

# Data Analysis 2: Fundamentals of Statistics

Instructor: Arieda Muço, Fall 2017

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# What Is Data Analysis 2 ?

- Introductory course to statistics
  - Hands-on Exercises
    - R, Stata
  - Understand the structure of data
    - How it is born, how it is stored, how it can be accessed
    - Common issues with real-life data
  - Produce, visualize and understand essentials statistics
    - Distributions of variables,
    - Statistics describing aspects of the distributions
  - Understand some fundamental statistical concepts
    - Expected value, standard deviation, etc.

# Data Analysis 2 will help

- Support better decisions in
    - Business
    - Public Policy
    - And many other fields
  - Formulate hypothesis and test them
  - Importance of random sampling
  - Understand how biases or dealing carelessly with data can misrepresent findings
  - Small versus large sample size
    - Touch upon the big data revolution
-

# Data Can Be More Useful Than Other Evidence

*What stats allows you to do is not take things at face value.  
The idea that I trust my eyes more than the stats, I don't buy  
that because I have seen magicians pull rabbits out of hats  
and I just know that rabbit's not in there*

Billy Beane, interview

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# Statistics Meaning

- The word statistics is derived from the Italian word "stato" which means "state" and refers to a person involved in the affairs of state. Therefore, statistics originally meant the collection of facts useful to the "statista".
- Statistics in this sense was used in 16-th century Italy, then spread to France, Holland, and Germany.
- Surveys of people and property actually began in ancient times.
- Oftentimes, the data are summarized, displayed in meaningful ways and analyzed.

# Statistics and Information

- Today, statistics is not restricted to information about the state but exceed the realm of human endeavor.
- Neither do we restrict ourselves to merely collecting numerical information called data.
- Statistics is the science of generalizing from the data.



# Information about the course

- 6 lectures
  - 150 mins each
  - in classroom
- 3 practice sessions
  - 100 mins each
  - in computer labs
- Exam during the last lecture
  - Friday, October 27

# Grading

- Class participation
  - Quizzes in all coming lectures
  - Will be graded and count for the final grade
  - 3 group assignments
- Exam
  - Written examination
- Grade
  - First, you pass the written examination which counts for 60% of the course
  - 15 % will be from the quizzes
  - 25 % will be from the group assignments

# Background material

- No main textbook
- Extensive support material (Handouts and codes)
  - Slides

# Software

- R
  - Used by statisticians, data scientists, and economists
  - Free
  - Steep learning curve
- Stata
  - Used mainly by economists
  - Need to buy license (installed in CEU labs)
  - Easier to learn
- We will provide codes for all applications we cover
  - R and Stata as well (when possible)

# Ceu Moodle

- Separate site for
  - Data Analysis 1
  - Data Analysis 2
  - Data Analysis 3
  - Etc.
- Handouts, lecture slide shows, other supporting material
- Homework assignments
  - Use the site to turn in your homework
- Register to the course
- Check the site regularly
- Read your CEU email regularly

# Take away

- Data Analysis at CEU Econ
  - is an integrated sequence of many 2 credit courses
- By completing them you can
  - do meaningful data analysis on your own
  - understand other people's data analysis
    - Unless it's very complicated

# Take away

- DA2 is a stepping stone
  - To the fundamental statistical concepts used throughout
  - You will complement your software skills with DA1
- Passing DA2 should not be hard
  - If you read the material provided
  - Do the problem sets

# Classes are mandatory

- You can skip 25% of the classes
- I will know from your quizzes who participated in class
- If you don't attend lose 15 points from quizzes
- The quizz is going to be at a random time during the class
  - Begining
  - Middle
  - End



# Plan for the rest of the lecture

- Measures of centrality
- Measures of dispersion
- Probability (Bayes theorem)
- Distributions

# IMPORTANT!!

- Stata track will be held on computer labs
- R- track please bring your own computers
  - Install R, I recommend R-studio

Time to dive into some statistics

# Summary statistics

- A statistic is a single measure of some attribute of a **sample** (e.g., its arithmetic mean value). It is calculated by applying a **function** (statistical **algorithm**) to the values of the items of the sample, which are known together as a **set of data**.
- Basic summary statistics are the most widely used statistics to describe certain aspects of data.

# The mean (Example)

- Assume that you are arranging a trip to Vienna for New Years, and you are looking at hotel prices on a major hotel website.
- Suppose also that your only constraint: You are interested in staying strictly less than two kilometres away from the city center.

Table 1: Hotel Prices less than 2 km from the city centre

Hotel Name	Rating	Stars	Price (huf)	Dist center km
Hotel Lamee	4.5	4	148127	.2
Hotel Topazz	4.3	4	148127	.2
CH- Wellness Apartments	3.7	3	78702	.6
Hilton Vienna am Stadtpark	4.3	4	155528	.8
Derag Livinghotel An der Oper	4.6	4	131825	.8
Le Meridien Wien	4.4	5	285718	.8
Palais Hansen Kempinski Vienna	4.8	5	168646	.9
Hilton Vienna Plaza	4.6	4	155528	.9
Das Capri - Ihr Wiener Hotel	4.5	3	133526	1.2
Citadella Residence Appartments Vienna	5	4	90883	1.2
Royal Resort Apartments Urania	4.3	3.5	117116	1.3
Ruby Sofie Hotel Vienna	4.3	3.5	54342	1.4
Royal Resort Apartments Blattgasse	3.4	3.5	59339	1.4
NH Wien City	4	4	80888	1.7
Magdas Hotel	4	2	35697	1.8

# The Mean

- Is computed as the average of the values in the data at hand

$$\bar{x} = \frac{\sum x_i}{n}$$

- The mean hotel price for New Years Eve, the mean height of the class

$\bar{x}$       Sample average

$E[x]$       Expected value

# The Mean (Properties)

- It changes in the same way if we transform the variable in a linear fashion.

$$\frac{\sum(x_i + a)}{n} = \overline{x + a} = \bar{x} + a$$

- If we multiply a variable with a number, say b, its mean value gets multiplied by the same number b

$$\frac{\sum(x_i b)}{n} = \overline{x \cdot b} = \bar{x} \cdot b$$



# The Median

- The median is the middle value of the distribution in the sense that exactly half of the observations have lower value and the other half have higher value
- To compute the median we sort the values of our variable from the lowest to the highest
- When we have even numbers in our data, the median is the mean of the two middle values
- The median is usually less subjective to extreme values we have in our data

# The Mode

- The mode is the most frequent value in your dataset
- Sometimes we might have more than a mode in our dataset.

# Percentiles

- The P-th percentile of your data is the value below which lie P% of the numbers in the data. The position of the P-th percentile is given by:

$$\frac{(n+1)P}{100}$$

- where n is the number of observations in your dataset
- Find the 50 and the 80 percentile of the following data point
  - 1, 1, 1, 1, 1, 2, 3, 3, 7

# Percentiles (Answer)

- The 80 percentile is given by

$$\frac{(9+1)80}{100} = 3$$

- which is the number in the 8-th position of your data
- Find the 50 and the 80 percentile of the following data point
  - 1, 1, 1, 1, 1, 2, 3, 3, 7

# Quartiles

- Certain percentiles have greater importance than others as they break the distribution of data into 4 groups
- Quartiles are the percentage points that break down the dataset into quarters; first quarter, second quarter, third quarter, and fourth quarter
- The first quartile is the 25% percentile. Is that point below which lie 1/4 of your data
  - 1, 1, 1, 1, 1, 2, 3, 3, 7

# Quartiles

- The median is the 50-th percentile and the second quartile
- The third quartile is called the 75th percentile. Is the point below which lie 75% of the data.
- We often call 25th percentile as the lower quartile and the 75th percentile as the upper quartile

# Interquartile Range

- The interquartile range is the difference between the third and the first quartile.
- In the previous example the interquartile range is the difference between  $3-1=2$

# Range

- The range is the difference between the largest and the smallest observation in your data point
- This is an extremely rare event as this are fake data. Most of the times range and interquartile range do not coincide.



# Variance

- The variance of the observations is the average squared deviation of the data points from their mean.

$$Var(x) = \frac{\sum (x_i - \bar{x})^2}{n}$$

# Standard Deviation

- The standard deviation is the (positive) squared root of the variance of the data points.

$$Std(x) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

# Variance and Standard Deviation: Properties

- When we add a number to a variable its variance and standard deviation remain the same. (Can you say why?)
- When we multiply a variable with a number the variance is multiplied by the square of the number, and the standard deviation is multiplied by the absolute value of that number. (Can you prove this?)

# Histograms

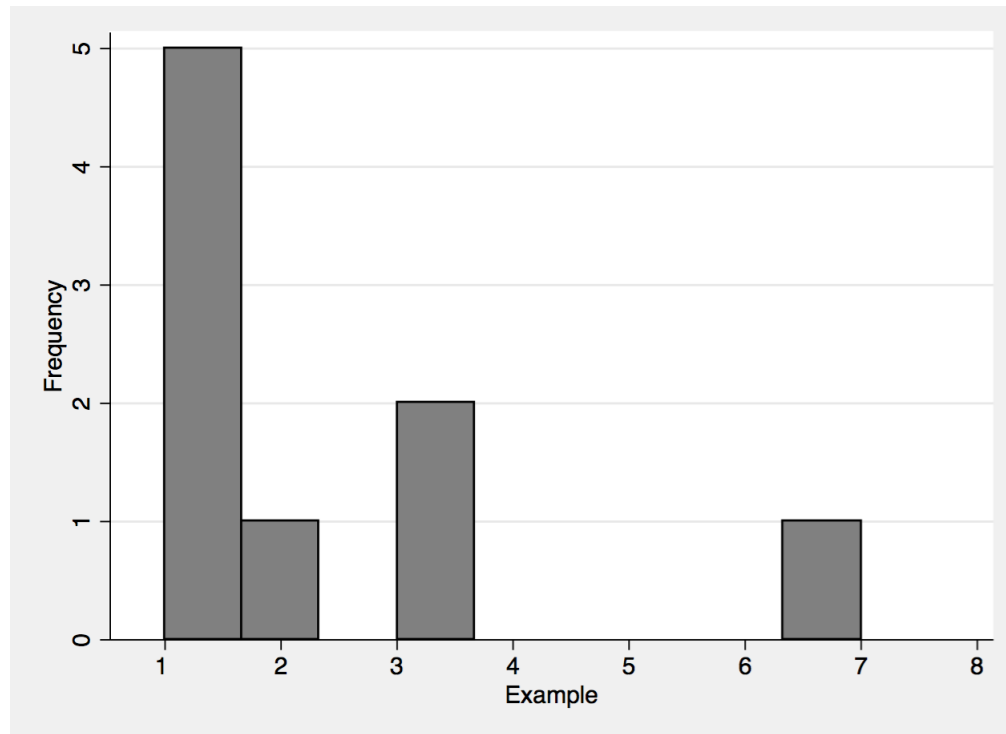
- An histogram is a plot made of bars of different heights.
- The height of each bar represents the frequency of values in each bar.
- Adjacent bars share sides.

# Histograms

- In our "silly" example from earlier on, how would the histogram look like?

# Histograms

- In our "silly" example from earlier on, how would the histogram look like?

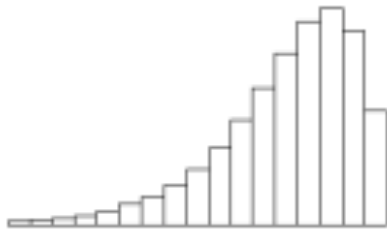


# Skewness

- Your data might be skewed in two ways; having a long left tail or having a long right tail.
  - In symmetric distributions mean = median (=mode)
- When it is skewed with a long right tail the mean is larger than the median.
- When it is skewed with a long left tail the mean is smaller than the median.

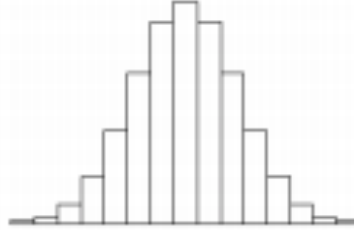
# Skewness

*Skewed Left*



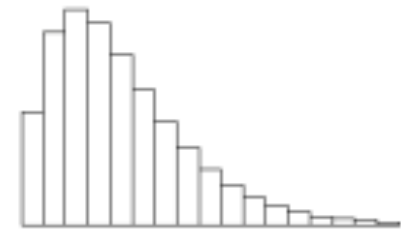
One Mode

*Symmetric*



Bell-Shaped

*Skewed Right*



One Mode



# The mean-median measure of skewness

- The mean -median measure of skewness captures this intuition, and it standardizes the mean-median difference by dividing it with the standard deviation.

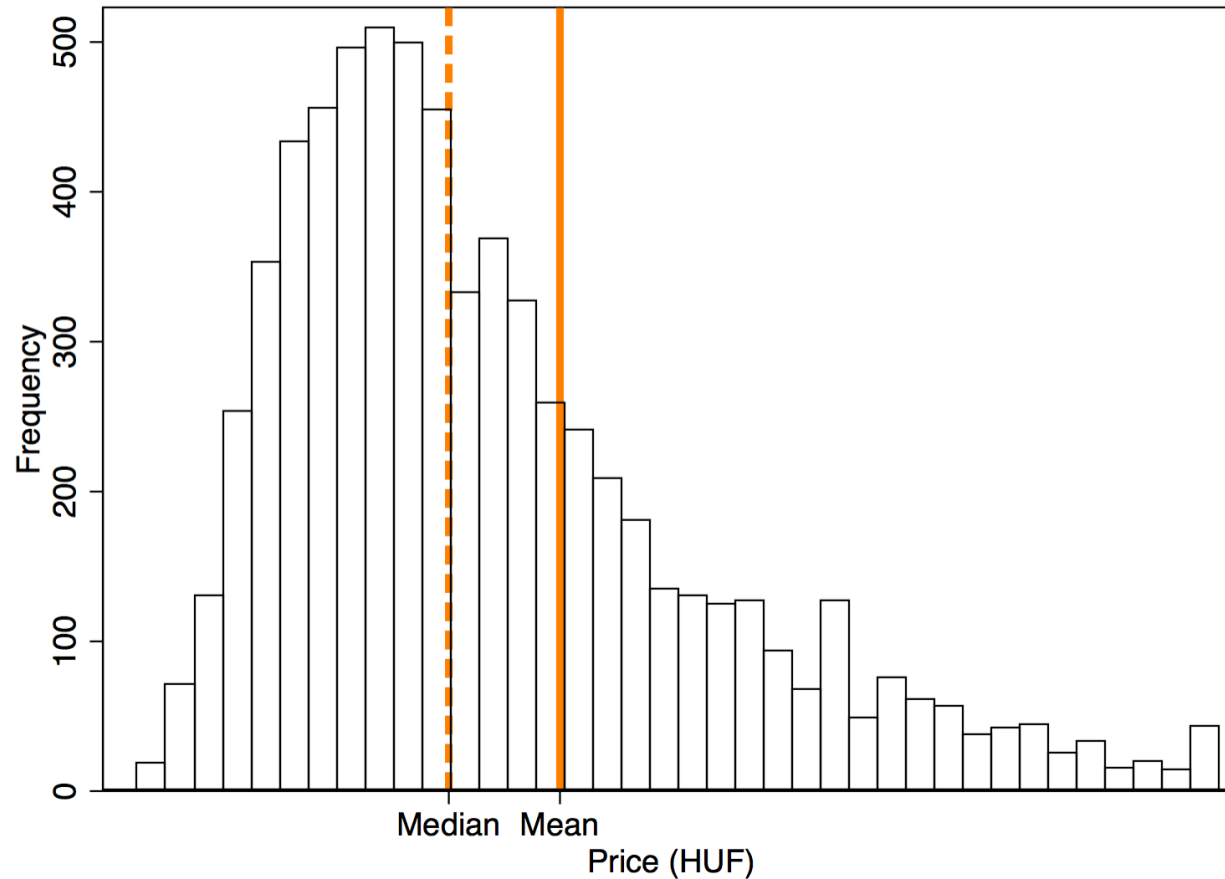
$$\frac{(\bar{x} - \text{med}(x))}{\text{Std}(x)}$$

# Can you tell me the type of skewness?

- In the price data, the mean-median statistics is of 0.249. What does this imply? Skewness to the right or the left?

$$\frac{(\bar{x} - \text{med}(x))}{\text{Std}(x)} = .249$$

# Skewness (Price Data)

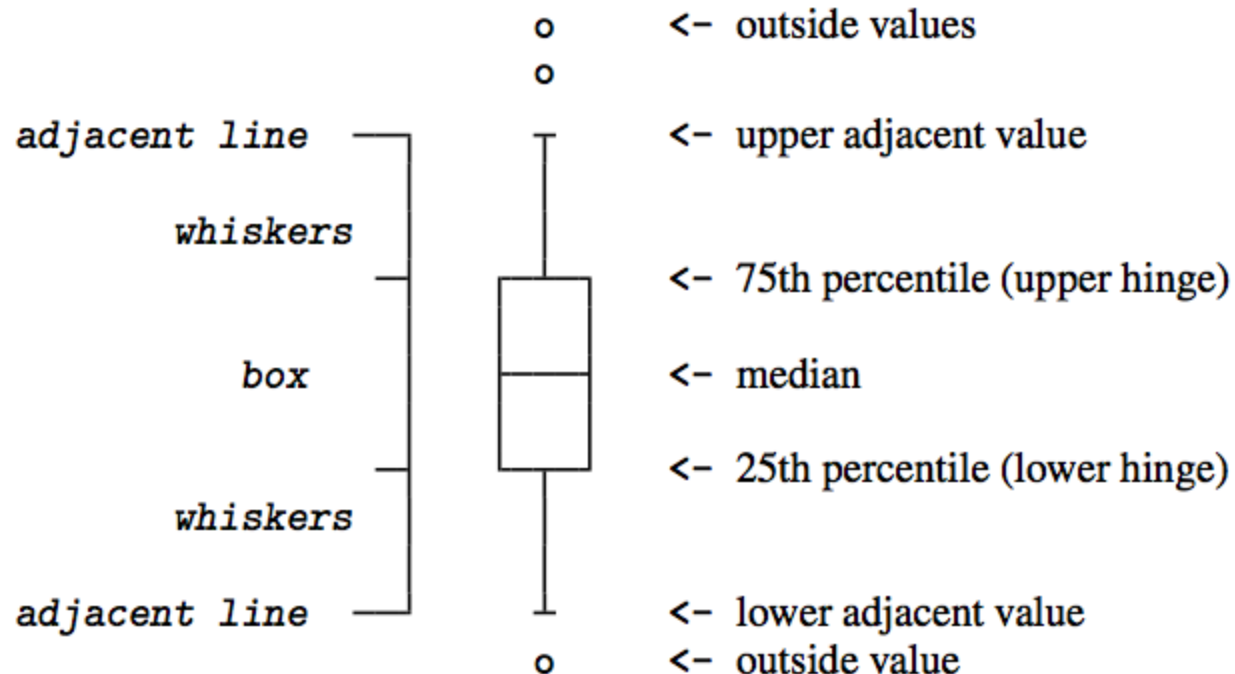


# Box Plot

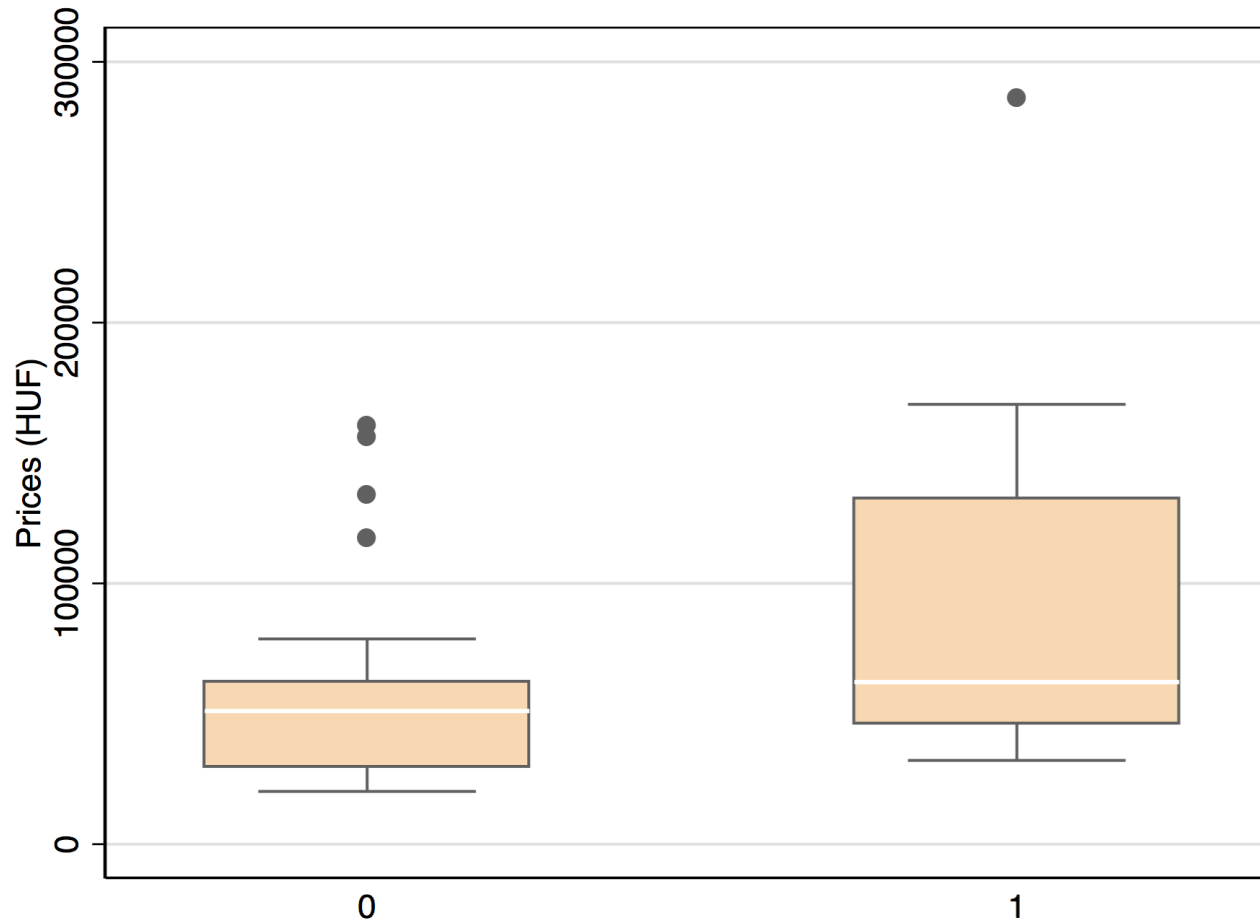
- The Box Plot is a combination of many statistics that we have already seen by now, namely:
  - The median of the data
  - The lower quartile
  - The third quartile
  - The smallest observation
  - The largest observation

The box plot conveys some important features of your data such as their skewness and show some of the quantiles in an explicit way

# Box Plot Example



# Trip to Vienna (Hotel Prices)



# Probability

- A probability is a measure of the likelihood of an event.

$$0 \leq p(event) \leq 1$$

- Probabilities are between zero and one. Sometimes we express them as a percentage.
- Joint probability is the probability that two events occur jointly

$$p(event1 \& event2)$$

- Probability that the event does not occur

$$p(\sim event)$$

# Independent Events

- Two event are said independent if their joint probability equals the product of their individual probabilities

$$p(event1 \& event2) = p(event1)p(event2)$$

- Examples?



# Conditional Probability

- Conditional probability is the probability of an event if another event happens

$$p(event1|event2) = \frac{p(event1 \& event2)}{p(event2)}$$

- Examples?

# Conditional Probability (of independent events)

- Conditional probability is the probability of an event if another event happens

$$p(event1|event2) = \frac{p(event1\&event2)}{p(event2)}$$

- In case the events are independent then:

$$\frac{p(event1\&event2)}{p(event2)} = \frac{p(event1)p(event2)}{p(event2)}$$

## Example (Trip to Vienna)

Suppose now that you allow the website to give a recommendation about a hotel in Vienna. In this case, the website will use all the hotels in the database.

	$\geq 2$ km	$< 2$ km	Total
$< 4$ stars	24	6	30
$\geq 4$ stars	23	9	32
Total	47	15	62

## Example (Trip to Vienna)

	$\geq 2$ km	$< 2$ km	Total
$< 4$ stars	24	6	30
$\geq 4$ stars	23	9	32
Total	47	15	62

- What is the unconditional probability that the hotel recommended from the website is 2 km or more distant from the city centre?

- Answer: 
$$\frac{24+23}{24+23+6+9} = .758$$

## Example (Trip to Vienna)

	$\geq 2$ km	$< 2$ km	Total
$< 4$ stars	24	6	30
$\geq 4$ stars	23	9	32
Total	47	15	62

- What is the conditional probability that the hotel recommended is within two km from the city center if it has at least four stars ?
- Answer:  $\frac{9}{23+9} = .281$

## Example (Trip to Vienna)

	$\geq 2$ km	$< 2$ km	Total
$< 4$ stars	24	6	30
$\geq 4$ stars	23	9	32
Total	47	15	62

- What is the joint probability of the website recommends a hotel less than two km from the city center and a hotel with at least 4 stars?

- Answer: 
$$\frac{9}{24+23+6+9} = .14$$

# Bayes Theorem

- Inverse conditional probabilities are two conditional probabilities, in which the role of the conditioning event and the conditional event are switched:

$$p(doped|positive)$$

- Two inverse conditional probabilities are related

$$p(event2|event1) = \frac{p(event1|event2)p(event2)}{p(event1)}$$

# Distributions

- All variables have a distribution. The distribution of a variable tells the number of times each possible value of the variable occurs in the data
- It is important to learn some theoretical distributions and their properties because it helps understand features of real data.
- Theoretical distributions are fully captured by a few parameters: these are statistics that determine the distributions

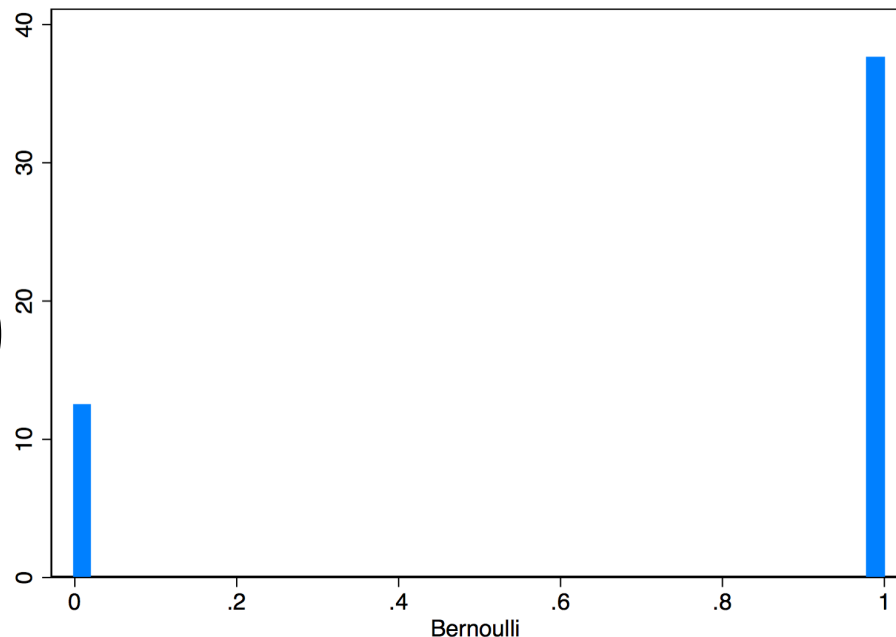


# Bernoulli Distribution

- The Bernoulli distribution is a theoretical distribution that we observe over and over: all zero-one variables are distributed Bernoulli.

$$\text{mean} = p$$

$$\text{var} = p(1 - p)$$

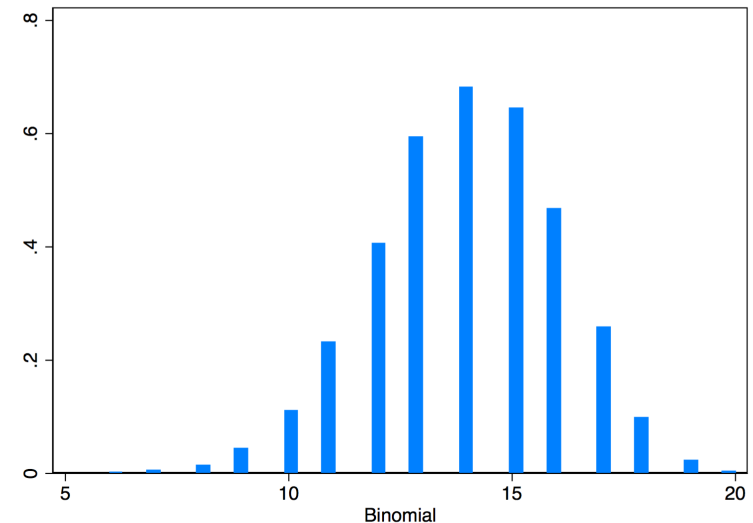


# Binomial Distribution

- The Binomial distribution is based on the Bernoulli distribution. A variable is distributed Binomial if it can be viewed as the sum of many independent Bernoulli variables with the same  $p$  parameter.

$$\text{mean} = np$$

$$\text{var} = np(1 - p)$$

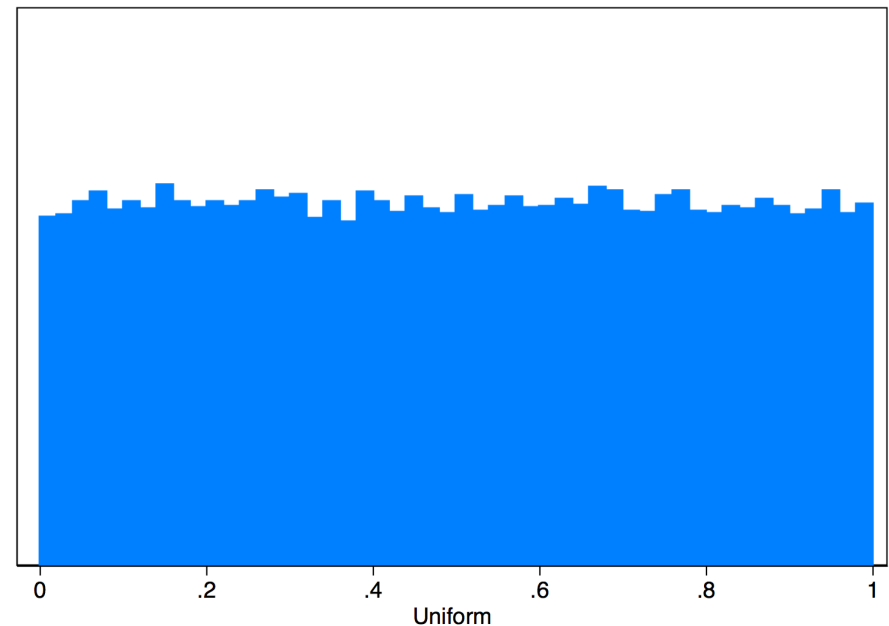


# Uniform

- The uniform distribution characterizes continuous variables with values that are equally likely to occur within a minimum value and a maximum value.

$$mean = \frac{a+b}{2}$$

$$var = \frac{(b-a)^2}{12}$$



# Normal Distribution

- It can be thought of as a generalization of the binomial with infinitely many Bernoulli variables added up

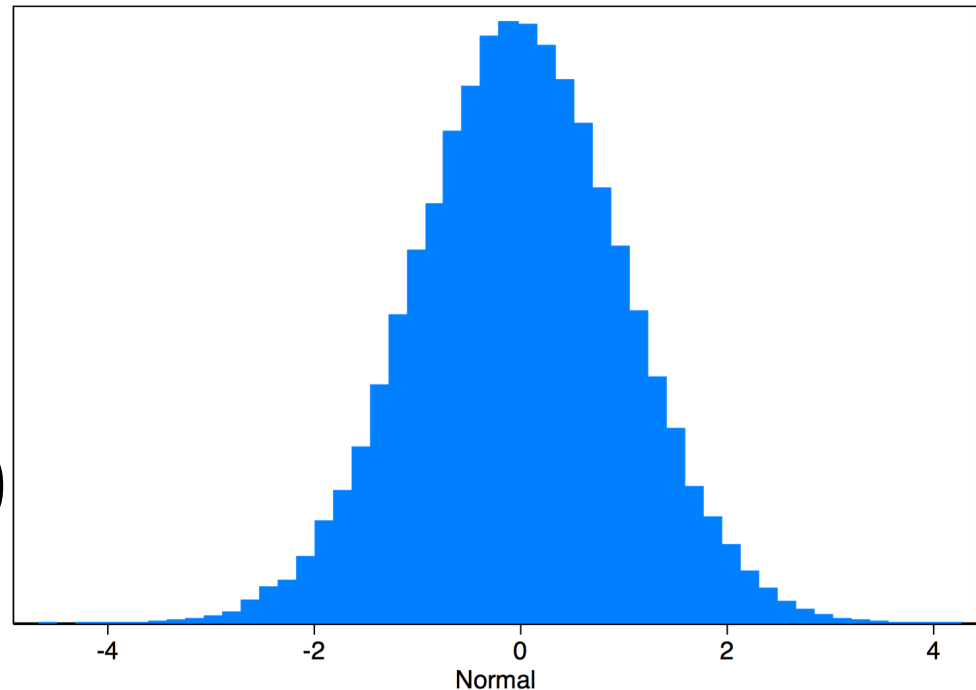
$$\text{mean} = \mu$$

$$\text{var} = \sigma^2$$

Standard Normal

$$\text{mean} = \mu = 0$$

$$\text{var} = \sigma^2 = 1$$

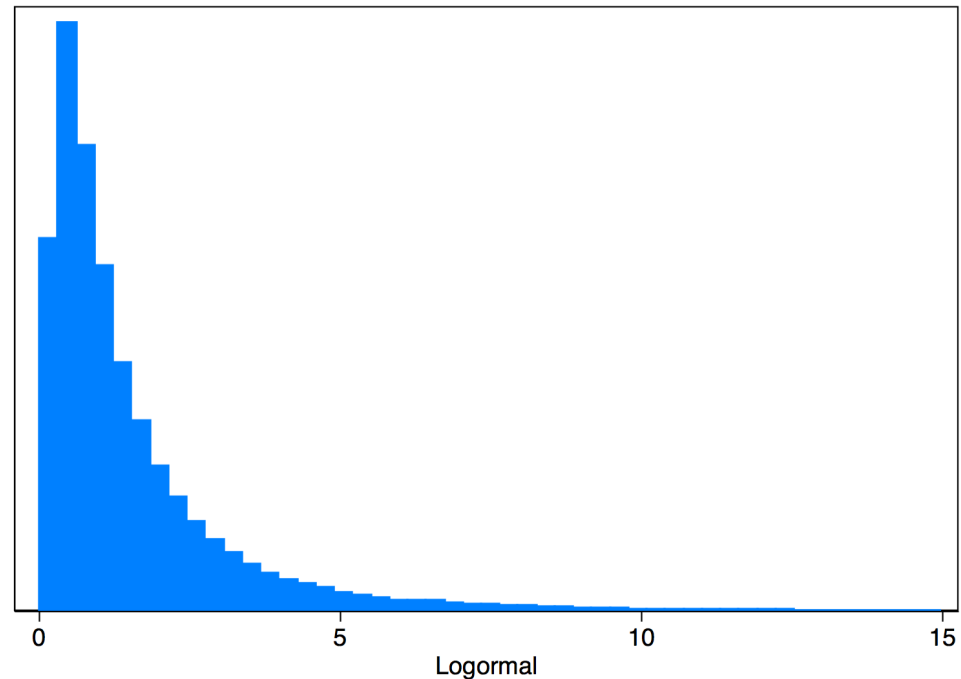


# Lognormal Distribution

- If we take a variable that is distributed normal  $\mathcal{X}$  and have the following transformation  $e^x$

$$mean = e^{(\mu + \frac{\sigma^2}{2})}$$

$$var = e^{(\mu + \frac{\sigma^2}{2})} e^{(\sigma^2 - 1)}$$



# See for yourself

<http://students.brown.edu/seeing-theory/distributions/>

Codes in R and Stata (which will be available in Moodle),  
change the parameters and see changes in the distribution

It's fun!!