DUMMIT–FOOTE EXERCISES

1. Chapter 1: Introduction to Groups

1.1. **Basic Axioms and Examples.**

2. Chapter 2: Subgroups

3. Chapter 3: Quotient Groups and Homomorphisms

4. Chapter 4: Group Actions

5. Chapter 5: Direct and Semidirect Products and Abelian Groups

6. Chapter 6: Further Topics in Group Theory

7. Chapter 7: Introduction to Rings

8. Chapter 8: Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains

9. Chapter 9: Polynomial Rings

10. Chapter 10: Introduction to Module Theory

11. Chapter 11: Vector Spaces

12. Chapter 12: Module Theory over Principal Ideal Domains

13. Chapter 13: Field Theory

14. Chapter 14; Galois Theory

15. Chapter 15: Commutative Rings and Algebraic Geometry

16. Chapter 16: Artinian Rings, Discrete Valuation Rings, and Dedekind Domains

17. Chapter 17: Introduction to Homological Algebra and Group Cohomology

18. Chapter 18: Representation Theory and Character Theory

18.1. **Linear Actions and Modules over Group Rings.**

18.2. **Wedderburn's Theorem and Some Consequeces.**

Date: January 6, 2025.

18.3. Character Theory and the Orthogonality Relations.

Problem 18.3.1. *Prove that* $\operatorname{tr} AB = \operatorname{tr} BA$ *for* $n \times n$ *matrices* A *and* B *with entries from any commutative ring.*

Problem 18.3.2.

Problem 18.3.3.

Problem 18.3.4.

Problem 18.3.5.

Problem 18.3.6. *Let* G *be a finite group. Let* $\phi : G \to \mathrm{GL}(V)$ *be a representation with character* ψ *over* $\mathbb{C}$. *Let* W *be the subspace* $\{v \in V \mid \phi(g)(v) = v \text{ for all } g \in G\}$ *of* V *fixed pointwise by all elements of* G. *Prove that* $\dim W = (\psi, \chi_1)$, *where* χ_1 *is the principal character of* G.

19. Chapter 19: Examples and Applications of Character Theory