Matice

28 September 2021 13:48

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{pmatrix}$$

m riadkov
n stĺpcov

$m=n \rightarrow$ ŠTVORCOVÁ MATICA

a_{ii}

$$\mathbb{I}=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & 1 & 0 \\ 0 & \cdots & 0 & 1\end{pmatrix}$$

STOPA $= tr = a_{11}+a_{22}+a_{33}+\dots+a_{nn}$

OPERÁCIE S MATICAMI:

1) SÚČET

$$A+B=\begin{pmatrix} \boxed{0} & -1 & 3 \\ 2 & 1 & 5 \\ -2 & 3 & -1\end{pmatrix}+\begin{pmatrix} \boxed{1} & 2 & -1 \\ 0 & 3 & 0 \\ 2 & 1 & 2\end{pmatrix}=\begin{pmatrix} 0+1 & -1+2 & 3+(-1) \\ 2+0 & 1+3 & 5+0 \\ -2+2 & 3+1 & -1+2\end{pmatrix}=\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 5 \\ 0 & 4 & 1\end{pmatrix}=\mathbb{C}$$

2) ROZDIEL

$$A-B=\begin{pmatrix} 0-1 & -1-2 & 3-(-1) \\ 2-0 & 1-3 & 5-0 \\ -2-2 & 3-1 & -1-2\end{pmatrix}=\begin{pmatrix} -1 & -3 & 4 \\ 2 & -2 & 5 \\ -4 & 2 & -3\end{pmatrix}=\mathbb{D}$$

3) NÁSOBIŤ SKALÁROM

$$3\cdot A=3\cdot\begin{pmatrix} 0 & -1 & 3 \\ 2 & 1 & 5 \\ -2 & 3 & -1\end{pmatrix}=\begin{pmatrix} 3\cdot 0 & 3\cdot(-1) & 3\cdot 3 \\ 3\cdot 2 & 3\cdot 1 & 3\cdot 5 \\ 3\cdot(-2) & 3\cdot 3 & 3\cdot(-1)\end{pmatrix}=\begin{pmatrix} 0 & -3 & 9 \\ 6 & 3 & 15 \\ -6 & 9 & -3\end{pmatrix}$$

4) SÚČIN

$$A\cdot B=\begin{pmatrix} 0 & -1 & 3 \\ 2 & 1 & 5 \\ -2 & 3 & -1\end{pmatrix}\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 0 \\ 2 & 1 & 2\end{pmatrix}=\begin{pmatrix} 6 & 0 & 6 \\ 12 & 12 & 8 \\ -4 & 4 & 0\end{pmatrix}=\mathbb{E}$$

stĺpcov A = # riadkov B

$e_{11}=0\cdot 1+(-1)\cdot 0+3\cdot 2=6$ $\quad e_{21}=2\cdot 1+1\cdot 0+5\cdot 2$ $\quad e_{31}=1\cdot(-2)+0\cdot 3+(-1)\cdot 2=$

$e_{12}=0\cdot 2+(-1)\cdot 3+3\cdot 1=0$ $\quad e_{22}=4+3+5$ $\quad e_{32}=2(-2)+3\cdot 3+(-1)\cdot 1$

$e_{13}=0\cdot(-1)+(-1)\cdot 0+3\cdot 2=6$ $\quad e_{23}=-2+0+10$ $\quad e_{33}=-2\cdot(-1)+3\cdot 0+(-1)\cdot 2=$

$$\boxed{A_{3\times 2}\cdot B_{2\times 6}=\mathbb{C}_{3\times 6}}$$

5) UMOCŇOVANIE

$0+(-2)+(-6)$

$$A^2=A\cdot A=\underbrace{\begin{pmatrix} 0 & -1 & 3 \\ 2 & 1 & -5 \\ -2 & 3 & -1\end{pmatrix}}\begin{pmatrix} 0 & -1 & 3 \\ 2 & 1 & -5 \\ -2 & 3 & -1\end{pmatrix}=\begin{pmatrix} -8 & 8 & 2 \\ 12 & -16 & 6 \\ 8 & 2 & -20\end{pmatrix}$$

$0\cdot 0+(-1)\cdot 2+3\cdot(-2)=0-2-6=-8$

IBA ŠTVORCOVÉ MATICE

6) TRANSPONOVANÁ M.

$A \cdots \begin{pmatrix} -2 & 3 & -1 \end{pmatrix} \begin{pmatrix} -2 & 3 & -1 \end{pmatrix} \begin{pmatrix} 8 & 2 & -20 \end{pmatrix}$ IBA ŠTVORCOVÉ MATICE

6) TRANSPONOVANÁ M.

$$A^T \qquad a_{ij} \to a_{ji} \qquad \begin{pmatrix} 0 & 2 & -2 \\ -1 & 1 & 3 \\ 3 & -5 & -1 \end{pmatrix}$$

7) INVERZNÁ M.

$$A^{-1} \qquad A^{-1} \cdot A = \mathbb{I} \qquad A \cdot A^{-1} = \mathbb{I}$$

$$A^{-1} = \left(\underbrace{\begin{matrix} 0 & -1 & 3 \\ 2 & 1 & -5 \\ -2 & 3 & -1 \end{matrix}}_{A} \middle| \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\mathbb{I}} \right) \overset{ERO_{1,2,3}}{\sim} \cdots \sim \left(\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\mathbb{I}} \middle| \boxed{A^{-1}} \right)$$

① a) $A + C$
$D + G$

b) $$5C - 7A + I = \begin{pmatrix} 25 & -5 & 10 \\ 0 & 5 & 15 \\ 45 & 20 & 0 \end{pmatrix} - \begin{pmatrix} 63 & -7 & 7 \\ 21 & -49 & 0 \\ 42 & -35 & 21 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -37 & 2 & 3 \\ -21 & 55 & 15 \\ 3 & 55 & -20 \end{pmatrix}$$

c) AC, $C \cdot A$, $A \cdot B$, $C \cdot B$, $D \cdot G$, $G \cdot D$, $A \cdot F$, $C \cdot F$

	R	S
A =	3	3
B =	3	2
C =	3	3
D =	2	2
E =	2	4
F =	3	1
G =	2	2

$A \cdot \boxed{2}$ $\quad A, C$

$B \cdot \boxed{3}$

⋮

d) $$D^2 - 2DG + G^2 = \begin{pmatrix} 2 & 7 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ 3 & 0 \end{pmatrix} - 2 \cdot \overbrace{\begin{pmatrix} 2 & 7 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} -7 & 6 \\ 9 & 1 \end{pmatrix}}^{\begin{pmatrix} 4 & 19 \\ 6 & 0 \end{pmatrix}} + \begin{pmatrix} -7 & 6 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} -7 & 6 \\ 9 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 25 & 14 \\ 6 & 21 \end{pmatrix} - \begin{pmatrix} 38 & 38 \\ 42 & 36 \end{pmatrix} + \begin{pmatrix} 103 & -36 \\ -54 & 55 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

f) $\underline{tr}(A \cdot A^T)$ $\qquad A^T = \begin{pmatrix} 9 & 3 & 6 \\ -1 & -7 & -5 \\ 1 & 0 & 3 \end{pmatrix}$ $\qquad A \cdot A^T = \begin{pmatrix} \; \end{pmatrix}$

② $R \cdot M \cdot S = T$ $\qquad M = ?$

$$\underbrace{\overset{R}{\begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 3 & 1 \end{pmatrix}} \overset{M}{\begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}}} \overset{S}{\begin{pmatrix} 3 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}} = \begin{pmatrix} 9 & -3 & 3 \\ 9 & 15 & 3 \\ 12 & -19 & 4 \end{pmatrix}$$

$$\begin{pmatrix} m_1 + 2m_3 & m_2 + 2m_4 \\ -m_1 + 4m_3 & -m_2 + 4m_4 \end{pmatrix}$$

$$\begin{pmatrix} m_1+2m_3 & m_2+2m_4 \\ -m_1+4m_3 & -m_2+4m_4 \\ 3m_1+m_3 & 3m_2+m_4 \end{pmatrix}$$

$$\begin{pmatrix} 3m_1+6m_3 & -m_2-2m_4 & m_1+2m_3 \\ -3m_1+12m_3 & m_2-4m_4 & -m_1+4m_3 \\ 9m_1+3m_3 & -3m_2-m_4 & 3m_1+m_3 \end{pmatrix} = \begin{pmatrix} 9 & -3 & 3 \\ 9 & 15 & 3 \\ 12 & -15 & 4 \end{pmatrix}$$

$$\begin{aligned} 3m_1+6m_3&=9 \\ -3m_1+12m_3&=9 \\ 9m_1+3m_3&=12 \end{aligned} \qquad \boxed{\begin{aligned} -m_2-2m_4&=-3 \\ m_2-4m_4&=15 \end{aligned}} \quad -3m_2-m_4=-19 \qquad \boxed{\begin{aligned} m_1+2m_3&=3 \\ -m_1+4m_3&=3 \end{aligned}} \quad 3m_1+m_3=4$$

$$\boxed{\begin{matrix} m_4=-2 & m_2=7 \\ m_3=1 & m_1=1 \end{matrix}}$$

$-m_2+4=-3$

$m_2=7$

$-6m_4=12$

$6m_3=6$

$m_1+2=3$

$m_1=1$

$$M=\begin{pmatrix} 1 & 7 \\ 1 & -2 \end{pmatrix}$$

(5)

$$A^{-1}=\left(\begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) (-2)R_2+R_1 \sim \left(\begin{array}{cc|cc} 4 & \boxed{5} & 1 & 0 \\ 0 & -1 & 1 & -2 \end{array}\right) R_1+5R_2 \sim \left(\begin{array}{cc|cc} 4 & 0 & 6 & -10 \\ 0 & -1 & 1 & -2 \end{array}\right) \begin{matrix} \frac{1}{4}R_1 \\ -R_2 \end{matrix} \sim \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{2} & -\frac{5}{2} \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$\underbrace{}_{\mathbb{I}} \quad \underbrace{}_{A^{-1}}$$

$$A \cdot A^{-1} = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{I} \checkmark$$

(4)

$x=?$

$y=?$

$$A=\begin{pmatrix} 1 & 2 & 0 \\ 0 & x & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad B=\begin{pmatrix} -1 & 0 & y \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$A \cdot B = \mathbb{I}$$

A^{-1}

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & x & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 & y \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & y-2 \\ x+1 & 1 & -x-1 \\ 0 & 0 & y-1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & y-3 \\ x+1 & 1 & -x-1 \\ 0 & 0 & y-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$y-2=0 \checkmark$

$x+1=0 \checkmark$

$-x-1=0$

$y-1=1 \checkmark$

$$\boxed{\begin{aligned} y&=2 \\ x&=-1 \end{aligned}}$$

(6)

$$S \cdot T = Z \quad / S^{-1} \text{ zleva}$$

$$\boxed{S^{-1} \cdot S}\, T = S^{-1} Z$$

$S \cdot T = Z \quad / S$ zľava

$\boxed{S^{-1}\cdot S}\, T = S^{-1}\cdot Z$

$T = S^{-1}\cdot Z$

$$S^{-1} = \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 1 & 4 & -2 & 0 & 1 & 0 \\ -1 & -3 & 2 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ R_2 - R_1 \\ R_3 + R_1 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ 2R_3 + R_2 \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right) \begin{array}{l} R_1 + R_3 \\ R_2 + R_3 \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right) \begin{array}{l} R_1 - R_2 \\ 1/2 \\ \\ \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 2 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{matrix}}_{S^{-1}} \\ 0 & 0 & 1 & \end{array}\right)$$

$$T = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 47 & 25 & 40 \\ 52 & 27 & 66 \\ -69 & -25 & -52 \end{pmatrix} = \begin{pmatrix} 2 & 23 & 14 \\ 23 & 2 & 14 \\ 1 & 2 & 2 \end{pmatrix}$$

$$S \cdot S^{-1} = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & -2 \\ -1 & -3 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

$$T = \begin{pmatrix} A & D & M \\ D & A & M \\ Z & A & A \end{pmatrix}$$

7) a)

$$\begin{pmatrix} 1 & 5 & 1 \\ 1 & 5 & -2 \\ 0 & 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 15 \\ 9 \\ 12 \end{pmatrix}$$

$$\mathbb{A} \cdot \mathbb{X} = \mathbb{B}$$

$A \cdot X = B \quad / A^{-1}$ zľava

$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$

$$\boxed{\mathbb{X} = \mathbb{A}^{-1} \cdot \mathbb{B}}$$

$$\mathbb{A}^{-1} = \left(\mathbb{A} \,\middle|\, \mathbb{I}\right) \sim \ldots \sim \left(\mathbb{I} \,\middle|\, \mathbb{A}^{-1}\right)$$