

# Dummit–Foote Exercises

January 6, 2025

## 1 Chapter 1: Introduction to Groups

### 1.1 Basic Axioms and Examples

### 1.2 Dihedral Groups

### 1.3 Symmetric Groups

### 1.4 Matrix Groups

**Problem 1.4.1.** *Prove that  $|\mathrm{GL}_2(\mathbb{F}_2)| = 6$ .*

### 1.5 The Quaternion Group

### 1.6 Homomorphisms and Isomorphisms

### 1.7 Group Actions

## 2 Chapter 2: Subgroups

### **3 Chapter 3: Quotient Groups and Homomorphisms**

## **4 Chapter 4: Group Actions**

## **5 Chapter 5: Direct and Semidirect Products and Abelian Groups**

## **6 Chapter 6: Further Topics in Group Theory**

## 7 Chapter 7: Introduction to Rings

## **8 Chapter 8: Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains**



## 9 Chapter 9: Polynomial Rings

## **10 Chapter 10: Introduction to Module Theory**

## **11 Chapter 11: Vector Spaces**

## **12 Chapter 12: Module Theory over Principal Ideal Domains**

## **13 Chapter 13: Field Theory**

## **14 Chapter 14; Galois Theory**

## **15 Chapter 15: Commutative Rings and Algebraic Geometry**

## **16 Chapter 16: Artinian Rings, Discrete Valuation Rings, and Dedekind Domains**



## **17 Chapter 17: Introduction to Homological Algebra and Group Cohomology**

## 18 Chapter 18: Representation Theory and Character Theory

### 18.1 Linear Actions and Modules over Group Rings

### 18.2 Wedderburn's Theorem and Some Consequences

### 18.3 Character Theory and the Orthogonality Relations

**Problem 18.3.1.** *Prove that  $\text{tr } AB = \text{tr } BA$  for  $n \times n$  matrices  $A$  and  $B$  with entries from any commutative ring.*

**Problem 18.3.2.**

**Problem 18.3.3.**

**Problem 18.3.4.**

**Problem 18.3.5.**

**Problem 18.3.6.** *Let  $G$  be a finite group. Let  $\phi : G \rightarrow \text{GL}(V)$  be a representation with character  $\psi$  over  $\mathbb{C}$ . Let  $W$  be the subspace  $\{v \in V \mid \phi(g)(v) = v \text{ for all } g \in G\}$  of  $V$  fixed pointwise by all elements of  $G$ . Prove that  $\dim W = (\psi, \chi_1)$ , where  $\chi_1$  is the principal character of  $G$ .*

## **19 Chapter 19: Examples and Applications of Character Theory**