{\kappa \eta_{-i} + \sqrt{1-\kappa^2}\varepsilon}{\sqrt{\beta_{-i}}},$$

where $\tilde{\kappa} \in (0, 1]$ and $\tilde{\kappa} > \kappa$.

Each trader believes that she is less exposed to correlated noise than other traders. Consequently, traders think they have the edge over other traders, as they expect to buy

[6] Ruling out short-selling is not central to the analysis, and allowing limited short-selling is straightforward.

[7] For $\kappa = 1$, prices are fully revealing which leads to well-known problems of inexistence as discussed in Grossman and Stiglitz (1980).

with greater probability when negatively correlated noise shocks depress the price below its fundamental value. In the following, expectations that suffer from *overconfidence* as in Assumption 1 are denoted by $\tilde{\mathbb{E}}\{\cdot\}$.

2.3 Trader's Problem

The problem of trader i is

$$\max_{\beta_i \geq 0} \tilde{\mathbb{E}} \left\{ \max_{x_i \in [0,2]} x_i(\pi(\theta) - P) | s_i, P \right\} - IA(\beta_i). \tag{P2.1}$$

Trader i takes two decisions. First, before trading takes place, trader i chooses information precision β_i subject to a convex information cost $IA(\beta_i)$ to maximize expected trading profits. Second, once prices are realized, traders use their private signal s_i and additionally learn from the price P to take the optimal buying decision subject to the position limit $x_i \in [0, 2]$.[8] Alternatively, traders can invest in a risk-less bond with unit return. Note that the expectation $\tilde{\mathbb{E}}(\cdot)$ misperceives the distribution of s_i according to Assumption 1.

2.4 Equilibrium

Trading The trader's problem is solved in reverse chronological order, starting with the trading decision. Due to linear preferences, the optimal demand of trader i taking information precision β_i as given is

$$x_i = \begin{cases} 0 & \tilde{\mathbb{E}}(\pi(\theta)|s_i, P) < P \\ \in [0, 2] & \tilde{\mathbb{E}}(\pi(\theta)|s_i, P) = P \\ 2 & \tilde{\mathbb{E}}(\pi(\theta)|s_i, P) > P. \end{cases} \tag{3}$$

Trader i buys up to the limit whenever she expects the risky asset to yield a higher return than the risk-less bond.

Information Acquisition Plugging (3) into Trader i's problem (P2.1) allows to

[8]The assumption of exogenously given position limits is standard in the literature to avoid unbounded demands by risk-neutral traders. This assumption is relaxed in section 5

write the information acquisition problem as

$$\max_{\beta_i \geq 0} \quad \widetilde{EU}(\beta_i, \beta) - IA(\beta_i) \tag{P2.2}$$

$$\widetilde{EU}(\beta_i, \beta) = \tilde{\mathbb{E}}\left\{\mathcal{P}(x_i = 2)(\pi(\theta) - P)\right\}, \tag{4}$$

where $\widetilde{EU}(\beta_i, \beta)$ denotes the expected trading profits depending on both the individual choice β_i and the symmetric choice of all other traders β. The probability of buying $\mathcal{P}(x_i = 2)$ stems from evaluating the expectations for the buying decision (3) over idiosyncratic noise η_i and the remaining expectations are taken over fundamental θ and correlated noise ε. Since the perceived probability of buying deviates from the true buying probability due to Assumption 1, also expected utility $\widetilde{EU}$ is distorted. As a result, traders expect to buy with a high probability when the price P is depressed due to a negative correlated noise shock ε. A higher β_i changes the likelihood of buying in a given state (θ, ε) as captured through the first-order condition for an interior solution $\beta_i > 0$,

$$\widetilde{MB}(\beta_i, \beta) = \tilde{\mathbb{E}}\left\{\frac{\partial \mathcal{P}(x_i = 2)}{\partial \beta_i}(\pi(\theta) - P)\right\} = IA'(\beta_i). \tag{5}$$

When choosing their information precision β_i, traders weigh the benefit of a possibly more profitable buying decision with the effort cost of acquiring more precise information. The marginal benefit also depends on the information precision of all other traders, β. Intuitively, the incentive to acquire information individually crucially depends on how efficient the market is already. In the extreme case, when the market is perfectly efficient ($\lim_{\beta\to\infty} P(\theta) = \pi(\theta)$), acquiring private information becomes worthless.

As will become clear shortly, the noise in the private signal and the information learned from the price P are correlated. In this case, a less noisy private signal may make s_i less informative about θ, as traders also use s_i to learn about the correlated noise ε. In turn, the information about ε can be used to better filter information about θ from the price. Since s_i is most informative about ε when trader i decides to put only infinitesimal effort in reducing noise in her signal, the following condition needs to be checked to rule out a corner solution,

$$EU(\beta_i^*, \beta) - IA(\beta_i^*) > \lim_{\beta_i \to 0} EU(\beta_i, \beta), \tag{6}$$

where β_i^* is the interior solution according to (5). In the following, ruling out the corner solution $\beta_i \to 0$ will be important for the existence of a symmetric equilibrium.

Market-Clearing As I verify ex-post, at the symmetric equilibrium ($\forall i : \beta_i = \beta$), traders buy two units whenever their private signal is above some threshold $\hat{s}(P)$. In other words, a higher realization of the private signal leads to a higher private valuation, and due to linear preferences, all traders with a private valuation higher than the price buy. Total demand is then equal to the mass of traders with a signal $s_i > \hat{s}(P)$ times the upper position limit:

$$D(\theta, P) = 2\left(1 - \Phi\left(\frac{\sqrt{\beta}}{\kappa}\left(\hat{s}(P) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon\right)\right). \tag{7}$$

For the market to clear, total demand must be equal to the normalized asset supply of one,

$$D(\theta, P) = 1. \tag{8}$$

The threshold $\hat{s}(P)$ can directly be derived from (8).

Price Signal Since there is a one-to-one relationship between the threshold $\hat{s}(P)$ and the price P, observing the price P is informationally equivalent to observing the public signal

$$z = \hat{s}(P) = \theta + \sqrt{\frac{1-\kappa^2}{\beta}}\varepsilon. \tag{9}$$

In the following, I will refer to z as the price signal, and expectations will condition on z rather than P. Note that the price signal z reveals the joint information set of traders by washing out idiosyncratic noise, $z = \int_0^1 s_i di$. If traders processed their private information correctly as in (2), they would disregard their private signal s_i after observing z. However, due to Assumption 1, traders think that their private signal s_i remains informative.[9]

Orthogonalized Signal As both the price signal z and private signal s_i contain correlated noise ε for $\tilde{\kappa} < 1$, observing $\{s_i, z\}$ is informationally equivalent to observing $\{\bar{s}_i, z\}$, where

$$\bar{s}_i = \theta + \frac{\tilde{\kappa}}{\sqrt{\beta_i}\left(1 - \sqrt{\frac{(1-\tilde{\kappa}^2)\beta}{(1-\kappa^2)\beta_i}}\right)}\eta_i \tag{10}$$

[9]Note that the price signal z was derived by assuming that traders have different private valuations. Since the public signal z aggregates all private information perfectly, rational traders would discard their private signal s_i and instead have homogenous valuations. Therefore, Assumption 1 avoids artificially breaking the indifference in the trading decision along the now uninformative private signal s_i to maintain the informative equilibrium.

is the orthogonalized signal, which has been cleaned from correlated noise ε. This confirms the initial conjecture that at the symmetric equilibrium ($\forall i : \beta_i = \beta$), a more positive realization of the private signal s_i also leads to a higher private valuation since $\bar{s}_i$ is increasing in η_i.

Uniqueness Finally, the price is equal to the private valuation of the marginal trader, who is indifferent between buying or not as in (3) and observed the private signal $s_i = z$. Therefore, trying to orthogonalize s_i with z as in (10) leaves the signal unchanged and the marginal trader conditions directly on s_i as if it was $\bar{s}_i$.[10] This result is captured in the following proposition.

Proposition 1. *Given $\beta > 0$, observing the price P is informationally equivalent to observing the signal $z \sim \mathcal{N}(\theta, \frac{1-\kappa^2}{\beta}\sigma_\varepsilon^2)$. In the unique equilibrium with $\forall i : \beta_i = \beta$, in which demand $x(s_i, P)$ is non-increasing in P, the price is equal to the valuation of the trader with the private signal $s_i = \hat{s}(P)$, leading to the price*

$$P(z) = \tilde{\mathbb{E}}\left\{\pi(\theta)|s_i = z, z\right\}. \tag{11}$$

As can be seen now, Assumption 1 does not only motivate trade and costly information acquisition but also distorts asset prices. Traders who suffer from stronger overconfidence perceive their private signal as being more independent of the price signal z and, therefore, put a larger weight on it when forming their valuations. This leads to a price distortion, as a rational trader would discard her private signal s_i after observing z, as s_i becomes fully uninformative.[11] As a result, the price will *overreact* to both negative and positive realizations of z.[12]

3 Equilibrium Characterization

3.1 Minimal Degree of Overconfidence

An often articulated critique of models with behavioral frictions is that the severity and persistence of behavioral deviations from the rational benchmark are unrealistic.

[10]More formally, the orthogonalization for (10) is done through $\bar{s}_i = \frac{s_i - az}{1-a}$ where $a = \sqrt{\frac{(1-\tilde{\kappa}^2)\beta}{(1-\kappa^2)\beta_i}}$. As $s_i = z$ for the marginal trader, it follows that $\bar{s}_i = s$.

[11]That s_i becomes uninformative after observing z is easy to see from the fact that at the symmetric equilibrium ($\forall i : \beta_i = \beta$) $s_i = z + \frac{\kappa}{\sqrt{\beta}}\eta_i$, i.e., s_i is a noisier version of z.

[12]See Albagli, Hellwig, and Tsyvinski (2021) for a more in-depth discussion of this distortion in a model with rational and noise traders.

Therefore, it is desirable to keep behavioral frictions to a minimum while maintaining a similar set of results. I investigate in the following how far $\tilde{\kappa}$ needs to deviate from κ for the equilibrium to exist. I first investigate a setting where agents receive a costless signal with fixed information precision β and consider then the case with costly information acquisition. Moreover, I conduct comparative statics on β.

3.1.1 Exogenous Information and The Rational Limit

If traders receive a costless signal with exogenous precision β, then any $\tilde{\kappa} > \kappa$ is sufficient to generate trade. This result follows directly from (10), as the orthogonalized signal $\bar{s}_i$ remains informative as long as each trader believes her private signal to be at least infinitesimally more idiosyncratic than other traders' signals.

How traders process their private signal s_i naturally has an effect on the market-clearing price. If traders' overconfidence increases, they will perceive their private signal as being more independent of the price signal z and, therefore, put a larger weight on it when forming their valuations. As traders become increasingly rational ($\tilde{\kappa} \to \kappa$), this distortion vanishes, and in this sense, asset prices become more efficient:

$$\lim_{\tilde{\kappa} \to \kappa} P(z) = \tilde{\mathbb{E}} \left\{ \pi(\theta) | z \right\}. \tag{12}$$

Asymptotically rational traders put an infinitesimal weight on their private signal, sufficient to generate trading but insufficient to distort asset prices. Consequently, the market price behaves as if the only source of information was the public signal z as in (9). Note that (12) is identical to the price that a rational market maker would set who observed all private signals $\{s_i\}$.

3.1.2 Endogenous Information and Private Information as a Signal of Correlated Noise

When traders instead need to acquire costly information, substantially stronger overconfidence is needed to guarantee the existence of an equilibrium. To see this, note that the private signal s_i serves a dual function. Whenever β_i is sufficiently low ($\beta_i < \hat{\beta} = \frac{1-\tilde{\kappa}^2}{1-\kappa^2}\beta$), a positive realization of s_i is more likely to stem from high correlated noise ε than a high fundamental θ. In this case, an increase in the private signal s_i can *decrease* the private valuation of trader i when holding the price signal z fixed.[13] This negative effect can

[13]In a first pass, a more positive realization of s_i gets translated into higher posterior beliefs about θ and ε. This new information about ε is used to filter the price signal z, which *decreases* the posterior

be seen from (10), which is decreasing in idiosyncratic noise η_i when β_i is sufficiently low. If β_i goes to zero, the signal $\bar{s}_i$ can be reformulated as

$$\lim_{\beta_i \to 0} \bar{s}_i = \theta - \frac{\tilde{\kappa}}{\sqrt{1-\tilde{\kappa}^2}} \sqrt{\frac{1-\kappa^2}{\beta}} \eta_i \tag{13}$$

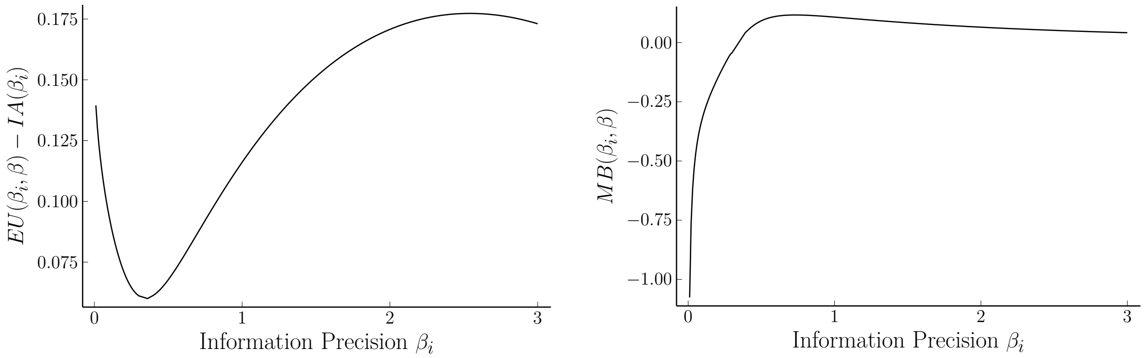
Even as s_i becomes an infinitely noisy signal of θ, it becomes more informative of correlated noise ε. This information can be used to filter the price signal z, which also contains correlated noise ε, thus yielding a more precise estimate of θ.

Therefore, traders face not only a trade-off between precision and cost when choosing β_i, but also between learning about ε and θ. In particular, initially increasing β_i can leave traders with *less* information about θ than they would learn if they chose $\beta_i \to 0$ to maximize their information about ε. The net effect of increasing β_i is only positive once it crosses the threshold $\hat{\beta}$.

The trade-off is highlighted in Figure 1. In the left panel, the expected utility (4) is initially decreasing in β_i and has a local maximum at $\beta_i \to 0$. The right panel shows that the marginal benefit (5) is initially negative and turns only positive for $\beta_i > \hat{\beta}$. Therefore, any symmetric equilibrium ($\forall i : \beta_i = \beta$) must be robust to traders free-riding on the public signal z ($\beta_i \to 0$) and using s_i to learn about the correlated noise ε. In this case, the conditions for the existence of the symmetric equilibrium as in (6) are fulfilled.

Note that $\beta \to 0$ cannot be an equilibrium, as the public signal z becomes infinitely noisy and free-riding on z impossible. As prices become uninformative about θ, the private signal s_i can no longer be used to filter correlated noise ε out of z, and traders switch to increasing the precision of their private signal. Formally, this can be seen from (13) becoming infinitely noisy for $\beta \to 0$, such that any $\beta_i > 0$ would yield a more informative signal. Therefore, if the marginal costs of acquiring the initial units of information are sufficiently low, a no-information equilibrium cannot exist. However, a symmetric equilibrium with $\beta > 0$ does not have to exist either. If free-riding on the price signal z is attractive at the symmetric equilibrium, no equilibrium exists at all.

estimate of θ through z. If s_i is sufficiently informative about ε (and uninformative about θ), this effect can dominate.



(a) Expected Utility

(b) Marginal Benefit

Figure 1: Expected Utility and Marginal Benefit for $\tilde{\kappa} < 1$.

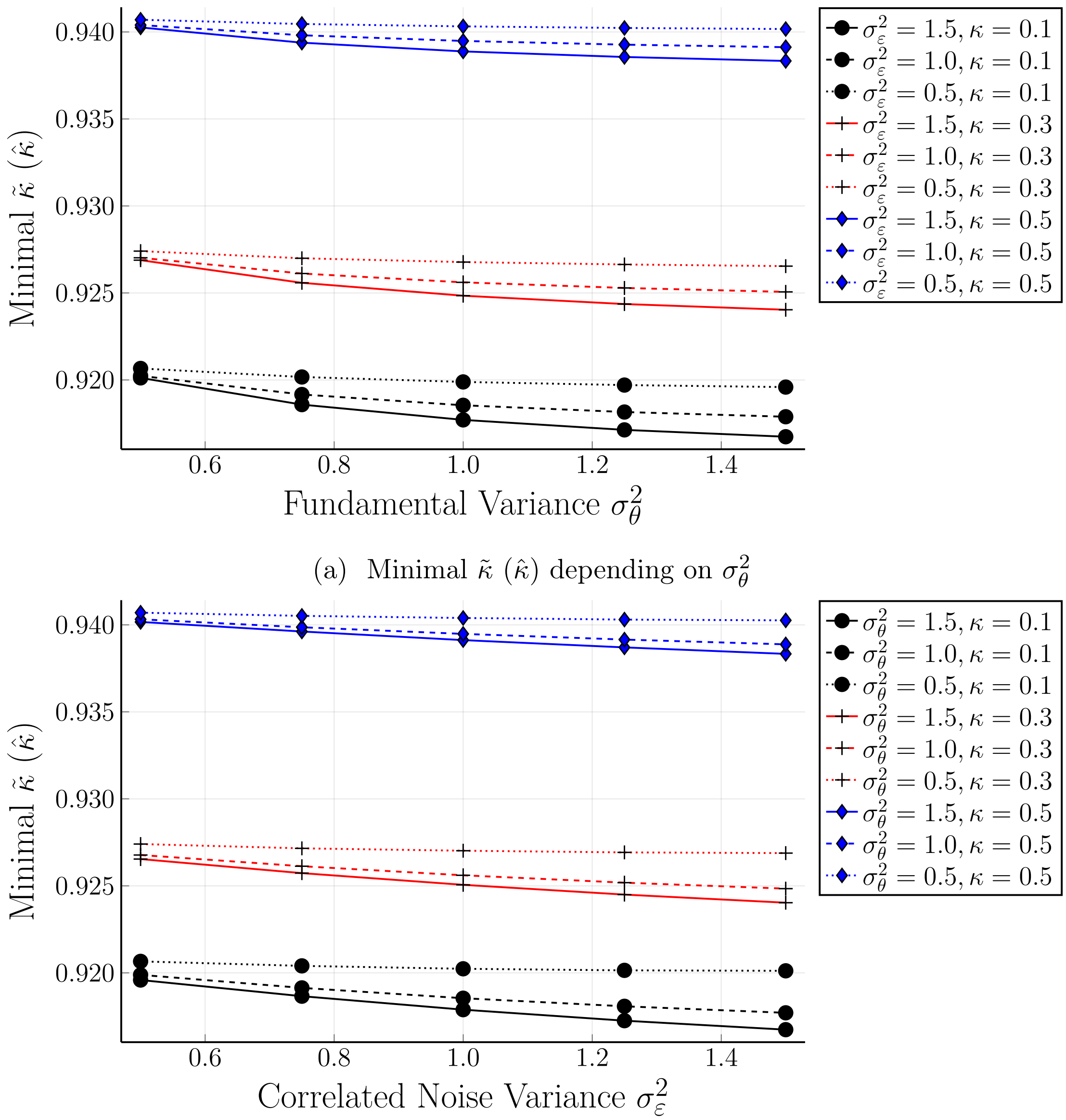
Notes: Expected utility $\widetilde{EU}(\beta_i, \beta)$ minus information acquisition costs $IA(\beta_i)$ and marginal benefit $\widetilde{MB}(\beta_i, \beta)$ of increasing β_i for a given level of market information precision β.

3.1.3 Minimal Overconfidence

As traders become increasingly overconfident ($\tilde{\kappa} \to 1$), they believe that their private signal s_i does not contain any correlated noise. Therefore, traders believe that increasing β_i always leads to more precise information on θ. As (13) shows, the closer $\tilde{\kappa}$ is to unity, the less valuable is the private signal for learning about correlated noise. Using continuity arguments, a symmetric equilibrium can exist whenever $\tilde{\kappa}$ is above some minimal value $\hat{\kappa} < 1$, as free-riding on the price signal z becomes less attractive as $\tilde{\kappa}$ increases.

Figure 2 shows how $\hat{\kappa}$, the minimal $\tilde{\kappa}$ for which the symmetric equilibrium exists, depends on κ, σ_θ^2, and σ_ε^2. The first result is that for different combinations of parameters, overconfidence needs to be substantial to incentivize information acquisition and deter free-riding on public information ($\hat{\kappa} >> \kappa$). Furthermore, for the parameters considered, $\hat{\kappa}$ decreases in σ_θ^2 and σ_ε^2. Figure 3 provides the corresponding equilibrium β for each combination of parameters for $\tilde{\kappa} = \hat{\kappa}$. The main takeaway is that β is increasing in uncertainty through higher σ_θ^2 and σ_ε^2. A lower κ increases the amount of correlated noise that enters the price as seen in (9), which makes acquiring precise information more attractive as the price deviates more strongly from its fundamental value.

Finally, the cost of information acquisition affects $\hat{\kappa}$ in two ways. For an individual trader, lower costs make it relatively more attractive to acquire more precise information in comparison to free-ride on the price signal z, lowering $\hat{\kappa}$. However, if

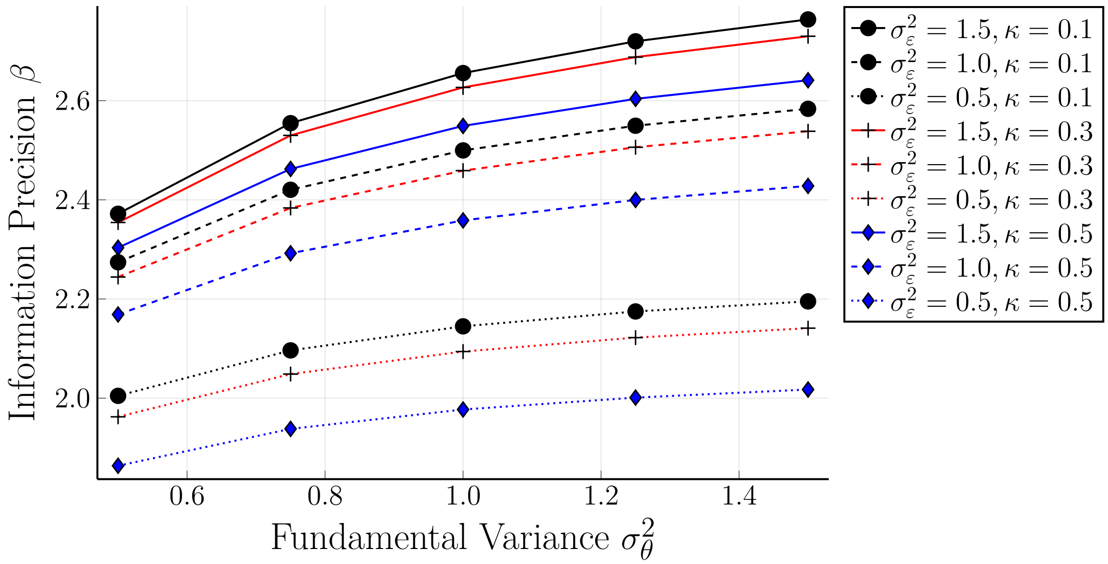


(a) Minimal $\tilde{\kappa}$ ($\hat{\kappa}$) depending on σ_θ^2

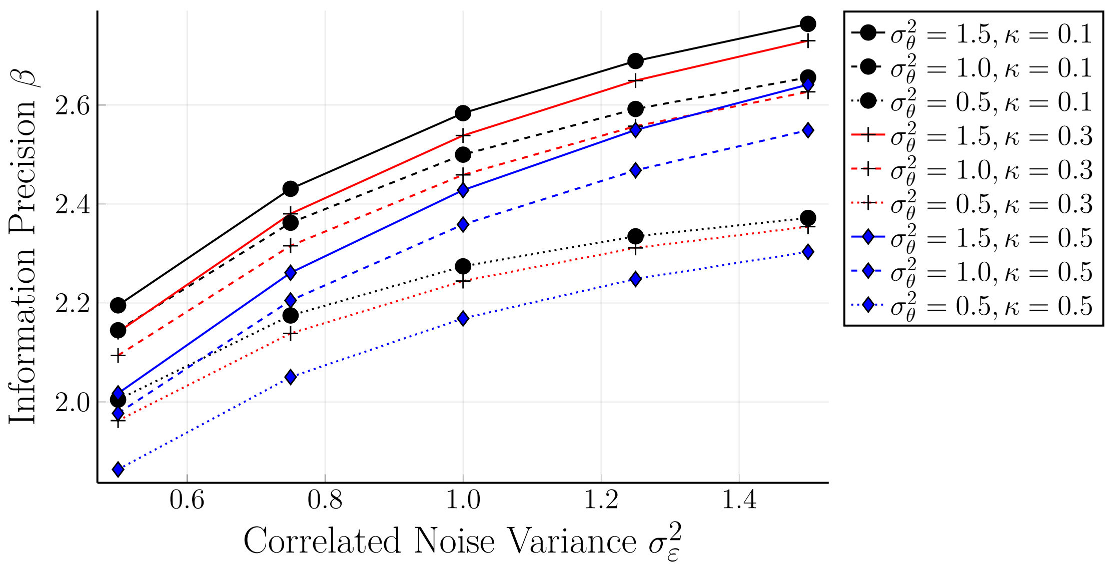
(b) Minimal $\tilde{\kappa}$ ($\hat{\kappa}$) depending on σ_ε^2

Figure 2: Comparative Statics for Minimal Overconfidence $\hat{\kappa}$.

Notes: Substantial amounts of overconfidence are needed when information acquisition is endogenous. The minimal $\tilde{\kappa}$ is falling in σ_θ^2 and σ_ε^2, but increasing in κ.



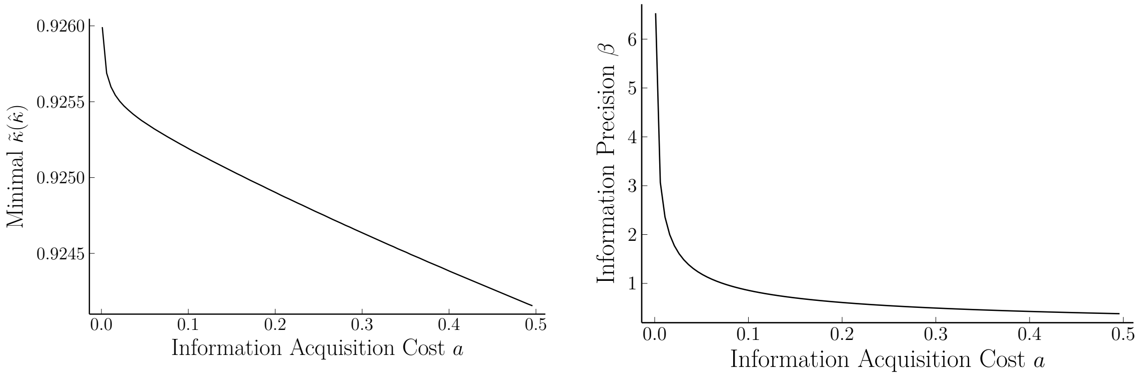
(a) Information precision β depending on σ_θ^2.



(b) Information precision β depending on σ_ε^2.

Figure 3: Comparative Statics for β at $\tilde{\kappa} = \hat{\kappa}$.

Notes: Equilibrium information precision β depending on σ_θ^2, σ_ε^2, and κ at the minimal $\tilde{\kappa}$ ($\hat{\kappa}$).



(a) Minimal $\tilde{\kappa}$ ($\hat{\kappa}$)

(b) Information Precision β

Figure 4: Comparative Statics on Information Acquisition Cost.

Notes: Define the information acquisition cost as $IA(\beta_i) = a\beta_i^b$. Then, the minimal $\tilde{\kappa}$ ($\hat{\kappa}$) is decreasing and information acquisition β is increasing in a. For this graph $\kappa = 0.3$, $\tilde{\kappa} = \hat{\kappa}$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $b = 2$.

all traders acquire more precise information, the whole market becomes more efficient, which lowers the perceived trading profits of each individual trader. Around the symmetric equilibrium ($\forall i : \beta_i = \beta$), an increase in β *decreases* the precision of $\bar{s}_i$ as in (10), but *increases* the precision of the information that free-riding traders can extract as in (13). This channel tends to increase $\hat{\kappa}$ if the marginal cost of information acquisition increases. In Figure 4, the second effect dominates, and $\hat{\kappa}$ is increasing in the marginal cost of information acquisition.

3.2 Information Acquisition and Overconfidence

Whereas overconfidence motivates information acquisition in the first place, it is less clear whether a marginal increase in overconfidence ($\tilde{\kappa}$) encourages further information acquisition or instead leaves traders satisfied with a less precise private signal. Note that $\tilde{\kappa}$ affects price informativeness only through the information choice β_i, as $\tilde{\kappa}$ has no direct influence on z and, therefore, on what an objective observer learns from the price. The following Lemma captures the main channel through which $\tilde{\kappa}$ affects the information acquisition decision.

Lemma 1. *Denote the orthogonalized signal as* $\bar{s}_i = a + \frac{\eta_i}{\zeta_i}$ *where*

$$\zeta_i = \frac{\sqrt{\beta_i}}{\tilde{\kappa}} \left(1 - \sqrt{\frac{(1-\tilde{\kappa}^2)\,\beta}{(1-\kappa^2)\,\beta_i}}\right).$$

Then,

(i) a higher β_i *increases the precision of* $\bar{s}_i$ *once* $\beta_i \geq \hat{\beta} = \frac{1-\tilde{\kappa}^2}{1-\kappa^2}\beta$*:*

$$\left.\frac{\partial \zeta_i}{\partial \beta_i}\right|_{\beta_i \geq \hat{\beta}} > 0.$$

(ii) at the symmetric equilibrium ($\forall i : \beta_i = \beta$)*, higher overconfidence* $\tilde{\kappa}$ *increases the precision of* $\bar{s}_i$*:*

$$\left.\frac{\partial \zeta_i}{\partial \tilde{\kappa}}\right|_{\beta_i = \beta} > 0.$$

(iii) higher overconfidence $\tilde{\kappa}$ *leads to smaller marginal effects of increasing* β_i *on the precision of* $\bar{s}_i$*:*

$$\frac{\partial^2 \zeta_i}{\partial \beta_i \partial \tilde{\kappa}} < 0.$$

The above Lemma shows that both an increase in $\beta_i \geq \hat{\beta}$ and $\tilde{\kappa}$ at the symmetric equilibrium lead to trader i believing that her private signal s_i became more informative about θ. An increase in β_i makes trader i's private signal s_i less noisy, whereas an increase in $\tilde{\kappa}$ makes trader i believe that s_i is less correlated to the price signal z, making information extraction about θ easier. However, a larger β_i increases the precision of trader i's private information by less the more overconfident (larger $\tilde{\kappa}$) trader i is. The reason is that an increase in β_i reduces the noise in s_i and at the same time lowers s_i's correlation with the price signal z, which both increase the precision of trader i's private information. However, if the correlation is already small, then the correlation-reducing effect is limited, decreasing the overall precision-increasing effect of choosing a higher β_i. The last point leads directly to the following proposition.

Proposition 2. *If expected trading profits* (4) *are concave in the precision of trader* i*'s private signal* $\bar{s}_i$ *around the symmetric equilibrium* ($\beta_i = \beta$)*, then the individual* *information choice* β_i *is decreasing in* $\tilde{\kappa}$*,*

$$\left.\frac{\partial \beta_i}{\partial \tilde{\kappa}}\right|_{\beta_i=\beta} < 0.$$

The intuition for Proposition 2 is that overconfidence $\tilde{\kappa}$ and information acquisition β_i are substitutes, which discourages information acquisition when overconfidence $\tilde{\kappa}$ is high. Formally, an increase in overconfidence $\tilde{\kappa}$ increases the precision of $\bar{s}_i$, which lowers the marginal benefit of further increasing its precision.[14] If the marginal effect of increasing β_i on $\bar{s}_i$'s information precision was constant, this alone would lead to a decrease in trader i's choice of β_i to realign the lower marginal benefit with the unchanged marginal cost. Additionally, Lemma 1 (iii) shows that the marginal effect of increasing β_i is *decreasing* in overconfidence $\tilde{\kappa}$, which amplifies the crowding-out effect of higher overconfidence on information acquisition.

Whereas this mechanism operates on the individual level, Figure 5 shows that the symmetric information choice β can also decrease when all traders become more overconfident at the same time. Therefore, markets in which traders suffer more from severe overconfidence can be less informative. This finding contrasts with the effect of overconfidence in models with risk-averse traders (e.g., Ko and Huang 2007; Peress 2014), where overconfident traders take more aggressive trading positions, making prices more informative.

3.3 Can Overconfidence Persist?

Another natural objection to models with behavioral frictions is that long-lived agents should eventually learn from their mistakes and become rational. In this case, traders would eventually learn the true composition of their private information if they repeatedly traded. Therefore, imperfect and possibly biased priors about noise composition in the private signal alone cannot sustain informed trading in this model.

To make this point, consider an infinite repetition of the static model in discrete time, $t = 1, 2, 3 \ldots \infty$. Long-lived traders can buy a risky asset every period. The properties of the asset and market are otherwise the same. Let trader i be uncertain about the composition of noise κ in her private signal (2). After observing $\{\theta_t, \varepsilon_t, s_{it}\}_{t=1,\ldots,T}$

[14] The marginal benefit of increasing the precision of $\bar{s}_i$ must eventually tend to zero, as the potential trading profits are bounded from above due to exogenous position limits.

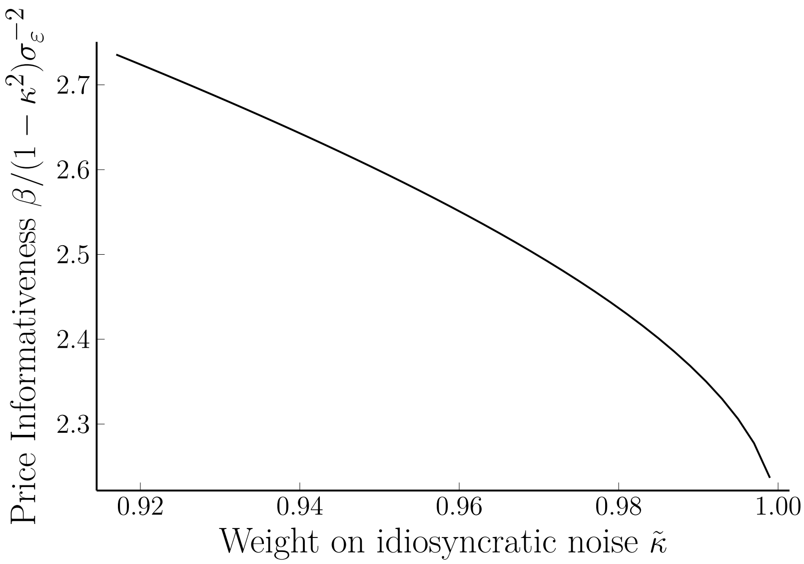


Figure 5: Information Precision Choice β and $\tilde{\kappa}$.

Notes: As traders become more overconfident, they find it less attractive to acquire precise information. The result is that price informativeness decreases in the market. For this figure, $\kappa = 0.3$ and $\hat{\kappa} = 0.92$.

for a long time, estimating

$$\sqrt{\beta_{it}}(s_{it} - \theta_t) = \alpha + \gamma\varepsilon_t + \nu_t \tag{14}$$

using OLS yields consistent estimates of α and γ. As traders gather more experience, the estimates converge to $\hat{\alpha} \to 0$ and $\hat{\gamma} \to \sqrt{1-\kappa^2}$. The error term $\nu_t = \kappa\eta_{it}$ is independent from ε_t by assumption.

The issue of overconfident traders learning from their mistakes can be circumvented methodologically by assuming that traders are short-lived and each period replaced by new overconfident traders. However, the literature shows that agents may not learn the truth under certain assumptions even after many periods, e.g., when agents have uncertainty about the underlying distribution of variables (Acemoglu, Chernozhukov, and Yildiz 2016).

Even if overconfident traders did not learn, a related argument is that they may not survive in the market as their mistakes are costly, which eventually deplete their wealth. Several papers suggest that less than fully rational traders can survive for a number of reasons (for an overview, see Dow and Gorton 2006). For example, arbitrageurs' unwillingness to trade aggressively against noise, e.g., due to short horizons (De Long et al. 1990), limits-to-arbitrage such as risk-aversion of arbitrageurs or imperfect information (Shleifer and Vishny 1997) or adjustments in the trading strategy by rational traders (Kyle and Wang 1997; Benos 1998). Moreover, overconfident or noise traders may use riskier strategies, which yield a higher return in the short-term (e.g., De Long et al. 1990). Finally, Hirshleifer and Luo (2001) argue that overconfident traders may be better able to exploit mispricing caused by noise or liquidity traders, allowing them survive in the long-run.

4 Applications

Studying the relationship between trading and information acquisition in financial markets and behavioral biases yields relevant insights in its own right, as such deviations from rationality receive increasing attention in financial settings over the last decades (for a survey, see Hirshleifer 2015). Apart from this direct interpretation, the model can be used to study more general settings and questions, in which overconfidence plays the role of a modeling device to motivate trading and information acquisition. As it turns out, the model adds tractability by simplifying the market-clearing condition, which is showcased in the example of trader heterogeneity and aggregate resource constraint. A larger extension is covered in the following section, in which traders have position limits that depend on the price and their trading capital.

4.1 Trader Heterogeneity

Not all traders in financial markets have access to the same information technologies. The most striking difference is between retail traders, who trade in their free time, and specialized hedge funds that may use elaborate machine learning algorithms and large quantities of data to inform their trading decisions. The model allows studying the effects of such heterogeneity in information technologies and their impact on market efficiency.

The setup is identical to before, except that two types of traders $j \in \{A, B\}$ are active in the market. The two groups are subject to group-specific sentiment shocks $\varepsilon^j \sim \mathcal{N}(0, \sigma^2_{\varepsilon^j})$ and information precisions β^j.[15] Moreover, the overconfidence assumption is

[15] Traders may also vary in how overconfident they are, as in Assumption 1. A more general case is covered in Section A.B.

simplified such that traders believe their signal to be fully idiosyncratic.[16]

Assumption 2 (***Overconfidence***)**.** *Trader* i *of group* j *believes the information structure to be*

$$s_i^j = \theta + \frac{\eta_i^j}{\sqrt{\beta_i^j}}$$
$$s_{-i}^j = \theta + \frac{\eta_{-i}^j + \varepsilon^j}{\sqrt{\beta_{-i}^j}}.$$

Following the same steps as before, demands by both groups clear the market,

$$D^A(\theta, \varepsilon^A, P) + D^B(\theta, \varepsilon^B, P) = 1, \tag{15}$$

where demand by group j can be derived by assuming that all traders in group j with a private signal $s_i^j > \bar{s}^j(P)$ buy one unit of the asset:

$$D^j(\theta, \varepsilon^j, P) = 1 - \Phi\left(\sqrt{\beta^j}\left(\hat{s}^j(P) - \theta\right) - \varepsilon^j\right). \tag{16}$$

Rearranging and applying the inverse of the standard normal cdf leads to the price signal as a function of the thresholds $\hat{s}^j(P)$

$$z^{Het} = \left(1 + \sqrt{\frac{\beta^B}{\beta^A}}\right)^{-1}\left[\hat{s}^A(P) + \sqrt{\frac{\beta^B}{\beta^A}}\hat{s}^B(P)\right] = \theta + \frac{\varepsilon^A + \varepsilon^B}{\sqrt{\beta^A} + \sqrt{\beta^B}} \tag{17}$$

The price signal z^{Het} has a similar form to the case with one group of traders as in (9). An increase in the information precision of either group reduces the correlated noise of both types in the price signal. Since the thresholds are linked through the market-clearing condition, they only signal information jointly. The uniqueness result of Proposition 1 also extends to this case, and the price is equal to the marginal trader's valuation of either group:

$$j \in \{A, B\} : \tilde{\mathbb{E}}^j\left\{\pi(\theta) | s_i^j = \hat{s}^j(P), z\right\} = P. \tag{18}$$

The relevant spillovers from one group of traders to another are the same as in

[16] This assumption guarantees that traders process their private signal only as being informative about θ but not about the correlated noise ε^j.

the model with a single trader type. If one group acquired more precise information (e.g., β^A increased), prices become more informative, and traders in group B adjust their information acquisition in response. If information precisions across groups are substitutes, then an increase of information acquisition in one group leads to decreased information acquisition in the other group. Similarly, in a model with one trader type, lower information acquisition costs may encourage information acquisition at the individual level. However, a more precise price signal due to more market-wide information acquisition dampens this effect.

A parsimonious setup entails one rational group A that is unaffected by correlated noise ($\sigma^2_{\varepsilon^A} \to 0$) and another boundedly-rational group B with overconfident beliefs as in Assumption 1 or 2. An appealing feature of this setup is that all traders behave identically ex-ante as everyone believes to be a member of group A. Moreover, this setup shows that not all traders need to be overconfident to motivate trade and information acquisition. Any particular split in the population between rational and overconfident traders is merely chosen to maintain the normality of the price signal z^{Het}.

The model allows studying various settings with heterogeneity between traders, which is relevant when some shocks affect one group more strongly than the other. For example, the recent abundance of data and more sophisticated algorithms may benefit more institutional investors. In contrast, retail investors may not be able to process data but participate due to overconfidence. Furthermore, including a group of rational traders may be attractive as it allows to compute a measure of trading profits and think about heterogeneity and inequality as in Mihet (2020), as rational traders would make profits at the expense of overconfident traders.

4.2 Aggregate Resource Constraints

Conventional models of noisy financial markets study two types of agents. First, rational traders with private information, who are limited in their ability to eliminate mispricing due to limits to arbitrage (Shleifer and Vishny 1997). Second, noise traders buy or sell assets randomly and keep prices from being fully revealing, providing an incentive to acquire information for rational traders. Whereas this approach can be used productively for partial equilibrium analysis, it can lead to difficulties when considering general equilibrium settings. In particular, noise traders add and remove resources from the economy, which is an unappealing feature when the economy is otherwise closed.

This problem can be avoided by letting traders invest in many markets and allowing for an endogenously determined interest rate as in Kantorovitch (2021). With boundedly-rational traders, a more positive aggregated realization of correlated noise increases the traders' demand. The corresponding price increase is dampened by a higher interest rate, which leads to a heavier discounting of future payoffs. In this way, prices can never exceed the traders' total resources, which allows using the model as a building block in macroeconomic models. Still, aggregate noise shocks can be a source of uncertainty, as traders do not necessarily have perfect information about aggregate shocks.[17]

5 Funding Constraints

The baseline model assumed exogenous position limits in terms of units of the asset, an assumption that can be found in many models with risk-neutral traders (Dow, Goldstein, and Guembel 2017; Albagli, Hellwig, and Tsyvinski 2021). In reality, however, traders have finite private capital, and margin requirements limit the funds that traders can raise for investment (Brunnermeier and Pedersen 2009).

The following presents a model with position limits that depend on private trading capital and the asset price while maintaining the normality of the price signal. After introducing the model, I focus on the role of disagreement in forming asset prices.

5.1 Model

In this model, heterogeneously informed traders are limited in their investment capacity by their private trading capital.[18] Limited trading capital by itself may depress asset prices akin to "cash-in-the-market"-pricing (Allen and Gale 1994), where the asset may trade below its fundamental value due a lack of liquidity. Paired with heterogeneous beliefs as in Fostel and Geanakoplos (2012) and Simsek (2021), price-dependent limits to traders' investment capacity will play an important role in determining asset prices *at all times*. For example, if traders have plenty of trading capital, optimists will buy up the whole asset supply, thereby inflating the price. In contrast, if traders have relatively little trading capital, optimists alone will not clear the market, and

[17] A similar result is derived in Albagli, Hellwig, and Tsyvinski (2017). In their setting, households split their savings between an informed hedge fund sector and a loss-making and randomly trading mutual fund sector, whereas in Kantorovitch (2021) households manage their investments autonomously.

[18] Abstracting away from borrowing simplifies the analysis while maintaining the main intuition. Traders' private wealth can be thought of as their maximal capital, including borrowing.

increasingly pessimistic traders need to buy to absorb the total asset supply, which depresses the market-clearing price. The model is introduced more formally in the following.

5.2 Traders

There are overlapping generations of traders indexed by $i \in [0, 1]$. Time is discrete and infinite. Traders live for two periods, are risk-neutral and patient. Trader i's utility function, who is born in period t is

$$U_{it} = \tilde{\mathbb{E}}_{it} \left\{ C_{it+1} \right\} - IA(\beta_{it}), \tag{19}$$

where C_{it+1} is trader i's consumption at the end of period $t+1$ and $IA(\beta_{it})$ are information acquisition costs for a given information precision β_{it}. When young, traders each receive wealth W, which they can use to buy assets or invest in a risk-less bond with return $R > 1$. Traders cannot short-sell.[19]

5.3 Assets

Each period a single perfectly dividable asset with monotonically increasing payoff function $\pi(\theta_t)$ is sold by the old to the young, where $\theta_t \overset{iid}{\sim} \mathcal{N}\left(\bar{\theta}, \sigma_\theta^2\right)$ determines the asset's fundamental value. The payoff is weakly positive for all realizations of θ_t, such that $\forall \theta_t \in \mathbb{R} : \pi(\theta_t) \geq 0$.[20] Additionally, I assume that the asset does not have a guaranteed payoff, such that $\lim_{\theta_t \to -\infty} \pi(\theta_t) = 0$.[21] Traders learn about the fundamental θ_t in period t, but the corresponding payoff $\pi(\theta_t)$ is realized at the beginning of period $t+1$ before the old sell the asset to the young.

[19] Restricting short-selling simplifies the analysis, but can be introduced as long as the volume of short-selling is constrained by the amount of trading capital.

[20] Ruling out negative payoffs does not substantially affect the results. Assets with negative payoffs can be studied as long as their payoff is positive in some states. In that case, some infinitely optimistic traders will always attribute a positive value to the asset, which will keep the asset price positive in all states.

[21] Ruling out safe payoffs allows to present the mechanism in the cleanest way.

5.4 Information Structure

Traders can exert effort to acquire a noisy signal of θ_t,

$$s_{it} = \theta_t + \frac{\eta_{it} + \varepsilon_t}{\sqrt{\beta_{it}}}, \tag{20}$$

where $\eta_{it} \overset{iid}{\sim} \mathcal{N}(0,1)$ is idiosyncratic noise, which is independently distributed among traders. In contrast, correlated noise $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ affects all traders equally. The precision of trader i's signal is captured by β_{it}. The correlated noise component ε_t can be interpreted as a form of sentiment which drives fluctuations in asset prices away from their fundamental value. To provide incentives for trading and information acquisition, traders are assumed to be overconfident.

Assumption 3 (***Overconfidence***)**.** *Trader i believes the information structure to be*

$$\begin{aligned} s_{it} &= \theta_t + \frac{\eta_{it}}{\sqrt{\beta_{it}}} \\ s_{-it} &= \theta_t + \frac{\eta_{-it} + \varepsilon_t}{\sqrt{\beta_{-it}}}. \end{aligned}$$

This simplified version of the overconfidence assumption guarantees that traders perceive the noise in their private signal s_{it} as fully independent of what they learn from the asset price ($\tilde{\kappa} = 1$ in the baseline model).[22] This overconfidence motivates traders to exert costly effort to increase their information precision, as they expect to be able to buy when asset prices are depressed due to a negative correlated noise shock.

5.5 Trader's Problem

Traders solve the following problem,

$$\max_{\beta_{it}} \quad \tilde{\mathbb{E}} \left\{ \max_{x_{it}} \tilde{\mathbb{E}} \left\{ x_{it} \left(\pi\left(\theta_t\right) + P_{t+1} - RP_t \right) | s_{it}, P_t \right\} \right\} + RW - IA\left(\beta_{it}\right) \tag{P2.3}$$

$$s.t. \quad x_{it} \in \left[0, \frac{W}{P_t}\right] \tag{21}$$

$$\beta_{it} \geq 0. \tag{22}$$

[22] This formulation is also used in Kantorovitch (2021).

The trader decides first on information precision β_{it}, which determines the precision of her private signal s_{it} and, therefore, the perceived ability to trade profitably. At the trading stage, trader i decides on how many units of the asset to buy conditional on realizations of the private signal s_{it} and asset price P_t. Different from the baseline model, the position limits in (21) now depend on both the price P_t and the available trading capital W. As a result, traders can buy at most W/P_t units of the asset.

5.6 Equilibrium

Trading The trader's problem is solved in reverse chronological order. First, taking the information precision β_{it} as given, the buying decision is

$$x\left(s_{it}, P_t\right) = \begin{cases} 0 & \text{if } \tilde{\mathbb{E}}\left\{\pi\left(\theta_t\right) + P_{t+1} | s_{it}, P_t\right\} < RP_t \\ \in \left[0, \frac{W}{P_t}\right] & \text{if } \tilde{\mathbb{E}}\left\{\pi\left(\theta_t\right) + P_{t+1} | s_{it}, P_t\right\} = RP_t \\ \frac{W}{P_t} & \text{if } \tilde{\mathbb{E}}\left\{\pi\left(\theta_t\right) + P_{t+1} | s_{it}, P_t\right\} > RP_t \end{cases}. \tag{23}$$

Trader i buys zero units if her valuation is below the price, is indifferent between buying or not when her valuation equals the price, and spends her whole wealth W if her valuation exceeds the price.

Information Acquisition Trader i chooses her information precision β_{it} to improve her ability to identify profitable trading opportunities. The first-order condition for the information production decision is obtained after plugging (23) into (P2.3). Evaluating expectations with respect to the realizations of idiosyncratic noise η_{it} leads to the probability of buying $\mathcal{P}\{x_{it} = \frac{W}{P_t}\}$. Taking the information acquisition decision of other traders as given ($\beta_{-it} = \beta_t$) and taking the partial derivative with respect to β_{it} leads to the marginal benefit of information acquisition,

$$\widetilde{MB}\left(\beta_{it}, \beta_t\right) = \tilde{\mathbb{E}}\left\{\frac{\partial \mathcal{P}\left\{x_{it} = \frac{W}{P_t}\right\}}{\partial \beta_{it}} \frac{W}{P_t}\left(\pi\left(\theta_t\right) + P_{t+1} - RP_t\right)\right\}. \tag{24}$$

The marginal benefit of acquiring information consists of three parts. The first is the change in the probability of buying in state $(\theta_t, \varepsilon_t)$ given information choices (β_{it}, β_t). The position size W/P_t determines the stake of trader i and scales any trading profits. Finally, trading profits given by the difference between the payoff plus the resale price $\pi\left(\theta_t\right) + P_{t+1}$ and the opportunity cost of buying RP_t.

Market-Clearing In the symmetric equilibrium ($\forall i : \beta_{it} = \beta_t$), traders spend their total trading capital W whenever their private signal is above some threshold, $\hat{s}\left(P_t, W\right)$. Normalizing the asset supply to one and summing up demands of all traders with a private signal about the threshold leads to the market-clearing condition,

$$\frac{W}{P_t}\left(1 - \Phi\left(\sqrt{\beta_t}\left(\hat{s}\left(P_t, W\right) - \theta_t\right) - \varepsilon_t\right)\right) = 1, \tag{25}$$

which allows solving for the threshold directly,

$$\hat{s}\left(P_t, W\right) = \theta_t + \frac{\varepsilon_t + \Phi^{-1}\left(1 - \frac{P_t}{W}\right)}{\sqrt{\beta_t}}. \tag{26}$$

The identity of the marginal trader now also depends on the ratio between price and wealth. If the price P_t is large relative to total wealth W, most traders need to buy to clear the market. Therefore, the marginal trader needs to be close to the bottom of the trader distribution, i.e., she must be pessimistic about the asset.

Price Signal Traders learn from the price P_t, which is equivalent to observing the noisy signal,

$$z_t = \hat{s}\left(P_t, W\right) - \frac{\Phi^{-1}\left(1 - \frac{P_t}{W}\right)}{\sqrt{\beta_t}} = \theta_t + \frac{\varepsilon_t}{\sqrt{\beta_t}}. \tag{27}$$

I call z_t the *price signal* and expectations condition on z_t instead of P_t. Note that although prices depend on trading capital W, the price signal is invariant to changes in W and remains always normally distributed.[23] The normality of the price signal is maintained due to the focus of a single trader type, which keeps the following analysis tractable.

Uniqueness The following proposition shows that the equilibrium is unique for a given symmetric information precision β_t. Moreover, the price P_t is equal to the valuation of the *marginal trader* who is just indifferent between buying or not buying and who observed the private signal $s_{it} = \hat{s}\left(P_t, W\right)$. Any trader who is more optimistic than the marginal trader ($s_{it} > \hat{s}\left(P_t, W\right)$) buys W/P_t shares, whereas more pessimistic traders invest their trading capital in the risk-less bond.

Proposition 3. *Given $\beta_t > 0$, observing P_t is equivalent to observing the signal $z_t \sim \mathcal{N}(\theta_t, \sigma_\varepsilon^2/\beta_t)$. In the unique equilibrium, in which demand $x\left(s_{it}, P_t, W\right)$ is non-increasing in P_t, the price is equal to the valuation of the trader with the private signal*

[23] I demonstrate in the Appendix A.C that normality is lost if the model is populated by rational and noise traders.

$s_{it} = \hat{s}\left(P_t, W\right)$, *leading to the price*

$$P\left(z_t, W\right) = \frac{1}{R}\tilde{\mathbb{E}}\left\{\pi\left(\theta_t\right) + P_{t+1} | s_{it} = \hat{s}\left(P_t, W\right), z_t\right\}. \tag{28}$$

The price reflects the beliefs of the marginal trader $\hat{s}\left(P_t, W\right)$ and the price signal z_t. If due to positive news the price signal z_t increases, then all traders become more optimistic, and the price must increase in response. However, an increase in the price requires a change in the identity of the marginal trader. Due to the price increase, the joint trading capital of the previous buyers is insufficient, and more pessimistic traders need to buy to clear the market. Consequently, the effect of an increase in z_t on the price P_t is *dampened*, because the marginal trader becomes *more pessimistic* (i.e., $\hat{s}(P_t, W)$ as in (26) is decreasing in P_t).

Resale Price For simplicity, I assume that θ_t and ε_t are *iid*, which leads to the expected resale price

$$\mathbb{E}\left\{P_{t+1}\right\} = \frac{1}{R-1}\mathbb{E}\left\{\tilde{\mathbb{E}}\left\{\pi\left(\theta_t\right) | s_{it} = \hat{s}\left(P_t, W\right), z_t\right\}\right\}. \tag{29}$$

For a given β_t, the expected resale price (29) is uniquely determined. Whereas the LHS is monotonically increasing in $\mathbb{E}\left\{P_{t+1}\right\}$, the right hand side is monotonically decreasing. An increase in the resale price increases the price today, which leads to a downward shift in the identity of the marginal trader. As a result, the now more pessimistic marginal trader values the payoff $\pi(\theta_t)$ less.

5.7 Equilibrium Characterization

This section investigates the relationship between disagreement and the average price level and shows that a high expected resale price discourages information acquisition.

5.7.1 Information Precision and Disagreement

Changes to the information precision β_t have two effects on the market-clearing price. First, more precise information lets traders put more weight on the price signal z_t and their private information. Therefore, a higher β_t makes the price react more strongly to changes in the price signal z_t. Second, the precision of private information determines the degree of disagreement between traders. If traders disagree more, any change in the identity of the marginal trader to maintain market-clearing will have a stronger effect on the price. In particular, disagreement among traders must be hump-shaped in information precision β_t as seen in Figure 6a. As traders share a common prior, they perfectly agree if they do not acquire private information ($\beta_t = 0$). Similarly, if traders acquired perfect information, they would all learn the truth. Therefore, disagreement must take its maximum for an intermediate value of information precision β_t.

These two effects of changes in information precision β_t do not wash out when averaging over different realizations of the price signal z_t as seen in Figures 6b and 6c. In these figures, the expected price $\mathbb{E}\{P_{t+1}\}$ is shown as a function of a constant information precision in all coming periods ($\forall s \geq t : \beta_s = \beta$). Although the price does not feature informational feedback (e.g., through a managerial decision based on z_t), the expected *price level* moves with the constant information precision β.[24]

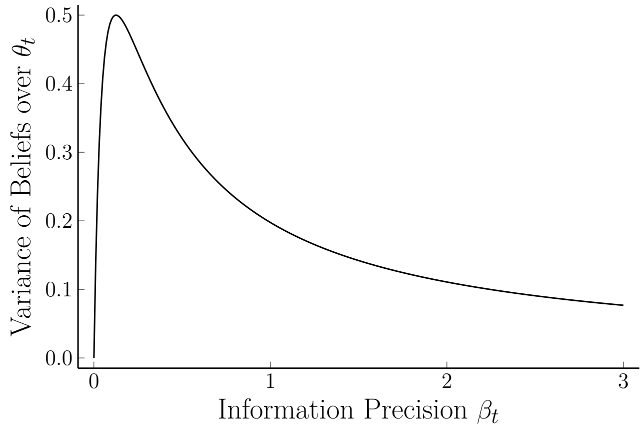
Whether the expected price is initially decreasing or increasing in β_t depends on how abundant or scarce trading capital is among traders. If traders have abundant resources to buy the asset, optimists ($\eta_{it} > 0$) will be able to buy up the market, which inflates the price. As traders acquire initial units of information, optimists become relatively more optimistic as traders disagree more intensely. Therefore, the price may increase initially in β_t as in 6b. In contrast, if trading capital is scarce, most traders need to buy to clear the market. Mechanically, the marginal trader must be relatively pessimistic ($\eta_{it} < 0$), and an initial increase in β_t depresses the price as seen in 6c. Conversely, a decrease in β_t may be associated with an increase in the expected price, as an uninformed trader would value the asset more highly than an informed yet pessimistic trader.

5.7.2 Information Acquisition and Resale Price

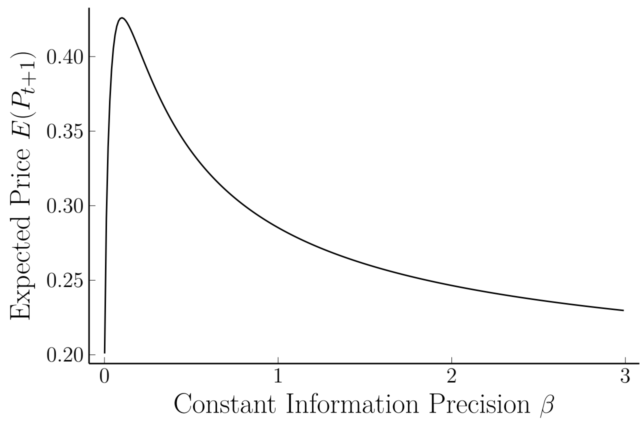
A high expected resale price can discourage information acquisition as it limits the trader's ability to exploit her information. Today's information helps forecast the payoff in the proximate periods but loses precision for the distant future if θ_t is not fully persistent. For example, an interest rate fall leads to a price increase primarily driven by less discounting on distant payoffs. As a result, traders must buy fewer units at a higher price, limiting their ability to speculate on proximate payoffs for which their information is most valuable.

Moreover, a higher resale price limits potential trading losses. From (23), trader i

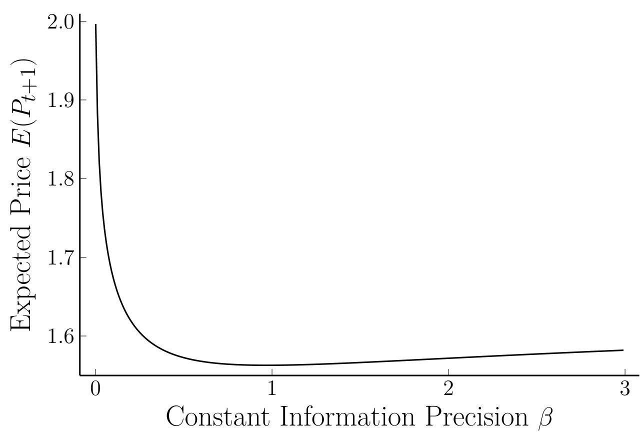
[24] The expected price would also depend on information precision β, if position limits were given in units of the asset, e.g., as in Albagli, Hellwig, and Tsyvinski (2021). The reason is a failure of the law of iterated expectations due to imperfect information aggregation, and its severity depends on the information precision β. This channel is also present here, but not of main interest.



(a) Disagreement is hump-shaped in β_t.



(b) Marginal traders are mostly optimists and trading capital is abundant.



(c) Marginal traders are mostly pessimists and trading capital is scarce.

Figure 6: Disagreement and Expected Price Depending on Information Precision.

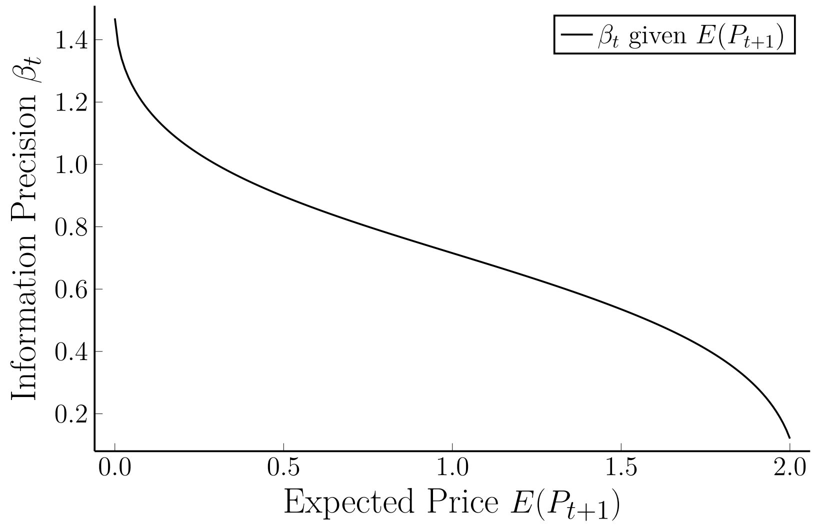


Figure 7: Information Precision Choice β_t is Decreasing in $\mathbb{E}\{P_{t+1}\}$.

buys whenever she expects to turn a profit, i.e., $\tilde{\mathbb{E}}\{\pi|s_{it}, P_t\} > RP - \mathbb{E}\{P_{t+1}\}$. However, an increase in the expected resale price $\mathbb{E}\{P_{t+1}\}$ translates to a less than one-for-one increase in P_t as the marginal trader's identity needs to adjust to maintain market-clearing. As a result, the next payoff stream $\pi(\theta_t)$ is valued by a more pessimistic trader and the RHS is decreasing in $\mathbb{E}\{P_{t+1}\}$ and trader i expects to turn a profit also for lower realizations of her private signal s_{it}. In the extreme case, as $\frac{1}{R}\mathbb{E}\{P_{t+1}\} \to W$, trader i finds it optimal to buy irrespective of her private information, leading to an information-insensitive buying decision and, therefore, no incentive for information acquisition.

Taken together, an increase in the expected resale price can discourage information acquisition, as seen in Figure 7. In the figure, the expected resale price is changed exogenously, and traders decide on their information precision β_{it} before trading takes place in period t.

5.7.3 Discussion

The presented model is stylized and seeks to capture the main mechanism linking disagreement through varying degrees of information acquisition and funding constraints, limiting the positions that traders can take. In the following, I discuss other topics that are not formally incorporated in the model yet relevant.

Interest Rates In this model, the interest rate R is exogenously given, although it is natural to assume that it is connected to the rate of return on investment. For example, when trading capital is scarce, expected returns should be high, and interest rates should increase. As a result, asset prices fall, and trading capital becomes relatively less scarce. Therefore, endogenously determined interest rates are an offsetting force to the relative scarcity or abundance of trading capital.

Funding Constraints and General Equilibrium Throughout, I have assumed that traders' trading capital W is constant. Suppose the described financial market is to be understood as a market for a single specific asset. In that case, liquidity cannot remain scarce indefinitely. At some point, traders will redistribute their capital towards illiquid markets to earn a premium. Instead, the asset is more abstract and representative of the whole stock market, the economy should accumulate capital over time as returns are high. In both cases, markets should adjust in the long-run and provide capital where its return is the highest.

Borrowing Constraints Borrowing between traders is a natural mechanism to redistribute trading capital from pessimistic to optimistic traders as in Simsek (2013). Such borrowing can avoid depressed prices due to a lack of trading capital in the hands of optimists, which could move asset prices closer to their objective valuation. The main complication in considering borrowing between traders is that the price signal z is normally distributed when trading capital is equally distributed among buyers, which is otherwise not guaranteed. One possibility is to consider an ex-ante security design problem, in which buyers issue and sell a junior tranche or credit default swaps to more pessimistic traders. As a result, buyers borrow from other traders and the same amount of trading capital.

Complexity Finally, the model sheds light on the incentives of asset originators in creating complex assets (Asriyan, Foarta, and Vanasco 2020) when investors are heterogeneously informed. Complex assets are more difficult to learn about, i.e., information acquisition costs are higher for such assets. If trading capital is sufficiently scarce (see Figure 6c), the model predicts that the asset originator may want to make information acquisition more costly, i.e., make the asset *more complex.* Such an increase in complexity can increase asset prices by reducing disagreement. On the other hand, if funding is abundant (see Figure 6b), an asset originator may prefer an intermediate level of information acquisition, which maximizes the price-inflating influence of optimistic traders. To achieve this goal, the asset originator may increase or decrease complexity, depending on the initial level of information acquisition.

6 Conclusion

I presented a model of financial markets with dispersed information similar to Albagli, Hellwig, and Tsyvinski (2021), in which overconfidence motivates trade and information acquisition. Traders overestimate the precision of their private signal, as they underestimate the correlation of their signal with the information that can be learned from the market-clearing price. I parametrize the degree of overconfidence, which governs the discrepancy between the true and perceived distribution of the private signal.

Whereas an infinitesimal amount of overconfidence is sufficient to generate trade when information is free, overconfidence must be substantial for equilibrium existence when information is costly. The reason is that the traders' private signals serve a dual function. The private signal is informative about the asset's fundamental and correlated noise, which affects the price. Information about the correlated noise can be used to better distinguish between price changes driven by fundamentals or noise.

As acquiring information reduces the noise in the signal, traders need to balance the cost and benefit of gathering more precise information and decide on learning about correlated noise or the fundamental. By not acquiring information, traders can choose to *free-ride* on the information acquisition of other traders. Since *free-riding* cannot be an equilibrium strategy, equilibrium existence requires that traders find information acquisition and learning about the fundamental sufficiently attractive. This is the case when traders believe their signal to be relatively uninformative about correlated noise, i.e., which is true for strongly overconfident traders.

I use the model to study several applications, for example, trader heterogeneity. Traders that suffer from varying degrees of overconfidence or have different levels of exposure to correlated noise can interact through the financial market while maintaining the normality of the price signal. This setup can be used to study the effects of technological changes (e.g., availability of new data sources and processing techniques) that disproportionally affect one group of traders (e.g., institutional investors in comparison to retail traders). Moreover, the model can be used as a building block in general equilibrium models, as shown in Kantorovitch (2021).

Finally, I study a setting in which traders' position limits depend on their available trading capital and the asset price. With such funding constraints, the effect of shocks on the asset price is dampened. For instance, positive news about the fundamental increase the asset price. As a result, more traders need to buy to clear the market. As the most optimistic traders buy first, these new buyers must be relatively more pessimistic than the previous buyers. As a result, the initial price increase is reduced. The model makes predictions on the relationship between disagreement and the asset price, as disagreement determines how extreme the beliefs of optimists or pessimists are. For instance, disagreement increases the asset price if price-setting traders are mostly optimists.

A Derivations

A.A Market-Clearing with One Trader Type

All traders with $s_i > \hat{s}(P)$ buy two units of the risky asset, such that

$$s_i > \hat{s}\left(P\right) \iff \eta_i > \frac{\sqrt{\beta}}{\kappa}\left(\hat{s}\left(P\right) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon \tag{30}$$

Given that $\eta_i \sim \mathcal{N}(0,1)$, the probability of buying can be written as

$$P(s_i > \hat{s}(P)) = 1 - \Phi\left(\frac{\sqrt{\beta}}{\kappa}\left(\hat{s}\left(P\right) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon\right) \tag{31}$$

Equating total demand to a normalized asset supply of one leads to the market-clearing condition, which allows to solve for $\hat{s}(P)$ directly

$$\begin{aligned}
2\left(1 - \Phi\left(\frac{\sqrt{\beta}}{\kappa}\left(\hat{s}\left(P\right) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon\right)\right) &= 1 \\
\iff \Phi\left(\frac{\sqrt{\beta}}{\kappa}\left(\hat{s}\left(P\right) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon\right) &= \frac{1}{2} \\
\iff \frac{\sqrt{\beta}}{\kappa}\left(\hat{s}\left(P\right) - \theta\right) - \frac{\sqrt{1-\kappa^2}}{\kappa}\varepsilon &= 0 \\
\iff \hat{s}\left(P\right) = \theta + \sqrt{\frac{1-\kappa^2}{\beta}}\varepsilon, &
\end{aligned} \tag{32}$$

which is also equal to the market signal z.

A.B Market-Clearing with Two Trader Types

Assume now instead that there are two groups of traders indexed by $j \in \{A, B\}$. The signal structure is as before but with group-specific weight on idiosyncratic noise κ^j, information precision β^j, and correlated noise shock ε^j. Furthermore, assume that the parameters for both groups are such that either both groups get more optimistic or pessimistic as their private signal increases. The former is the case when the private signal is sufficiently more informative about the fundamental θ than correlated noise ε. The latter is true for the opposite case.

The market-clearing condition is

$$\sum_{j\in\{A,B\}} D^j(\theta,\varepsilon^j,P)=1 \tag{33}$$

In the case when the traders' private signals are more informative about θ, all traders above the threshold $\hat{s}^j(P)$ buy,

$$D^j(\theta,\varepsilon^j,P)=1-\Phi\left(\frac{\sqrt{\beta^j}}{\kappa^j}\left(\hat{s}^j(P)-\theta\right)-\frac{\sqrt{1-(\kappa^j)^2}}{\kappa^j}\varepsilon^j\right). \tag{34}$$

In the other case, when the private signal is more informative about ε than θ, all traders with a signal below $\hat{s}^j(P)$ buy,

$$D^j(\theta,\varepsilon^j,P)=\Phi\left(\frac{\sqrt{\beta^j}}{\kappa^j}\left(\hat{s}^j(P)-\theta\right)-\frac{\sqrt{1-(\kappa^j)^2}}{\kappa^j}\varepsilon^j\right). \tag{35}$$

In both cases, rearranging (33) and applying the inverse of the standard normal cdf leads after some algebra to the price signal

$$\begin{aligned} z &= \left(1+\sqrt{\frac{\beta^B}{\beta^A}}\frac{\kappa^A}{\kappa^B}\right)^{-1}\left[\hat{s}^A(P)+\sqrt{\frac{\beta^B}{\beta^A}}\frac{\kappa^A}{\kappa^B}\hat{s}^B(P)\right] \\ &= \theta+\frac{\sqrt{1-(\kappa^A)^2}\varepsilon^A+\frac{\kappa^A}{\kappa^B}\sqrt{1-(\kappa^B)^2}\varepsilon^B}{\sqrt{\beta^A}+\sqrt{\beta^B}\frac{\kappa^A}{\kappa^B}}, \end{aligned} \tag{36}$$

Finally, since marginal traders in both groups must be indifferent between buying or not, the thresholds can be derived from

$$\mathbb{E}^j(\pi(\theta)|s_i^j=\hat{s}^j(P),P)=P. \tag{37}$$

A.C Limited Trading Capital and Noise Trading

In contrast to section 5, assume that traders receive a signal of the form

$$s_{it} = \theta_t + \frac{\eta_{it}}{\sqrt{\beta_{it}}}, \tag{38}$$

where $\eta_{it} \sim \mathcal{N}(0,1)$ is idiosyncratic noise. Additionally, noise traders demand a random number $\frac{W}{P_t}\Phi(u_t)$ of assets where $u_t \sim \mathcal{N}(0,\sigma_u^2)$. Following the same steps as before, the market-clearing condition is

$$\frac{W}{P_t}\left(1 - \Phi\left(\sqrt{\beta_t}\left(\hat{s}\left(P_t, W\right) - \theta_t\right)\right)\right) + \frac{W}{P_t}\Phi(u_t) = 1, \tag{39}$$

where the threshold can be derived as

$$\hat{s}\left(P_t, W\right) = \theta + \frac{\Phi^{-1}\left(1 - \frac{P_t}{W} + \Phi(u_t)\right)}{\sqrt{\beta}}. \tag{40}$$

From here it is evident that the threshold $\hat{s}\left(P_t, W\right)$ is not a linear function of normally distributed random variables and therefore itself not a normally distributed signal of θ.

A.D Proofs

Proof of Proposition 1. The proof is identical to the proof of Proposition 1 in Kantorovitch (2021) or the proof to Proposition 1 in Albagli, Hellwig, and Tsyvinski (2021). The only difference is the information structure. It is sufficient to show that at the symmetric equilibrium ($\forall i : \beta_i = \beta$), a more positive realization of s_i also leads to a higher private valuation. Since the public signal z as in (12) and the private signal s_i as in (2) both contain correlated noise ε, s_i can be orthogonalized to derive a signal that is independent of ε,

$$\bar{s}_i = \frac{s_i - \sqrt{\frac{(1-\tilde{\kappa}^2)\beta}{(1-\kappa^2)\beta_i}}z}{1 - \sqrt{\frac{(1-\tilde{\kappa}^2)\beta}{(1-\kappa^2)\beta_i}}} = \theta + \frac{\tilde{\kappa}}{\sqrt{\beta_i}\left(1 - \sqrt{\frac{(1-\tilde{\kappa}^2)\beta}{(1-\kappa^2)\beta_i}}\right)}\eta_i. \tag{41}$$

Therefore, observing $\{s_i, z\}$ is equivalent to observing $\{\bar{s}_i, z\}$. It follows that traders with a more positive realization of $\bar{s}_i$ indeed have a higher valuation, as their posterior on θ is increasing in $\bar{s}_i$. □

Proof of Lemma 1. Write the orthogonalized signal as

$$\bar{s}_i = a + \frac{\eta_i}{\zeta_i} \tag{42}$$

where

$$\zeta_i = \frac{\sqrt{\beta_i}}{\tilde{\kappa}} \left(1 - \sqrt{\frac{1-\tilde{\kappa}^2}{1-\kappa^2}} \sqrt{\frac{\beta}{\beta_i}} \right). \tag{43}$$

The results are then straightforward to show. (i) follows from

$$\frac{\partial \zeta_i}{\partial \beta_i} = \frac{1}{2\beta_i \tilde{\kappa}} > 0. \tag{44}$$

(ii) can be derived as

$$\begin{aligned}
\left.\frac{\partial \zeta_i}{\partial \tilde{\kappa}}\right|_{\beta_i=\beta} &= \sqrt{\beta} \left(\frac{\partial}{\partial \tilde{\kappa}} \frac{1}{\tilde{\kappa}} - \frac{\partial}{\partial \tilde{\kappa}} \sqrt{\frac{\tilde{\kappa}^{-2}-1}{1-\kappa^2}} \right) \\
&\propto -\frac{1}{\tilde{\kappa}^2} - \frac{\left(\tilde{\kappa}^{-2}-1\right)^{-\frac{1}{2}} \left(-2\tilde{\kappa}^{-3}\right)}{2\sqrt{1-\kappa^2}} \\
&= \frac{1}{\tilde{\kappa}^3 \sqrt{\tilde{\kappa}^{-2}-1}\sqrt{1-\kappa^2}} - \frac{1}{\tilde{\kappa}^2} \\
&= \frac{1}{\tilde{\kappa}^2} \left(\frac{1}{\sqrt{1-\tilde{\kappa}^2}\sqrt{1-\kappa^2}} - 1 \right) \\
&> 0.
\end{aligned} \tag{45}$$

Finally, (iii) stems simply from

$$\frac{\partial^2 \zeta_i}{\partial \beta_i \partial \tilde{\kappa}} = -\frac{1}{2\beta_i \tilde{\kappa}^2} < 0 \tag{46}$$

□

Proof of Proposition 2. Starting from a symmetric equilibrium ($\beta_i = \beta$), an increase in β_i cannot make trader i worse off. Given that the orthogonalized signal (10) becomes more precise as $\beta_i \geq \beta$ increases, traders can always add noise to their signal to maintain the same signal precision. However, it must be that $\beta_i \to \infty$ provides the highest level of utility, as trader i can then realize all trading profits, which is unattainable with a noisy signal. Therefore, the marginal benefit of increasing $\beta_i \geq \beta$ must be weakly positive everywhere and strictly positive somewhere.

Assume that expected trading profits as in (4) are concave in ζ_i around the symmetric equilibrium $\beta_i = \beta$. Moreover, a higher β_i increases expected trading profits around the symmetric equilibrium as $\zeta_i > 0$ and $\frac{\partial \zeta_i}{\partial \beta_i} > 0$ as in Lemma 1 (i). Then, an increase in $\tilde{\kappa}$ also increases ζ_i as in Lemma 1 (ii), which decreases the marginal benefit of increasing ζ_i further holding β_i constant due to the concavity of expected trading profits. Reinforcing this effect, the marginal effect of increasing β_i on ζ_i is also lower after an increase in $\tilde{\kappa}$ as follows from Lemma 1 (iii).

Taking these results together, if $\tilde{\kappa}$ increased for trader i, then her marginal benefit of increasing β_i must decrease around $\beta_i = \beta$. As a result, trader i chooses a lower information precision β_i. □

Proof of Proposition 3. The proof is identical to the proof of Proposition 1 in Kantorovitch (2021) or the proof to Proposition 1 in Albagli, Hellwig, and Tsyvinski (2021). The only difference is that traders' positions now depend on the price P_t and trading capital W. Therefore, it has to be verified that the price P_t is increasing in the price signal z_t. The proof begins in the following.

There must be a threshold $\hat{s}\left(P_t, W\right)$ such that all traders with $s_{it} \geq \hat{s}\left(P_t, W\right)$ find it profitable to buy $\frac{W}{P_t}$ units of the risky asset and otherwise abstain from buying. Different from before, the threshold also depends on trading capital W, as traders may be able to buy different quantities of the asset depending on how much capital they have and how expensive the asset is.

The price is equal to the valuation of the marginal trader as in (23) who is just indifferent between buying or not

$$P_t = \frac{1}{R} \tilde{\mathbb{E}} \left\{ \pi(\theta_t) + P_{t+1} | s_{it} = \hat{s}\left(P_t, W\right), P_t \right\}. \tag{47}$$

The price is now the solution to the implicit function above, as the price determines the threshold $\hat{s}\left(P_t, W\right)$ on the right-hand-side. This monotone demand schedule leads to total demand

$$D(\theta_t, \varepsilon_t, P_t, W) = \frac{W}{P_t} \left(1 - \Phi \left(\sqrt{\beta_t} \left(\hat{s}\left(P_t, W\right) - \theta_t \right) - \varepsilon_t \right) \right), \tag{48}$$

where $\Phi(\cdot)$ is the standard-normal cdf. Equalizing total demand with a normalized supply of one leads to the market-clearing condition

$$\frac{W}{P_t} \left(1 - \Phi \left(\sqrt{\beta_t} \left(\hat{s}\left(P_t, W\right) - \theta_t \right) - \varepsilon_t \right) \right) = 1, \tag{49}$$

with the unique solution

$$\hat{s}\left(P_t, W\right) = \theta_t + \frac{\varepsilon_t + \Phi^{-1}\left(1 - \frac{P_t}{W}\right)}{\sqrt{\beta_t}}. \tag{50}$$

As is evident now, the threshold cannot be expressed purely in terms of the fundamental shock θ_t, correlated noise ε_t, and information precision β_t. Therefore, it is also not possible to express the price P_t explicitly.

Nonetheless, the price is uniquely pinned down by the threshold $\hat{s}\left(P_t, W\right)$ and P_t is equal to the trading capital of all traders with a signal above the threshold. The price signal z_t can be extracted from the threshold,

$$z_t = \hat{s}\left(P_t, W\right) - \frac{\Phi^{-1}\left(1 - \frac{P_t}{W}\right)}{\sqrt{\beta_t}} = \theta_t + \frac{\varepsilon_t}{\sqrt{\beta_t}}. \tag{51}$$

The price is also invertible with respect to z_t. Consider an increase in z_t, which also increases the valuation of the marginal trader and the price P_t. However, the previous buyers are not able to clear the market anymore. Therefore, $\hat{s}\left(P_t, W\right)$ needs to shift down, accommodating the higher price P_t. It follows that there is a bijective mapping between the threshold $\hat{s}\left(P_t, W\right)$ and the price signal z_t as shown above. Therefore, the price P_t is invertible in z_t.

It follows that observing P_t is equivalent to observing $z_t \sim \mathcal{N}\left(\theta_t, \beta_t^{-1}\sigma_\varepsilon^2\right)$. Traders treat the signal z_t and their private signal $s_{it} \sim \mathcal{N}\left(\theta_t, \beta_{it}^{-1}\right)$ as mutually independent. Conditioning on the price signal z_t allows to rewrite the price as

$$P_t = \frac{1}{R}\tilde{\mathbb{E}}\left\{\pi(\theta_t) + P_{t+1} | s_{it} = \hat{s}\left(P_t, W\right), z_t\right\}. \tag{52}$$

where posterior expectations of trader ij are given by

$$\theta_t | s_{it}, z_t \sim \mathcal{N}\left(\frac{\sigma_\theta^{-2}\bar{\theta} + \beta_{it}s_{it} + \beta_t\sigma_\varepsilon^{-2}z_t}{\sigma_\theta^{-2} + \beta_{it} + \beta_t\sigma_\varepsilon^{-2}}, \frac{1}{\sigma_\theta^{-2} + \beta_{it} + \beta_t\sigma_\varepsilon^{-2}}\right). \tag{53}$$

The rest of the proof follows the proof of Proposition 1 in Kantorovitch (2021) or the proof to Proposition 1 in Albagli, Hellwig, and Tsyvinski (2021). □

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