Illia Ponomarov
Skup.-I

(Pr) 1 diferenciali

$\cos\left(\frac{9\pi}{20}\right)$ $\qquad x = \frac{9\pi}{20}, \; x_0 = 1$

$f(x) = \cos(x)$

$f'(x) = -\sin x$

$f(x)_k = f(x_0) + f'(x_0)(x - x_0)$

$f(x)_k = \cos(x) - \sin x\,(x - 1) =$

$\Rightarrow f(1) = \cos(1) - \sin(1)\left(\frac{9\pi}{20} - 1\right)$

$f(1) = \cos(1) - \sin(1) \cdot \frac{9\pi}{20} - \sin(1) \cdot (-1) =$

$= 0 - \frac{\pi}{2} \cdot \frac{9\pi}{20} + 1 \cdot \frac{\pi}{2} = \underline{-\frac{9\pi^2}{40} + \frac{\pi}{2}}$

PR 2 Taylorov polynom

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$f(x) = (x^2+1) \cdot \ln(x^2+1) \quad , x_0 = 0$

$f(0) = (0+1) \cdot \ln(0+1) = 1 \cdot 0 = 0$

1) $f'(x) = (x^2+1)' \cdot \ln(x^2+1) + (x^2+1)(\ln(x^2+1))' =$

$= 2x\ln(x^2+1) + (x^2+1) \cdot \frac{2x}{x^2+1} =$

$= 2x\ln(x^2+1) + 2x$

$f'(0) = 2 \cdot 0 \cdot \ln(1) + 2 \cdot 0 = 0$

2) $f''(x) = 2\ln(x^2+1) + \frac{2x}{x^2+1} \cdot 2x + 2$

$f''(0) = 2\ln(1) + \frac{2 \cdot 0}{1} \cdot 2 \cdot 0 + 2 =$

$= 2 \cdot 0 + 2 = 2$

3) $f'''(x) = 2 \cdot \ln(x^2+1) + \frac{4x^2}{x^2+1} + 2 =$

$= \frac{4x^3+12x}{(x^2+1)^2} \qquad f'''(0) = \frac{0+0}{1^2} = 0$

4) $f''''(x) = \frac{(4 \cdot 3x^2+12)(x^2+1)^2 - (4x^3+12x) \times 2(x^2+1) \cdot 2x}{((x^2+1)^2)^2} = \frac{-4x^4 - 24x^2 + 12}{(x^2+1)^3}$

$f''''(0) = \frac{-4 \cdot 0 - 24 \cdot 0 + 12}{(0+1)^3} = \frac{12}{1} = 12$

$T = 0 + 0(x-0)^1 + \frac{2}{2!}(x-0)^2 + \frac{0}{3!}(x-0)^3 +$

$+ \frac{12}{4!}(x-0)^4$

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$$\lim_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}+\lim_{x\to 1}\left(\frac{1}{2\ln x}-\frac{1}{x^2-1}\right)$$

1) $\lim_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1} \overset{"\frac{0}{0}"}{\underset{L'H}{=}}$ ~~lim~~ ~~4cos4x~~

$=\lim_{x\to 0}\frac{\sin 4x}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=\lim_{x\to 0}\frac{\sin 4x(\sqrt{x+1}+1)}{x+1-1}=$

$=\lim_{x\to 0}\frac{\overset{1=}{\sin 4x}(\sqrt{x+1}-1)}{x}=\lim_{x\to 0}4(\sqrt{x+1}+1)=$

$=\lim_{x\to 0}4(\sqrt{0+1}+1)=4\cdot 2=8$

2) $\lim_{x\to 1}\left(\frac{1}{2\ln x}^{\backslash x^2-1}-\frac{1}{x^2-1}^{\backslash 2\ln x}\right)=\lim_{x\to 1}\frac{x^2-1-2\ln x}{2\ln x\cdot(x^2-1)}\overset{"\frac{0}{0}"}{\underset{L'H}{=}}$

$=\lim_{x\to 1}\frac{\frac{2x^2-2}{x}}{2x-\frac{2}{x}+4x\cdot\ln x}=\lim_{x\to 1}\frac{\frac{2x^2-2}{x}}{\frac{2x^2-2+4x^2\cdot\ln x}{x}}=$

$=\lim_{x\to 1}\frac{2x^2-2}{2(x^2-1+2x^2\ln x)}=\lim_{x\to 1}\frac{\cancel{2}(x^2-1)}{\cancel{2}(x^2-1+2x^2\cdot\ln x)}=$

$=\lim_{x\to 1}\frac{x^2-1}{x^2-1+2x^2\cdot\ln x}\overset{"\frac{0}{0}"}{=}\lim_{x\to 1}\frac{2x}{4x+4x\cdot\ln x}=\lim_{x\to 1}\frac{\cancel{2x}}{\cancel{2x}(2+2\ln x)}=$

$=\lim_{x\to 1}\frac{1}{2+2\ln x}=\lim_{x\to 1}\frac{1}{2+2\ln 1}=\frac{1}{2}$

$\Rightarrow$ PR3 $\Rightarrow$ P3

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3) $8^{1/2} - \frac{1}{2} = \frac{16-1}{2} = \frac{15}{2}$

PR6

$\int e^{2x} \cdot \sin(e^x)\, dx$ $\qquad t = e^x$

$t = e^x$ $\qquad dx = \frac{1}{e^x}$

$\int \underline{t^2} \cdot \underline{\sin(t)} \cdot \frac{1}{t} = \int \frac{t}{v} \cdot \frac{\sin(t)}{u}$

$u = \sin(t) \qquad u' = \cos(t)$

$v' = t \qquad v = \frac{t^2}{2}$

$\sin(t) \cdot \frac{t^2}{2} - \int \cos(t) \cdot \frac{t^2}{2} =$

$= \sin(t) \cdot \frac{t^2}{2} - \frac{1}{2} \int \frac{\cos(t)}{v} \cdot \frac{t^2}{u}$

$u = t^2, \quad u' = 2t$

$v' = \cos(t) \quad v = \sin(t)$

$\sin(t) \cdot \frac{t^2}{2} - \frac{1}{2}\left(t^2 \cdot \sin t - \int \frac{\sin}{v} \cdot \frac{2t}{u}\right)$

$u = 2t \qquad u' = 2$

$v = \sin t \quad v' = -\cos t$

$\sin(t) \cdot \frac{t^2}{2} - \frac{1}{2}\left(t^2 \cdot \sin t - 2\int \cos(t)\right) =$

$= \sin(e^x) \cdot \frac{e^{2x}}{2} - \frac{1}{2}\left(e^x \cdot \sin e^x - 2 \cdot \cos(e^x)\right)$

PR 4

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$$f(x) = x - \frac{x}{x+1}$$

1) $D(f) \in R \setminus \{-1\}$

2) $f(-x) = -x - \left(\frac{-x}{-x+1}\right) \neq -f(x)$ Ani parna, ani nepárna

je spojitá, pretože $D \in R$, neperiodická

3) $x = 0 \iff f(0) = 0 - \frac{0}{0+1} = 0 - 0 = \underline{0}$

$y = 0 \iff f \quad x^{x+1} - \frac{x}{x+1} = 0;$

$\frac{x^2 + x - x}{x+1} = 0; \quad \frac{x^2}{x+1} = 0$

$\underline{(0;0)}$

4) ABBS $\nexists$, pret. $D \in R$

ASS $\quad y = ax - b$

$a = \lim_{x \to \infty} \frac{x - \frac{x}{x+1}}{x} = \lim_{x \to \infty} \frac{x^2}{\frac{x+1}{x}} = \lim_{x \to \infty} \frac{x}{x+1} = \frac{\infty}{\infty} =$

$= \lim_{x \to \infty} \frac{x}{x(1+\frac{1}{x})} = \lim_{x \to \infty} \frac{1}{1+\frac{1}{x}} =$

$= 1$

b) $\lim_{x \to \infty} x - \frac{x}{x+1} - x = \lim_{x \to \infty} -\frac{x}{x+1} \overset{LH}{=} \frac{1}{1} = -1$

$y = -1$

PR 4 =>

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(5) PR 4

$f(x) = x - \frac{x}{x+1}$

(1)

$f'(x) = 1 - \frac{x'(x+1) - x(x+1)'}{(x+1)^2} = 1 - \left(\frac{x+1-x}{(x+1)^2}\right) =$

(2)

$= 1 - \frac{1}{(x+1)^2}$; $1 - \frac{1}{(x+1)^2} = 0$

je

(3) x

$(x+1)^2 = 1$

$x + 1 = \pm 1$

$\begin{cases} x+1 = -1 \\ x+1 = +1 \end{cases}$ $\begin{cases} x = -1-2 \\ x = 0 \end{cases}$ $\begin{cases} x = -2 \\ x = 0 \end{cases}$

$- \quad + \quad - \quad +$

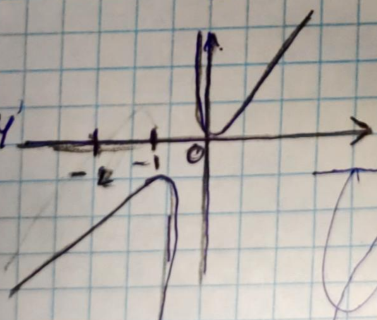
$-2 \quad -1 \quad 0$

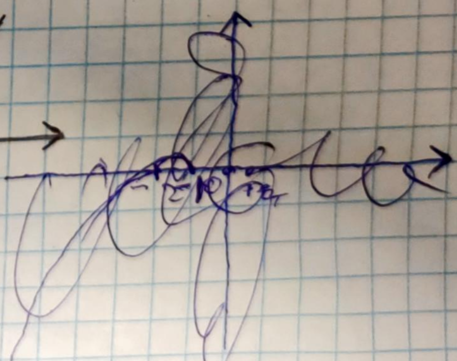
(4)

~~f(-2) = ...~~

$f(-2) = -2 - \left(\frac{-2}{-2+1}\right) = -4$

$f(0) = 0 - \frac{0}{0+1} = 0$





Lok. MAX [-4; -2]

Lok. Min [0; 0]

(6)

$f''(x) = 1 - \frac{x}{(x+1)^2} = -\frac{x(x+1)^2 - x(x+1)^2}{((x+1)^2)^2} =$

$= \frac{-x^2 + 2x + 1 - x \cdot 2(x+1)}{((x+1)^2)^2} = -\left(\frac{x^2 + 2x + 1 - 2x^2 - 2x}{((x+1)^2)^2}\right) =$

$= -\left(\frac{-x^2 + 1}{((x+1)^2)^2}\right) = \frac{x^2 - 1}{((x-1)^2)^2}$

⟹ PR 4

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6

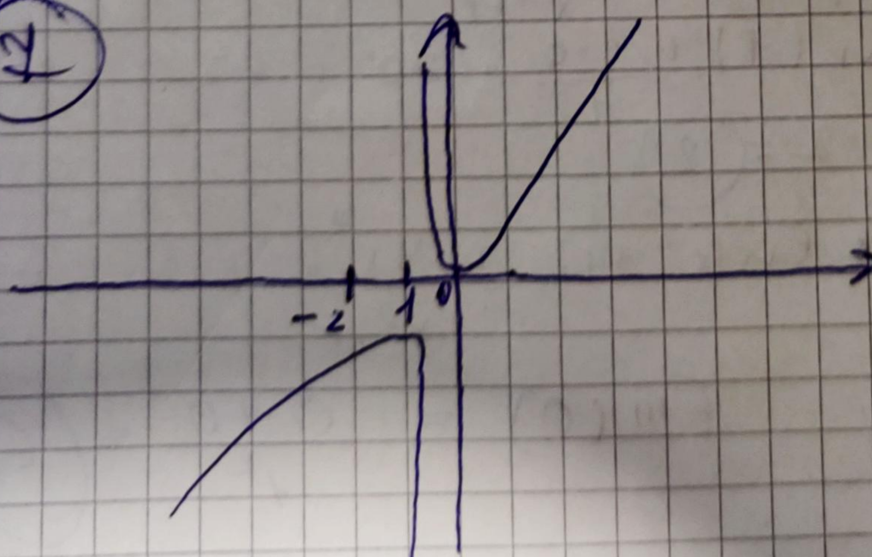
$$f'(x) = 1 + \frac{1}{(x+1)^2}$$

$$f''(x) = \frac{2}{(x+1)^3}$$

$$\frac{2}{(x+1)^3} = 0$$

$x \in \emptyset$

7



RR 5 Skup I Illia Ponomarov

a) $\sum_{n=1}^{\infty} \frac{n}{(2n+1)^n}$ b) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

a) $\lim_{x \to \infty} \sqrt[n]{\frac{n}{(2n+1)^n}} = \frac{\sqrt[n]{n}}{2n+1} = \frac{1}{\infty} = 0 < 1$

Konverguje

b) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$$\int_1^{\infty} \frac{1}{t^2+1} = \operatorname{arctg}(1) - \frac{\pi}{4}$$

$\frac{\pi}{4}$ Konverguje

$\Rightarrow$ PR5 $\Rightarrow$ P3

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3) $8^{\frac{1}{2}} - \frac{1}{2} = \frac{16-1}{2} = \frac{15}{2}$

PR6

$\int e^{2x} \cdot \sin(e^x)\,dx$ $t = e^x$

$t = e^x$ $dx = \frac{1}{e^x}$

$\int \underline{t^2} \cdot \underline{\sin(t)} \cdot \frac{1}{t} = \int \frac{t}{v} \cdot \frac{\sin(t)}{u}$

$u = \sin(t)$ $u' = \cos(t)$

$v' = t$ $v = \frac{t^2}{2}$

$\sin(t) \cdot \frac{t^2}{2} - \int \cos(t) \cdot \frac{t^2}{2} =$

$= \sin(t) \cdot \frac{t^2}{2} - \frac{1}{2} \int \frac{\cos(t)}{v} \cdot \frac{t^2}{u}$

$u = t^2$, $u' = 2t$

$v' = \cos(t)$ $v = \sin(t)$

$\sin(t) \cdot \frac{t^2}{2} - \frac{1}{2}\left(t^2 \cdot \sin t - \int \frac{\sin}{v} \cdot \frac{2t}{u}\right)$

$u = 2t$ $u' = 2$

$v = \sin t$ $v' = -\cos t$

$\sin(t) \cdot \frac{t^2}{2} - \frac{1}{2}\left(t^2 \cdot \sin t - 2 \int \cos(t)\right) =$

$= \sin(e^x) \cdot \frac{e^{2x}}{2} - \frac{1}{2}\left(e^x \cdot \sin e^x - 2 \cdot \cos(e^x)\right)$

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PR 7

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$$\int_6^7 \underbrace{\ln(x^2-7x+10)}_{u}\cdot \underbrace{1}_{v} dx$$

$x^2-7x+10 \,|\, x^2+10$

$u = \ln(x^2-7x+10)$ $\quad u' = \frac{2x-7}{x^2-7x+10}$

$v' = 1$

$$\begin{array}{r|l} x^2-7x+10 & x^2+10 \\ -\ (x^2+10) & 1 \\ \hline -7x+20 & \end{array}$$

$v = x$

$$\ln(x^2-7x+10)\cdot x - \int^5 \frac{2x^2-7x}{x^2-7x+10} =$$

$$= \ln(x^2-7x+10)\cdot x - \int_6^5 \frac{x^2-7x+10+2x^2-7x-x^2+7x+10}{x^2-7x+10} =$$

$$= \ln(x^2-7x+10)\cdot x - \int_6^5 1 + \frac{2x^2+10}{x^2-7x+10} =$$

$$= \ln(x^2-7x+10)x - x\cdot\int_6^5 \left(-\frac{7x+20}{x^2-7x+10}\right)$$

PR 8 Obsah

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$f(x) = x^2 + 4x$, $g(x) = \frac{x^2}{2}$, $h(x) = -x$

1) $x^2 + 4x = -x$

$x^2 + 4x + x = 0$

$x^2 + 5x = 0$

$x(x+5) = 0$

$x_1 = 0 \quad x_2 = -5$

$a = -5$

$\frac{x^2}{2} = -x$

$\frac{x^2}{2} + x^{2} = 0$

$\frac{x^2 + 2x}{2} = 0$

$x(x+2) = 0$

$x_1 = 0$, $x_2 = \cancel{0}\,2$

$b = -2$

$$P_1 = \int_{-5}^{-2} x^2 + 4x = \frac{x^3}{3} + \frac{\cancel{4}^{2}x^2}{\cancel{2}} \Big|_{-5}^{-2} =$$

$$= \left(-\frac{125}{3} + 50\right) - \left(-\frac{8}{3} + 8\right) = \left(\frac{-125 + 150}{3}\right) + \frac{8 + 24}{3} =$$

$$= \frac{25}{3} + \frac{32}{3} = \frac{56}{3}$$

$$P_2 = \int_{-5}^{-2} \frac{x^2}{2} = \frac{1}{2}\int_{-5}^{2} x^2 = \frac{1}{2} \cdot \frac{x^3}{3} \Big|_{-5}^{2} = \frac{x^3}{6} \Big|_{-5}^{2} =$$

$$= -\frac{125}{6} + \frac{8}{6} = -\frac{117}{6}$$

$$P_3 = P_1 - P_2 = \frac{56}{3} + \frac{117}{6} = \frac{112 + 117}{6} = \frac{229}{6}$$