Kombinatorika-úvod

09 November 2021 11:41

② $0,2,4,5,6,8 \rightarrow$ 6 číslic

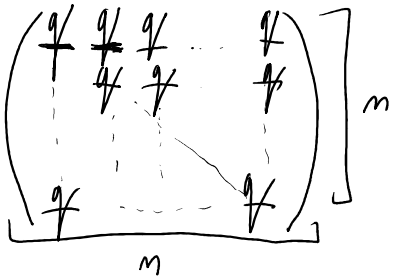
$5 \cdot 6 \cdot 6 \cdot 6 \cdot 2$ môžeme opakovať cifry

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ (0) nemôžeme

$4 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ (5)

0,3m

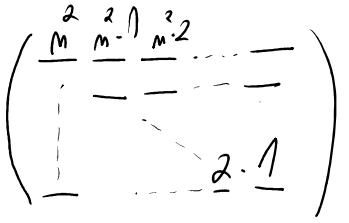
③ $\{0,1,2,\dots,q-1\}$



m^2 pozícií

q^{m^2}

aspoň m^2 prvkov



$\binom{q}{m^2} \cdot m^2! = \frac{q!}{(q-m^2)!\,m^2!} \cdot m^2! = \frac{q!}{(q-m^2)!}$

④ abcd

$|b-c|=2$

$\{0,1,2,\dots,9\}$

b c

0 2

1 3

2 4

3 5

4 6

5 7

6 8

7 9

c b

$8 \times 2 = 16$

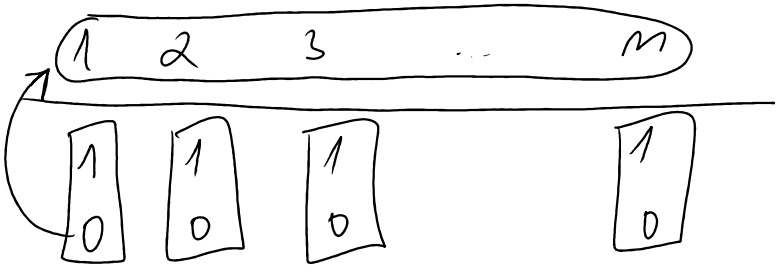
9 · 10

16

$9 \cdot 16 \cdot 10$

⑤ $\{1,2,\dots,m\}$ s nepárnymi číslami

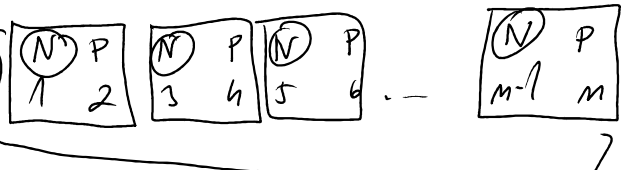
2^m



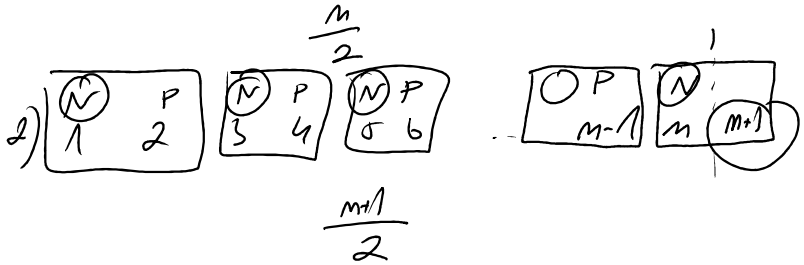
$2 \cdot 2 \cdot 2 \cdot \dots \cdot 2 \rightarrow 2^m = \underbrace{2 \cdot 2 \cdot 2 \cdot \dots \cdot 2}_{m}$

1) m párne $\frac{m}{2}$ $2^{\frac{m}{2}}$

2) m nepárne $\frac{m+1}{2}$ $2^{\frac{m-1}{2}}$



2) m nepárne $\frac{m+1}{2}$ $2^{\frac{m+1}{2}}$ // 1 2 | 3 4 | 5 6 | ... | m-1 m



① (A,B) $A \subseteq B \subseteq \{1,2,\dots,m\}$

$m=1$ $A \subseteq B \subseteq \{1\}$

$B=\{1\} \to A=\{1\} \vee A=\emptyset$ $3=3^1$

$B=\emptyset \to A=\emptyset$

$m=2$ $A \subseteq B \subseteq \{1,2\}$

$B=\{1,2\} \to A=\{1,2\} \vee A=\{1\} \vee A=\{2\} \vee A=\emptyset$

$B=\{1\} \to A=\{1\} \vee A=\emptyset$

$B=\{2\} \to A=\{2\} \vee A=\emptyset$ $9=3^2$

$B=\emptyset \to A=\emptyset$

3^m

$(A;B)$

VŠEOB. $A \subseteq B \subseteq \{1,2,\dots,m\}$

→ $A=\emptyset$ $B=2^m$ → $\binom{m}{0}\cdot 2^{m-0}$ +

→ $A=\{1\}$ PREV $B=2^{m-1}$ → $\binom{m}{1}\cdot 2^{m-1}$ +

→ $A=\{2\}$ PREV $B=2^{m-2}$ → $\binom{m}{2}\cdot 2^{m-2}$

→ $A=\{1,2,\dots,m\}$ $B=1$ → $\binom{m}{m}\cdot 2^{m-m}$

$x=2$

$y=1$

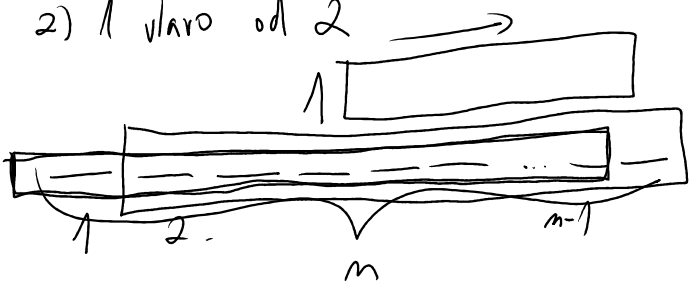
$$\sum_{k=0}^{m}\binom{m}{k}\cdot 2^{m-k}\cdot 1^{k}=(2+1)^m=3^m$$

BINOMICKÁ VETA: $\sum_{k=0}^{m}\binom{m}{k}\cdot x^{m-k}\cdot y^{k}=(x+y)^m$

⑧ 1) $\{1,2,\dots,m\}$

$$\binom{m}{k}\cdot k!=\frac{m!}{(m-k)!\cdot k!}\cdot k!=\frac{m!}{(m-k)!}$$

2) 1 vľavo od 2

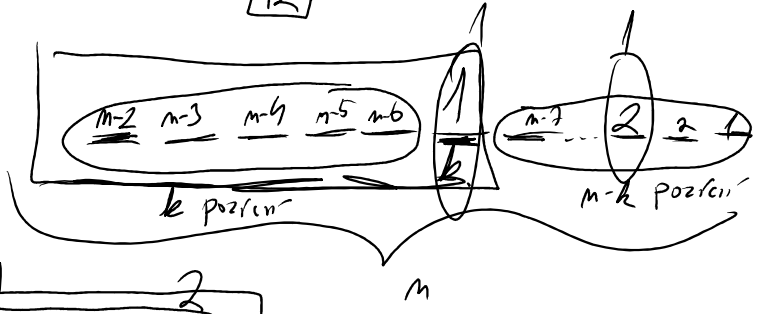


1 je na k-tej poz.

$$\sum_{k=1}^{m-1}\binom{n-k}{1}\cdot(n-2)!$$

1 vľavo od 2

12



⑨ P Q R S T U V W X Y Z → 8 prvkov

STU

$8!$

⑩ $n \cdot \underbrace{(n-1)\cdot(n-1)\cdot\ldots\cdot(n-1)}_{k-1} = n\cdot(n-1)^{k-1}$ n písmen

k

⑪ $\{0,1\}$

párny # jednotiek

k

$\binom{k}{0}+\binom{k}{2}+\binom{k}{4}+\ldots+\binom{k}{k}$ k je párne

(umiestnenia 1)

$\binom{k}{1}+\binom{k}{3}+\binom{k}{5}+\ldots+\binom{k}{k}$ k je nepárne

(umiestnenia 0)

⑫ 29 → 10 ž → 19 m

5 čl. komisia

a) $\binom{28}{5}$ b) $\binom{10}{1}\cdot\binom{19}{4}$ c) $\binom{10}{1}\cdot\binom{14}{4}+\binom{10}{1}\cdot\binom{17}{3}\cdot\binom{2}{1}$

BEZ NOVÁKA BEZ KÁVOVÁ

S PRÁVE 1

d) $\binom{10}{1}\binom{19}{4}+\binom{10}{2}\binom{15}{3}+\binom{10}{3}\binom{19}{2}+\binom{10}{4}\binom{15}{1}+\binom{10}{5}$

⑳ a) $(\sqrt[3]{3}+\sqrt[5]{5})^{15}=\underbrace{\binom{15}{0}\cdot(\sqrt[3]{3})^{0}(\sqrt[5]{5})^{15}}_{0.}+\underbrace{\binom{15}{1}(\sqrt[3]{3})^{1}(\sqrt[5]{5})^{14}}_{1.}+\ldots+\underbrace{\binom{15}{15}\cdot(\sqrt[3]{3})^{15}\cdot(\sqrt[5]{5})^{0}}_{15.}$

1 5^1 2.

$NSN(3,5)=15$

2