Špeciálne grupy

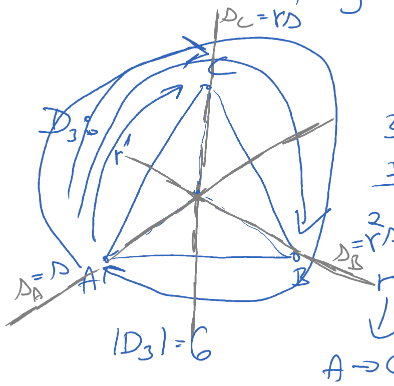
07 December 2021 11:43

$(M, *)$

⇒ DIHEDRÁLNA GRUPA D_n

$|D_n| = 2n$ → n rotácií
→ n symetrií os.

$D_n = \langle r, s \mid r^n = e, s^2 = e, rs = sr^{-1} \rangle$



$|D_3| = 6$

3 rotácie

$\frac{360°}{n}$ $\frac{360°}{3} = 120°$

$r^1, r^2, r^3 = e$

r^1: A → C, C → B, B → A

r^2: A → B, C → A, B → C

r^3: A → A, B → B, C → C

3 symetrie

$D_3 = \langle r, s \mid r^3 = e, s^2 = e, rs = sr^{-1} \rangle$

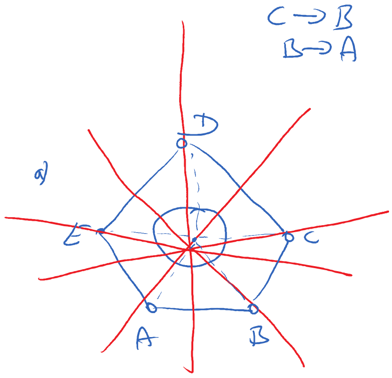
$D_3 = \{e, r^1, r^2, s, rs, r^2 s\}$

s: A → A, C → B, B → C

rs: C → C, A → B, B → A

$r^2 s$: B → B, A → C, C → A

① a)



$|D_5| = 10$ → 5 rotácií
→ 5 symetrií

$\frac{360°}{5} = 72°$

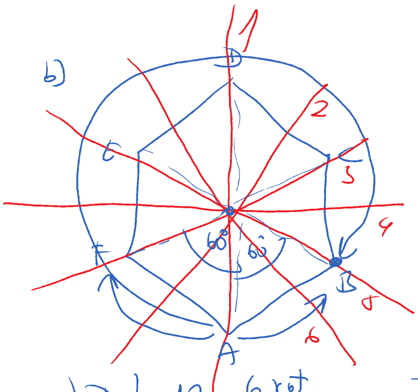
r^1 (72°) = 5
r^2 (144°) = 5
r^3 (216°) = 5
r^4 (288°) → 5
r^5 (360°) e → 1

$s_A = s$ = 2
$s_B = r^4 s$ = 2
$s_C = r^3 s$ = 2
$s_D = r^2 s$ = 2
s_E rs = 2

A → D → B → E → C → A
A → C → E → B → D → A

A → B → C → D → E → A

b)



$|D_6| = 12$ 6 rot.
6 rot.

$r^1 = 6$
$r^2 = 3$
$r^3 = 2$
$r^4 = 3$
$r^5 = 6$
$e = 1$

$\frac{360°}{6} = 60°$

s, rs, $r^2 s$, $r^3 s$, $r^4 s$, $r^5 s$ } 2

$r^5 = r^{-1}$
$r^4 = r^{-2}$

② a) $D_4 = \langle r, s \mid r^4 = e, s^2 = e, rs = sr^{-1} \rangle$

$r^3 s^4 r^{-1} s^5 r s r s$

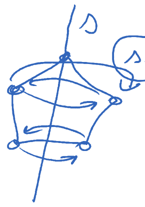
$r^3 s r^4 r^4 s r s$ (e e)

$r^3 s \boxed{rs}$ → $r^3 s s r^{-1} = r^3 r^{-1} = r^2$

$r^{-1} r^5 = r^4$

$s^4 s s^2 2 = ee = e$

b) $D_7 = \langle r, s \mid r^7 = e, \boxed{s^2 = e} \boxed{r^1 s = sr^{-1}} \rangle$



$ss = e$

$s r^9 s^5 r^{-3} r^5 s r s^5 r^4$

$s r^9 s r^2 s \boxed{rs} r^4$

$s r^9 s r^2 s s r^{-1} r^4$

$s r^9 s r r^3$

$s^5 = s^2 s^2 s$ (e e)

$s r^2 s r^5$

$r^3 s \boxed{rs} \to r^3 \underbrace{ss}_{e} r^{-1} = r^3 r^{-1} = r^2$

$srsrsrsr$

sr^2sr^3

sr^2sr^3

sr^2sr^5

$sr\boxed{rs}r^5$

sr^{-1}

$ssr^{-1}r^{-1}r = r^3$

e

c) $D_8 = \langle r, s \mid r^8 = e, s^2 = e, rs = sr^{-1} \rangle$

$rsr^2sr^4r^{11}sr^{-2}s^3r^2$

r^5

s

$s^{-1} = s$

③ IZOMORFIZMUS GRÚP:

$(M_1, *)$ $(M_2, \circ)$

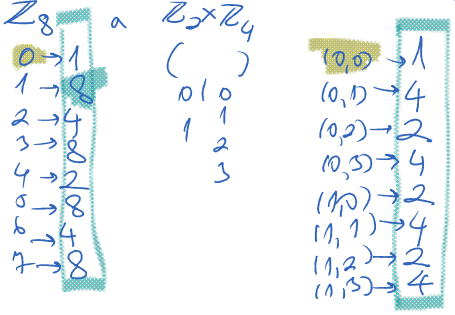
homomorfizmus:

φ je bijekcia

$\varphi(x * y) = \varphi(x) \circ \varphi(y)$

$\exists \{6, 9, 12, 13, 18, 21, 24\}$

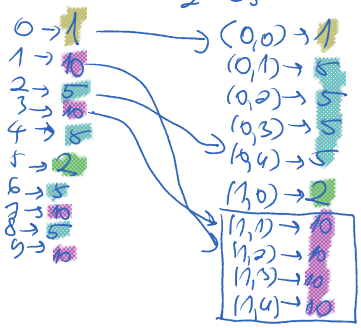
a) $\mathbb{Z}_8$ a $\mathbb{Z}_2 \times \mathbb{Z}_4$



$(1,0) * (1,0) = (0,0)$

nie sú izomorfné

b) $\mathbb{Z}_{10}$ a $\mathbb{Z}_2 \times \mathbb{Z}_5$



$\varphi(x+y) = \varphi(x) + \varphi(y)$

$\varphi(0) = (0,0)$

$\varphi(1) = (1,2)$

$\varphi(2) = (0,4)$

$\varphi(3) = (1,1)$

$\varphi(4) = \ldots$

$\varphi(1+1) = \varphi(1) + \varphi(1) = (1,2) + (1,2) = (0,4)$

$\varphi(2+1) = \varphi(2) + \varphi(1) = (0,4) + (1,2) = (1,1)$

④ $\{e, (12), (345), (354), (12)(345), (12)(354)\}$

S_n $(abcde)$

$(12) \cdot \boxed{(12)} = (1)(2)(3)(4)(5)$

$(345) \cdot \underbrace{(345)}_{(354)}$

dc cb

dcba

ba

$(abcdefg) \circ (gfedcba) = (a)(b)(c)(d)\ldots$

5

4 G 3

⑤ S_6

$(ab) \circ (ab) = (a)(b)$ 2

$(abc)(abc)(abc) = (a)(b)(c)$ 3

$(abcd)$ 4

$(ab)(cde)$ 6

$(ab)(cdef)$ 4