Determinanty

05 October 2021 11:42

MATICE TRANSFORMÁCIE:

S – MATICA SÚRADNÍC BODOV

T – MATICA TRANSFORMÁCIE

$$T \cdot S = N$$

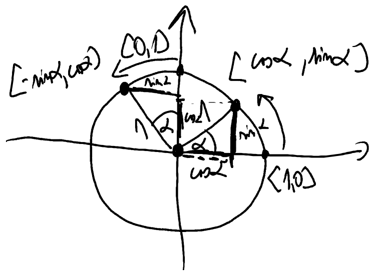
I. POSUVNUTIE: $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$

$$\underbrace{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}}_{T} \cdot \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ 1 & 1 & \dots & 1 \end{pmatrix} = \begin{pmatrix} x_1+a & x_2+a & \dots & x_n+a \\ y_1+b & y_2+b & \dots & y_n+b \\ 1 & 1 & \dots & 1 \end{pmatrix}$$

II. ŠKÁLOVANIE:

$$\begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \end{pmatrix} = \begin{pmatrix} cx_1 & cx_2 & \dots & cx_n \\ dy_1 & dy_2 & \dots & dy_n \end{pmatrix}$$

III. OTOČENIE:



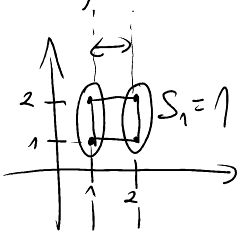
$$T = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

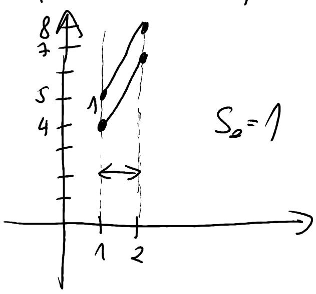
① $(1;1), (2;1), (1;2), (2;2)$

a) $(0,0)$ $\alpha = 60°$

$$T = \begin{pmatrix} 1/2 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1/2 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{pmatrix} \qquad T \cdot S = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$$

b) $T = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

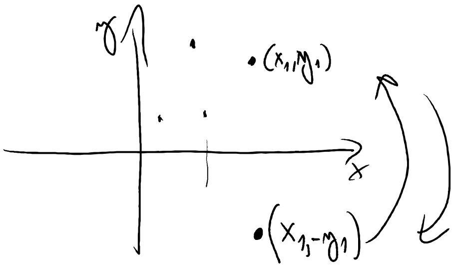




$S_2 = 1$

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 4 & 7 & 5 & 8 \end{pmatrix}$$

②



$$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

DETERMINANTY:

1x1

$(3) \quad D = |3| = 3$

2x2

$$\begin{vmatrix} 7 & -4 \\ 6 & -5 \end{vmatrix} = +(7\cdot(-5)) - (6\cdot(-4)) = -35+24 = -11$$

3x3

$$\begin{vmatrix} 3 & -1 & 5 \\ 6 & 1 & 4 \\ 0 & 5 & 1 \end{vmatrix}\begin{matrix} 3 & -1 \\ 6 & 1 \\ 0 & 5 \end{matrix} = +(3\cdot1\cdot1)+(-1\cdot4\cdot0)+(5\cdot6\cdot5)-(0\cdot1\cdot5)-(5\cdot4\cdot3)-(1\cdot6\cdot(-1)) =$$
$$= 3+0+150-0-60+6 = 99$$

$a=4 \; b=\sqrt5 \quad a=2 \; b=\sqrt7$

$$|B| = \begin{vmatrix} 4+\sqrt5 & 2-\sqrt7 \\ 2+\sqrt7 & 4-\sqrt5 \end{vmatrix} = (16-5)-(4-7) = 14$$

$a^2-b^2=(a-b)(a+b)$

$$|F| = \begin{vmatrix} 1 & 4 & 5 \\ 3 & 2 & 4 \\ 5 & 3 & 1 \end{vmatrix}\begin{matrix} 1 & 4 \\ 3 & 2 \\ 5 & 3 \end{matrix} = 1\cdot2\cdot1+4\cdot4\cdot5+5\cdot3\cdot3-5\cdot2\cdot5-3\cdot4\cdot1-1\cdot3\cdot4 = 2+80+45-50-12-12 = 53$$

VLASTNOSTI:

ERO 1 → zmení znamienko D

ERO 2 → výsledný D bude k-násobný

ERO 3 → nemení D

→ nulový riadok/stĺpec $D=0$

→ matica v △ tvare $D = a_{11} a_{22} \dots a_{nn}$

→ $|I|=1$

→ $|A^T|=|A|$

$\boxed{\det(A\cdot B)=\det(A)\cdot\det(B)}$

$\det(K^{-1})$

$\det(K)\cdot\det(K^{-1}) = \det(K\cdot K^{-1}) = \det(I) = 1$

$\det(K)\cdot\det(K^{-1})=1$

$k\cdot\det(K^{-1})=1$

$\det(K^{-1}) = \frac{1}{k}$

(4)

$S_3 - S_2$

a) $\begin{vmatrix} 7 & 2 & -3 \\ 5 & -1 & 8 \\ -4 & 5 & 1 \end{vmatrix} \neq \begin{vmatrix} 7 & 2 & -5 \\ 5 & -1 & 9 \\ -4 & 5 & -4 \end{vmatrix}$

(5) $\det(A)=0$

$S_4 + \frac{1}{2}S_2$

$$\begin{pmatrix} 13 & 4 & 7 & -2 \\ 12 & 6 & 0 & -3 \\ 11 & 0 & 8 & 0 \\ 10 & -8 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} & & & 0 \\ & & & 0 \\ & & & 0 \\ & & & 0 \end{pmatrix}$$

$\det = 0$

(6) $K = \begin{pmatrix} k & l & m \\ n & o & p \\ q & r & s \end{pmatrix} \quad \det(K) = -4$

a) $\det(3K) = \begin{vmatrix} 3k & 3l & 3m \\ 3n & 3o & 3p \\ 3q & 3r & 3s \end{vmatrix}\begin{matrix} \cdot3 \\ \cdot3 \\ \cdot3 \end{matrix} = 3(3\cdot(3\cdot(-4))) = 27\cdot(-4) = -108$

b) $\det(2K^{-1}) = 2\cdot2\cdot2\left(-\frac{1}{4}\right) = -\frac{8}{4} = -2$

c) $\det((2K)^{-1}) = \frac{1}{8\cdot(-4)} = -\frac{1}{32}$

$\det(K)\cdot\det(K^{-1}) = \det(K\cdot K^{-1}) = 1$

$$\det(K)\cdot\det(K^{-1}) = \det(\underbrace{K\cdot K^{-1}}_{I}) = 1$$

$$k\cdot\det(K^{-1}) = 1$$

$$\boxed{\det(K^{-1}) = 1/k}$$

$0\cdot(-4)$

4x4

$$\begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & 1 & 0 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 2 & 1 \end{vmatrix} = 2\cdot(-1)^{3+1}\begin{vmatrix} 0 & 2 & 3 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{vmatrix} + 1\cdot(-1)^{3+2}\cdot\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 3 & 2 & 1 \end{vmatrix} + 1\cdot(-1)^{3+3}\begin{vmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 3 & 2 & 1 \end{vmatrix} + 3\cdot(-1)^{3+4}\cdot\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 3 & 2 & 2 \end{vmatrix}$$

$$|P| = \begin{vmatrix} 3 & 5 & 1 & 2 \\ 1 & 4 & -2 & 1 \\ 4 & 7 & 0 & 0 \\ 3 & 9 & 0 & 0 \end{vmatrix} = 3\cdot(-1)^{1+4}\begin{vmatrix} 5 & 1 & 2 \\ 4 & -2 & 1 \\ 7 & \boxed{0} & \boxed{0} \end{vmatrix} + 9\cdot(-1)^{2+4}\cdot\begin{vmatrix} 3 & 1 & 2 \\ 1 & -2 & 1 \\ 4 & \boxed{0} & \boxed{0} \end{vmatrix} + 0 + 0 =$$

$$= (-3)\cdot(7+28) \quad + (9)\cdot(4+16) = (-3)\cdot 35 + 9\cdot 20 = 75$$

(8) a) $(6,0)$ $(3,1)$ $(-1,-2)$

$$S = \pm\frac{1}{2}\begin{vmatrix} 6 & 3 & -1 \\ 0 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

(9)

$$V = \pm\frac{1}{6}\begin{vmatrix} 1 & 4 & -1 & 5 \\ -1 & 0 & -1 & 7 \\ 2 & 2 & -2 & 4 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

(10) $U = \begin{pmatrix} 4 & 2 & 1 \\ 4 & -1 & 0 \\ 1 & -2 & -1 \end{pmatrix}$

$$U^{-1} = \frac{1}{\det(U)}\cdot \operatorname{adj}(U)$$

$$A = \begin{pmatrix} 1 & 4 & -7 \\ 0 & -5 & 10 \\ 1 & 4 & -12 \end{pmatrix}$$

MATICA ALGEBRAICKÝCH DOPLNKOV

$a_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 0 \\ -2 & -1 \end{vmatrix} = 1$

$a_{21} = (-1)^3\begin{vmatrix} 2 & 1 \\ -2 & -1 \end{vmatrix} = (-1)\cdot(-2+2)$

$a_{12} = (-1)^3\begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} = (-1)\cdot(-4) = 4$

$a_{22} = (-1)^4\begin{vmatrix} 4 & 1 \\ 1 & -1 \end{vmatrix} = -4-1 = -5$

$a_{13} = (-1)^4\begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} = -8+1 = -7$

$a_{23} = (-1)^5\begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} = (-1)\cdot(-8-2) = +10$

$a_{31} = (-1)^4\begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 1$

$a_{32} = (-1)^5\begin{vmatrix} 4 & 1 \\ 4 & 0 \end{vmatrix} = 4$

$a_{33} = (-1)^6\begin{vmatrix} 4 & 2 \\ 4 & -1 \end{vmatrix} = (-4)+(-8)$

$$\operatorname{adj} = (A)^T = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -5 & 4 \\ -7 & 10 & -12 \end{pmatrix}$$

CRAMEROVO PRAVIDLO

(11)

a) $3x + 2y + 6z = \boxed{8}$
$-x \quad + z = \boxed{0}$
$6x + y - 2z = \boxed{12}$

$$A = \begin{pmatrix} 3 & 2 & 6 \\ -1 & 0 & 1 \\ 6 & 1 & -2 \end{pmatrix} \qquad |A| = D = \begin{vmatrix} 3 & 2 & 6 \\ -1 & 0 & 1 \\ 6 & 1 & -2 \end{vmatrix} = 12-6-3-4 = -1$$

$6x+y-2z=\boxed{12}$ $\quad |6 \quad 1 \quad -2| \quad |6 \quad 1 \quad -2| \quad =-1$

$A_x = \begin{pmatrix} \boxed{8} & 2 & 6 \\ 0 & 0 & 1 \\ 12 & 1 & -2 \end{pmatrix}$ $\quad A_y = \begin{pmatrix} 3 & \boxed{8} & 6 \\ -1 & 0 & 1 \\ 6 & 12 & -2 \end{pmatrix}$ $\quad A_z = \begin{pmatrix} 3 & 2 & \boxed{8} \\ -1 & 0 & 0 \\ 6 & 1 & 12 \end{pmatrix}$

$D_x = 24-8=16$ $\quad D_y = 48-36-16-72 = -76$ $\quad D_z = -8+24=16$

$x = \frac{D_x}{D} = \frac{16}{-1} = -16$ $\quad y = \frac{D_y}{D} = \frac{-76}{-1} = 76$ $\quad z = \frac{D_z}{D} = \frac{16}{-1} = -16$

$(-16; 76; -16)$

Sk: Č1 = — = P1

Č2 = — = P2

Č3 = — = P3