① a) $(3x-y+2z)^6$ $\boxed{x^2 y z^3}$

MULTINOMICKÁ VETA:

$$(x_1+x_2+\dots+x_m)^n = \sum_{k_1+k_2+\dots+k_m=n} \boxed{\binom{n}{k_1,k_2,\dots,k_m} x_1^{k_1} x_2^{k_2} \dots x_m^{k_m}}$$

$x^2 y z^3$

$(3x-y+2z)^6$

$$\boxed{\begin{matrix} m=3 & x_1=3x \\ n=6 & x_2=-y \\ & x_3=2z \end{matrix}}$$

$k_1=2$ $k_2=1$ $k_3=3$

$$\binom{6}{2,1,3} x_1^2 x_2^1 x_3^3 = \frac{6!}{2!\,1!\,3!}(3x)^2(-y)^1(2z)^3 =$$

$$= \frac{6!}{2!\,1!\,3!}\, 3^2 x^2 (-1)^1 y\, 2^3 z^3 = -4320\, x^2 y z^3$$

$60\cdot(9\cdot 8\cdot(-1)) = 60\cdot(-72)$

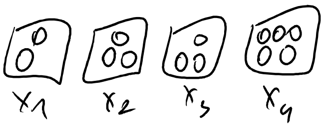
b) $(a-2b+4c+d)^7$ $a^2b^2c^2d$

$$\binom{7}{2,2,2,1} x_1^2 x_2^2 x_3^2 x_4^1 = \frac{7!}{2!\,2!\,2!}(a)^2(-2b)^2(4c)^2 d = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{1}}\, a^2\, 4b^2\, 16c^2 d =$$

$$= 40\,320\, a^2b^2c^2d$$

$x_1=a$ $x_2=-2b$ $x_3=4c$ $x_4=d$

② a) $x_1+x_2+x_3+x_4=13$ $0\le x_n\le 13$




$|||\underbrace{000}_{13}$ $(0,0,0,13)$

$\binom{13+3}{3}$ 00|0 00|0 0000|000

$(3,4,3,3)$

$(2,5,3,3)$

$\underbrace{00}_{x_1}|\underbrace{000}_{x_2}|\underbrace{000}_{x_3}|\underbrace{00000}_{x_4}$

$\underbrace{0000000000000}_{13}\underbrace{|||}_{3}$

$\underbrace{0\,0\,0}_{x_1}\ \underset{1}{|}\ \underbrace{0\,0\,0\,0}_{x_2}\ \underset{1}{|}\ \underbrace{0\,0\,0}_{x_3}\ \underset{1}{|}\ \underbrace{0\,0\,0}_{x_4}$ $(2,3,5,3)$

$\binom{16}{3}$

b) $x_1+x_2+x_3+x_4<13 \to \boxed{x_1+x_2+x_3+x_4\le 12}$

$\binom{12+4}{4} = \binom{16}{4}$

000|000|00000|0

$\underbrace{000}_{x_1} | \underbrace{000}_{x_2} | \underbrace{00000}_{x_3} | \underbrace{0}_{x_4} |$

$\binom{12+4}{4} = \binom{16}{4}$

$0\,0 | 0\,0 | 0\,0\,0 | 0\,0 | 0\,0\,0$

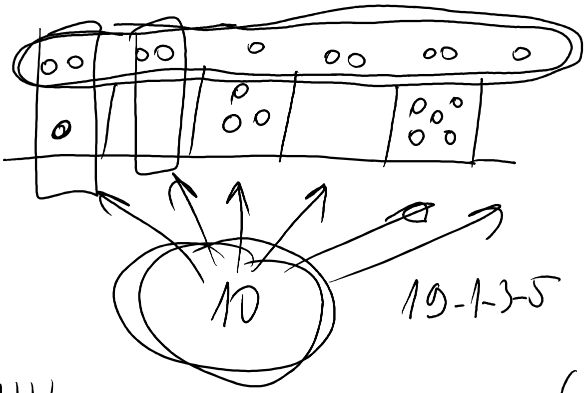
$(2,2,3,2)$

$|||| \underbrace{0000 \ldots 0}_{12}$

$(0,0,0,0)$

③ $x_1+x_2+x_3+x_4+x_5+x_6 = 19$

$x_1 \geq 1$
$x_3 \geq 3$
$x_5 \geq 5$



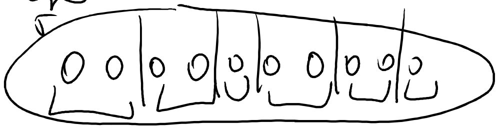
$19-1-3-5$

$x_1+x_2+x_3+x_4+x_5+x_6 = 10$

$(3,2,4,2,7,1)$

$\underbrace{0000000000}_{10} |||||$

$\binom{10+5}{5}$



④ 9 nápojov
21 ľudí

a) $9\;9\;9\;9\;9 \ldots 9\;9\;9 = 9^{21}$

b)

| 5 | 6 | 1 | 0 | 1 | 1 | 1 | 1 | 5 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

21

$\underbrace{000 \ldots 00}_{21} ||||||||$

$\binom{21+8}{8}$

⑥ a) POLOOBLAČNO → 11 p

1

11 10 9 2 1

(6) a) POLOOBLAČNU $\rightarrow$ 11 p

$$\frac{11!}{4!\,2!}$$

11 10 9 ... 2 1

POLOOBLAČNO
POLOOBLAČNO

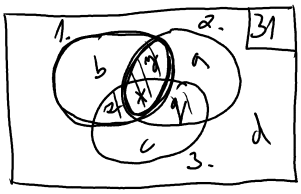
4!

P _ L _ _ _ BLAČN _
4 3 2 1

6p

b) $$P = \frac{\binom{7}{6}\cdot 6!}{\binom{11}{6}\cdot 6!}$$ PRIAZNIVÉ / VŠETKY

(7)

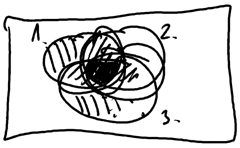



$20 = b+y+x+z$ $\quad 10 = x+y$
$16 = a+y+x+q$ $\quad 8 = z+x$
$15 = z+x+q+c$ $\quad 9 = x+q$
$31 = a+b+c+x+y+z+q$

a) $x = ?$

$|1|+|2|+|3| - |1\wedge 2| - |2\wedge 3| - |1\wedge 3| + |1\wedge 3\wedge 2| = |1\vee 2\vee 3|$

$20+16+15-10-9-8+|1\wedge 2\wedge 3| = 31$




$x = 7$

b) $y+z+q = ?$ $\quad q=2$ $\quad z=1$ $\quad y=3$ $\quad y+z+q = 6$

c) $a+b+c = ?$ $\quad 31-7-6$

(14) a) $a_n = 7a_{n-1} - 12a_{n-2}$ $\quad a_1 = -3$, $a_2 = 3$

$R^n = 7R^{n-1} - 12R^{n-2}$ $\quad n=2$

$\boxed{R^2 = 7R^1 - 12R^0} \rightarrow R^2 - 7R + 12 = 0$

$(R-4)(R-3) = 0$ $\quad R_1 = 4$ $\quad R_2 = 3$

$a_n = \alpha\cdot R_1^n + \beta\cdot R_2^n$

$a_n = \alpha\cdot 4^n + \beta\cdot 3^n$

$n=1$ $\quad a_1 = \alpha\cdot 4^1 + \beta\cdot 3^1$
$n=2$ $\quad a_2 = \alpha\cdot 4^2 + \beta\cdot 3^2$

$-3 = 4\alpha + 3\beta \quad |\cdot(-4)$
$3 = 16\alpha + 9\beta$

$15 = -3\beta$
$\boxed{-5 = \beta}$

$3 = 16\alpha - 45$
$48 = 16\alpha$
$\boxed{3 = \alpha}$

$a_n = 3\cdot 4^n - 5\cdot 3^n$

VIACNÁSOBNÝ KOREŇ: $a_n = n^0 \alpha \cdot R_1^n + n^1 \cdot \beta \cdot R_2^n + n^2 \cdot \gamma \cdot R_3^n + \dots$