

Algebra a diskrétna matematika

Príklady na precvičenie

1. týždeň

Gaussovou eliminačnou metódou riešte dané sústavy lineárnych rovníc.

1.
$$\begin{aligned}x + y - z &= 10 \\10x + 4y + z &= -12 \\3x + 2y + z &= -6\end{aligned}$$

3.
$$\begin{aligned}6x - y + 2z &= 21 \\3x + 2y - 3z &= -3 \\5x + y - z &= 10\end{aligned}$$

5.
$$\begin{aligned}x - 4y + z &= 18 \\2x - 7y - 2z &= 4 \\4x - 13y - 8z &= -24\end{aligned}$$

7.
$$\begin{aligned}8x + 3y + 10z &= 17 \\2x - y + 4z &= -4 \\5x - 2y - z &= -5\end{aligned}$$

2.
$$\begin{aligned}x + 3y - 2z &= 16 \\7x - y + 3z &= 16 \\4x + 2y - z &= 16\end{aligned}$$

4.
$$\begin{aligned}2x - 3y + 5z &= 14 \\3x + y - 2z &= 0 \\4x - 2y + 2z &= 10\end{aligned}$$

6.
$$\begin{aligned}x + 4y - z &= 0 \\3x + y - 2z &= 0 \\2x + 9y + 2z &= 0\end{aligned}$$

8.
$$\begin{aligned}3x + 4y + 25z &= 3 \\5x + y + 10z &= 7 \\7x + 6y &= 5\end{aligned}$$

9.
$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 2 \\x_1 + 2x_2 + 3x_3 + 4x_4 &= 7 \\x_1 - x_2 + x_3 - x_4 &= 0 \\-x_1 + 2x_2 - 3x_3 + 4x_4 &= 5\end{aligned}$$

10.
$$\begin{aligned}5x_1 - x_2 + 2x_3 + 10x_4 &= 0 \\-x_1 + 2x_2 + 6x_3 - 5x_4 &= 9 \\9x_1 + 12x_2 + 5x_3 + 13x_4 &= 1 \\-2x_1 - 5x_2 - 20x_3 + 5x_4 &= 7\end{aligned}$$

11.
$$\begin{aligned}7x_1 - x_2 - 10x_3 - 2x_4 &= 2 \\x_1 + x_2 - 6x_3 + x_4 &= 8 \\3x_1 - 5x_2 + x_3 - x_4 &= 0 \\4x_1 - x_2 + 2x_3 - x_4 &= 2\end{aligned}$$

12.
$$\begin{aligned}x_1 + 4x_2 - 4x_3 + x_4 &= 2 \\x_1 + 3x_2 + x_3 - x_4 &= 13 \\2x_1 - x_2 + x_3 + 3x_4 &= 5 \\4x_1 + x_2 + 3x_3 + 6x_4 &= 12\end{aligned}$$

$$\begin{aligned}
13. \quad & 7x_1 + 3x_2 + 6x_3 - x_4 = 0 \\
& 3x_1 + 2x_2 + x_3 + x_4 = 0 \\
& -x_1 + x_2 - x_3 + 2x_4 = 0 \\
& 3x_1 + 2x_2 - 2x_3 + 2x_4 = 0
\end{aligned}$$

$$\begin{aligned}
14. \quad & x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = -2 \\
& 3x_2 + 2x_3 + x_4 + 6x_5 = -11 \\
& 3x_2 + 2x_3 + 4x_4 - 4x_5 = -15 \\
& 2x_1 + 4x_2 + 6x_3 - x_4 + 25x_5 = -7
\end{aligned}$$

$$\begin{aligned}
15. \quad & 2x_1 + x_2 - x_3 - x_4 = 3 \\
& 3x_1 - 2x_2 + 2x_3 - 5x_4 = 1 \\
& -x_1 - x_2 + x_3 = -2 \\
& 5x_1 - x_2 + x_3 - 6x_4 = 4
\end{aligned}$$

$$\begin{aligned}
16. \quad & 2x_1 - x_2 + 3x_3 - 3x_4 = -7 \\
& 4x_1 + x_2 - 7x_3 + x_4 = 5 \\
& -x_1 + 2x_2 - 2x_3 - x_4 = -11 \\
& 5x_1 - 3x_2 - x_3 + 2x_4 = 22
\end{aligned}$$

$$\begin{aligned}
17. \quad & x + 2y + z + u - v = 3 \\
& x - y - z + 4u + v = -2 \\
& 2x + y + 2z + 3u - v = 2 \\
& -x + 2y + 4z - v = 3 \\
& 3x - 2y - z - u - v = -1
\end{aligned}$$

$$\begin{aligned}
18. \quad & 5x_1 + x_2 - x_3 + x_4 + x_5 = -6 \\
& 3x_1 + 2x_2 + x_3 - x_4 = 10 \\
& x_1 - x_2 - x_3 - x_4 + x_5 = 8 \\
& -2x_1 + x_2 + 2x_3 + x_5 = 6 \\
& 3x_2 + x_3 + 2x_4 + 2x_5 = -13
\end{aligned}$$

$$\begin{aligned}
19. \quad & x + y + z + 2u - v = 2 \\
& 4x + 2y - 6z + u - v = 4 \\
& 2x - y - 7z + u - v = -8 \\
& x + 4y + 10z + 5u - 2v = -2 \\
& -4y - 8z + u - v = 12
\end{aligned}$$

$$\begin{aligned}
20. \quad & x_1 + x_2 + x_3 + 2x_4 - x_5 = 2 \\
& -2x_1 + x_2 + 7x_3 - x_4 + x_5 = -8 \\
& x_1 + 4x_2 + 10x_3 + 5x_4 - 2x_5 = -2 \\
& 4x_1 + 2x_2 - 6x_3 + x_4 - x_5 = 4 \\
& x_1 + 2x_3 + 6x_4 - 3x_5 = 10
\end{aligned}$$

$$\begin{aligned}
21. \quad & x_1 + 7x_2 + 5x_3 + 2x_4 = 4 \\
& x_1 - 2x_2 - x_3 - x_4 = 5 \\
& 3x_1 - 2x_2 + x_3 - x_4 = 13 \\
& 2x_1 + 9x_2 + 8x_3 + 3x_4 = 7 \\
& x_1 + 5x_2 + 3x_3 + x_4 = 5
\end{aligned}$$

$$\begin{aligned}
22. \quad & x_1 - x_2 + x_3 - x_4 - x_5 = 2 \\
& x_2 - x_3 - x_4 = 0 \\
& 4x_1 - 5x_2 + 3x_3 - 3x_4 - 4x_5 = 8 \\
& 2x_1 - 4x_2 - 2x_5 = 4 \\
& 7x_1 - 10x_2 + 4x_3 - 4x_4 - 7x_5 = 14
\end{aligned}$$

$$\begin{aligned}
23. \quad & x_1 + x_2 - x_5 - x_6 = 2 \\
& 2x_1 + x_2 - x_3 - 2x_5 - x_6 = 4 \\
& -x_1 + x_2 + 2x_4 - x_5 - 3x_6 = 8 \\
& 3x_1 - x_2 - 5x_3 + x_4 - 4x_5 = 11 \\
& 5x_1 + 2x_2 - 2x_3 - x_4 - 4x_5 - 2x_6 = 5 \\
& x_1 - x_2 - 3x_3 + x_4 - 2x_5 = 7
\end{aligned}$$

$$\begin{aligned}
24. \quad & x_1 + x_2 - x_3 + x_4 + x_5 + x_6 = 10 \\
& x_2 + x_3 - x_5 = 20 \\
& x_1 - x_2 + x_6 = 5 \\
& 2x_1 + x_4 + x_5 + x_6 = 0 \\
& 5x_1 + 3x_2 - 4x_3 + x_6 = 45
\end{aligned}$$

$$\begin{aligned}
25. \quad & x_1 + x_7 = 0 \\
& x_2 + x_3 + x_7 = 1 \\
& x_1 + x_2 + x_5 + x_7 = 2 \\
& x_1 + x_2 + x_3 = 3 \\
& x_1 + x_2 + x_5 + x_6 = 4 \\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5 \\
& x_1 + x_3 + x_4 + x_5 + x_6 - x_7 = 6
\end{aligned}$$

Príklad 26. Určte, ktoré matice sú v redukovanom tvare.

a)

$$\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 16 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 10 & 0 \end{pmatrix}$$

c)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

d)

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

e)

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$