IE 303 Modeling and Methods in Optimization

The Computing Project

Bilkent University
Department of Industrial Engineering
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Introduction and Motivation

This assignment will count as your homework grade. (10% overall) Even though modeling optimization projects can be tricky, solving the problem with a solver is an issue on its own for many applications, but especially for those involving integer variables. The goal of this computing project is to make students get used to state-of-the-art solvers, in particular, to `gurobipy`.

This assignment must be completed in groups of two. Write a *TYPED* report including your answers, methodology, and, if any, assumptions, and please convert it to a *.pdf* file. Do not include any code in your *.pdf* file. Instead, place your written material and `gurobipy` python codes in a zip file and submit it with the name format below. Send separate python scripts for each question.

NameSurname_NameSurname_303Project.zip

Then, please **send your .zip files to** bora.cetin@bilkent.edu.tr**!** The deadline for this assignment is **12.05.2023, Sunday, 23:59**. Your *.zip* file should include one *.pdf* file (your typed report) and four python scripts, including your models in `gurobipy` for each question. Good luck!

Grading Scheme

In addition to the questions below, you are getting 10% of your grade from formatting. Late submission grades are calculated according to the formula below. Let $\mathcal{G}$ be your original grade. Let x be the amount of late submission in hours. Then your final grade will be calculated as

$$\mathcal{G}'(x) = \begin{cases} \mathcal{G}e^{-\frac{x}{40\ln 8}} & \text{if } 0 \leq x \leq 24 \\ 0 & \text{if } x > 24 \end{cases}$$

So if you are about 24 hours late, then you lose approximately about 20% of your project grade. Submissions that are more than 24 hours late will not be accepted.

Question 0 (Install `gurobipy`)

(5pts) Install `gurobipy` (Gurobi Optimization Python API) to your computers. You should solve problems below via `gurobipy`.

Question 1 (Warm-Up)

(15pts) Consider the integer programming model below.

$$\begin{aligned} \textbf{maximize} \quad & 78x_1 + 63x_2 + 94x_3 \\ \textbf{subject to} \quad & 2x_1 + x_2 + 4x_3 \leq 51 \\ & x_1 + 3x_2 + 2x_3 \leq 41 \\ & x_1 + x_2 \geq 14 \\ & x_1, x_2, x_3 \in \mathbb{Z}_+ \end{aligned}$$

a) Solve it using `gurobipy`. Report the optimum solution.

b) Now, solve the LP relaxation of this problem. How far are you from the integer optimum solution? If your solution to this part is not an integer solution, add a valid inequality that is also a cutting plane to this problem, cutting away the LP relaxation optimum solution. You can use any method to find such a cut, but in your report you should show that your inequality is indeed *valid*.

Question 2 (The Locked Treasure Grid)

(30pts) You've found a map to a treasure hidden within a mysterious grid. The grid is composed of locked cells, each has a specific number of keys to collect. You start at the top-left corner of the grid with a certain number of keys. Each cell in the grid either gives you more keys or requires keys to move through. It means some cells might even take away keys!

- You can only go right or down.
- You can only visit one cell at a time.

Your goal is to navigate from the top-left corner to the bottom-right corner while maximizing the number of keys owned in the bottom-right corner. Let $\mathcal{K}$ be the matrix containing the number of keys you gain (or lose) when a cell is visited.

a) Formulate an Integer Programming Problem to solve the problem described above.

b) Using the model you have formulated in part *a*), solve the following instance.

$$\mathcal{K} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & -1 \\ 2 & 1 & 3 \\ 6 & -1 & 4 \end{bmatrix}$$

Question 3 (Wars in Lombardy)

(40pts) The year is 1423 and the storm is gathering around Lombaria! The ambitious Filippo Maria Visconti, Duke of Milan, have attacked you the fellow Venetians! You and your mercenary friends Tuna, Kaan, Ali, Kutup, Özgün and Muhittin are stuck in Lombardy, Italy. You need to somehow save the people of Lombardy from the cruel Duke. You are currently located in Pavia, and you can rescue people to come with you along your journey in Lombardy. Due to the logistic reasons, each of you have the a capacity to rescue 82 people during your route. Since the Duke is fast and furious, you only have time to visit 7 cities in total.

But you are lucky, all of you are Medieval IE/OR specialists! So, your job is to gather as many people as possible in order to collaborate and survive against the Milanese. In other words, you need to come up with a route that maximizes the number of people rescued along your route.

Rules

- From a city, you can only go to a neighboring city.
- You need to start from Pavia.
- The route is finished once you visit the 7^{th} city.

The region of Lombardy has 12 cities in total that can be seen in Figure 1.

Figure 1: Map of the region of Lombardy, Italy

Each city has a number of people residing in it as follows:

City	Population	City	Population
Pavia	86	Sondrio	18
Lodi	23	Lecco	34
Milan	326	Cremona	36
Varese	89	Mantua	41
Como	60	Brescia	127
Monza/Brianza	86	Bergamo	111

We also have some additional conditions. Ali knows someone named Sena in Milan, who must be rescued since she is the only medical scientist left in Lombardy. Moreover, Kutup knows a new companion of his, Bora, with a capacity to rescue 78 more people, who is living right next to the Lake Como. Özgün knows a person in Bergamo, Yiğit, who is among the population of Bergamo. Özgün informs you that if Yiğit is to come across with his 36 friends in Cremona, Yiğit is going to persuade them to leave your party and they will rush to the aid of the Venetians in the battlefield against the Milanese, so they will NOT be rescued in the end.

Solve this problem using *Mixed Integer Linear Programming* and report your solution after implementing it in `gurobipy`. Clearly indicate the visited cities in your reports.