AI Planning
Exercise Sheet 7

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Exercise 7.1

(a) $\Pi' = \{V, I, O, \gamma\}$ with

- $V = \{above\text{–}a, above\text{–}b, above\text{–}c, below\text{–}a, below\text{–}b, below\text{–}c\}$
  $\mathscr{D}_{above\text{–}\Upsilon} = \{A, B, C, n\} \setminus \{\Upsilon\}$
  $\mathscr{D}_{below\text{–}\Upsilon} = \{A, B, C, t\} \setminus \{\Upsilon\}$
  where $\Upsilon \in \{A, B, C\}$

- $I(a) = 1$ for $a \in \{below\text{–}b = t, above\text{–}b = A, above\text{–}a = n,$
  $below\text{–}c = t, above\text{–}c = n\}$
  $I(a) = 0$ else

- $O = \{move\text{–}X\text{–}Y\text{–}Z, move\text{–}X\text{–}Table\text{–}Z, move\text{–}X\text{–}Y\text{–}Table\}$
  $move\text{–}X\text{–}Y\text{–}Z = \langle(below\text{–}X = Y) \land (above\text{–}X = n) \land (above\text{–}Z = n),$
  $(above\text{–}Y := n) \land (below\text{–}X := Z)\rangle$
  $move\text{–}X\text{–}Table\text{–}Z = \langle(below\text{–}X = t) \land (above\text{–}X = n) \land (above\text{–}Z = n),$
  $(below\text{–}X := Z)\rangle$
  $move\text{–}X\text{–}Y\text{–}Table = \langle(below\text{–}X = Y) \land (above\text{–}X = n),$
  $(above\text{–}Y := n) \ \land (below\text{–}X := t)\rangle$
  for pair-wise distinct $X, Y, Z \in \{A, B, C\}$

- $\gamma = (above\text{–}c = B) \land (above\text{–}a = C)$

And the addition[1] that every $above|below[:]= \Upsilon$ with $\Upsilon \in \{A, B, C\}$ implies its counterpart (e.g. $above\text{–}A[:]= B$ also tests/sets $below\text{–}B[:] = A$).

(b) The induced propositional planning task Π'' is the (regular) planning task $\Pi'' = \langle A', I', O', \gamma\rangle$, where

- $A' = \{(above\text{–}a, B), (abova\text{–}a, C), (above\text{–}a, n), (above\text{–}b, A), (above\text{–}b, C), (above\text{–}b, n),$
  $(above\text{–}c, A), (above\text{–}c, B), (above\text{–}c, n), (below\text{–}a, B), (below\text{–}a, C), (below\text{–}a, t),$
  $(below\text{–}b, A), (below\text{–}b, C), (below\text{–}b, t), (below\text{–}c, A), (below\text{–}c, B), (below\text{–}c, t)\}$

- $I'((v, d)) = 1 iff I(v) = d$
  $I'((v, d)) = 1$ for $(v, d) \in \{below\text{–}b, n), (above\text{–}b, A), (above\text{–}a, n), (below\text{–}c, t), (above\text{–}c, n)\}$

[1] to make this a bit less verbose and better readable

- $O' = \{move\text{–}X\text{–}Y\text{–}Z, move\text{–}X\text{–}Table\text{–}Z, move\text{–}X\text{–}Y\text{–}Table\}$
  $move\text{–}X\text{–}Y\text{–}Z = \langle (below\text{–}X, Y) \wedge (above\text{–}X, n) \wedge (above\text{–}Z, n),$
  $(above\text{–}Y, n) \wedge \neg(above\text{–}Y, X) \wedge$
  $(below\text{–}X, Z) \wedge \neg(below\text{–}X, Y)\rangle$
  $move\text{–}X\text{–}Table\text{–}Z = \langle (below\text{–}X, t) \wedge (above\text{–}X, n) \wedge (above\text{–}Z, n),$
  $(below\text{–}X, Z) \wedge \neg(below\text{–}X, t)\rangle$
  $move\text{–}X\text{–}Y\text{–}Table = \langle (below\text{–}X, Y) \wedge (above\text{–}X, n),$
  $(above\text{–}Y, n) \wedge \neg(above\text{–}Y, X) \wedge (below\text{–}X, t \wedge \neg(below\text{–}X, Y))\rangle$

- $\gamma = (above\text{–}c, B) \wedge (above\text{–}a, C)$

(c) To show: There is an isomorphism between Π'' and Π, therefore Π' and Π are equivalent.
$f : S \mapsto S''$, with $f(s) =$ replace all valiables $X\text{–}on\text{–}Y$ in s with
$(above\text{–}y, X) \wedge (below\text{–}x, Y)$ and all
$Z\text{–}clear$ in s with $(above\text{–}z, n)$
$g : O \mapsto O''$, with $g(o) =$ replace all valiables $X\text{–}on\text{–}Y$ in o with
$(above\text{–}y, X) \wedge (below\text{–}x, Y)$ and all
$Z\text{–}clear$ in o with $(above\text{–}z, n)$

Since both functions are just a relabeling of variables they have the required properties.

Exercise 7.2

(a) Since both h_1 and h_2 include the blank tile, $h_1 + h_2$ is not admissible.
Proof by counterexample:
$n = 1$, $m = 2$

$$s = \begin{matrix} t_0 & t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 & t_7 \\ t_8 & t_9 & t_{10} & t_{11} \\ t_{12} & t_{13} & b & t_{14} \end{matrix} \quad \alpha_2\text{-view} : \begin{matrix} t_0 & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & b & - \end{matrix} \quad \alpha_1\text{-view} : \begin{matrix} - & t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 & t_7 \\ t_8 & t_9 & t_{10} & t_{11} \\ t_{12} & t_{13} & b & t_{14} \end{matrix}$$

$h^*(s) = 1$
$h_1(s) + h_2(s) = 1 + 1 = 2$
$2 > 1$, therefore $h_1 + h_2$ is not admissible.

(b) Goal awareness: for every goal state s_γ of the full puzzle, $h_3(s_\gamma) = 0$ and $h_4(s_\gamma) = 0$. Hence $h_3 + h_4$ is goal aware.
Consistency: since a tile is accounted for in at most one of $\{h_3, h_4\}$, each step necessary to reach the goal counts as either 1 or 0 in $h_3 + h_4$. Therefore $h_3 + h_4$ is consistent.
Since it is goal aware and consistent it is admissible.