Gaussova eliminačná metóda

21 September 2021 14:24

ERO1 – VÝMENA 2 RIADKOV

ERO2 – PRENÁSOBENIE RIADKU NENULOVÝM Č.

ERO 3 – PRIČÍTANIE ČÍSLA RIADKU K INÉMU

1) $x+y-z=10$
$10x+4y+z=-12$
$3x+2y+z=-6$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 10 & 4 & 1 & -12 \\ 3 & 2 & 1 & -6 \end{array}\right) \begin{matrix} \\ R_2-10R_1 \\ R_3-3R_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 0 & -6 & 11 & -112 \\ 0 & -1 & 4 & -36 \end{array}\right) 6R_3-R_2 \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 0 & -6 & 11 & -112 \\ 0 & 0 & 13 & -104 \end{array}\right)$$

$x+4+8=10$
$\boxed{x=-2}$

$-6y+11(-8)=-112$
$-6y=88-112$
$\boxed{y=4}$

$0x+0y+13z=-104$
$13z=-104$
$\boxed{z=-8}$

$\boxed{[-2;4;-8]}$

SKÚŠKA: $\check{C}_1=-2+4+8=10=P_1$ ✓
$\check{C}_2=-20+16-8=-12=P_2$ ✓
$\check{C}_3=-6+8-8=-6=P_3$ ✓

$-112+88$

REDUKOVANÝ TVAR:

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 0 & -6 & 11 & -112 \\ 0 & 0 & 13 & -104 \end{array}\right) \cdot\left(\frac{1}{13}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 0 & -6 & 11 & -112 \\ 0 & 0 & 1 & -8 \end{array}\right) \begin{matrix} R_1+R_3 \\ R_2-11R_3 \end{matrix} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -6 & 0 & -24 \\ 0 & 0 & 1 & -8 \end{array}\right) R_1-R_2 \cdot\left(-\frac{1}{6}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -8 \end{array}\right)$$

5) $x-4y+z=18$
$2x-7y-2z=4$
$4x-13y-8z=-24$

$$\left(\begin{array}{ccc|c} 1 & -4 & 1 & 18 \\ 2 & -7 & -2 & 4 \\ 4 & -13 & 8 & -24 \end{array}\right) \begin{matrix} \\ R_2-2R_1 \\ R_3-4R_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & -4 & 1 & 18 \\ 0 & 1 & -4 & -32 \\ 0 & 3 & -12 & -96 \end{array}\right) R_3-3R_2 \sim \left(\begin{array}{ccc|c} 1 & -4 & 1 & 18 \\ 0 & 1 & -4 & -32 \\ 0 & 0 & 0 & 0 \end{array}\right) R_1+4R_2 \sim$$

$\boxed{z=\lambda;\ \lambda\in\mathbb{R}}$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -15 & -110 \\ 0 & 1 & -4 & -32 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\boxed{x=-110+15\lambda}$
$\boxed{y=-32+4\lambda}$

SKÚŠKA: $\check{C}_1=-110+15\lambda-(-32+4\lambda)4+\lambda=-110+15\lambda+128-16\lambda+\lambda=18=P_1$ ✓
$\check{C}_2=-220+30\lambda+224-28\lambda-2\lambda=4=P_2$ ✓
$\check{C}_3=-440+60\lambda+416-52\lambda-8\lambda=-24=P_3$ ✓

$[-110+15\lambda;-32+4\lambda;\lambda;\ \lambda\in\mathbb{R}]$

10) $5x_1-x_2+2x_3+10x_4=0$
$-x_1+2x_2+6x_3-5x_4=9$
$9x_1+12x_2+5x_3+13x_4=1$
$-2x_1-5x_2-20x_3+5x_4=7$

$$\left(\begin{array}{cccc|c} 1 & -2 & -6 & 5 & -9 \\ 0 & 9 & 32 & -15 & 45 \\ 0 & 30 & 59 & -32 & 82 \\ 0 & -9 & -32 & 15 & -11 \end{array}\right) \begin{matrix} \\ \\ \\ R_4+R_2 \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & -2 & -6 & 5 & -9 \\ 0 & 9 & 32 & -15 & 45 \\ 0 & 30 & 59 & -32 & 82 \\ 0 & 0 & 0 & 0 & 34 \end{array}\right)$$

$0=34$

$$\left(\begin{array}{cccc|c} 0 & 0 & 34 & 0 & 0 \end{array}\right)$$

$34x_3=0 \Rightarrow x_3=0$

14) $x_1+2x_2+3x_3+4x_4+5x_5=-2$
$3x_2+2x_3+x_4+6x_5=-11$
$3x_2+2x_3+4x_4-4x_5=-15$
$2x_1+4x_2+6x_3-x_4+25x_5=-7$

$$\left(\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & -2 \\ 2 & 4 & 6 & -1 & 25 & -7 \\ 0 & 3 & 2 & 1 & 6 & -11 \\ 0 & 3 & 2 & 4 & -4 & -15 \end{array}\right) \begin{matrix} \\ R_2-2R_1 \\ \\ \\ \end{matrix} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & -2 \\ 0 & 0 & 0 & -9 & 15 & -3 \\ 0 & 3 & 2 & 1 & 6 & -11 \\ 0 & 3 & 2 & 4 & -4 & -15 \end{array}\right) \begin{matrix} \cdot 1/3 \sim \\ \\ \\ R_4-R_3 \end{matrix}$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & -2 \\ 0 & 0 & 0 & -3 & 5 & -1 \\ 0 & 3 & 2 & 1 & 6 & -11 \\ 0 & 0 & 0 & 3 & -10 & -4 \end{array}\right) \begin{array}{l} \\ \\ \\ R_2+R_4 \end{array} \sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & -2 \\ 0 & 0 & 0 & 0 & -5 & -5 \\ 0 & 3 & 2 & 1 & 6 & -11 \\ 0 & 0 & 0 & 3 & -10 & -4 \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & -2 \\ 0 & 3 & 2 & 1 & 6 & -11 \\ 0 & 0 & 0 & 3 & -10 & -4 \\ 0 & 0 & 0 & 0 & -5 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \boxed{x_5 = 1}$$

$3x_4 - 10 = -4$

$\boxed{x_4 = 2}$

$\boxed{x_3 = A;\ A \in \mathbb{R}}$

$x_1 - \frac{4}{3}A - \frac{38}{3} + 3A + 8 + 5 = -2$

$x_1 = \frac{4-9}{3}A + \frac{38-45}{3}$

$\boxed{x_1 = -\frac{7}{3} - \frac{5}{3}A}$

$3x_2 + 2A + 2 + 6 = -11$

$\boxed{x_2 = \frac{-2A}{3} - \frac{19}{3}}$

SKÚŠKA: $L_1 = -\frac{7}{3} - \frac{5}{3}A - \frac{4}{3}A - \frac{38}{3} + 3A + 8 + 5 = -15 + 8 + 5 = -2 = P_1$ ✓

$L_2 = -2A - 19 + 2A + 2 + 6 = -11 = P_2$ ✓

$L_3 = -2A - 19 + 2A + 8 - 4 = -15 = P_3$ ✓

$L_4 = \left(-\frac{7}{3} - \frac{5}{3}A\right)2 + 4\left(-\frac{2}{3}A - \frac{19}{3}\right) + 6A - 2 + 25 = -\frac{14}{3} - \frac{10}{3}A - \frac{8}{3}A - \frac{76}{3} + 6A - 2 + 25 = -30 - 2 + 25 = -7 = P_4$ ✓

$\left[-\frac{7}{3} - \frac{5}{3}A;\ -\frac{2}{3}A - \frac{19}{3};\ A;\ 2;\ 1;\ A \in \mathbb{R}\right]$