# Dummit–Foote Exercises

January 9, 2025

# 1 Chapter 1: Introduction to Groups

## 1.1 Basic Axioms and Examples

## 1.2 Diherdral Groups

## 1.3 Symmetric Groups

## 1.4 Matrix Groups

**Problem 1.4.1.** $|\mathrm{GL}_2(\mathbb{F}_2)| = 6$.

**Problem 1.4.2.**

**Problem 1.4.3.**

**Problem 1.4.4.**

**Problem 1.4.5.**

**Problem 1.4.6.**

**Problem 1.4.7.** *For a prime $p$, $|\mathrm{GL}_2(\mathbb{F}_p)| = p^4 - p^3 - p^2 + p$.*

## 1.5 The Quaternion Group

## 1.6 Homomorphisms and Isomorphisms

## 1.7 Group Actions

# 2 Chapter 2: Subgroups

# 3 Chapter 3: Quotient Groups and Homomorphisms

# 4 Chapter 4: Group Actions

# 5 Chapter 5: Direct and Semidirect Products and Abelian Groups

# 6 Chapter 6: Further Topics in Group Theory

# 7 Chapter 7: Introduction to Rings

## 7.1 Basic Definitions and Examples

**Problem 7.1.1.** *Let $R$ be a ring with 1. Then $(-1)^2 = 1$ in $R$.*

**Problem 7.1.2.** *Let $R$ be a ring with 1. If $u \in R$ is a unit, then $-u$ is also a unit.*

# 8 Chapter 8: Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains

# 9 Chapter 9: Polynomial Rings

# 10 Chapter 10: Introduction to Module Theory

# 11 Chapter 11: Vector Spaces

# 12 Chapter 12: Module Theory over Principal Ideal Domains

# 13 Chapter 13: Field Theory

# 14 Chapter 14; Galois Theory

# 15 Chapter 15: Commutative Rings and Algebraic Geometry

# 16 Chapter 16: Artinian Rings, Discrete Valuation Rings, and Dedekind Domains

# 17 Chapter 17: Introduction to Homological Algebra and Group Cohomology

# 18 Chapter 18: Representation Theory and Character Theory

## 18.1 Linear Actions and Modules over Group Rings

## 18.2 Wedderburn's Theorem and Some Consequeces

## 18.3 Character Theory and the Orthogonality Relations

**Problem 18.3.1.** *Prove that* $\operatorname{tr} AB = \operatorname{tr} BA$ *for* $n \times n$ *matrices* $A$ *and* $B$ *with entries from any commutative ring.*

**Problem 18.3.2.**

**Problem 18.3.3.**

**Problem 18.3.4.**

**Problem 18.3.5.**

**Problem 18.3.6.** *Let* $G$ *be a finite group. Let* $\phi : G \to \mathrm{GL}(V)$ *be a representation with character* $\psi$ *over* $\mathbb{C}$*. Let* $W$ *be the subspace* $\{v \in V \mid \phi(g)(v) = v \text{ for all } g \in G\}$ *of* $V$ *fixed pointwise by all elements of* $G$*. Prove that* $\dim W = (\psi, \chi_1)$*, where* $\chi_1$ *is the principal character of* $G$*.*

# 19 Chapter 19: Examples and Applications of Character Theory