Dummit–Foote Exercises

January 6, 2025

- 1 Chapter 1: Introduction to Groups
- 1.1 Basic Axioms and Examples

2 Chapter 2: Subgroups

3 Chapter 3: Quotient Groups and Homomorphisms

4 Chapter 4: Group Actions

5	Chapter 5: Direct and Semidirect Products and Abelian Groups

6 Chapter 6: Further Topics in Group Theory

7 Chapter 7: Introduction to Rings

8	Chapter 8: Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains

9 Chapter 9: Polynomial Rings

10 Chapter 10: Introduction to Module Theory

11 Chapter 11: Vector Spaces

12 Chapter 12: Module Theory over Principal Ideal Domains

13 Chapter 13: Field Theory

14 Chapter 14; Galois Theory

15 Chapter 15: Commutative Rings and Algebraic Geometry

16 Chapter 16: Artinian Rings, Discrete Valuation Rings, and Dedekind Domains

17 Chapter 17: Introduction to Homological Algebra and Group Cohomology

18 Chapter 18: Representation Theory and Character Theory

- 18.1 Linear Actions and Modules over Group Rings
- 18.2 Wedderburn's Theorem and Some Consequeces
- 18.3 Character Theory and the Orthogonality Relations

Problem 18.3.1. Prove that $\operatorname{tr} AB = \operatorname{tr} BA$ for $n \times n$ matrices A and B with entries from any commutative ring.

Problem 18.3.2.

Problem 18.3.3.

Problem 18.3.4.

Problem 18.3.5.

Problem 18.3.6. Let G be a finite group. Let $\phi: G \to \operatorname{GL}(V)$ be a representation with character ψ over \mathbb{C} . Let W be the subspace $\{v \in V \mid \phi(g)(v) = v \text{ for all } g \in G\}$ of V fixed pointwise by all elements of G. Prove that $\dim W = (\psi, \chi_1)$, where χ_1 is the principal character of G.

19 Chapter 19: Examples and Applications of Character Theory