AI Planning
Exercise Sheet 4

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Exercise 4.1

For easy readability let the tiles be referred to as b_1, b_2, w_1 and w_2 and the empty cell be referred to as e. Furthermore, let the actions move and jump be denoted as $m_c(t)$ and $j_c(t)$ respectively where c is the destination cell$\in \{1, 2, 3, 4, 5\}$ and t is the tile that is being relocated.
As an example, the initial state is:
b_1, b_2, w_1, w_2, e
If we then apply $j_5(b_2)$ we reach:
b_1, e, w_1, w_2, b_2

(a) Let $\lceil o \rceil$ denote the state s reached by applying the operation $o \in \{m_c(t), j_c(t)\}$.
$f(\lceil m_5(w_2) \rceil) = 1 + 4 = 5$
$f(\lceil j_5(w_1) \rceil) = 1 + 4 = 5$
$f(\lceil j_5(b_2) \rceil) = 2 + 2 = 4$
Apply $j_5(b_2)$ which results in s_1:
b_1, e, w_1, w_2, b_2
$f(\lceil m_2(b_1) \rceil) = 3 + 2 = 5$
$f(\lceil m_2(w_1) \rceil) = 3 + 2 = 5$
$f(\lceil j_2(w_2) \rceil) = 3 + 2 = 5$
$\lceil j_2(b_2) \rceil = I \in \mathit{closed}$
Apply $m_2(b_1)$ which results in s_2:
e, b_1, w_1, w_2, b_2
Apply $m_2(w_1)$ which results in s_3:
b_1, w_1, e, w_2, b_2
Apply $j_2(w_2)$ which results in s_4:
b_1, w_2, w_1, e, b_2
Expanding on s_2:
$\lceil m_1(b_1) \rceil = s_1 \in \mathit{closed}$
$f(\lceil j_1(w_1) \rceil) = 4 + 1 = 5$
$f(\lceil j_1(w_2) \rceil) = 5 + 1 = 6$
Expanding on s_3:
$f(\lceil j_3(b_1) \rceil) = 4 + 1 = 5$
$f(\lceil m_3(w_1) \rceil) = 4 + 2 = 6$
$f(\lceil m_3(w_2) \rceil) = 4 + 2 = 6$
$f(\lceil j_3(b_2) \rceil) = 4 + 3 = 7$

Expanding on s_4:
$f(\lceil j_4(b_1)\rceil) = 5 + 0 = 5$

Since h is goal aware and the minimum cost of an operator is 1 we're done at this point. There may be other solutions but none with a cost of less than 5. The resulting plan is: $j_5(b_2), j_2(w_2), j_4(b_1)$ with a total cost of 5 and a final state:
e, w_2, w_1, b_1, b_2

(b) Suppose we loosened the rules of our puzzle in the following way: there are infinitely many empty cells left of the leftmost b_i and right of the rightmost w_j. Furthermore jumps over more than 2 tiles are allowed while the calculation of cost follows the original rule but never exceeds 2.
In this relaxed setting the puzzle can always be solved at a cost of

$$h_r^*(s) = \sum_{i=1}^{\text{number of black tiles}} b_i \times \text{number of white tiles right of } b_i$$

Since this is a less restrictive setting $h_r^*(s) \leq h^*(s)$. Furthermore $h_r^*(s)$ is a more general variant of $h(s)$. In other words, for the specific case I we can say $h_r^*(s) = h(s)$ and therefore $h(s) \leq h^*(s)$.

Exercise 4.2

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