Dummit–Foote Exercises

January 6, 2025

1 Chapter 1: Introduction to Groups

1.1 Basic Axioms and Examples

2 Chapter 2: Subgroups

3 Chapter 3: Quotient Groups and Homomorphisms

4 Chapter 4: Group Actions

5 Chapter 5: Direct and Semidirect Products and Abelian Groups

6 Chapter 6: Further Topics in Group Theory

7 Chapter 7: Introduction to Rings

8 Chapter 8: Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains

9 Chapter 9: Polynomial Rings

10 Chapter 10: Introduction to Module Theory

11 Chapter 11: Vector Spaces

12 Chapter 12: Module Theory over Principal Ideal Domains

13 Chapter 13: Field Theory

14 Chapter 14; Galois Theory

15 Chapter 15: Commutative Rings and Algebraic Geometry

16 Chapter 16: Artinian Rings, Discrete Valuation Rings, and Dedekind Domains

17 Chapter 17: Introduction to Homological Algebra and Group Cohomology

18 Chapter 18: Representation Theory and Character Theory

18.1 Linear Actions and Modules over Group Rings

18.2 Wedderburn's Theorem and Some Consequeces

18.3 Character Theory and the Orthogonality Relations

Problem 18.3.1. *Prove that* $\operatorname{tr} AB = \operatorname{tr} BA$ *for* $n \times n$ *matrices* A *and* B *with entries from any commutative ring.*

Problem 18.3.2.

Problem 18.3.3.

Problem 18.3.4.

Problem 18.3.5.

Problem 18.3.6. *Let* G *be a finite group. Let* $\phi : G \to \mathrm{GL}(V)$ *be a representation with character* ψ *over* $\mathbb{C}$. *Let* W *be the subspace* $\{v \in V \mid \phi(g)(v) = v \text{ for all } g \in G\}$ *of* V *fixed pointwise by all elements of* G. *Prove that* $\dim W = (\psi, \chi_1)$, *where* χ_1 *is the principal character of* G.

19 Chapter 19: Examples and Applications of Character Theory