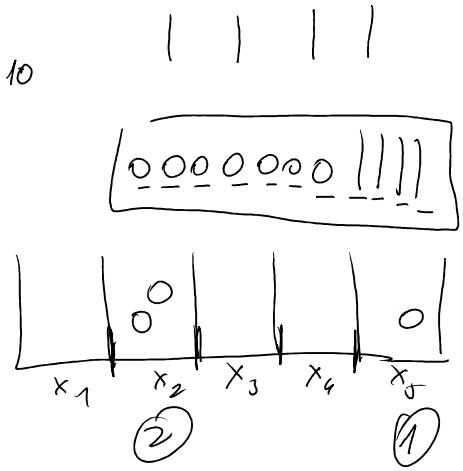
Algebraické štruktúry

30 November 2021 11:14

$x_1+x_2+x_3+x_4+x_5=10$

$x_2 \geq 2$

$x_5 \geq 1$



$\binom{11}{4} = \frac{11!}{(11-4)!\cdot 4!}$

0 00|000|00|00|0

3 3 2 2 0

BINÁRNE OPERÁCIE

$a \diamond b = c \qquad M \times M \to M$

KOMUTATÍVNA: $\forall a,b \in M:\ a \diamond b = b \diamond a$

ASOCIATÍVNA: $\forall a,b,c \in M:\ (a \diamond b) \diamond c = a \diamond (b \diamond c)$

DISTRIBUTÍVNY ZÁKON: $\diamond, *$ $\forall a,b,c \in M:\ a \diamond (b * c) = (a \diamond b) * (a \diamond c)$

② a) $\mathbb{Z}, *$

$a * b = (a+b)^2$

KOMUT.: $a*b = b*a$

$a*b = (a+b)^2 = (b+a)^2 = b*a$ ✓

$a*b = \times = b*a$

$b*a = \times$

$(5^2+1)^2 = 26^2$

$(3+3^2)^2 = 12^2$ ✗

ASOCIAT.: $(a*b)*c = a*(b*c)$

$(a*b)*c = ((a+b)^2+c)^2$

$a*(b*c) = (a+(b+c)^2)^2$

$a=5,\ b=2,\ c=1$

b) $a \ominus b = a+b-6$

KOMUT.: $a \ominus b = b \ominus a$

$a \ominus b = a+b-6 = b+a-6 = b \ominus a$ ✓

$a \ominus (b \ominus c) = a+(b+c-6)-6$

ASOC.: $(a \ominus b) \ominus c = (a+b-6)+c-6 =$

$= a+b-6+c-6 = a+(b+c-6)-6 =$

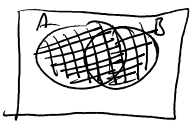
$= a \ominus (b \ominus c)$ ✓

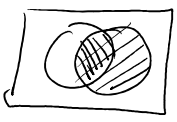
① $S = \{1,2,3\}$

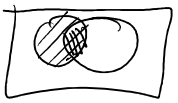
M

$\forall A,B \in M:\ A \ominus B = (A \cup B) - (A \cap B)$

KOM.: $A \ominus B = (A \cup B) - (A \cap B) = (B \cup A) - (B \cap A) = B \ominus A$ ✓

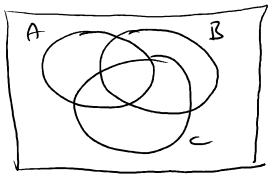


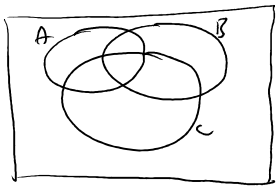




ASOCIAT.: $(A \ominus B) \ominus C = ((A \cup B) - (A \cap B)) \ominus C = (((A \cup B) - (A \cap B)) \cup C) - (((A \cup B) - (A \cap B)) \cap C)$ ✗

$A \ominus (B \ominus C) = A \ominus ((B \cup C) - (B \cap C)) = (A \cup ((B \cup C) - (B \cap C))) - (A \cap ((B \cup C) - (B \cap C))) = F$

$E =$ 

$F =$ 

ALGEBRAICKÉ ŠTRUKTÚRY:

- $(M, *)$ → GRUPOID
- GRUPOID $(M, *)$ S ASOC. BO $*$ → POLOGRUPA
- POLOGRUPA $(M, *)$ S NEUTRÁLNYM PRVKOM e → MONOID

  $e \in M$; $\forall x \in M \quad e * x = x * e = x$
- MONOID $(M, *)$, V KTOROM $\exists$ INVERZY → GRUPA

③ $(\mathbb{Q}, *)$

a) $a * b = \frac{a+b}{7}$ $(\mathbb{Q}, *)$ aspoň GRUPOID

$\frac{a+b}{49} + \frac{c}{7} =$ ——

→ ASOC.: $(a*b)*c = \frac{\left(\frac{a+b}{7}\right)+c}{7} = \frac{\frac{a+b+7c}{7}}{7} = \frac{a+b+7c}{49}$ ✗

$a = 1$, $b = 2$, $c = 3$

$a*(b*c) = \frac{a+\left(\frac{b+c}{7}\right)}{7} = \frac{7a+b+c}{49}$

b) $(\mathbb{Q}, \ominus)$

$a \ominus b = a + b - ab$ $(\mathbb{Q}, \ominus)$ ✓ GRUPOID

→ ASOC.: $(a \ominus b) \ominus c = (a+b-ab) + c - (a+b-ab)\cdot c = a+b-ab+c-ac-bc+abc$ ✓✓

$a \ominus (b \ominus c) = a + (b+c-bc) - a(b+c-bc) = a+b+c-bc-ab-ac+abc$

$(a \ominus b) \ominus c = (a+b-ab) + c - (a+b-ab)\cdot c = a+b+c-bc-ab-ac+abc =$

$= a + (b+c-bc) - a(b+c-bc) = a \ominus (b \ominus c)$ ✓ POLOGRUPA

→ NEUTRÁLNY PRVOK: $e \ominus x = x \ominus e = x$

$e \ominus x = e + x - e \cdot x = x$

$e = 0$ $\quad x + 0 - 0 \cdot x = x$ ✓

$x + 0 - x \cdot 0 = x$

$\frac{0}{1} \in \mathbb{Q}$ MONOID

→ INVERZNÉ PRVKY: $x \ominus x^{-1} = e$

$1 \ominus x^{-1} = e$

$1 + x^{-1} - 1 \cdot x^{-1} = 0$

$\frac{-x}{1-x}$ $\quad x = 1$

→ INVERZNÉ PRVKY: $x \oplus x^{-1} = e$

$x + x^{-1} - x \cdot x^{-1} = 0$

$x^{-1}(1-x) + x = 0$

$x^{-1} = \frac{-x}{(1-x)}$

$\frac{-x}{1-x}$ $\quad x = 1$ ✗

$1 + x^{-1} - 1 \cdot x^{-1} = 0$

$1 + x^{-1} - x^{-1} \neq 0$

$1 \neq 0$

$x + x^{-1} - x \cdot x^{-1} = 0$

$x^{-1} - x \cdot x^{-1} = -x$

$x^{-1}(1-x) = -x \quad / : (1-x)$

$x^{-1} = \frac{-x}{1-x}$

$x + x^{-1} \neq e$

$x \neq 1$

⑤ $110 \oplus 111 = 11111$

⑫ a) $x + x = 6 \pmod 7$

$2x = 6 \quad / : 2 \pmod 7$

$x = 3$

$NSD(2, 7) = 1$

$x = 7k + 3$

3, 10, 17

$\underline{a} = \underline{b} \pmod k$

$k \mid a - b$

$a \mod k = e$

$e \in \{0, 1, \ldots, k-1\}$

$a : k = c + e$

$b : k = d + e$

$x + x = 6 \pmod 7$

$2x : 7$ zv. a

$6 : 7$ zv. a

$7 \mid 2x - 6$

$k_2 + x - (l + x) = m$